Comparison of PI/PID/PIDD2 Controllers for Higher Order Processes *

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Abstract: PI, PID and PIDD2 controllers are compared for higher order processes. Tuning rules with robustness constraints are first proposed in this paper for PI, PID and PIDD2 controllers. Then the tuned rules are applied to the benchmark higher order processes and a three-tank system. It is shown that higher order derivatives can improve the control performance for processes with small time delays, while it is of no significant performance improvement for processes with large delays.

Keywords: PID control, PID tuning, higher order processes.

1. INTRODUCTION

Proportional-Integral-Derivative (PID) loops are by far the most common feedback control mechanism for industrial processes (Astrom and Hagglund, 1995; Ang et al., 2005; Vilanova and Visioli, 2012). The three parameters of a PID controller have a very clear connection with the performance of the system, so it can be easily tuned online. However, with the increasing complexity of industrial processes and the increase of various uncertainties in the plants, the control performance of traditional PID control may not be satisfactory due to its specific structure (Sung and Lee, 1996).

On the other hand, many modern control techniques were tried to replace the PID control. Although the advanced control techniques have contributed to the improvement of the control performance, they are seldom found in practice due to the implementation, tuning and maintenance issues. When a process is already up and running, the trial-anderror design can be more convenient than the advanced control alternatives that require taking the process offline for tests. And even when the advanced control technique theoretically would provide improved performance, the extra effort and expense required may not be worth it. Therefore, more than 90% of the controllers in feedback control are still of PID type (Astrom and Hagglund, 1995, 2001).

To improve the performance of conventional PID controllers with minimal control structure modification, PID plus second-order derivative (PIDD2) or Proportional Integral Derivative Accelerated (PIDA) controllers are investigated (Anwar et al., 2018; Jitwang et al., 2019; Huba et al., 2020). It is shown that PIDD2/PIDA controllers can provide better performance in automatic voltage regulation (AVR) and automatic generation control (AGC) systems than conventional PID controllers (Khakpour and Mirabbasi, 2015; Zhao et al., 2019; Mokeddem and Mirjalili, 2020; sai Kalyan and Suresh, 2021; Izci et al., 2023). Higher order PIDD2D3 (Simanenkov et al., 2017) and PID with higher order filter (Divakar and Kumar, 2022) are also proposed to improve the control performance of conventional PID controllers.

Though a PIDD2 controller can provide better performance than a conventional PID controller, the tuning of its parameters becomes more complex, and it is more sensitive to noisy environment. This paper will propose tuning formulae for PI, PID and PIDD2 controllers for highorder processes with robustness constraint. The tuning rules can make sure that PI, PID and PIDD2 controllers have the same robustness measure so that fair comparison can be made. It is shown that higher order derivatives can improve the control performance for processes with small time delays, while it is of no significant performance improvement for processes with large delays.

2. TUNING CRITERIA

For a general higher order process

$$P(s) = \frac{N(s)}{D(s)}e^{-\theta s} \tag{1}$$

The following controllers will be discussed in the paper.

PI controller:

$$K(s) = K_p + K_i/s \tag{2}$$

PID controller:

$$K(s) = K_p + K_i/s + K_ds \tag{3}$$

PIDD2 controller:

$$K(s) = K_p + K_i/s + K_d s + K_{d2} s^2$$
(4)

2.1 Performance index

To evaluate the closed-loop load disturbance attenuation performance, the integral of time absolute error (ITAE) is considered, which is defined as:

^{*} Supported by the Starting Research Fund of North China University of Technology.

$$ITAE = \int_{0}^{\infty} t|e(t)|dt$$
(5)

where e(t) = r(t) - y(t). In this paper, we will investigate the case when the disturbance enters at the process input (load disturbance).

2.2 Robustness index

The robustness of a control system is one of the most important issues in controller design, because a model is always inaccurate in some sense. Two well-known measures for robustness measure are:

$$M_s = \left\|S\right\|_{\infty} = \max_{\omega} \left|\frac{1}{1 + L(j\omega)}\right| \tag{6}$$

$$M_p = \|T\|_{\infty} = \max_{\omega} \left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \tag{7}$$

where L(s) = P(s)K(s) is the open-loop transfer function. M_s is a good measure of system robustness against the low and mid-frequency uncertainties, and M_p is a good measure of system robustness against the mid- and high frequency uncertainties.

