Anti-windup scheme for PI temperature control of an open-loop unstable chemical reactor

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Abstract: The problem of designing a proportional-integral (PI) temperature control with anti-windup (AW) scheme for an open-loop (OL) unstable chemical reactor is addressed. The aim is to improve the systematization of application-oriented designs employed in industrial practice. The combination of nonlinear dynamics and industrial control ideas yields a PI controller: (i) with back-calculation (BC) AW scheme, (ii) with assurance of robust closed-loop (CL) operation with criterion to choose control gain and limits, and (iii) that outperforms existing AW with BC plus conditional integration (CI) scheme. The proposed approach is illustrated and tested with a representative example through numerical simulations.

Keywords: PI control, anti-windup, saturated control, process control, unstable systems

1. INTRODUCTION

Continuous exothermic chemical reactors are important processes where a wide variety of products and intermediate materials are manufactured in the chemical and petrochemical industries. The reactors can range from spatially lumped stirred tanks to spatially distributed tubular reactors. Due to the combination of linear transport with nonlinear reaction mechanisms this kind of processes have open-loop (OL) dynamics with typical global-nonlinear behavior, such as steady-state (SS) multiplicity, bifurcation, limit cycling and parametric sensitivity [Upal et al., 1974]. In spite of its simplicity against their spatially distributed tubular counterparts, the continuous stirred tank reactors (CSTRs) capture the essence of exothermic reactors with strongly nonlinear behavior. Due to the large ratio of heat production to conversion rate, the safe design and operation of the heat exchange system is of paramount importance. The design or redesign of these reactors to manufacture sophisticated materials, sensitive to temperature changes, or of commodities in an efficient manner, amounts to attain a suitable compromise between product quality regulation, robustness and control effort. In general, this means pushing the process to operation in regions of nonlinear behavior, with more demand on the monitoring and control schemes.

By far, industrial chemical reactors are operated with proportional-integral (PI) temperature control schemes with anti-windup (AW) protection for control saturation handling [Shinskey, 1990]. The advantages of the PI control are its simplicity, but their drawback is that the development and maintenance relies heavily in per process experience, insight and extensive testing with simulation followed by pilot to industrial scaling up. In particular, this is the case of a PI controller equipped with AW scheme, in the understanding that control saturation in OL fragilely stable or unstable reactors can induce performance degradation or even generation of unproductive extinction or catastrophic ignition straneous SSs [Alvarez et al., 1991, Chen and Chang, 1985].

The state of the art on PI control with AW schemes can be seen elsewhere, ([Åström and Hägglund, 2006, Visioli, 2006], and references therein) and here it suffices to say that: (i) the problem has been addressed with a diversity of techniques for an ample range of processes that include OL unstable processes, and (ii) the scheme with back-calculation (BC) plus conditional integration (CI) have been favorable compared against a set of eight different techniques [Visioli, 2006]. Basically, the task of designing a PI plus AW ($PI + AW$) controller for stable or unstable systems is performed in two steps. First, on the basis of a unstable first order plus time delay (UFOPTD) model identified with relay or step inputs, and in the absence of control saturation, standard tuning rules [Internal Model Control (IMC), Ziegler-Nichols, and so on] are applied to choose the proportional-integral gain pair in order to attain a suitable transient versus control effort behavior [O'Dwyer, 2009]. Then, the AW reset time is adjusted to avoid performance degradation due to saturation, different techniques and recommendations in how to tune it exists. It must be pointed out that the key choose of control limits, especially for OL unstable systems, is not an explicit part of the $PI + AW$ control design procedures and techniques.

These considerations motivate the present study on the control of exothermic CSTRs with different $PI + AWs$, with emphasis in the improvement of the systematization

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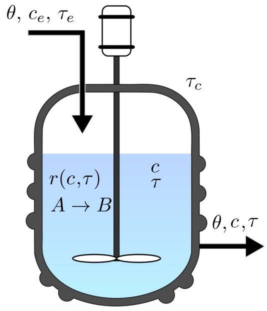

Fig. 1. Continuous stirred-tank reactor

and performance of the design by reducing the tuning and testing effort beyond a per process basis. The idea is to exploit, as much as possible, the accumulated knowledge and insight on the nonlinear behavior of chemical reactors, to obtain a PI controller with: (i) BC-AW scheme, (ii) guarantee of robust closed-loop (CL) stability accompanied by criteria to choose control gains and limits, and (iii) better behavior that the ones of existing schemes. The proposed approach is illustrated with a simulated example.

2. CONTROL PROBLEM

Consider the jacketed CSTR depicted in Fig. 1, where a reactant is converted in product through a first-order exothermic reaction with Arrhenius temperature dependency, and heat removal is perform with a coolant jacket. For standard modeling assumptions [Aris, 2013], the reactor dynamics are the dimensionless mass and heat balances

$$\dot{c} = \bar{\theta}(\bar{c}_e - c) - c\alpha(\tau) + \tilde{f}_c(c; \tilde{\theta}, \tilde{c}_e), \quad c(0) = c_o \tag{1a}$$

$$\dot{\tau} = \bar{\theta}(\bar{\tau}_e - \tau) - \bar{\upsilon}(\tau - u) + c\alpha(\tau) + \tilde{f}_\tau(\tau; \tilde{\theta}, \tilde{\tau}_e, \tilde{\upsilon}), \ \tau(0) = \tau_o \tag{1b}$$

