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Data-driven fractional PID control: application to DC motors in flexible joints

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Abstract: Recent advances in data-driven (or model-free) control have permitted to enhance the closed loop behavior of linear and especially nonlinear systems using very simple control structures. As a result, unknown or badly known dynamics are compensated and disturbances are rejected without any learning or on-line identification procedure. However, the ultra-local phenomenological models on which this control technique rely have not yet exploited the fractional nature of many processes and the nonlocal nature of the fractional integrodifferential operators. In this paper, fractional derivatives are used in the so called model free control structure in order to explore the advantages they provide in terms of robustness and dynamic response. Fractional and integer order data driven PIDs will be compared for a DC motor in a robot flexible joint control application in a simulation environment.

Keywords: PID control, fractional control, data-driven control

1. INTRODUCTION

PID controllers are still by far the most popular feedback design in industry (Aström and Hägglund, 2006). The reasons for the popularity of this technique can be found both in the simplicity of the control structure and in the physical meaning of control parameters that allow operators to easily tune and maintain the controlled systems.

However, the behavior of process is frequently modified due to the change of operating condition, the process ageing, the faulty behavior of a particular actuator and the non-linearity of the systems to be controlled. Therefore, it is necessary either to readjust the control parameters - a huge literature exist on adaptive control (Aström and Wittenmark, 1989)- or to somehow model the aforementioned uncertainty or nonlinearities in order to maintain the desired control performance with any model based control strategy.

Most adaptive control techniques and methodologies are typically assumed that the structure of the system is known linear and the parameters may be unknown or slow time-varying. However, for complex practical systems, the structure of the plant is often difficult to determine and the parameters are hard to identify, which make the adaptive control designing and applications questionable.

Besides, robust control techniques attempt to quantify the amount of uncertainty a certain model possess in order to find a compensator able to fulfill the control requirements all through the parametric uncertainty region. The main drawback of these model based approaches is that a precise mathematical description of the plant is not always available or it is too costly and time consuming.

Data-driven control approaches focus on designing controllers merely using input and output measurement data of a plant. Since these approaches do not require a model of a plant in controller designing, the modelling process and the theoretical assumptions on the dynamics of the plant disappear. Several data-driven controller tuning techniques have appeared in the literature recently. Iterative feedback tuning (Hjalmarsson et al., 1998), iterative correlation-based tuning (Karimi et al., 2004), iterative unfalsified control (Safonov and Tsao, 1997), iterative learning control (Ahn et al., 2007) and virtual reference feedback tuning (Campi et al., 2002) are examples of data-driven controller tuning techniques, where one or more experiments are needed in each iteration.

A new family of controllers has arisen in the last years that is based on the data-driven control paradigm in a different way: it uses input/output data -in a sort of feedforward controller- to compensate the effects of unmodelled dynamics that more simple control structures -like PID- are not able to properly handle. Active disturbance rejection control (Han, 2009) treats the discrepancy betweeen the real system and a non-physical differential phenomenological model to reject disturbances in real time. To achieve such a task, an extended state observer is needed, and therefore the simplicity of PID becomes a burden in tuning the associated observers. A rather similar model-free approach permits to fast and properly compensate uncertainty and unmodelled dynamics while keeping simplicity. These techniques, initiated in (Fliess et al., 2006), propose an algebraic framework to deal with fast numerical

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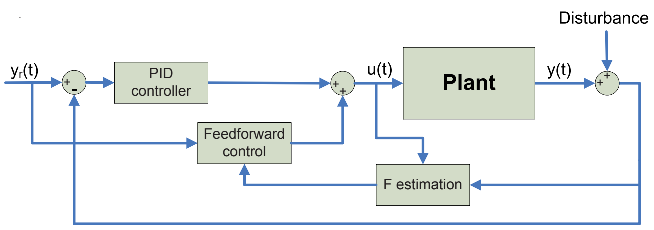


Fig. 1. Data-driven control scheme.

derivatives estimation, and thereafter, model free control design. However, the ultra-local phenomenological models on which this control technique rely have not yet exploited the fractional nature of many processes and the nonlocal nature of the fractional integrodifferential operators. In this connection, a preliminary work is presented in this paper to explore the interest of the additional parameter that a fractional differential model introduces.

