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Study on Model Free Frequency Estimation for Vibration Suppression PIS Control

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Abstract: Many industrial control systems such as compressors, magnetic synchronous motors, etc. generate periodic disturbances, and it makes strict control difficult. Model-based control methods using disturbance observers can suppress these disturbances but have some problems such as modeling errors, system fluctuations, and noise effects. Proportional-Integral-Sinusoidal (PIS) control is another method with a simple structure, and its effectiveness is guaranteed by the internal model principle. However, PIS controller is required to estimate a disturbance frequency. This study proposes a model-free frequency estimation mechanism based on extremum seeking control and suppresses periodic disturbances by PIS control using the estimated results. The effectiveness is verified through the numerical example of the two inertia model.

Keywords: PIS control, extremum seeking control, periodic disturbance, internal model principle, sinusoidal internal model

1. INTRODUCTION

Mechanical systems such as compressors and magnetic synchronous motors generate periodic disturbances. Typical methods to suppress periodic disturbances are model-based control using a disturbance observer and Proportional-Integral-Sinusoidal (PIS) control etc.. Disturbance observer is generally designed using a controlled object model and used to generate the input that cancels out the disturbance. Therefore, there are problems such as worsening of estimation accuracy due to modeling errors and system fluctuations, and enhancement of the high-frequency noise. Although adaptive methods to update model parameters are effective for system fluctuations, higher-order dynamics not included in the model may make the system destabilize (C. Rohrs et al. (1985); M. Bodson (2004)).

In contrast, PIS control has a simple structure with a S compensator and a PI controller. The internal model principle guarantees the convergence of control error to zero even if a periodic disturbance is included in the control system (S. Fukuda et al. (2001),Y. Nakamura et al. (2020)). However, the internal model requires the frequency information of the periodic disturbance, and if there is an error, the effectiveness of the S compensator is reduced. Therefore, it is necessary to estimate the frequency of periodic disturbances. S. Takagi et al. (2019) proposed PIS control with an automatic frequency estimation mechanism for a pneumatic vibration isolator. However, since the periodic disturbance is shaped into a square wave by a schmitt trigger, the amplitude of the disturbance must be known and constant. This makes it difficult to apply to general applications.

For the periodic disturbance identification, Q. Liu et al. (2023) proposed an extremum seeking control (K. B. Ariyur et al. (2003); M. Krstic et al. (2000)) based parameter identification method. The extremum seeking control is a gradient method that extracts gradient information of an unknown function without using a mathematical model and adaptively searches for and maintains an extreme value. Q. Liu et al. estimated the frequency and amplitude of a periodic disturbance without using a model of the controlled object and periodic disturbance.

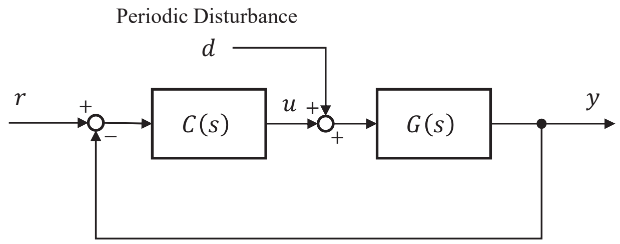

Fig. 1. Control system with periodic disturbance

This study proposes an automatic frequency estimation PIS control based on extremum seeking control. The proposed method estimates the frequency of the periodic disturbance without using any model of the controlled object and the periodic disturbance. The estimated frequency is used in the sinusoidal internal model of PIS control to suppress the periodic disturbance. The effectiveness of this method is verified through numerical simulation of the two-inertia system model.

2. PROBLEM SETTING

The control system with periodic disturbance is shown in Fig. 1. Assume that a controlled object is a SISO system, and $r(t), u(t), y(t)$ and $d(t)$ are the reference value, the control input, the plant output, and the periodic disturbance, respectively. $C(s)$ and $G(s)$ are the controller and the controlled object, and s is the Laplace operator. In this case, the output $Y(s)$ becomes

$$Y(s) = \frac{G(s)C(s)}{1+G(s)C(s)}R(s) + \frac{G(s)}{1+G(s)C(s)}D(s), \quad (1)$$

where $Y(s) = \mathcal{L}[y(t)], R(s) = \mathcal{L}[r(t)]$ and $D(s) = \mathcal{L}[d(t)]$. In this paper, it is assumed that the amplitude and frequency information of the periodic disturbance are unknown.