$$\alpha(\tau) = \mathrm{e}^{a_r - \epsilon/\tau}, \quad a_r = \ln Da, \quad \tilde{\theta}^- \le \tilde{\theta}(t) \le \tilde{\theta}^+,$$
$$\tilde{c}_e^- \le \tilde{c}_e(t) \le \tilde{c}_e^+, \quad \tilde{\tau}_e^- \le \tilde{\tau}_e(t) \le \tilde{\tau}_e^+, \quad \tilde{\upsilon}^- \le \tilde{\upsilon}(t) \le \tilde{\upsilon}^+$$

c (or τ) is the reactant concentration (or temperature) state, $y = \tau$ is the measured temperature output, $u \in U$ is the manipulated jacket coolant temperature (τ_c), $\bar{\theta}$ is the nominal flow rate, $\bar{c}_e$ (or $\bar{\tau}_e$) is the nominal inlet concentration (or temperature), $\bar{\upsilon}$ is the nominal (heat transfer-to-convection) Stanton number, $\alpha(\tau)$ is the Arrhenius function, ϵ is the (activation energy-to-adiabatic temperature rise) Arrhenius number and a_r is computed with the reaction rate-to-convection Damkholer number Da. The functions $\tilde{f}_c$ and $\tilde{f}_\tau$ reflect the effect of the flow ($\tilde{\theta}$), feed concentration ($\tilde{c}_e$) and temperature ($\tilde{\tau}_e$), as well as Stanton number $\tilde{\upsilon}$ bounded fluctuations, where the notation $\tilde{(\cdot)} = (\cdot) - \bar{(\cdot)}$, is used, and $\bar{(\cdot)}$ means nominal value. The vector $\boldsymbol{d} \in D$ i of exogenous input-disturbances is defined as

$$\boldsymbol{d} = (\tilde{\theta}, \tilde{c}_e, \tilde{\tau}_e, \tilde{\upsilon})^T \tag{2}$$

and U and D and are bounded spaces.

Over its parameter space, the errorless Reactor (1) [with $\tilde{f}_c = \tilde{f}_\tau = 0$] has regions of simple and complex nonlinear OL dynamics, including monostability, bistability and limit cycling delimited by saddle-node and Hopf bifurcation manifolds [Upal et al., 1974]. Simple dynamics means with a unique robust attractor, and complex dynamics means with phenomena that occur "in the large" such as SS multiplicity and limit cycling [Hubbard and West, 1991]. Robustness means structural stability, when small parameter changes do not change the geometry of the dynamics [Hirsch et al., 2012].

As representative case example let us regard the bistable case of Reactor (1) with parameters [Aris, 2013]

$$\bar{\theta} = \bar{c}_e = \bar{\upsilon} = 1, \ \bar{\tau}_e = \bar{\tau}_c = 1.75, \ \epsilon = 50, \ a_r = 25 \tag{3}$$

and three-SS set ($\boldsymbol{x} = [c, \tau]^T$, $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u)$)

$$\bar{\boldsymbol{x}}_I = (0.0089, 2.206), \ \bar{\boldsymbol{x}}_S = (0.5, 2.0), \ \bar{\boldsymbol{x}}_E = (0.964, 1.768)$$

with stable ignition (or extinction) $\bar{\boldsymbol{x}}_I$ (or $\bar{\boldsymbol{x}}_E$), and unstable saddle $\bar{\boldsymbol{x}}_S$. The corresponding statics are

$$\boldsymbol{f}(\bar{\boldsymbol{x}}_i, \bar{u}) = \boldsymbol{0}, \ \bar{y}_i = \boldsymbol{c}_y \bar{\boldsymbol{x}}_i, \ i = E, S, I, \ \bar{\boldsymbol{x}}_s := \bar{\boldsymbol{x}}, \ \boldsymbol{c}_y \bar{\boldsymbol{x}}_S := \bar{y} \tag{4}$$

where $\bar{\boldsymbol{x}}$ is the prescribed SS for CL operation, and $\bar{y}$ the corresponding temperature setpoint.

2.1 PI control with anti-windup schemes

Here we present two common AW techniques that are used in industry to avoid degradation of CL performance under saturation.

Controller: The structure of a PI controller with AW scheme is as follows

$$u = \bar{u} + PI(\psi) + AW(v) \tag{5a}$$

$$PI(\psi) = -k_p \left(\psi + t_I^{-1} \int_0^t \psi ds \right), \ \psi = y - \bar{y} \tag{5b}$$

$$u_s = \mathrm{sat}(u) = \begin{cases} u^+ & \text{if } u > u^+ \\ u & \text{if } u_- \le u \le u^+ \\ u^- & \text{if } u < u_- \end{cases} \tag{5c}$$

$$\boldsymbol{k}_c = (\boldsymbol{k}_g, \boldsymbol{l}), \ \boldsymbol{k}_g = (k_p, t_I, t_a)^T, \ \boldsymbol{l} = (u^-, u^+) \tag{5d}$$

where PI is the proportional-integral control operator with proportional gain k_p and reset time t_I, AW is the AW operator, u_s is the saturation of the calculated control signal u, and u^- (or u^+) is the lower (or upper) control limit; $\boldsymbol{k}_c$ contains the five adjustable control parameters (5d): three gains ($\boldsymbol{k}_g$), and two control limits ($\boldsymbol{l}$).

