Design of a Data-Oriented PID controller for a Two Degree of Freedom Control System

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Abstract: Virtual Reference Feedback Tuning (VRFT) and Fictitious Reference Iterative Tuning (FRIT) are the data-oriented tuning schemes for directly designing feedback controllers. In these schemes, controller parameters are tuned to achieve the desired response without any system identifications. Furthermore, these schemes have been extended to the two-degree of freedom (2DOF) control systems in recent years. The conventional design schemes of 2DOF controllers are a two-step tuning method. The first step is to design a feedback controller to satisfy the desired disturbance response. The second step is to tune the feed-forward controller parameters to achieve the desired reference response. Consider these methods are complicated, this paper provides a one-step tuning scheme for a 2DOF control system based on data-oriented method. According to the proposed scheme, the reference of the complementary sensitivity function is introduced, and the least squares method can be applied to calculate the optimal controller parameters. Finally, the effectiveness of the proposed scheme is verified by using a numerical example.

Keywords: Two-degree of freedom control, Fictitious exogenous signal, VRFT, FRIT, Data-oriented control

1. INTRODUCTION

In recent years, the scheme to directly calculate control parameters from operational data has attracted the attention. Campi et al. (2002), Kaneko (2013), and Kano et al. (2011) have been proposed the Virtual Reference Feedback Tuning (VRFT) method, Fictitious Reference Iterative Tuning (FRIT) method, and Extended FRIT (E-FRIT) method, respectively. These schemes can be calculated the controller parameters without any system identifications, and are extended for two-degree of freedom (2DOF) control system. See Lecchini et al. (2002); Sakata et al. (2011); Ogawa and Kano (2016).

2DOF control system can obtain two desired characteristics which are the disturbance and reference response. However, the algorithm of designing 2DOF controller is complicated because it is a two-step manner to achieve the aforementioned desired characteristics. The algorithm is as follows:

[Step 1]
Feedback controller parameters are calculated by minimizing the evaluation function relating to the disturbance response.

[Step 2]
Feedforward controller parameters are calculated by minimizing the evaluation function relating to the reference response.

In this paper, the desired reference model and complementary sensitivity function are introduced to adjust the controller parameters from a set of closed-loop data. According to the proposed scheme, the algorithm is more straightforward than the conventional scheme because controllers are designed by using a new one-step tuning manner with one evaluation function. Besides, the proposed scheme has the feature that the least squares method can be applied to design feedforward and feedback controllers simultaneously. Finally, the effectiveness of the proposed scheme is verified by numerical examples.

2. DESIRED REFERENCE RESPONSE AND DISTURBANCE RESPONSE IN 2DOF CONTROL SYSTEM

2DOF control system is shown in Fig. 1. C_r is the feedforward controller, C_e is the feedback controller, G is the controlled object in In Fig. 1. $r(t), e(t), y(t), u(t), d(t)$ are the reference signal, control error, input, output, and disturbance, respectively. It is assumed that the disturbance $d(t)$ is known. In Fig. 1, the output $y(t)$ is expressed by the following equation.

$$y(t) = \frac{G(z^{-1})\{C_e(z^{-1}) + C_r(z^{-1})\}}{1 + G(z^{-1})C_e(z^{-1})} r(t) + \frac{G(z^{-1})}{1 + G(z^{-1})C_e(z^{-1})} d(t) \qquad (1)$$

Next, the reference trajectory $y_m(t)$ in Fig. 1 can be expressed as the reference target response $G_{mr}r(t)$ and the reference disturbance response $G_{md}d(t)$. In addition,

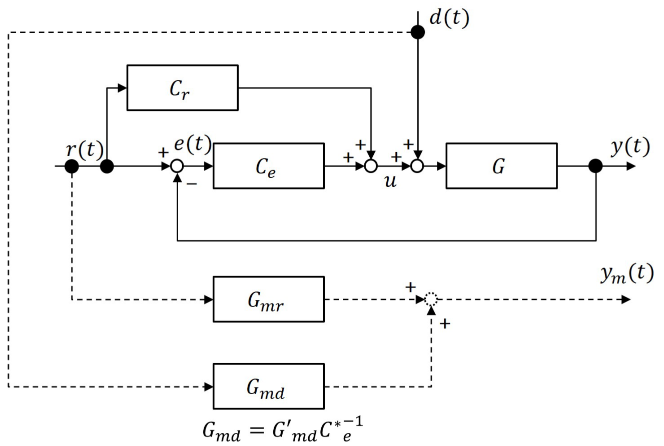

Fig. 1. Block diagram of a two-degree of freedom control system.