3. AUTOMATIC FREQUENCY ESTIMATION PIS CONTROL

This chapter describes the principle of PIS control and the automatic estimation mechanism of the disturbance frequency based on the extremum seeking control.

3.1 PIS Control

PIS control is the method consisting of a PI controller, which is commonly used in servo systems, and the S (Sinusoidal) compensator. Let each transfer functions in Fig. 1 $C(s) = \frac{N_C(s)}{D_C(s)}$, $G(s) = \frac{N_G(s)}{D_G(s)}$, $R(s) = \frac{N_R(s)}{D_R(s)}$ and $D(s) = \frac{N_D(s)}{D_D(s)}$. From the final value theorem, the steady-state error of the control system is given as

$$\begin{aligned}\lim_{s\to0} sE(s) &= \lim_{s\to0} s(R(s) - Y(s)) \\ &= \lim_{s\to0} s\left(\frac{R(s)}{1+G(s)C(s)} - \frac{G(s)D(s)}{1+G(s)C(s)}\right) \\ &= \lim_{s\to0} \frac{s}{\Delta(s)}\left(\frac{D_G(s)D_C(s)N_R(s)}{D_R(s)} - \frac{N_G(s)D_C(s)N_D(s)}{D_D(s)}\right),\end{aligned} \quad (2)$$

$$(\Delta(s) = D_G(s)D_C(s) + N_G(s)N_C(s)).$$

$\frac{1}{\Delta(s)}$ is represented as equation (3), which satisfies the following *Assumption 1*,

$$\frac{1}{\Delta(s)} = \frac{1}{(s-\mu_1)(s-\mu_1)...(s-\mu_m)}. \quad (3)$$

Assumption 1. The control system is stable and satisfies

$$R_e(\mu_i) < 0, \qquad (i = 1, 2, ..., m). \quad (4)$$

Let the denominator polynomial $D_C(s)$ of the controller have common factors with $D_R(s)$ and $D_D(s)$ as

$$D_C(s) = D'_C(s)D_R(s)D_D(s). \quad (5)$$

Substituting equation (5) into equation (2) gives

$$\begin{aligned}\lim_{s\to0} sE(s) = \lim_{s\to0} \frac{s}{\Delta(s)}\big(&D_G(s)D'_C(s)D_D(s)N_R(s) \\ &- N_G(s)D'_C(s)D_R(s)N_D(s)\big).\end{aligned} \quad (6)$$

As known as the internal model principle (B. A. Francis et al. (1976)), equation (6) guarantees that the steady-state error is reduced to zero.

For example, if the reference value $r(t)$ is a step signal and the periodic disturbance is a sine wave $d(t) = A_m \sin(\omega_d t)$, the controller is designed to make the steady-state error zero as

$$C(s) = K_P + \frac{K_I}{s} + K_S\frac{s}{s^2+\omega_d^2}, \quad (7)$$

where K_P, K_I, and K_S are proportional gain, integral gain, and sine wave gain, respectively.

