Data-Driven PID Control Tuning for Disturbance Rejection in a Hierarchical Control Architecture ⋆

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Abstract: This work presents some guidelines for tuning PID controllers in order to increase robustness within a hierarchical control structure focused on load disturbance rejection, in which the process' mathematical model is unknown. The proposed structure consists in two control loops: an inner PID control layer tuned using only data collected from the process, whose set point signal is governed by an outer predictive control layer, with the purpose of increasing closed-loop performance and enabling the specification of constraints. Some simulation results are presented, in which it is shown that the appropriate tuning of the PID controller allows the outer loop to correctly predict the inner loop behavior and therefore provide better disturbance rejection than the data-based tuned PID alone.

Keywords: Data-driven control, PID, predictive control, disturbance rejection.

1. INTRODUCTION

Disturbance rejection is a major source of concern in industrial control loops, occasionally being more important than set point tracking. The literature on model-based control methods emphasizing disturbance rejection is well consolidated (Chen and Seborg, 2002; Shamsuzzoha and Lee, 2007; Szita and Sanathanan, 1996).

In terms of modern data-based control methods, the majority of them were developed for addressing the reference tracking problem, such as Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002). Literature on data-driven methods for disturbance rejection or attenuation is still relatively small: in Jeng and Ge (2016), an adaptation of the VRFT method is used, where the reference model choice takes into account the desired disturbance dynamics. Two and three degrees of freedom VRFT-tuned controllers are presented in Guardabassi and Savaresi (1997) and Rojas et al. (2011), where feedforward control structures are employed. In Eckhard et al. (2018), the Virtual Disturbance Feedback Tuning (VDFT) is presented, which proposes a data-driven model-matching method for tuning a fixed structure controller, in a similar fashion as in VRFT.

The VDFT method assumes that the load disturbance signal cannot be measured and is able to provide in closed-loop a disturbance behavior close to – or, under certain ideal conditions, equivalent to – the desired one, represented by a disturbance reference model. Although disturbance rejection is improved by the method, reference tracking performance may be harmed. A manner of improving overall closed-loop performance is to implement an outer advanced control loop, acting as a set point governor for the inner VDFT-tuned loop. A similar hierarchical control structure was presented in Piga et al. (2017), where the outer predictive control layer assumes the desired reference model as the actual model of the inner linear parameter-varying control layer, designed to match a desired closed-loop model. The work aims to manage reference tracking and does not address the scenario in which the assumption is not valid, i. e. the reference model is not achievable by the inner control structure.

This work aims to adapt the hierarchical control structure from Piga et al. (2017) for disturbance rejection and to propose adaptations in the inner controller design so as to increase closed-loop robustness, anticipating scenarios when the reference model is not achievable. Employing the VDFT method to tune the inner controller, the disturbance reference model is partly loosened in a flexible formulation of the VDFT, allowing a tighter matching of the desired model. Also, an iterative experimental project is suggested in order to improve cost function shaping, despite a sub-optimal filter choice.

This work is divided in the following manner: section 2 presents the problem formulation and the proposed control structure. Section 3 describes the design steps for the inner PID control, the proposed adaptations for increasing robustness and the outer predictive control. Finally, in section 4, a simulation example is exploited in order to illustrate the proposed control approach, and section 5 presents some conclusions from the work.

2. PROBLEM FORMULATION

Consider a discrete-time linear time-invariant SISO system described as

$$y(t) = G(q)u(t) + \nu(t) \tag{1}$$

⋆ This work has been supported by CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico.

where $u(t)$ is the process' input, $y(t)$, the process' output, $\nu(t)$, the output noise, which is characterized as $\nu(t) = H(q)e(t)$ with $e(t)$ a white-noise process with variance λ_e^2, and $G(q)$, its minimum-phase transfer function, which is assumed to be unknown. Also, q is the discrete shift operator ($qx(t) = x(t+1)$). It is also assumed that, in closed-loop, the process' input is susceptible to load disturbances $d(t)$, as can be seen in the inner loop in Fig. 1.