A combination of M_s and M_p is more appropriate (Tan et al., 2006). Define

$$M := \begin{bmatrix} (I+PK)^{-1} & (I+PK)^{-1}P \\ K(I+PK)^{-1} & K(I+PK)^{-1}P \end{bmatrix}.$$
 (8)

and

$$\Delta := \begin{bmatrix} \Delta_1 & 0\\ 0 & \Delta_2 \end{bmatrix}. \tag{9}$$

where Δ_1 and Δ_2 are uncertainties with compatible dimension. Then $\mu_{\Delta}(M)$ is a measure of system robustness (Tan et al., 2006). It is shown that for a single-loop control system

$$\varepsilon := \mu_{\Delta}(M) = \sup_{\omega} (\|S\|_{\infty} + \|T\|_{\infty})$$
(10)

thus ε is a combination of M_s and M_p , and is a good robustness measure for a single-loop control system. The larger ε is, the weaker the robustness is.

So the parameters of the PI,PID,PIDD2 controllers can be found by solving the following optimization problem:

$$\begin{cases}
\min_{K} \text{ ITAE} \\
\text{s.t. } \varepsilon < \gamma
\end{cases}$$
(11)

where γ is a given robustness requirement, and ITAE is computed for load disturbance.

3. APPROXIMATION OF HIGHER ORDER MODELS

First-order processes with deadtime (FOPDT) models are usually used to tune PI and PID controllers.

$$P_1(s) = \frac{k}{Ts+1}e^{-\tau s}$$
(12)

There are many methods to approximate a higher order process with FOPDT models, e.g., half-rule method (Skogestad and Grimholt, 2012). Any of them can be used to obtain the FOPDT model. In this paper, a simple and effective frequency domain approximation method is proposed. The main idea of this method is to approximate the actual plant P(s) with the FOPDT model $P_1(s)$ at the bandwidth.

The procedure to find an FOPDT model via the bandwidth method is as follows.

• Compute the steady-state gain of the process P(0). The process gain can be obtained as

$$k = P(0) \tag{13}$$

• Compute the bandwidth ω_B of P(s), which is the frequency that the magnitude of the $P(j\omega)$ equals to $1/\sqrt{2}P(0)$. Then the constant T can be obtained as

$$T = 1/\omega_B \tag{14}$$

• The time delay can be estimated as

$$\tau = (\angle P(j\omega_B) - \pi/4)/\omega_B \tag{15}$$

where $\angle P(j\omega_B)$ is the phase of P(s) at the bandwidth, and $\pi/4$ is the phase of $\frac{1}{Ts+1}$ at the bandwidth ω_B .

Making the FOPDT model approximate the original higher order system at the bandwidth will guarantee the robustness measure below the bandwidth frequency, while other methods have no such property.

To tune PIDD2 controllers, we will consider second-order processes with deadtime (SOPDT) models.

$$P_2(s) = \frac{k}{(T_1s+1)(T_2s+1)}e^{-\tau s}$$
(16)

To obtain a SOPDT model $P_2(s)$ (16), choose the inverse of the smallest pole of P(s) as T_1 , and then approximate $P(s)(T_1s+1)$ with a FOPDT model $k/(T_2s+1)e^{-\tau s}$.

4. TUNING RULES WITH ROBUSTNESS CONSTRAINT

Any algorithm (e.g. genetic algorithm (GA), particle swam optimization (PSO), etc.) can be used to solve the optimization problem (11). But it is hard to find a tuning rule for FOPDT or SOPDT models for any given robustness measure. However, the tuning rules can be found for some special models, and tuning rules can thus be obtained for the given robustness measure.