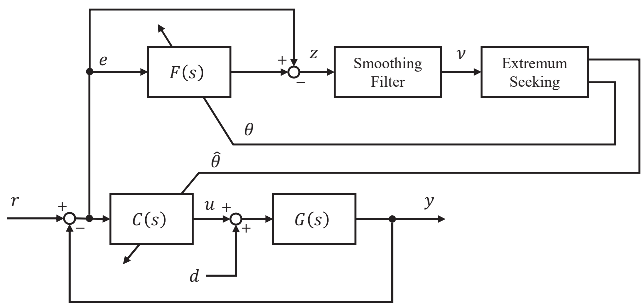

Fig. 2. Automatic frequency estimation mechanism

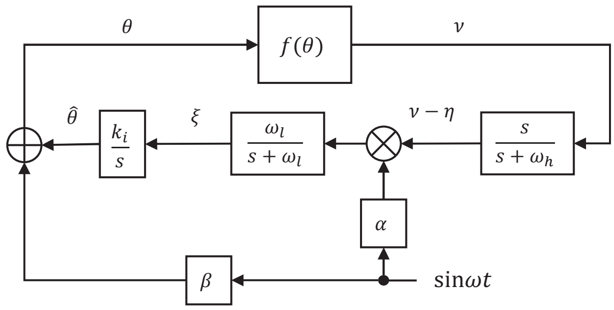

Fig. 3. Block diagram of extremum seeking controller

On the other hand, according to equation (7), the sinusoidal internal model requires the frequency information ω_d of periodic disturbances. In general, the pre-estimated frequency value $\hat{\omega}_d$ is used for the internal model. If the error between $\hat{\omega}_d$ and ω_d is large, the effectiveness of the S compensator decreases.

3.2 Disturbance frequency estimation based on extremum seeking control

This paper proposes the new automatic frequency estimation mechanism shown in Fig. 2 based on the method which Q. Liu et al. (2023) proposed. In Fig. 2, $F(s)$ is a bandpass filter given as

$$F(s) = \frac{2\zeta\omega_a s}{s^2 + 2\zeta\omega_a s + \omega_a^2}, \quad (8)$$

where ζ is the damping ratio and ω_a is the center frequency. The smoothing filter is a full-wave rectification type. The difference $z(t)$ between the control error $e(t)$ and the bandpass filter output is taken absolute value as $|z(t)|$. $|z(t)|$ is fed into the following low pass filter

$$F_s(s) = \frac{1}{T_l s + 1}, \quad (9)$$

where T_l is the time constant of the low-pass filter. At the center frequency $\omega_a = \omega_d$, the gain and phase of the control error $e(t)$ matches those of the bandpass filter output, so $\nu(t)$ is minimized. The extremum seeking control finds the center frequency ω_a of the bandpass filter that minimizes the smoothed signal $\nu(t)$.

The basic structure of the extremum seeking controller is shown in Fig. 3. The extremum seeking controller consists of a high-pass filter, a low-pass filter, an integrator, a multiplier, and an adder. ω_h, ω_l are the cutoff frequencies of the high-pass filter and low-pass filter, respectively, $k_i (k_i < 0)$ is the integral gain, $\sin\omega t \, (\omega > \omega_l, \omega_h)$ is the perturbation signal, $\alpha (\alpha > 0)$, $\beta (\beta > 0)$ are the gains of the perturbation signal.

Let $\nu(t)$ in the equation (9) be obtained from the following function

$$f(\theta(t)) = \nu^* + \frac{f''}{2}(\theta^* - \theta(t))^2, \tag{10}$$

where any C^2 class function could be locally regarded as a quadratic function and approximated to the equation (10). Also, when $f(\theta(t))$ is a downwardly convex function, $f'' > 0$. ν^* is the minimum value of $f(\theta(t))$, and θ^* is the unknown frequency of the periodic disturbance. $\theta(t)$ is the input to $f(\theta(t))$ and given as

$$\theta(t) = \hat{\theta}(t) + \beta \sin \omega t. \tag{11}$$

When $f(\theta(t))$ is minimum, $\theta(t) = \theta^*$. Make the following assumption,

Assumption 2. ν^*, θ^* are constants.

Under this assumption, the estimation error $\tilde{\theta}(t)$ is represented with the estimated value $\hat{\theta}(t)$ and θ^* as

$$\tilde{\theta}(t) = \theta^* - \hat{\theta}(t). \tag{12}$$

Substituting equation (12) into equation (11) gives

$$\theta(t) - \theta^* = \beta \sin \omega t - \tilde{\theta}(t). \tag{13}$$

$\nu(t)$ is expanded to

$$\begin{aligned}\nu(t) &= \nu^* + \frac{f''}{2}(\tilde{\theta}(t) - \beta \sin \omega t)^2 \\ &= \nu^* + \frac{\beta^2 f''}{4} - \frac{\beta^2 f'' \cos 2\omega t}{4} \\ &\quad - \beta f'' \tilde{\theta}(t) \sin \omega t + \frac{f'' \tilde{\theta}^2(t)}{2}.\end{aligned} \tag{14}$$

The purpose of the extremum seeking control is to find θ^* that takes the minimum value of the function $f(\theta(t))$. This means $\tilde{\theta}(t) \to 0$ as well. In equation (14), the 4th and 5th terms include information on $\tilde{\theta}(t)$. Therefore, the information of $\tilde{\theta}(t)$ is extracted by perturbation signal and filter processing.