Fig. 1 shows the hierarchical control structure, where $w(t)$ is the reference signal that should be tracked by $y(t)$. The outer predictive control layer is responsible for managing the inner reference control $r(t)$ delivered to the inner control layer, represented by $\mathcal{M}$, where $C(q,\rho)$ is a controller parametrized in ρ and designed for disturbance rejection. The choice of using an MPC strategy in the outer control loop is motivated by the need to achieve high closed-loop performance and handle constraints (Piga et al., 2017). The purpose of this control strategy is to handle disturbance rejection in a high performance scheme, while still coping with reference tracking and input/output constraints, and considering only data measured from the plant. In order to do so, the idea is to design both controllers $C(q,\rho)$ and MPC in a two-step procedure without deriving a process model $G(q)$.

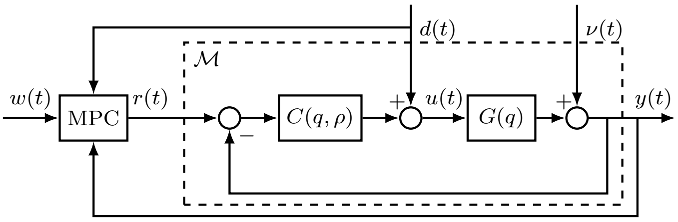

Fig. 1. Hierarchical structure diagram.

Now one of the main challenges of the proposed control structure is to provide the outer model-based control loop with a sufficiently exact model so as to represent the inner loop behavior. With this intention, this work presents a series of design choices to tune the inner controller through a model matching problem using only measured data so it results in a closed-loop behavior $\mathcal{M}$ very similar to the desired one.

3. CONTROLLER DESIGN

The guidelines for the design of the inner loop and outer loop controllers are described in this section.

3.1 Data-Driven PID controller

The Virtual Disturbance Feedback Tuning method consists in an one-shot data-driven approach, which aims to solve a model matching problem for disturbance rejection (Eckhard et al., 2018). This problem considers a linearly parametrized discrete-time controller $C(q,\rho)$

$$C(q,\rho) = \rho^T \bar{C}(q), \tag{2}$$

where $\rho = [\,\rho_1 \ \dots \ \rho_n\,]^T$ is the parameters vector and $\bar{C}(q)$ is an $n-$vector of transfer functions in q. The choice of $\bar{C}(q)$ and n will determine the controller class employed $\mathcal{C} = \{C(q,\rho), \rho \in P \subseteq \mathbb{R}^n\}$. Considering a PID controller class, (2) can be rewritten as

$$C(q,\rho) = [\,\rho_1 \ \rho_2 \ \rho_3\,] \begin{bmatrix} \dfrac{q^2}{q(q-1)} \\ \dfrac{q}{q(q-1)} \\ \dfrac{1}{q(q-1)} \end{bmatrix}. \tag{3}$$

In closed-loop, the expression for the system's response is given as

$$y(t) = Q(q,\rho)d(t) + T(q,\rho)r(t) + S(q,\rho)\nu(t) \tag{4}$$

with

$$T(q,\rho) \triangleq \frac{C(q,\rho)G(q)}{1 + C(q,\rho)G(q)}, \tag{5}$$

$$Q(q,\rho) \triangleq \frac{G(q)}{1 + C(q,\rho)G(q)}, \tag{6}$$

$$S(q,\rho) \triangleq \frac{1}{1 + C(q,\rho)G(q)}. \tag{7}$$

Let $y_d(t)$ be the desired closed-loop response to a given disturbance $d(t)$, and define $Q_d(q)$, the desired transfer function, as in $y_d(t) = Q_d(q)d(t)$. Now the usual model matching problem consists in determining the vector of parameters ρ, which minimizes

$$\begin{aligned} &\min_\rho J^{DM}(\rho) \\ &J^{DM}(\rho) \triangleq \sum_t \{[Q(q,\rho) - Q_d(q)]d(t)\}^2 . \end{aligned} \tag{8}$$