4.1 PI tuning

For pure delay models,

$$P(s) = e^{-\tau s} \tag{17}$$

Integral controller can be computed for the pure delay model for a given robustness measure ε as

$$K_i = (1.321 - 1.743\varepsilon^{-0.885})/(\tau k) \tag{18}$$

thus a PI controller for FOPDT model (12) can be obtained as $(Ts + 1)\frac{K_i}{s}$ for the given robustness measure ε ,

i.e., For FOPDT model, tuning rule for PI controller is as follows:

$$K_i = (1.321 - 1.743\varepsilon^{-0.885})/(\tau k) K_p = TK_i$$
(19)

It is found that when the normalized delay $L := \tau/T$ of the FOPDT model is less than 0.5, the above rule has sluggish load disturbance rejection ability, thus to improve the performance, the following rules are found for FOPDT model with L < 0.5.

$$K_i = (0.3562 + 0.1426L^{-1.508})/(Tk)$$

$$K_p = (0.145 + 0.2826L^{-0.9878})/k$$
(20)

for robustness measure $\varepsilon = 2.0$, and

$$K_i = (0.5278 + 0.1899L^{-1.688})/(Tk)$$

$$K_p = (0.18 + 0.4184L^{-0.97711})/k$$
(21)

for robustness measure $\varepsilon = 2.5$.

4.2 PID tuning

PI controller can be computed for the pure delay model (17) for a given robustness measure ε as

$$K_i = (1.133 - 1.675\varepsilon^{-1.476})/\tau$$

$$K_p = (0.1747 + 0.0003069\varepsilon^{6.031})$$
(22)

thus a PID controller for FOPDT model can be obtained as $(Ts + 1)(K_p + \frac{K_i}{s})$ for the robustness measure ε , i.e., for FOPDT model (12), tuning rule for PID controller is as follows:

$$K_{i} = (1.133 - 1.675\varepsilon^{-1.476})/(\tau k)$$

$$K_{d} = (0.1747 + 0.0003069\varepsilon^{6.031})T/k$$

$$K_{p} = K_{d}/T + TK_{i}$$
(23)

As for the PI case, it is found that when the normalized delay of the FOPDT model is less than 1, the above rule has sluggish load disturbance rejection ability, thus to improve the performance, the following rules are found for FOPDT model with L < 1.

$$K_i = (0.2367 + 0.3898L^{-1.492})/(Tk)$$

$$K_p = (0.2309 + 0.5155/L)/k$$

$$K_d = 0.264T/k$$
(24)

for robustness measure $\varepsilon = 2.0$, and

$$K_i = (0.1365 + 0.6036L^{-1.554})/(Tk)$$

$$K_p = (0.2981 + 0.6328L^{-1.019})/k$$

$$K_d = 0.32T/k$$
(25)

for robustness measure $\varepsilon = 2.5$.

4.3 PIDD2 tuning

For SOPDT model (16), PIDD2 controller can be tuned by first tuning a PID controller $K_1(s)$ for FOPDT model $\frac{k}{T_1s+1}e^{-\tau s}$ via the above rules, and then form the PIDD2 controller by $K_1(s)(T_2s+1)$.

Ideal PID and PIDD2 controllers are tuned for FOPDT and SOPDT processes. It is well-known that derivative action is sensitive to measurement noise, so in practice filters should be used to implement the ideal PID and PIDD2 controllers. In this paper the observer-based PID structure (Tan et al., 2022) will be used to implement the ideal PID and PIDD2. For simplicity, details are omitted here.

5. SIMULATION FOR BENCHMARK SYSTEMS

In this section, the benchmark higher order systems (Astrom and Hagglund, 2000) are used to test the applicability of the proposed tuning method.

$$G_1(s) = \frac{1}{(s+1)^n}, (n = 4, 8, 20)$$
(26)

$$G_2(s) = \frac{1}{(s+1)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)},$$
 (27)
(\alpha = 0.2, 0.4, 0.8)

The parameters of PI, PID and PIDD2 controllers are given in Table 1 and 2 under robustness measure $\varepsilon = 2.0$.