The first AW technique is the well-known BC one [Åström and Hägglund, 2006], with structure

$$AW_{bc}(v) = t_a^{-1} \int_0^t v ds, \ v = u_s - u \tag{6}$$

where t_a is the anti-windup reset time, typically chosen as $t_a \le t_I$. This scheme is the most used technique and the rationale behind its functioning is that the integration of the difference between the applied control signal and the computed one, when saturation is active, compensates the accumulated error in the integral term of the PI controller. The second one is a combination of BC with CI (BC-CI) [Visioli, 2006] which combines the normal behavior of BC and logic conditions to switch off the integral term with the aim to avoid stopping integration at the beginning of the transient response when saturation is due to the proportional term, and at the same time to allow the decrease of the value of the anti-windup reset time in order to have smaller overshot. These combination provides better behavior under saturation specially for setpoint changes operation, and its structure is given by

$$AW_c(v,\psi) = \begin{cases} AW(v) & \text{if } C_I \text{ is met} \\ 0 & \text{otherwise} \end{cases} \tag{7a}$$

$$C_I : v \neq 0,\ \tilde{u} \times \psi > 0,\ \begin{cases} \psi > \psi_o \text{if } \ \psi_n > \psi_o \\ \psi < \psi_o \text{if } \ \psi_n < \psi_o \end{cases} \tag{7b}$$

$$\tilde{u} = u - \bar{u},\ \psi_o = y_o - \bar{y},\ \psi_n = y_n - \bar{y} \tag{7c}$$

C_I is the variable-structure integration condition, ψ_o (or ψ_n) is the old (or new) setpoint in deviation variable (with respect to the nominal value $\bar{y}$ (4)) and the anti-windup reset time must be chosen as $t_a = 0.03t_I$.

Tuning and functioning: To illustrate the tuning and performance of the two introduced $PI + AW$ controllers consider Reactor (1) with a delay $L = 0.045$, and the standard procedures for tuning [O'Dwyer, 2009, Visioli, 2006] discussed in the Introduction of this work. First, the nonlinear model (1) must be approximated with an UFOPTD model with the following structure,

$$\dot{y} = \lambda_x[y + Ku(t - t_D)], \quad y(0) = y_0, \quad \lambda_x = t_x^{-1} \tag{8}$$

For the case example a relay control-based test (with parameters $h = 1$, $\epsilon_r = 0.001$) [Bajarangbali et al., 2014] was used, and on the basis of the input-output signals obtained in the test the following parameters were identified: $t_x = 0.288$, $t_D = 0.047$, $K = 0.2564$. Then, considering no saturation, from a extensive collection of available IMC tuning rules for UFOPTD systems [O'Dwyer, 2009], the ones proposed by [Chidambaram et al., 1998] were selected because of its excellent performance (suitable compromise between transient response and control effort), thus the the gain pair (k_p, t_I) was computed as

$$k_p = \frac{1}{K}\left[4.656 - 13.05\frac{t_D}{t_x} + 1.436\frac{t_D}{t_x}\right], \quad t_I = 25t_x - 27t_D$$

Finally, for the PI with BC-AW ($PI + AW_{bc}$) technique the reset time is chosen as $t_a = t_I$, and for the PI with BC-CI-AW ($PI + AW_c$) technique $t_a = 0.03t_I$ [Visioli, 2006]. The resulting set of gains for the $PI + AW_{bc}$ controller (5), (6) is $\boldsymbol{k}_{g1}$ and for the case $PI + AW_c$ controller (5), (7) is $\boldsymbol{k}_{g2}$, with numerical values

$$\boldsymbol{k}_{g1} = (10.644, 5.199, 5.199),\ \boldsymbol{k}_{g2} = (10.644, 5.199, 0.156) \tag{9}$$

A simulation to test the performance and robustness of both controllers was performed. The control task is to steer the process output from the initial condition $y_0 = 1.95$ to the reference $r = \bar{y} = 2$ (the OL unstable SS) and then perform a step change to $r = \bar{y} = 2.033$ in order to change the quality product (a change from $\bar{c} = 0.5$ to 0.4 of reactant concentration), and then a step change in the inlet temperature $\tilde{\tau}_e$ was applied. The CL behavior is presented in Figs. 2 and 3 with two control limit pairs

$$\boldsymbol{l}_b = (1.67, 1.86), \qquad \boldsymbol{l}_g = (1.6, 1.9) \tag{10}$$

showing that: (i) with $\boldsymbol{l}_b$ (an unsuccessful trial) the reactor reaches a extinction-type straneous SS $\bar{\boldsymbol{x}} \neq \bar{\boldsymbol{x}}_S$ attractor with significant temperature offset (Fig. 2), and (ii) with $\boldsymbol{l}_g$ drawn (after 4 to 10 trials) by trial and error the reactor reaches the prescribed temperature setpoint with an intermediate temperature-composition SS $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}_S$ (Fig. 3). These simulations shows the importance of an adequate selection of control limits. The above considerations motivate the scope of this study: the derivation of a formal control gain-limit criterion to preclude CL