The ideal controller, which exactly solves the model-matching problem in (8), is given by

$$C_d(q) = Q_d^{-1}(q) - G^{-1}(q). \tag{9}$$

However as seen in (6), the term $Q(q,\rho)$ depends on the unknown process' transfer function $G(q)$. Instead, the VDFT method proposes an alternative to (8) based on data collected from the process. In order to do so, consider that a set of measured input and output data $Z_N = \{u(t), y(t)\}$, with $t = 1, \dots, N$, is available. From these data, a virtual control signal is calculated as $\bar{u}_c(t) = u(t) - Q_d^{-1}(q)y(t)$. Now the VDFT problem consists in identifying the controller $C(q,\rho)$, whose input $-y(t)$ results in an output $\bar{u}_c(t)$, i.e.

$$\begin{aligned} &\min_\rho J^{VD}(\rho) \\ &J^{VD}(\rho) \triangleq \sum_{t=1}^{N} \{K(q)[\bar{u}_c(t) + C(q,\rho)y(t)]\}^2 \end{aligned} \tag{10}$$

where $K(q)$ is a pre-filter.

Since the controller is linearly parametrized, (10) can be solved using the least squares method resulting in

$$\hat{\rho} = \left[\sum_{t=1}^{N} \phi(t)\phi(t)^T\right]^{-1} \left[\sum_{t=1}^{N} \phi(t)\bar{u}_{cK}\right] \tag{11}$$

where

$$\phi(t) \triangleq \bar{C}(q)K(q)y(t) \tag{12}$$

$$\bar{u}_{cK} \triangleq K(q)\bar{u}_c(t) \tag{13}$$

The solution of the least squares problem (11) exists and is unique if the input signal $u(t)$ is persistently exciting (pe) of order n (Bazanella et al., 2012). Considering a noise-free framework, in the case when $C_d(q) \in \mathcal{C}$, i.e. $\exists\, \rho_d$ such that $C(q,\rho_d) = C_d(q)$, the solution given in (11) results in ρ_d, independently of $K(q)$. As a consequence, $J^{VD} = J^{MD} = 0$ and the obtained closed-loop behavior is identical to the desired one, $Q(q,\rho_d) = Q_d(q)$ and $T(q,\rho_d) = C(q,\rho_d)Q_d(q)$. The prediction model for the inner control loop used in the MPC layer is therefore exact. When signals are corrupted by noise, an instrumental variable approach is used (Eckhard et al., 2018). When $C_d(q) \in \mathcal{C}$, an unbiased estimate $\hat{\rho}_{IV}$ is obtained through (11), such that $E[\hat{\rho}_{IV}] = \rho_d$.

In contrast, when $C_d(q) \notin \mathcal{C}$, not only is the ideal situation $J^{DM} = 0$ not achievable, but also the minimum of J^{VD} results different than that of J^{DM}. The VDFT method proposes choosing the filter $K(q)$ properly so that $J^{VD} \approx J^{DM}$ and therefore minimizing J^{VD} corresponds to minimizing J^{DM}. In this work, new approaches are yet developed so as to reduce J^{DM} even more and to enable the use of $Q_d(q)$ and $\hat{T}_d(q) \triangleq Q_d(q)C(q,\hat{\rho})$ as models of the inner loop by the MPC.

3.2 Design of filter $K(q)$

The choice of filter $K(q)$ is detailed in Eckhard et al. (2018). Its purpose is to approximate the minima from (8) and (10), when there is no controller $C(q,\rho)$ within $\mathcal{C}$ that satisfies $J^{DM} = 0$. In this case, it is shown that the optimal choice of $K(q)$ must satisfy

$$|K(e^{j\omega})|^2 = |Q_d(e^{j\omega})Q(e^{j\omega},\rho)|^2 \frac{\Phi_d(e^{j\omega})}{\Phi_y(e^{j\omega})} \tag{14}$$

where $\Phi_y(e^{j\omega})$ is the spectrum of the measured output signal and $\Phi_d(e^{j\omega})$, the spectrum of the disturbance signal one wishes to reject.

A practical choice of $K(q)$ is suggested as $K(q) = Q_d(q)$, considering some assumptions are satisfied (Eckhard et al., 2018):

Assumption 1. $|Q(e^{j\omega},\hat{\rho})| \approx |Q_d(e^{j\omega})|$, i.e. the closed-loop performance obtained with VDFT is close to the desired one;

Assumption 2. $y(t)$ and $u(t)$ are collected in closed-loop, with a controller $C(q,\rho_0)$;

Assumption 3. $|Q(e^{j\omega},\rho_0)| \approx |Q_d(e^{j\omega})|$, i.e. the initial closed-loop performance is close to the desired one.