G_{mr} and G_{md} are the reference target response model and the reference disturbance response model, respectively.

$$y_m(t) = G_{mr}(z^{-1})r(t) + G_{md}(z^{-1})d(t) \tag{2}$$

Here, $C_e^*(z^{-1})$ is introduced as the feedback controller that satisfies the given reference disturbance response, and $G_{md}(z^{-1})$ is expressed by the following equation:

$$\begin{aligned} G_{md}(z^{-1}) &= \frac{G(z^{-1})}{1+G(z^{-1})C_e^*(z^{-1})} \\ &= G'_{md}(z^{-1}){C_e^*}^{-1}(z^{-1}), \end{aligned} \tag{3}$$

where

$$G'_{md}(z^{-1}) = \frac{G(z^{-1})C_e^*(z^{-1})}{1+G(z^{-1})C_e^*(z^{-1})} \tag{4}$$

In other words, $G'_{md}(z^{-1})$ is a complementary sensitivity function and the equation (2) can be written by using $G'_{md}(z^{-1})$ and $C_e^*(z^{-1})$ as follows.

$$y_m(t) = G_{mr}(z^{-1})r(t) + G'_{md}{C_e^*}^{-1}(z^{-1})d(t) \tag{5}$$

In this paper, by specifying the reference target response model $G_{mr}(z^{-1})$ and the reference complementary sensitivity function $G'_{md}(z^{-1})$, 2DOF controllers $C_r^*(z^{-1})$ and $C_e^*(z^{-1})$ are designed in a one-step manner based on a set of closed-loop data.

3. ONE-STEP TUNING SCHEME FOR A TWO-DEGREE OF FREEDOM CONTROL SYSTEM

3.1 Description of a Controlled object and controllers

The controlled object is given by the following equation.

$$A(z^{-1})y(t) = z^{-(d+1)}B(z^{-1})u(t) + \xi(t) \tag{6}$$

$$\left.\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_n z^{-n_a} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_m z^{-n_b} \end{aligned}\right\} \tag{7}$$

In equation (6), ξ (t) shows the Gaussian white noise with the average 0 and variance σ_ξ^2. z^{-1} is a time shift operator and it is shown as $z^{-1}y(t) = y(t-1)$. n_a and n_b are the order of $A(z^{-1})$ and $B(z^{-1})$, respectively. In addition, k is the time-delay ($k \geq 0$), and in this paper, the time-delay is assumed to be known.

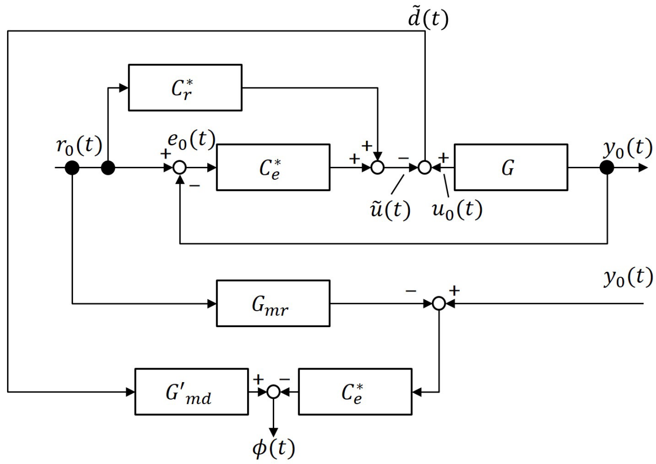

Fig. 2. Block diagram of the proposed control system.

In this paper, the feedback controller $C_e(z^{-1})$ and feed-forward controller $C_r(z^{-1})$ are designed by the following I-PD controller and P controller, respectively. The input $u(t)$ can be described by the following equation.

$$u(t) = K_{Ie}\frac{e(t)}{\Delta} - K_{Pe}y(t) - K_{De}\Delta y(t) + K_{Pr}r(t), \tag{8}$$

where K_{Pe}, K_{Ie}, and K_{De} are the proportional gain, integral gain and differential gain included in $C_e(z^{-1})$, respectively. Also, K_{Pr} is the proportional gain contained in $C_r(z^{-1})$.