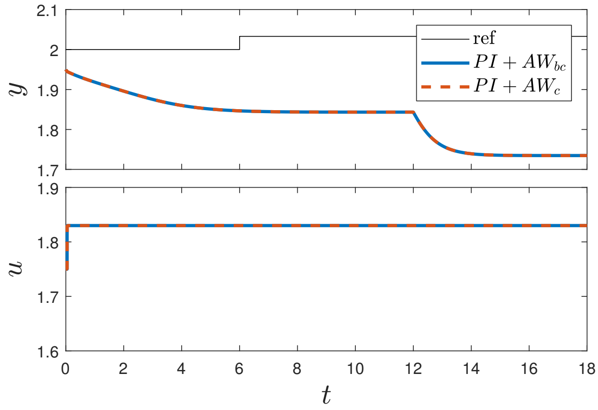

Fig. 2. CL reactor output y, concentration c, and control u behavior with $PI + AW_{bc}$, [(5),(6)] and $PI + AW_c$ [(5),(7)] with gains $\boldsymbol{k}_{g1}$ and $\boldsymbol{k}_{g2}$ in (9), respectively, and limits $\boldsymbol{l}_b$ (10)

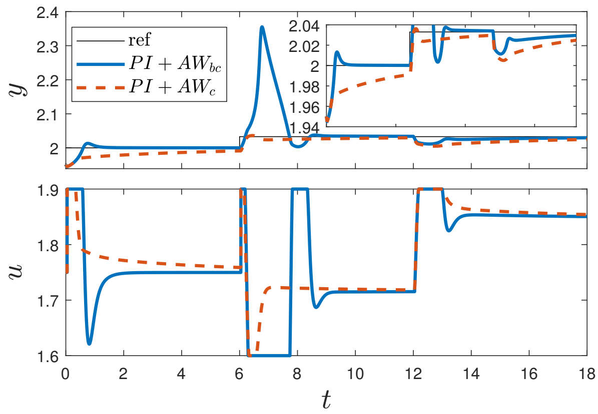

Fig. 3. CL reactor output y, concentration c, and control u behavior with $PI + AW_{bc}$, [(5),(6)] and $PI + AW_c$ [(5),(7)] with gains $\boldsymbol{k}_{g1}$ and $\boldsymbol{k}_{g2}$ (9), respectively, and limits $\boldsymbol{l}_g$ (10)

straneous attractors in the CL reactor with $PI + AW_{bc}$ and/or $PI + AW_c$.

2.2 Problem statement

Our problem consists in improving the industrial-type $PI + AW_{bc}$ (5), (6) and/or $PI + AW_c$ (5), (7) control designs to regulate the OL bistable reactor (1) about its unstable steady state $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}_S$ (4) in spite of load (input and/or setpoint) disturbances, including: (i) a priori (before simulation and/or testing) assurance of offsetless regulation with robust CL stability in terms of control gain $\boldsymbol{k}_g$ and limits $\boldsymbol{l}$ (5d), (ii) control scheme [preferably $PI + AW_{bc}$ (5), (6)] as simple as possible, (iii) transparent, easy-to-apply and systematic tuning procedure for the gain-limit five-entry vector $\boldsymbol{k}_c$ (5d), and (iv) improved compromise between output regulation speed, robustness and control effort.

3. CONTROLLER SYNTHESIS

As point of departure for the controller synthesis the OL statics are analyzed with a bifurcation analysis which led to information that will be used later for the selection of control limits. An input-output bifurcation map (IOBM) for the OL system (1) will be constructed as in [Alvarez et al., 1991], for this aim consider the OL statics (4) written as

$$\bar{\theta}(\bar{c}_e - c) - r(c,\tau) + \tilde{f}_c(c;\tilde{\theta},\tilde{c}_e) = 0 \tag{11a}$$

$$\bar{\theta}(\bar{\tau}_e - \tau) - \bar{\upsilon}\,(\tau - u) + c\alpha(\tau) + \tilde{f}_\tau(\tau;\tilde{\theta},\tilde{\tau}_e,\tilde{\upsilon}) = 0 \tag{11b}$$

solving (11a) for c we obtain

$$c = m(\tau) + d_c(\bar{y};\boldsymbol{d}), \quad m(\tau) = \frac{\bar{\theta}\bar{c}_e}{\bar{\theta} + \alpha(\tau)}$$

$$d_c(\bar{y};\boldsymbol{d}) = \frac{(1+\tilde{\theta})\tilde{c}_e + \left[\bar{c}_e\alpha(\tau)/(\bar{\theta}+\alpha(\tau))\right]\tilde{\theta}}{\bar{\theta} + \alpha(\tau) + \tilde{\theta}}$$

and substituting in (11b) considering no fluctuation in exogenous vector input $\boldsymbol{d}$ we obtain the OL IOBM, given by

$$\mathcal{O}:\ u = \mu_o(\tau, \bar{\boldsymbol{d}}) \tag{12}$$

where

$$\mu_o(u,\tau,\boldsymbol{d}) = [-\bar{\theta}(\bar{\tau}_e - \tau) + \bar{\upsilon}\tau - [m(\tau) + d_c(\bar{y};\boldsymbol{d},\tau]\alpha(\tau) - \tilde{f}_\tau(\tau;\tilde{\theta},\tilde{\tau}_e,\tilde{\upsilon})]/\bar{\upsilon}$$

and its perturbed versions

$$\mathcal{O}^+ :\ u = \mu_o(\tau,\boldsymbol{d}^-), \quad \mathcal{O}^- :\ u = \mu_o(\tau,\boldsymbol{d}^+) \tag{14}$$