3.3 Choice of $Q_d(q)$

Depending on the process' characteristics, e.g. the presence of time-delay and non-minimum phase zeros, and the controller class $\mathcal{C}$ employed, the chosen reference model may not be achievable. However, a sensible choice of $Q_d(q)$ allows the function J^{VD} to be driven closer to zero.

Although $C_d(q)$ does not necessarily belong to $\mathcal{C}$, it is advisable that $C_d(q)$ be close to the controller class as much as possible so a good closed-loop behavior can be achieved. In Szita and Sanathanan (1996), some theorems are presented in order to choose the reference model so the ideal controller for a model matching problem is proper and the resulting closed-loop, internally stable. The developments are mostly dependent on previous knowledge of the process, and data-based strategies have not yet been developed to handle this design issue. The following example illustrates the problem of obtaining a proper $C_d(q)$ such that $C_d(q) \in \mathcal{C}$.

Example 1. Consider the process

$$G(q) = \frac{0.01}{q-0.95} \tag{15}$$

a PID controller class and a desired reference model for step disturbance rejection

$$Q_d(q) = \frac{\alpha(q-1)}{(q-0.9)^2} \tag{16}$$

with α, a free parameter. Considering (9), the only value of α for which the ideal controller $C_d(q)$ is causal is $\alpha = 0.01$, the process' numerator constant, in which case

$$C_d(q) = \frac{15(q-0.9333)}{q-1} \tag{17}$$

is a PI controller and $C_d(q) \in \mathcal{C}$, with $\rho = [15, -14, 0]^T$.

Take the following expression, linking the ideal controller and the reference model:

$$Q_d(q) = \frac{G(q)}{1 + C_d(q)G(q)} = \frac{nG(q)dC_d(q)}{dG(q)dC_d(q) + nG(q)nC_d(q)} \tag{18}$$

where $G(q) \triangleq \frac{nG(q)}{dG(q)}$, with $nG(q)$ and $dG(q)$ coprime, and $C_d(q) \triangleq \frac{nC_d(q)}{dC_d(q)}$, with $nC_d(q)$ and $dC_d(q)$ coprime as well. In this work, the controller has a fixed PID structure, therefore the only degrees of freedom in (18) lie on the denominator, provided by $nC_d(q)$, and thus the numerator of $Q_d(q)$ is fixed (by the fixed controller denominator) and partly unknown (by the unknown process numerator). In the example shown above, it is possible to notice how the numerator of $G(q)$ should appear in the numerator of $Q_d(q)$, so that the controller $C_d(q)$ belongs to the class $\mathcal{C}$.

Besides, assuming that $G(q)$ is strictly proper and $C_d(q)$, proper, the relative degree of $Q_d(q)$ is given as

$$\begin{aligned} \Gamma[Q_d(q)] &= \Gamma[G(q)] - \Gamma[1 + C_d(q)G(q)] \\ &= \Gamma[G(q)] - \min\{0, \Gamma[C_d(q)G(q)]\} \\ &= \Gamma[G(q)] \end{aligned} \tag{19}$$

with $\Gamma[\cdot]$ is the relative degree operator (Gonçalves da Silva et al., 2018a). Therefore $Q_d(q)$ should have necessarily the same relative degree as $G(q)$, once $\Gamma[C_d(q)G(q)] > 0$.

An approach to tackle the problem of choosing $Q_d(q)$ is proposed in the following subsection, with a flexible adaptation of the VDFT method. Since usually performance requirements are specified by setting the desired poles of $Q_d(q)$, the denominator of $Q_d(q)$ is set accordingly, while the numerator of $Q_d(q)$ is partly loosened.

3.4 VDFT flexible formulation

In this hierarchical formulation, the desired dynamics translated in $Q_d(q)$ is not essential for the closed-loop overall performance, since an outer advanced control is used. Instead, the main concern lies in the inner control loop design: to obtain a controller $C(q,\rho)$ which minimizes the model matching problem (8) and also provides the outer loop with a representative model of the inner loop.