First, apply a high-pass filter to $\nu(t)$ and multiply by $\alpha \sin \omega t$ to obtain $\xi(t)$ as

$$\begin{aligned}\xi(t) = &-\frac{\alpha\beta f'' \tilde{\theta}(t)}{2} - \frac{\alpha\beta^2 f''}{8} \sin 3\omega t + \frac{\alpha\beta^2 f''}{8} \sin \omega t \\ &+ \frac{\alpha\beta f''}{2} \cos 2\omega t + \frac{\alpha f'' \tilde{\theta}^2(t)}{2} \sin \omega t,\end{aligned} \tag{15}$$

where $\tilde{\theta}^2(t)$ in the fifth term is ignored as it is sufficiently small because of the square of the error. High-frequency components are attenuated by the low-pass filter and the integrator, and $\xi(t)$ is approximated to

$$\xi(t) \approx -\frac{\alpha\beta f''}{2} \tilde{\theta}(t). \tag{16}$$

Since $\dot{\tilde{\theta}}(t) = -\dot{\hat{\theta}}(t)$, $\dot{\tilde{\theta}}(t)$ and its solution are given as

$$\dot{\tilde{\theta}}(t) = \frac{k_i \alpha\beta f''}{2} \tilde{\theta}(t), \tag{17}$$

$$\tilde{\theta}(t) = \tilde{\theta}_0 \exp\left(\frac{k_i \alpha\beta f''}{2} t\right), \tag{18}$$

where $\tilde{\theta}_0 = \theta^* - \hat{\theta}_0$ is the initial estimation error and $\hat{\theta}_0$ is the initial estimation value. Therefore, if $k_i \alpha\beta f'' < 0$, it becomes $\tilde{\theta}(t) \to 0$. Utilizing $\hat{\theta}(t)$ as the estimated disturbance frequency $\hat{\omega}_d$.

Table 1. List of two inertia model parameters

Parameter	Notation	Unit
Motor torque	τ_M	$[\mathrm{N \cdot m}]$
Motor angular velocity	ω_M	[rad/s]
Motor moment of inertia	J_M	$[\mathrm{kg \cdot m^2}]$
Motor viscous damping coefficient	D_M	$[\mathrm{N \cdot m/(rad/s)}]$
Load torque	τ_L	$[\mathrm{N \cdot m}]$
Load angular velocity	ω_L	[rad/s]
Load moment of inertia	J_L	$[\mathrm{kg \cdot m^2}]$
Load viscous damping coefficient	D_L	$[\mathrm{N \cdot m/(rad/s)}]$
Stiffness	k_s	$[\mathrm{N \cdot m/(rad/s)}]$
Torsion torque	τ_s	$[\mathrm{N \cdot m}]$