These curves determines the SS multiplicity of the OL systems and from its geometric representation (Fig. 4) we can find the control bifurcation values, denoted by u_* (or u^*) as the the minimum (or maximum) values of the curve $\mathcal{O}^+$ (or $\mathcal{O}^-$) at the temperature τ_* (or τ^*), i.e.,

$$u_* = \min_\tau \mu_o(\tau,\boldsymbol{d}^+),\ u^* = \max_\tau \mu_o(\tau,\boldsymbol{d}^-) \tag{15}$$

This bifurcation values will be used after for the selection of control limits for the $PI + AW$ controller (5).

3.1 Closed-loop dynamics

The application of the PI controller (4), in state space form (considering that u does not reach its limits)

$$\dot{\iota} = -\frac{k_p}{t_I}(\tau - \bar{y}),\ \iota(0) = \iota_o, \quad u_s = \mu_s(\tau,\iota) \tag{16a}$$

$$\mu_s(\tau,\iota) = \mathrm{sat}[\mu(\tau,\iota)], \quad \mu(\tau,\iota) = \bar{u} - k_p(\tau - \bar{y}) - \iota \tag{16b}$$

to the reactor (1) yields the CL system

$$\dot{c} = \bar{\theta}(\bar{c}_e - c) - r(c,\tau) + \tilde{f}_c(c;\tilde{\theta},\tilde{c}_e), \quad c(0) = c_o \tag{17a}$$

$$\dot{\tau} = \bar{\theta}(\bar{\tau}_e - \tau) - \bar{\upsilon}\,[\tau - \mu_s(\tau,\iota)] + c\alpha(\tau) + \tilde{f}_\tau(\tau;\tilde{\theta},\boldsymbol{d}) \quad \tau(0) = \tau_o \tag{17b}$$

$$\dot{\iota} = -\frac{k_p}{t_I}(\tau - \bar{y}), \qquad \iota_0 = \iota_o \tag{17c}$$

on the basis of the SSs analysis of the CL system it is possible to identify restriction on control limits in order to compensate disturbances and avoid the induction of straneous attractors.

3.2 Necessary control limits pair condition

Here, necessary conditions for the control limits of the $PI+AW$ controller are derived in order to attain offsetless regulation. For this aim. For given $\boldsymbol{d}$ (2), the CL statics (17) are

$$\bar{\theta}(\bar{c}_e - c) - r(c,\tau) + \tilde{f}_c(c;\tilde{\theta},\tilde{c}_e) = 0 \tag{18a}$$

$$\bar{\theta}(\bar{\tau}_e - \tau) - \bar{\upsilon}\,[\tau - \mu_s(\tau,\iota)] + c\alpha(\tau) + \tilde{f}_\tau(\tau;\tilde{\theta},\tilde{\tau}_e,\tilde{\upsilon},) = 0 \tag{18b}$$

$$-\frac{k_p}{t_I}(\tau - \bar{y}) = 0 \tag{18c}$$

whit unique-robust solution

$$\bar{\boldsymbol{x}}_c = (c,\tau,\iota)^T, c = m(\tau) + d_c(\bar{y};\boldsymbol{d}),\ \tau = \bar{y},\ \iota = d_\iota(\bar{y};\boldsymbol{d}) \tag{19}$$

$$d_\iota(\bar{y};\boldsymbol{d}) = \Big[\alpha(\bar{y})d_c(\bar{y},\boldsymbol{d}) + \tilde{\boldsymbol{f}}_\tau(\bar{y},\boldsymbol{d})\Big]$$

and control value

$$\bar{u}_\infty = \bar{u} - d_\iota(\bar{y};\boldsymbol{d}) \tag{20}$$

As expected [Shinskey, 1990], (20) says that the static integral action achieves offsetless output regulation ($\tau = \bar{y}$) by compensating, in a feedforward-like manner, the effect of the constant load disturbance $\boldsymbol{d}$ on the regulated output y. Let us denote by $\boldsymbol{d}^+$ (or $\boldsymbol{d}^-$) the combinations of disturbance limits that pose the largest (or lowest) heat exchange rate $\bar{\upsilon}(\bar{y} - u_\infty^-)$ [or $\bar{\upsilon}(\bar{y} - u_\infty^-)$]

$$\boldsymbol{d}^+ = (\tilde{\theta}^+,\tilde{c}_e^+,\tilde{\tau}_e^+,\tilde{\upsilon}^+)^T,\ \boldsymbol{d}^- = (\tilde{\theta}^-,\tilde{c}_e^-,\tilde{\tau}_e^-,\tilde{\upsilon}^-)^T \tag{21}$$

From these expressions, control (20) (with only integral action at play) is bounded from below and above as follows

$$u_\infty^-(\boldsymbol{d}) \le \bar{u}_\infty \le u_\infty^+(\boldsymbol{d}) \tag{22}$$

$$u_\infty^-(\boldsymbol{d}^+) := \bar{u} + d_\iota(\bar{y},\boldsymbol{d}^-),\ d_\iota(\bar{y},\boldsymbol{d}^-) < 0,$$
$$u_\infty^+(\boldsymbol{d}^-) := \bar{u} + d_\iota(\bar{y},\boldsymbol{d}^+),\ d_\iota(\bar{y},\boldsymbol{d}^+) > 0$$

Summarizing, the feedforward-like controller (with only integral action) is bounded from below and above (22), and consequently, the lower-upper control bound pair $[u_\infty^-(\boldsymbol{d}^-), u_\infty^-(\boldsymbol{d}^+)]$ conditions are necessarily (but not sufficiently) so that the $PI + AW$ controller attain offsetless regulation with unique CL SS $\bar{\boldsymbol{x}}_c$ (19), when control saturation is present.