The VDFT tuning method is hence adapted in a flexible formulation, in which the numerator of the reference model is parametrized as

$$Q_d(q,\eta) = \eta^T \bar{F}(q) \tag{20}$$

where $\eta = [\,\eta_1 \, \dots \, \eta_p\,]^T$ is a vector of p parameters, and $\bar{F}(q)$, a p-vector of rational transfer functions. In this manner, the poles of $Q_d(q,\eta)$ are still fixed for the desired dynamics, yet the numerator remains free.

The VDFT problem in (10) should now be solved regarding the variables η and ρ,

$$\min_{(\rho,\eta)\neq(0,0)} J^{VDf}(\rho,\eta)$$

$$J^{VDf}(\rho,\eta) \triangleq \sum_{t=1}^{N}\{K(q)[Q_d(q,\eta)(u(t)+C(q,\rho)y(t)) - y(t)]\}^2, \tag{21}$$

where $\bar{u}_c(t)$ was substituted by $u(t) - Q_d(q,\eta)^{-1}y(t)$ in (10) and the cost function was multiplied by $Q_d(q,\eta)$. The solution of (21) is given iteratively, as developed in Campestrini et al. (2011) for the VRFT method applied to nonminimum phase systems. The least squares problem is solved for η and ρ in an alternate fashion, for each iteration i, as in

$$\eta^i = \arg\min_{\eta} J^{VDf}(\rho^{i-1},\eta) \tag{22}$$

$$\rho^i = \arg\min_{\rho} J^{VDf}(\rho,\eta^i) \tag{23}$$

and initial values for ρ (or η) should be provided. Since data is collected in closed-loop for the VDFT method, it is suggested that $C(q,\rho^0)$ be chosen as the original controller $C(q,\rho_0)$ (Gonçalves da Silva et al., 2018b).

Regarding the sub-optimality of choosing filter $K(q) = Q_d(q)$, yet another adaptation is proposed. A first experiment is conducted with whichever $C(q,\rho_0)$ is operating in the loop, resulting in a sub-optimal controller $C(q,\rho_1)$ and $Q_d(q,\eta_1)$. In order to respect Assumption 3, this work suggests the user to perform a second experiment in closed-loop, now with $C(q,\rho_1)$. With the new collected data, tuning should be improved and the resulting controller should provide an even smaller cost J^{DM}, that is, $J^{DM}(\rho_1) < J^{DM}(\rho_0)$. A similar approach is used in Bazanella et al. (2008), where cost function shaping is performed employing intermediate reference models in a cautious control approach.

3.5 Advanced controller

An advanced control layer, based on a classic Generalized Predictive Control (GPC) (Clarke et al., 1987) has been chosen. The GPC allows the user to achieve high closed-loop performance in a straightforward formulation and also takes into account input and output constraints. The controller provides at each instant t a set of future N_r optimal control actions by minimizing a chosen objective function over a future-time horizon $t \in [t+1, t+N_y]$.

Consider a SISO system described as an ARX model

$$A(q)y(t) = B(q)r(t) + D(q)d(t) + e(t) \tag{24}$$

where $r(t)$ is the input signal, $y(t)$, the output, $d(t)$, the disturbance, $e(t)$, a white noise signal, $A(q)$, $B(q)$ and $D(q)$ are polynomials in q^{-1}.

The corresponding j-step ahead optimal predictor can be derived in a similar fashion as done in Camacho and Bordons (2007) for a CARIMA model, and is expressed as

$$\hat{y}(t+j|t) = S_j(q)\Delta r(t+j-1) + f(t+j) \tag{25}$$

where $\hat{y}(t+j|t)$ represents the j-step ahead prediction of $y(t)$ considering data up until instant t, $S_j(q)$ is a polynomial in q whose coefficients are the process' step response coefficients relative to input $r(t)$. Here, it was considered that the future disturbance signal is unknown (i.e. $\Delta d(t+j) = 0, \forall\, j > 0$). In addition, the free response $f(t+j)$ can be obtained recursively as