3.2 One-step tuning scheme using the fictitious exogenous signal

In this paper, based on the idea of VRFT and FRIT which are proposed by Campi et al. (2002) and Kaneko (2013), the controllers $C_r^*(z^{-1})$ and $C_e^*(z^{-1})$ are designed to achieve the reference trajectory $y_m(t)$ by using the fictitious exogenous signal $\widetilde{d}(t)$. A schematic diagram of the proposed scheme is shown in Fig. 2. $r_0(t), e_0(t), u_0(t)$, and $y_0(t)$ in Fig. 2 are preliminarily obtained closed-loop data which are the reference signal, control error, input, and output, respectively. $\widetilde{d}(t)$ and $\widetilde{u}(t)$ are the fictitious exogenous signal and input signal, respectively which are indicated by the following equation.

$$\begin{aligned} \widetilde{u}(t) = \; & K_{Ie}^* \frac{e_0(t)}{\Delta} - K_{Pe}^* y_0(t) \\ & - K_{De}^*\Delta y_0(t) + K_{Pr}^* r_0(t) \end{aligned} \tag{9}$$

$$\widetilde{d}(t) = u_0(t) - \widetilde{u}(t), \tag{10}$$

where $K_{Pe}^*, K_{Ie}^*, K_{De}^*$, and K_{Pr}^* is the PID gains that minimizes the following evaluation function J.

$$J = \frac{1}{N}\sum_{t=0}^{N} \phi^2(t), \tag{11}$$

where N shows the number of data. J mentioned above is an evaluation function that takes into account the fictitious exogenous signal based on the idea of VRFT.

$\phi(t)$ in equation (11) is defined as follows to apply the least squares method.

$$\begin{aligned}\phi(t) &= G'_{md}(z^{-1})\widetilde{d}(t) - \{y_0(t) - G_{mr}(z^{-1})r_0(t)\}\frac{K^*_{Ie}}{\Delta} \\ &= G'_{md}(z^{-1})u_0(t) - G'_{md}(z^{-1})\{K^*_{Ie}\frac{e_0(t)}{\Delta} \\ &\quad -K^*_{Pe}y_0(t) - K^*_{De}\Delta y_0(t) + K^*_{Pr}r_0(t)\} \\ &\quad -\{y_0(t) - G_{mr}(z^{-1})r_0(t)\}\frac{K^*_{Ie}}{\Delta}\end{aligned} \quad (12)$$

The evaluation function J can be expressed by using the equation (12) as follows:

$$J = \frac{1}{N}(\boldsymbol{v} - \Phi\boldsymbol{\theta})^{\mathrm{T}}(\boldsymbol{v} - \Phi\boldsymbol{\theta}), \quad (13)$$

where

$$\boldsymbol{v} = [G'_{md}(z^{-1})u_0(0), G'_{md}(z^{-1})u_0(1), \cdots, G'_{md}(z^{-1})u_0(N)]^{\mathrm{T}} \quad (14)$$

$$\Phi = [\boldsymbol{\psi}(0), \boldsymbol{\psi}(1), \cdots, \boldsymbol{\psi}(N)]^{\mathrm{T}} \quad (15)$$

$$\boldsymbol{\psi}(t) = \begin{bmatrix} -G'_{md}(z^{-1})y_0(t) \\ \dfrac{G'_{md}(z^{-1})e_0(t) + y_0(t) - G_{mr}(z^{-1})r_0(t)}{\Delta} \\ -G'_{md}(z^{-1})\Delta y_0(t) \\ G'_{md}(z^{-1})r_0(t) \end{bmatrix}^{\mathrm{T}} \quad (16)$$

$$\boldsymbol{\theta} = [K^*_{Pe}, K^*_{Ie}, K^*_{De}, K^*_{Pr}]^{\mathrm{T}} \quad (17)$$

Therefore, the PID gains can be calculated by applying the least squares method of the following equation.

$$\boldsymbol{\theta} = (\Phi^{\mathrm{T}}\Phi)^{-1}\Phi^{\mathrm{T}}\boldsymbol{v} \quad (18)$$

In this paper, the design of a data-oriented 2DOF control system has been discussed. Furthermore, it is possible to apply the proposed scheme to the one degree of freedom control system. Specifically, feedback controller parameters can be adjusted by setting $K_{Pr} = 0$ using the same algorithm. Besides, the equation (12) is described based on the I-PD control law. In the case of the PID control law, it should be noted that it is expressed by the following equation instead of equation (12).