3.3 Control limits for closed-loop stability

Here, necessary and sufficient conditions on the control gain and limits of the $PI + AW$ controller are derived, in order to attain offsetless regulation with unique CL SS. For this aim, consider the CL dynamics (17), that can be written as follows

$$u = \mu_o(\tau,\bar{\boldsymbol{d}}), \quad u_s = \mu_s(\tau,\iota), \quad -\frac{k_p}{t_I}(\tau - \bar{y}) = 0 \tag{23}$$

the first equation in (23) is the OL IOBM (12) and the second one produce the CL curve

$$\mathcal{C}:\ u = \mu_s(\tau,\iota) \tag{24}$$

the third equation in (23) results from the integral term and ensures ofsetless regulation to the prescribed SS if condition (22) are met. According to this rationale, the CL SSs are given by the intersections of the curve $\mathcal{C}$ and $\mathcal{O}$ when there is no disturbances, in other case the SSs solutions are given by the intersections of the perturbed versions $\mathcal{C}^-$ (or $\mathcal{C}^+$) with $\mathcal{O}^-$ (or $\mathcal{O}^+$). Since the nominal reactor is bistable, its OL curve $\mathcal{O}$ is a cubicoid with three roots, one per SS, and the control curve $\mathcal{C}$ is a straight line with negative slope (proportional to the gain k_p) over $[u^-, u^+]$. In order to ensure a unique CL SS, a unique robust intersection between curves $\mathcal{C}$ and $\mathcal{O}$ is needed and also that the control limit u^+ (or u^-) is chosen above (or below) the bifurcation value u^* (or u_*) and the proportional gain k_p is grater than the slope k_p^* at the SS point of the open loop curve $\mathcal{O}$. This geometric global-nonlinear result is stated next in proposition form.

Proposition 1. The CL system (17) of the reactor (1) with $PI+AW$ control (5) has as unique robust SS $\bar{\boldsymbol{x}}_c$ (19) with offsetless regulated output $\tau = \bar{y}$ if and only if the proportional control gain k_p is chosen sufficiently large and the control limit u^- (or u^+) are chosen sufficiently small (or large) so that the open loop curve $\mathcal{O}$ and CL curve $\mathcal{C}$ have one robust intersection at the temperature-control point $(\bar{y}, \bar{u})$, and the perturbed curves $\mathcal{O}^+$ (or $\mathcal{C}^+$) and $\mathcal{O}^-$ (or $\mathcal{C}^-$) have a unique intersection at the point $(\bar{y}, u_\infty^+)$ [or $(\bar{u}, u_\infty^+)$], i.e.,

$$\text{(i)}\, k_p > k_p^* + \varepsilon_k,\ \text{(ii)}\, u < u_* - \varepsilon_*,\ \text{(iii)}\, u^+ > u^* + \varepsilon^* \square \quad (25)$$

In condition (25), the positive constant triplet $(\varepsilon_k, \varepsilon_*, \varepsilon^*)$ ensure the robustness of th intersection between $\mathcal{O}^- \cap \mathcal{C}^-$ and $\mathcal{O}^+ \cap \mathcal{C}^+$. The constant k_p^* is defined as

$$k_p^* = \gamma'(\bar{\tau}) - \bar{\lambda}_\tau > 0,\ \gamma = m(\tau)\alpha(\tau),\ \bar{\lambda}_\tau = \eta'(\tau) = \bar{\theta} + \bar{\upsilon}$$

and is the negative slope of the cubicoid $u = \mu_o(\tau, \boldsymbol{d})$.

4. CONTROL TUNING

In this section, the tuning (discussed in Section 2) of the $PI + AW$ control scheme (5) is improved, by formulating the three adjustable gains (k_p, t_I, and t_a) in terms of only two adjustable, transparent and easy to tune parameters. For this aim, recall the model (1) of the rector and rewrite it in the u-parametric differential-algebraic form

$$\dot{c} = \bar{\theta}(\bar{c}_e - c) - c\alpha(\tau) + \tilde{f}_c(c; \tilde{\theta}, \tilde{c}_e), \quad c(0) = c_o \quad (26a)$$

$$\dot{\tau} = \lambda_m \tau + \bar{\upsilon} u + d_m, \quad \tau(0) = \tau_o, \quad y = \tau \quad (26b)$$

$$d_m = f_m(c, \tau, \tilde{\theta}, \tilde{\tau}_e, \tilde{\upsilon}) \quad (26c)$$

where

$$\lambda_m := k_p^* > 0,$$

$$f_m(c, \tau, \tilde{\theta}, \tilde{\tau}_e, \tilde{\upsilon}) := \bar{\theta}(\bar{\tau}_e - \tau) - \bar{\upsilon}\tau + c\alpha(\tau) + \tilde{f}_\tau(\tau; \tilde{\theta}, \tilde{\tau}_e, \tilde{\upsilon},)$$

This model is the point of departure to construct the proposed $PI + AW$ controller.