$$f(t{+}j{+}1) = q(1{-}A(q))f(t{+}j) + B(q)r(t{+}j) + D(q)d(t{+}j) \tag{26}$$

with $r(t+k) = r(t-1)$ and $d(t+k) = d(t)\,\forall k \geq 0$ and $f(t) = y(t)$. The vector of N_r future control signal variations is obtained at each instant t as a solution of the following constrained problem:

$$\Delta R = \arg\min_{\Delta r} J^{PC} \quad \text{s.t. } g(\Delta r) \leq b \tag{27}$$

where J^{PC} is a classical quadratic cost function

$$J^{PC} = \sum_{j=1}^{N_y}[\hat{y}(t{+}j|t) - w(t{+}j)]^2 + \sum_{j=1}^{N_r}\lambda[\Delta r(t{+}j{-}1)]^2 \tag{28}$$

for reference tracking and penalizing control effort, in which λ is a weighting factor. Only the first element of ΔR is applied to the process, and the procedure is repeated at the next instant.

4. SIMULATION RESULTS

To illustrate the control approach presented in this paper, a SISO model identified from a pilot plant (Fig. 2) is considered. Tank 1 level should be controlled to achieve a desired liquid level through the manipulation of the valve's opening. Also, the valve's opening is susceptible to load disturbances. The system has therefore one input $u(t)$, the valve's opening (given in %), one output $y(t)$, the Tank 1 level (cm), and a disturbance signal $d(t)$ (%).

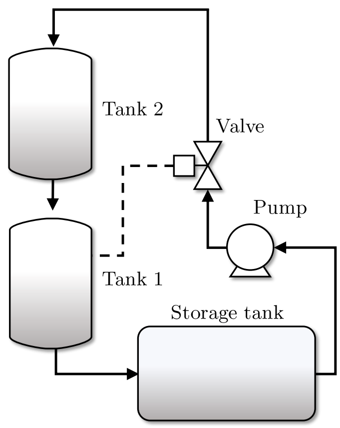

Fig. 2. Schematic diagram of the system.

The model was obtained through a grey-box identification method and resulted in the second order discrete-time model

$$G(q) = \frac{3.9725 \times 10^{-5}}{(q-0.9943)(q-0.9926)} \tag{29}$$

with sampling time $T_s = 1\,\text{s}$. Note that (29) is a linear model which represents the system's behavior around a given operation point, here chosen as $y_{op} = 30\,\text{cm}$ and $u_{op} = 85\,\%$.

To design the controller, two operational constraints are considered: the valve's opening has a range of $[0 - 100]\,\%$ and the valve's opening variation should not surpass $5\,\%$ per second. The latter is necessary since the real actuator cannot respond ideally to aggressive control signals.

The system is considered to be operating in a closed-loop with the following PID controller

$$C(q, \rho_0) = \frac{43.147(q - 0.9692)(q - 0.9961)}{q(q - 1)} \tag{30}$$

obtained through the VRFT method with reference model chosen as $T_d(q) = \frac{0.0001}{(q-0.99)(q-0.99)}$ for set point tracking.

In order to increase disturbance rejection performance, the flexible VDFT approach is used to tune the inner PID controller, using as desired disturbance model

$$Q_d(q, \eta) = [\,\eta_1\ \ \eta_2\,] \begin{bmatrix} q \\ 1 \end{bmatrix} F(q) \tag{31}$$

with

$$F(q) = \frac{(q - 1)}{(q - 0.99)^3 q} \tag{32}$$

The reference model in (31) and (32) is chosen so that $\Gamma[Q_d(q, \eta)] = \Gamma[G(q)] = 2$. It is supposed here that the relative degree of the process can be easily obtained from its time response and that such information is available.

Closed-loop data, using $C(q, \rho_0)$, was collected applying a pulse of amplitude $1\,\%$ and duration of $1300\,\text{s}$ to the disturbance signal $d(t)$ at instant $200\,\text{s}$. Using the flexible VDFT method, with 500 iterations, a first pair of vectors $(\hat{\rho}_1, \hat{\eta}_1)$ was tuned, which resulted in a $J^{DM}(\hat{\rho}_1) = 0.0028$ (calculated with $d(t)$ an $1300\,\text{s}$-long unitary step signal). To improve this value, a second closed-loop experiment, using $C(q, \hat{\rho}_1)$, was performed, whose input/output data can be seen in Fig. 3.