The responses of the benchmark systems under the tuned PID controllers are shown in Figure 1 and 2 with a step reference at t = 1s and a step input disturbance at appropriate time, and the relevant indexes for the disturbance attenuation are given in Table 1 and 2.

Table 1. Parameters of PID controllers for G_1

n = 4	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.441	0.192	_	_	39.07	1.99	1.40
PID	0.833	0.317	0.607	-	19.05	1.99	1.35
PIDD2	1.529	0.511	1.519	0.501	10.25	1.97	1.28
n = 8	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.243	0.073	-	-	231.58	1.96	1.40
PID	0.537	0.103	0.647	_	144.26	2.02	1.46
PIDD2	0.691	0.123	1.167	0.602	112.07	2.01	1.45
n = 20	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.129	0.024	_	_	1927.1	1.95	1.39
PID	0.376	0.034	1.037	-	1181.1	2.00	1.47
PIDD2	0.429	0.037	1.491	1.338	1047.3	2.00	1.48

Table 2. Parameters of PID controllers for G_2

$\alpha = 0.2$	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	1.424	1.717	-	-	0.820	1.95	1.28
PID	2.607	3.894	0.274	-	0.285	2.00	1.19
PIDD2	20.48	41.61	2.712	0.054	0.0124	1.92	1.16
$\alpha = 0.4$	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.763	0.715	-	_	3.43	1.98	1.35
PID	1.368	1.302	0.305	-	1.60	1.97	1.25
PIDD2	5.156	4.961	1.572	0.116	0.25	2.04	1.23
$\alpha = 0.8$	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.464	0.268	-	-	20.30	1.99	1.40
PID	0.865	0.444	0.457	-	9.95	1.98	1.34
PIDD2	1.825	0.800	1.348	0.323	4.48	1.97	1.28

From the relevant indexes in Table 1 and 2, for higher order process G_1 , PIDD2 can achieve better disturbance rejection performance for small n compared with PID. But as n increases, the benefit of PIDD2 over PID is not obvious, as shown in Figure 1. For process G_2 , as α decreases, the benefit of PIDD2 over PID becomes obvious, as shown in Figure 2. In all the cases, PID and PIDD2 are superior to PI controllers.

The parameters of the FOPDT model approximation for G_1 are:



Fig. 1. Closed-loop responses of G_1

$$T = 2.299, \tau = 1.967, L = 0.856, G_1(n = 4)$$

$$T = 3.324, \tau = 5.16, L = 1.552, G_1(n = 8)$$
 (28)

$$T = 5.325, \tau = 15.6, L = 2.930, G_1(n = 20)$$

and for G_2 ,

$$T = 1.01, \tau = 0.1057, L = 0.1046, G_2(\alpha = 0.1)$$

$$T = 1.0394, \tau = 0.2255, L = 0.217, G_2(\alpha = 0.2) \quad (29)$$

$$T = 1.7316, \tau = 1.4071, L = 0.8126, G_2(\alpha = 0.8)$$

It is clear that higher order derivatives can improve the control performance for processes with small time delays (normalized delay L < 1), while it is of no significant performance improvement for processes with large delays (normalized delay L > 1), especially if there is measurement noise.



Fig. 2. Closed-loop responses of G_2

Further, consider the following high-order systems discussed in Visioli (2005).

$$G_{3}(s) = \frac{(15s+1)^{2}(4s+1)(2s+1)}{(20s+1)^{3}(10s+1)^{3}(5s1)^{3}(0.5s+1)^{3}}$$

$$G_{4}(s) = \frac{(-0.3s+1)(0.08s+1)}{(2s+1)(s+1)(04s+1)(0.2s+1)(0.05s+1)^{3}}$$

$$G_{5}(s) = \frac{(-45s+1)}{(20s+1)^{3}(18s+1)^{3}(5s+1)^{3}} \cdot \frac{(4s+1)}{(10s+1)^{2}(16s+1)(14s+1)(12s+1)}$$
(30)

The parameters of PI, PID and PIDD2 controllers tuned for the three systems are given in Table 3 under robustness measure $\varepsilon = 2.0$. The responses of the three systems under the tuned PID controllers are shown in Figure 3 with a step reference at t = 1s and a step input disturbance at appropriate time, and the relevant indexes for the disturbance attenuation are given in Table 3.