4.1 Model for tuning

Following [Schaum et al., 2015], in the reactor model (26) drop the concentration dynamics (26a) and the algebraic element (26c) to get the model

$$\dot{\tau} = \lambda_m \tau + \bar{\upsilon} u + d_m, \quad \tau(0) = \tau_o, \quad \dot{d}_m \approx 0$$

for control design, with exogenous unmeasured input d_m. In deviation variables, this model becomes

$$\dot{\psi} = \lambda_m \psi + \bar{\upsilon}\tilde{u} + \iota_m, \quad \psi(0) = \psi_0, \quad |\dot{\iota}_m/\iota_m| \approx \lambda_m \ll \omega \quad (27)$$

$$\psi = y - \bar{y},\ \tilde{u} = u - \bar{u},\ \iota_m = d_m - \bar{d}_m,\ \bar{d}_m = f_m(\bar{c}, \bar{\tau}, \boldsymbol{d})$$

where ι_m is: (i) a slow-varying (with respect to the value ω to be precised) unmeasured exogenous input signal, (ii) time-wise determined by the measured signal pair $(y, u)(t)$, and (iii) reconstructible arbitrarily fast (up to measurement noise), through a dynamic linear observer. The proposed model, compared with the conventional UFOPTD (8), does not require an identification procedure to determine its parameters since the information of the detailed model is used and only approximated values of λ_m (obtained with a Taylor series expansion) and $\bar{\upsilon}$ [obtained from model (1) parameters] are needed. In order to attain for the unknown nonlinearities the proposed model condensates all its effects in an reconstructible signal which is a smarter and more efficient mechanisms than only approximating nonlinear behavior with a time delay.

4.2 PI with back calculation AW scheme

The enforcement of the robustly stable linear output regulation dynamics

$$\dot{\psi} = -(k - \lambda_m)\psi, \quad \psi_0(0) = \psi_0, \quad k > \lambda_m + \varepsilon_\lambda$$

on the dynamic model (27) followed by solution for u of the resulting algebraic equation yields the linear output-feedback controller

$$\tilde{u} = -(k\psi + \iota_m)/\bar{\upsilon} \quad (28)$$

On the other hand, on the basis of the OL simplified model (27), set the (improper) observer

$$\dot{\hat{\iota}}_m = \omega\left[\dot{\psi} - (\lambda_m \psi + \bar{\upsilon}\tilde{u} + \hat{\iota}_m)\right], \quad \hat{\iota}_m(0) = \hat{\iota}_{mo}$$

and apply the coordinate change $\chi = \hat{\iota}_m - \omega\psi$ to obtain the first-order proper load observer

$$\dot{\chi} = -\omega\chi - \omega\left[(\omega + \lambda_m) + \bar{\upsilon}\tilde{u}\right], \quad \chi(0) = \chi_o, \quad \hat{\iota}_m = \chi + \omega\psi \quad (29)$$

The combination of the saturated version of controller (28) with the load observer (29) yields the linear output feedback controller in IMC form

$$\dot{\chi} = -\omega\chi - \omega\left[(\omega + \lambda_m)\psi + \bar{\upsilon}\tilde{u}_s\right], \quad \chi(0) = \chi_o$$

$$\tilde{u}_s = \text{sat}\left\{-\left[(k + \omega)\psi + \chi\right]\right\}$$

which can be rewritten as

$$\dot{\chi} = -\omega\chi - \omega\left[(\omega + \lambda_m)\psi + \bar{\upsilon}\tilde{u}\right] - \omega\bar{\upsilon}(\tilde{u}_s - \tilde{u}),\ \chi(0) = \chi_o$$

$$\tilde{u}_s = \text{sat}\left\{-\left[(k + \omega)\psi + \chi\right]\right\}$$

substitution of the calculated controller (28) yields

$$\dot{\chi} = \omega(k - \lambda_m)\psi - \omega\bar{\upsilon}\tilde{v},\ \tilde{v} = \tilde{u}_s - \tilde{u},\ \tilde{u}_s = -(k + \omega)\psi - \chi$$

which in $PI + AW$ form is given by

$$\tilde{u} = -\frac{k + \omega}{\bar{\upsilon}}\left\{\psi + \left[\frac{\omega(k - \lambda_m)}{k + \omega}\int_0^t \psi\, ds\right]\right\} + \omega\int_0^t \tilde{v}\, ds \quad (30a)$$

$$\tilde{u}_s = sat(\tilde{u}), \qquad \tilde{v} = \tilde{u}_s - \tilde{u} \quad (30b)$$

which is the $PI + AW_{bc}$ controller (5), (6) with the following choice of proportional gain and integral and anti-windup reset times

$$k_p = \frac{k + \omega}{\bar{\upsilon}} > k_p^* + \varepsilon_k,\ t_I = (k + \omega)/\left[\omega(k - \lambda_m)\right] \quad (31a)$$

$$t_a = \omega^{-1}, \quad \omega \approx n_\omega k, \quad n_\omega \in [3, 20] \quad (31b)$$

Compared with the standard design (5) with three adjustable parameters, the proposed $PI+AW$ scheme underlain by two-adjustable parameters: the convergence speeds k, of the prescribed CL dynamics, and ω of the estimator. The choice $\omega \approx n_\omega k$ states that the load estimation dynamics is from three to twenty times faster than the prescribed CL output regulation dynamics.