$$\phi(t) = G'_{md}(z^{-1})\widetilde{d}(t) - \{y_0(t) - G_{mr}(z^{-1})r_0(t)\} \cdot\frac{\Delta K^*_{Pe} + K^*_{Ie} + \Delta^2 K^*_{De}}{\Delta} \quad (19)$$

4. NUMERICAL EXAMPLES

The effectiveness of the proposed scheme is verified by numerical examples. The controlled object is given as the continuous-time system by the following equation.

$$G(s) = \frac{K}{1+Ts}e^{-Ls}, \quad (20)$$

where T, K, and L are the time constant, system gain, and time-delay, respectively. In this simulation, these system parameters are set to $T = 10, K = 0.5$, and $L = 0$. The following equation is obtained by discretizing the equation (20) with the sampling time $T_s = 1$ [s].

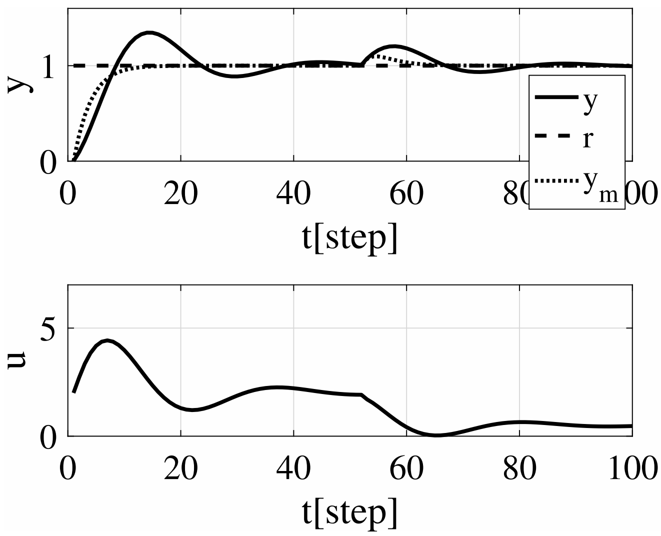

Fig. 3. Closed-loop data.

$$y(t) = -0.9048y(t-1) + 0.0476u(t-1) \quad (21)$$

The reference response model $G_{mr}(s)$ and reference complementary sensitivity function $G'_{md}(s)$ are given by the following equation based on the binomial model.

$$G_{mr}(s) = \frac{1+\beta\sigma s}{\left(1+\frac{1}{n}\sigma s\right)^n}e^{-L_m s} \quad (22)$$

$$G'_{md}(s) = \frac{1}{\left(1+\frac{1}{n}\sigma s\right)^n}e^{-L_m s}, \quad (23)$$

where σ is a coefficient related to the rise time, and L_m is the estimated time-delay. In this simulation, these setting parameters are set as $\sigma = 5$, $L_m = 0$, and $n = 2$, respectively. The coefficient β is set to $\beta = 0.4$ based on the reference which is proposed by Shigemasa et al. (1988). The equations (22) and (23) are discretized with $T_s = 1$ [s] as the reference model.

First of all, the following PID gains are applied to the control system to obtain the initial data.

$$K_{Pe} = 1.0, K_{Ie} = 1.0, K_{De} = 1.0, K_{Pr} = 1.0 \quad (24)$$

In addition, the disturbance $d(t)$ is given as follows:

$$d(t) = \begin{cases} 0 & (t < 50) \\ 1.5 & (t \geq 50) \end{cases}. \quad (25)$$

Fig. 3 shows the control result by using the PID gains of equation (24). In Fig. 3, the system output $y(t)$ does not track the reference trajectory $y_m(t)$.