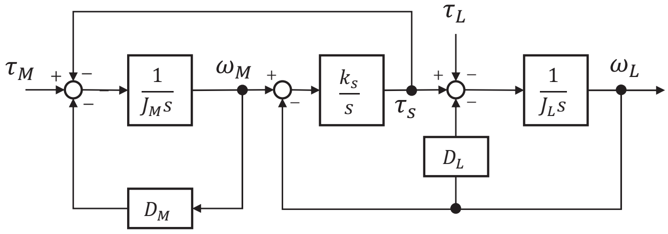

Fig. 4. Block diagram of two inertia model

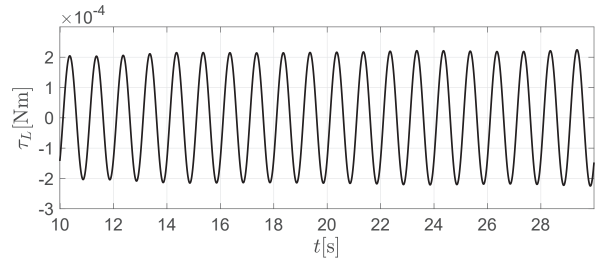

Fig. 5. Sinusoidal disturbance

4. NUMERICAL SIMULATION

The effectiveness of the proposed method is verified by applying it to a two-inertial system model. The block diagram of the two-inertia system model is shown in Fig. 4. The definitions of each symbol are provided in Table 1, and each model parameter values are set as follows: $J_M = 8.5 \times 10^{-6}, J_L = 3.0 \times 10^{-5}$. Stiffness k_s and Viscous damping coefficient D_M, D_L have each variation range that implies a modeling error such that $k_s = [5.0 \times 10^{-4}, 3.0 \times 10^{-3}], D_M = [1.0 \times 10^{-5}, 3.0 \times 10^{-5}], D_L = [1.0 \times 10^{-5}, 3.0 \times 10^{-5}]$.

In the following, $\omega_M(t)$ and $\tau_M(t)$ are denoted as $y(t)$ and $u(t)$. From Fig. 4, the transfer functions from $y(t)$ to $u(t)$ and from $\tau_L(t)$ to $y(t)$ are as follows:

$$\begin{aligned}\frac{Y(s)}{U(s)} &= \frac{s\left(J_L s^2 + D_L s + k_s\right)}{\left(J_M s^2 + D_M s + k_s\right)\left(J_L s^2 + D_L s + k_s\right) - k_s^2} \\ &= G(s),\end{aligned} \tag{19}$$

$$\frac{Y(s)}{T_L(s)} = G(s) \frac{k_s}{\left(J_L s^2 + D_L s + k_s\right)}, \tag{20}$$

where $T_L(s) = \mathcal{L}[\tau_L(t)]$.

In this simulation, constant value control is applied with the reference value $r(t) = 0\,\mathrm{rad/s}$ for $y(t)$. The controlled object is subjected to a sinusoidal load torque with a frequency of $\omega_d = 6.3\,\mathrm{rad/s}$ shown in Fig. 5. Therefore, $G(s)$ in equation (19) is the controlled object, and from equation (20), the periodic disturbance transfer function $D(s)$ is given as

Table 2. List of design parameters

Parameters	Notation	
Initial value of $\hat{\theta}$	$\hat{\theta}_0$	1.0
Amplitude of perturbation signal	α	2.5
Amplitude of perturbation signal	β	6.0
Integral gain	k_i	2.0
Frequency of perturbation signal	ω	1.0
Cut-off frequency of high-pass filter	ω_h	0.3
Cut-off frequency of low-pass filter	ω_h	0.3
Design parameter of smoothing filter	T_l	10

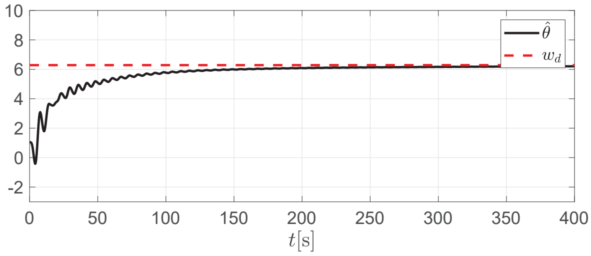

Fig. 6. Estimation result for disturbance frequency

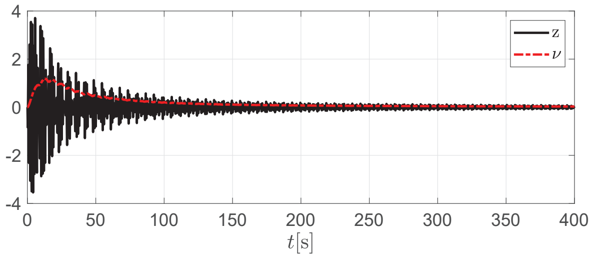

Fig. 7. Trajectory of smoothing filter output

$$D(s) = \frac{k_s}{(J_L s^2 + D_L s + k_s)} T_L(s). \tag{21}$$

The control parameters in equation (7) are as follows: $K_P = -1.7 \times 10^{-3}, K_I = 1.1 \times 10^{-3}, K_S = 1.5$. The bandpass filter is given below, and other parameters are shown in Table 2,

$$F(s) = \frac{0.2\pi\theta s}{s^2 + 0.2\pi\theta s + \theta^2}. \tag{22}$$

First, the estimation result by the automatic frequency estimation mechanism is shown in Fig. 6. From Fig. 6, the estimated value $\hat{\theta}(t)$ converges to the true value of the disturbance frequency ω_d. The smoothing filter output $\nu(t)$ is shown in Fig. 7. From Fig. 7, $z(t)$ is oscillatory because it means the difference between the control error $e(t)$ and the bandpass filter output. Since extremum seeking control is a gradient method, it is necessary to smooth $z(t)$. It is confirmed that $\nu(t)$ decreases as $\hat{\theta}(t)$ converges.