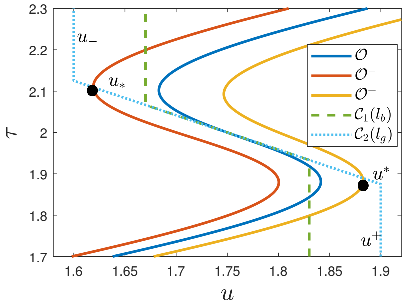

Fig. 4. Nominal OL $\mathcal{O}$, its perturbed versions $\mathcal{O}^-$ and $\mathcal{O}^+$ and control curves $\mathcal{C}_1(\boldsymbol{l}_b)$, and $\mathcal{C}_2(\boldsymbol{l}_g)$

5. CONTROL FUNCTIONING

Here, the functioning of the proposed controller $PI+AW_p$ (28) is presented through the case example with numerical simulation, and compared with the $PI + AW_c$ controller (5), (7) discussed in Section 2. In Fig. 4 are presented the nominal IOBM and its perturbed versions. From this geometric representation the bifurcation values $(u_*, u^*) = (1.619, 1.885)$ are founded. Also in Fig. 4, are presented two versions of the saturated control curve $\mathcal{C}_1(\boldsymbol{l}_b)$ (or $\mathcal{C}_2(\boldsymbol{l}_g)$), both with proportional gain $k_p = 16.8$, but control limit $\boldsymbol{l}_b$ (or $\boldsymbol{l}_g$) from (10). In the first (or second) case the control limits were chosen so that Proposition 1 is violated (or fulfilled), in the light of admissible disturbance sizes. Fig. 4 explains why the PI+AW controllers of Section 2 failed -Fig. 2-, (or functioned) (-Fig. 3-): because the gain limit conditions of Proposition 1 were violated (or met). When the condition of Proposition 1 are met the curves $\mathcal{O}$ and $\mathcal{C}_2(\boldsymbol{l}_g)$ in Fig. 4 have one robust intersection. Otherwise when the upper limit condition is violated ($\mathcal{C}_1(\boldsymbol{l}_b)$ in Fig. 4) there are three intersections with two straneous stable attractors (in Fig. 2 the extinction one was reached).

The application the tuning procedure (31) yielded, after two to three refinement trials aver to adjustable parameter [$k = 2$ and $\omega = 2.5$ in (31)] the gains:

$$(k_p, t_I, t_a) = (16.800, 0.486, 0.007)$$

The functioning of the proposed controller $PI+AW_p$ (30) and its $PI + AW_c$ (5), (7) counterpart, with gains $\boldsymbol{k}_{g2}$ in (9) are presented in Fig. 5, when the reactor is subjected to sequenced temperature initial condition, setpoint and feed temperature step input disturbances. As it can be seen, the proposed $PI + AW_p$ controller outperforms the $PI + AW_c$, with faster and smoother response and less wasteful control action.

6. CONCLUSIONS

The $PI+AW$ scheme for an OL unstable chemical reactor has been upgraded with: (i) a more systematic construction, (ii) better performance in the sense of compromise between output regulation speed and smoothness, disturbance rejection and setpoit tracking capability, and control effort. The design includes: (i) assurance of robust CL stability, (ii) a detailed model-based criterion to choose control limits, (iii) simple and transparent tuning rules to choose the three control parameters (proportional gain as well as integral reset time of the $PI + AW$ scheme). The gain tuning scheme was developed on the basis of an observer-control realization of the $PI + AW$ controller, constructed with an unstable linear model driven by an unmeasured-observable exogenous input disturbance. The design exploited the characteristics of the global-nonlinear reactor behavior. The proposed design was applied to a representative reactor example, finding that an adequately tuned PI with BC AW outperforms its PI with BC or BC-CI AW counterparts tuned with existing approaches.

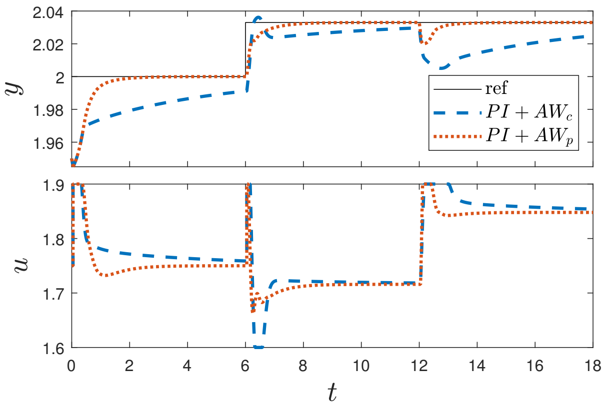

Fig. 5. Control functioning of proposed approach $PI + AW_p$ and controller $PI + AW_c$ using control limits $\boldsymbol{l}_g$

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