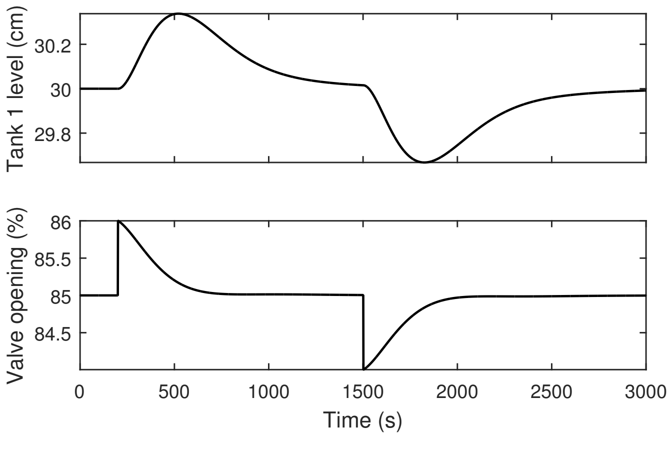

Fig. 3. Closed-loop input and output data to a step disturbance signal.

The flexible VDFT was once more solved, and the reference model obtained for this second round is

$$Q_d(q, \hat{\eta}_2) = \frac{7.2979 \times 10^{-5}(q - 1)(q - 0.4618)}{q(q - 0.99)^3} \tag{33}$$

and the following controller has been tuned:

$$C(q, \hat{\rho}_2) = \frac{433.89(q^2 - 1.985q + 0.9849)}{q(q - 1)} \tag{34}$$

which results in a closed-loop disturbance response $Q(q, \hat{\rho}_2)$ very similar to the desired one $Q_d(q, \hat{\eta}_2)$, with $J^{DM}(\hat{\rho}_2) = 1.4959 \times 10^{-5}$, calculated with data from Fig. 4.

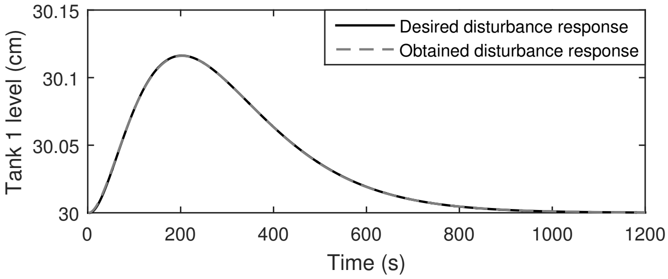

Fig. 4. Comparison between $Q_d(q, \hat{\eta}_2)$ and $Q(q, \hat{\rho}_2)$.

In the next step of the control design, the constrained GPC problem is specified in the following manner:

$$\Delta R = \arg\min_{\Delta r} J^{PC} \tag{35}$$

s.t.

$$-5 \leq C(q, \hat{\rho}_2)(r(t) - y(t)) + d(t) - u(t-1) \leq 5 \tag{36}$$

$$0 \leq C(q, \hat{\rho}_2)(r(t) - y(t)) + d(t) \leq 100 \tag{37}$$

where the constraint in (36) takes into account the real actuator's rate $\Delta u(t)$ limitation and (37) considers the valid range of the input signal $u(t)$. Also, the prediction and control horizons were chosen as $N_y = N_r = 10$ and the control effort weight was taken as $\lambda = 1$. Moreover, $Q_d(q, \hat{\eta}_2)$ and $\hat{T}_d(q) \triangleq C(q, \hat{\rho}_2) Q_d(q, \hat{\eta}_2)$ were used as model for the inner-loop behavior.

The closed-loop results of the hierarchical control structure can be seen in Fig. 5. For comparison, Fig. 5 also shows the closed-loop behavior for the VRFT controller $C(q, \rho_0)$ and also the flexible VDFT controller $C(q, \hat{\rho}_2)$. The simulation has reproduced the saturation effects on the control signal and its variation.