Table 3. Parameters of PID controllers for G_3 , G_4 , G_5

G_3	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.387	0.0103	_	_	13105	1.99	1.40
PID	0.759	0.017	9.95	-	6401.9	2.01	1.38
PIDD2	1.368	0.026	24.32	149	3582.8	1.99	1.35
G_4	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.545	0.225	-	-	30.53	1.99	1.39
PID	0.976	0.376	0.641	_	15.21	1.96	1.32
PIDD2	2.222	0.766	2.043	0.578	5.86	1.97	1.36
G_5	K_p	K_i	K_d	K_{d2}	ITAE	ε	M_s
PI	0.095	0.0021	_	_	24832	2.05	1.44
PID	0.329	0.0030	8.793	-	15225	2.10	1.51
PIDD2	0.393	0.0033	14.34	156.5	12886	2.15	1.54

The parameters of the FOPDT model approximation for G_3 , G_4 and G_5 are:

$$T = 37.69, \tau = 36.76, L = 0.975, (G_3)$$

$$T = 2.428, \tau = 1.679, L = 0.691, (G_4)$$
(31)

$$T = 45.15, \tau = 179, L = 3.967, (G_5)$$

thus PIDD2 can improve the performance of PID for G_3 and G_4 , but no significant improvement for G_5 , as shown in Figure 3.

6. PRACTICAL VALIDATION

In this section, a three-tank system is utilized as a platform to validate the proposed method. The inlet flow rate feeding the upper tank is manipulated by a pump output, while the liquid drains freely through the bottom of the upper tank to the middle and lower tank. The liquid drains freely through the bottom of the lower tank to a pool, where the pump takes the liquid. The objective is to adjust the pump output to maintain the liquid level in the lower tank at set point. The valve position at the middle tank acts as a disturbance to the tank process.

The three-tank process can be identified as

$$P(s) = \frac{0.066}{(49s+1)^3} \tag{32}$$

It can be approximated with FOPDT model

$$P_1(s) = \frac{0.066}{96.11s + 1}e^{-60.5s} \tag{33}$$

and SOPDT model

$$P_2(s) = \frac{0.066}{(73.66s+1)(49s+1)}e^{-27.8s}$$
(34)

Then according to tuning formula, the following controllers can be tuned with $\varepsilon = 2.0$. For PI controller, the parameters are

$$K_p = 9.085, K_i = 0.0945 \tag{35}$$

For PID controller, the parameters are

$$K_p = 15.92, K_i = 0.160, K_d = 384.4$$
 (36)

For PIDD2 controller, the parameters are

$$K_p = 43.43, K_i = 0.392, K_d = 1481, K_{d2} = 14440$$
 (37)



Fig. 3. Closed-loop responses of G_3 - G_5

Observer-based PID (Tan et al., 2022) with $b_0 = 10^{-6}$, $\bar{\omega}_o = 0.2$ and $\alpha = 1$ are used to implement the ideal PID and PIDD2 controllers.

Since the FOPDT model has a normalized delay L = 0.63, it is predicted that PIDD2 controller have the best disturbance rejection performance, which can be verified with the responses of the three-tank processes under the tuned controllers shown in Figure 4. It is clear that the PIDD2 and PID controllers can improve the performance of the PI controller, but the pump output of the PIDD2 is not smooth, so in practice, the overall performance of PID is the best for the three-tank process.

7. CONCLUSION

PI, PID and PIDD2 tuning rules were derived for any given robustness measure, and the tuned rules were applied to the benchmark higher order processes and a three-





(b) Pump output

Fig. 4. Closed-loop responses of the three-tank process

tank system. It was shown that higher order derivatives can improve the control performance for processes with small time delays, while it is of no significant performance improvement for processes with large delays. In practical control, filters or observers must be tuned simultaneously with the higher order PID controller parameters, which will be investigated in the future.

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