Finally, Control results are compared between PI control, PI control with disturbance observer and the proposed method. The disturbance observer has nominal parameters $k_s = 1.0 \times 10^{-3}, D_M = 2.3 \times 10^{-5}, D_L = 1.5 \times 10^{-5}$. Fig. 8 shows the control results up to 200 s. From Fig. 8, both PI control with the disturbance observer and the proposed method suppresses the effects of the disturbance. The disturbance observer is more effective until the estimated value converges to the true value, but the proposed method is more effective when the estimated value converges to the true value. This may be due to the influence of modeling errors for the disturbance observer.

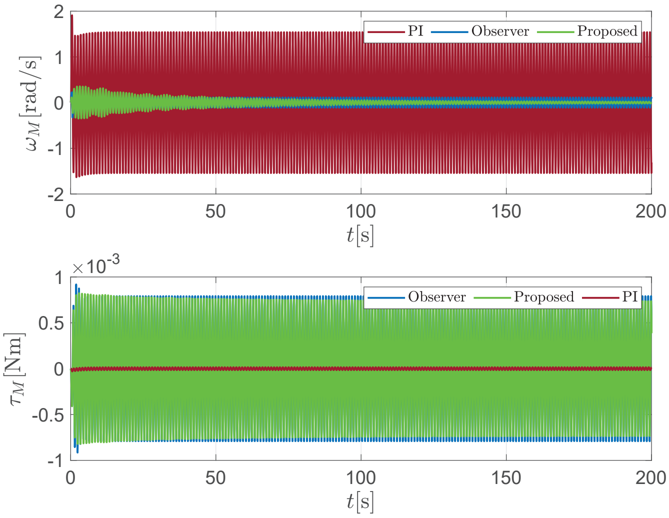

Fig. 8. Control result of PI controller and proposed method

5. CONCLUSIONS

In this paper, the disturbance frequency estimation method based on the extremum seeking control is proposed to construct the sinusoidal internal model for the PIS controller. Its effectiveness is verified by applying the proposed method to the two-inertia system model. As a result, the estimated frequency converged to the true value, and the influence of disturbances was suppressed.

REFERENCES

C. Rohrs, L. Valavani, M. Athans and G. Stein. Robustness of Continuous-Time Adaptive Control Algorithms in the Presence of Unmodeled Dynamics. *IEEE Transactions on autmatic control* 1985.

M. Bodson. Rejection of periodic disturbances of unknown and time-varying frequency. *International Journal of Adaptive Control and Signal Processing* 2004.

S. Fukuda and T. Yoda. A novel current-tracking method for active filters based on a sinusoidal internal model [for PWM inverters]. *IEEE Transactions on Industry Applications* 2001.

Y. Nakamura, H. Akagawa and S. Wakui. Flow disturbance attenuation for pneumatic anti-vibration apparatuses with a Sinusoidal compensator and vibration transmissibility analysis. *Mechanical Engineering Journal* 2020.

S. Takagi, Y. Kurusu and S. Wakui. Disturbance suppression of pneumatic anti-vibration apparatuses using automatic frequency estimation PIS control. *Transactions of the Japan Society of Mechanical Engineers* 2019.

Q. Liu, X. Huo, K. Z Liu, M. Chu and A. Wu. Extremum seeking-based online identification of spatially cyclic disturbances for rotating systems. *Preprints of the 22nd IFAC World Congress* 2023.

K. B. Ariyur, M. Krstic and H-H. Wang. Real-Time Optimization by Extremum-Seeking Control. *John Wiley Sons, Inc. ,* 2003.

M. Krstic and H-H. Wang. Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica* 2000.

B. A. Francis and W. M. Wonham. The internal model principle control theory. *Automatica* 1976.