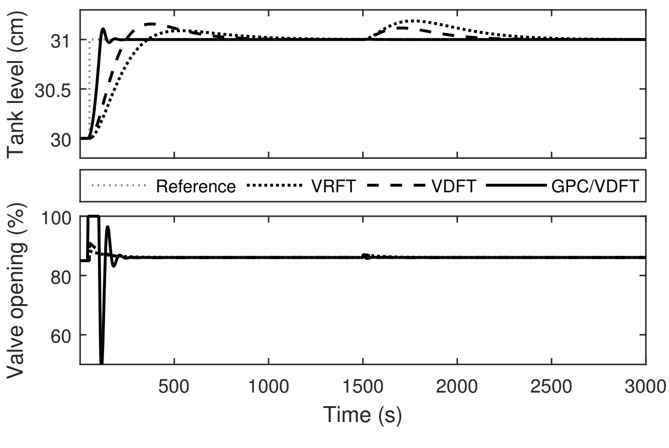

Fig. 5. Comparison of the closed-loop responses obtained with different controller design approaches.

Considering a cost function defined as

$$J \triangleq \sum_{t=1}^{N} [w(t) - y(t)]^2 \tag{38}$$

the VRFT tuned PID results in $J = 181.4826$, the VDFT tuned PID, in $J = 84.2106$ and the hierarchical control structure with inner loop using VDFT, $J = 25.8531$, calculated with data shown in Fig. 5.

As expected, the joint structure with data-based tuned PID and GPC results in an altogether better performance: in disturbance rejection, but also in reference tracking. The success of the control structure depends mostly on whether the PID tuning achieves the desired closed-loop response. The choice of letting part of the reference model free to be identified plays a major role in that success, allowing the GPC layer to correctly predict and actuate over the process.

For comparison, assume now that the inner PID loop is tuned with the classical VDFT method, for a fixed reference model of

$$Q_d(q) = \frac{1 \times 10^{-4}(q-1)}{(q-0.99)^3}. \tag{39}$$

The resulting PID controller is

$$C(q, \hat{\rho}_3) = \frac{-27.582(q-1.089)(q-0.9959)}{q(q-1)} \tag{40}$$

and Fig. 6 shows the obtained closed-loop response $Q(q, \hat{\rho}_3)$ compared to the chosen $Q_d(q)$, resulting in $J^{DM}(\hat{\rho}_3) = 0.8450$, which is considerably larger than the $J^{DM}(\hat{\rho}_2)$ index achieved with the flexible formulation.

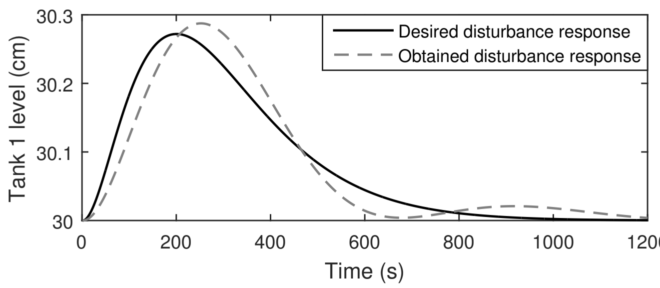

Fig. 6. Comparison between $Q_d(q, \hat{\eta}_3)$ and $Q(q, \hat{\rho}_3)$.

The models $Q_d(q)$ and $\hat{T}_d(q) = C(q, \hat{\rho}_3)Q_d(q)$ fail to represent the real closed-loop. As a consequence, during the quadratic programming solution the problem becomes infeasible due to constraints inconsistencies and the GPC layer thus fails to provide a well-behaved response to the constrained control problem.

5. CONCLUSION

This paper has introduced directions in the tuning of a restricted order controller, so the resulting closed-loop behavior highly resembles the desired reference model for disturbance rejection. The concepts developed are particularly interesting for use within a two-layer control structure, in which the outer loop is designed as a model-based predictive control and is based on the chosen disturbance reference model for the inner loop, therefore disregarding any *a priori* knowledge on the process' model. As a consequence, the performance of the MPC layer depends majorly on whether the achieved behavior matches the desired specification in the VDFT-tuned control loop. Future extensions to the research should include improving the MPC layer robustness, enlarging the scope to multivariable systems and implementing the developed control structure in a real-time application.

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