Next, Fig. 4 shows the control result of applying the proposed scheme using the closed-loop data of Fig. 3. At this time, the next PID gains are calculated.

$$\begin{aligned}&[\beta = 0.4] \\ &K_{Pe} = 9.57, K_{Ie} = 2.28, K_{De} = 0.0, K_{Pr} = 3.52\end{aligned} \quad (26)$$

In the Fig. 4, because the system output $y(t)$ can track the reference target response and the reference disturbance

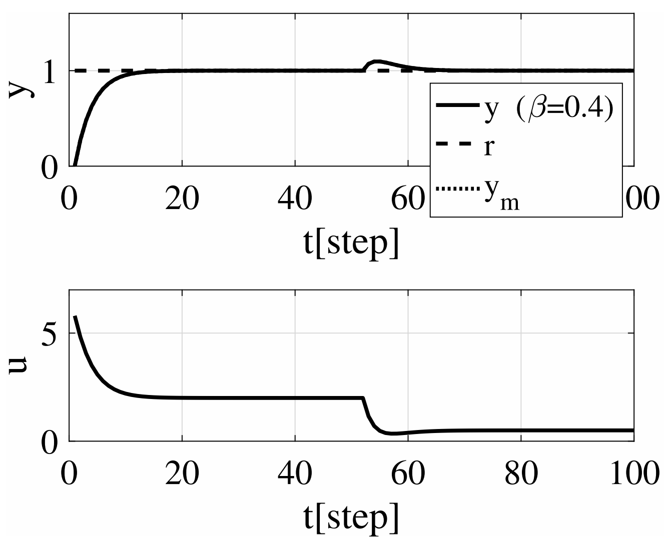

Fig. 4. Control results by using the proposed scheme ($\beta = 0.4$).

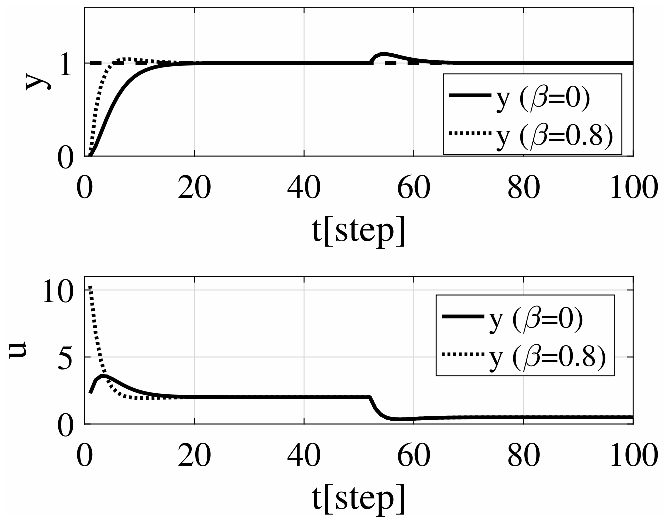

Fig. 5. Control results by using the proposed scheme ($\beta = 0, \beta = 0.8$).

response, the effectiveness of the proposed scheme can be verified.

Finally, Fig. 5 shows the control results when β included in the equation (22) is set as $\beta = 0$ and $\beta = 0.8$. At this time, each PID gains are calculated by using the proposed scheme as follows.

$$[\beta = 0]$$
$$K_{Pe} = 9.57, K_{Ie} = 2.28, K_{De} = 0.0, K_{Pr} = 0.0 \tag{27}$$
$$[\beta = 0.8]$$
$$K_{Pe} = 9.57, K_{Ie} = 2.28, K_{De} = 0.0, K_{Pr} = 8.03 \tag{28}$$

In Fig. 5, it is possible to specify the reference response arbitrarily by changing the zeros with the value of β. From the equation (22) and (23), in the case of $\beta = 0$, the reference response model $G_{mr}(s)$ and the reference complementary sensitivity function $G'_{md}(s)$ coincide with each other. In other words, the reference trajectory $y_m(t)$ can be satisfied by using only feedback controller without feedforward controller. Therefore, in the equation (27), $K_{Pr} = 0$ is calculated appropriately by the proposed scheme.

Moreover, even if the value of β is different from Fig. 5, the same disturbance response is obtained because $G'_{md}(s)$ does not depend on β.

5. CONCLUSIONS

In this paper, a one-step adjustment method of 2DOF controller based on a set of closed-loop data. According to the proposed scheme, the least squares method can be applied to design feedforward and feedback controllers simultaneously. Specifically, the reference signal and the fictitious exogenous signal are utilized to adjust the PID gains to satisfy the desired reference response and complementary sensitivity function. The effectiveness of the proposed scheme is verified from the system output tracks the reference trajectory using numerical examples. Moreover, the reference response can be designed independently of disturbance response is confirmed.

The future works are to examine the influence of time-delay, noise, and unknown disturbance and also consider an extension of the proposed scheme to the recursive least squares method for the time-varying system.

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