The rest of the paper is organized as follows. Section 2 will be devoted to present the key features of the model-free (or data-driven) control strategy used in this work. Since the ultra-local phenomenological models on which this control technique rely do not exploit the fractional nature of many process, Section 3 presents a generalization of the aforementioned data driven PID control. The advantages of such generalization are shown in Section 4, where a robot flexible joint control application is presented. Finally some concluding remarks and future work are drawn in Section 5.

2. DATA-DRIVEN PID CONTROLLERS

As given in Fliess and Join (2009), consider a nonlinear finite-dimensional SISO system

$$\Phi(t, y, \dot{y}, \dots, y^{(\iota)}, u, \dot{u}, \dots, u^{(\kappa)}) = 0$$

where Φ is a sufficiently smooth function of its arguments. Assume that for some integer n, $0 < n \leq \iota$, $\frac{\partial \Phi}{\partial y^{(n)}} \not\equiv 0$. The implicit function theorem yields then locally

$$y^{(n)} = \Upsilon(t, y, \dot{y}, \dots, y^{(n-1)}, y^{(n+1)}, \dots, y^{(\iota)}, u, \dot{u}, \dots, u^{(\kappa)})$$

This equation becomes by setting $\Upsilon = F + \alpha u$:

$$y^{(n)} = F + \alpha u \tag{1}$$

where

- $\alpha \in \mathbb{R}$ is a constant parameter, which is chosen in such a way that F and αu are of the same magnitude,
- F is determined thanks to the knowledge of u, α, and of the estimate of $y^{(n)}$. It plays the role of a nonlinear back-box identifier.

The differentiation order n has been defined until present as an integer and most often as $n = 1$ or $n = 2$. If the simplest case is considered ($n = 1$), the term $F(t)$ reads at each sample time

$$F(t_k) = \hat{\dot{y}}(t_k) - \alpha u(t_{k-1}) \tag{2}$$

Likewise, when the second derivative ($n = 2$) is used in the local model, the estimator F becomes

$$F(t_k) = \hat{\ddot{y}}(t_k) - \alpha u(t_{k-1}) \tag{3}$$

The model locality implies that (1) is only valid in a very short period of time (one sampling period). If the sampling rate is high enough with respect to the system time constant, model (1) accurately represents the system dynamics, and therefore the desired behavior can be obtained, as illustrated in Figure 1, by merging an inversion of (1) and a classical PID controller

$$u(t) = \frac{1}{\alpha}\left(y_r^{(n)}(t) - F(t)\right) + K_P e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt} \tag{4}$$

where y_r is a the reference trajectory, $e = y_r - y$ is the tracking error, K_P , K_I , $K_D \in \mathbb{R}$ are suitable gains.

The main advantage of this approach is that it uses a standard PID controller structure, but it is able to take into account, without any modeling procedure, the unknown parts of the system. As a consequence, Fliess and co-workers (Fliess and Join, 2008) called this model-free technique intelligent PID (i-PID) control. However, in order to avoid confusing statements for the intelligent control community, we have preferred to keep calling this technique in the generic way it has been done before: data-driven (DD) PID control.

Remark 1. This algebraic techniques for data driven PID control have already been successfully implemented in several applications, where the advantages with respect with standard PID is clearly highlighted (cf. e.g. (Villagra et al., 2009), (Villagra and Balaguer, 2011), (Villagra and Herrero-Perez, 2011)).

Remark 2. Note that since successive derivatives of the reference trajectory are required in this approach, the set-point variation has to be transformed into a sufficiently smooth trajectory (see (Villagra et al., 2010) for more details on how to transform a step into an equivalent smoothed trajectory).

3. AN EXTENSION TO FRACTIONAL DERIVATIVE ORDERS

Even though DD-PID provide a remarkable behavior to compensate unmodelled dynamics and reject disturbances Fliess et al. (2011), all the applications carried out until present within this framework have been limited to the simplest differential orders ($n = 1$ or $n = 2$). The main goal of this preliminary work is to explore the advantages of introducing fractional derivatives in the ultra-local model presented in equation (1).

3.1 Fractional derivatives

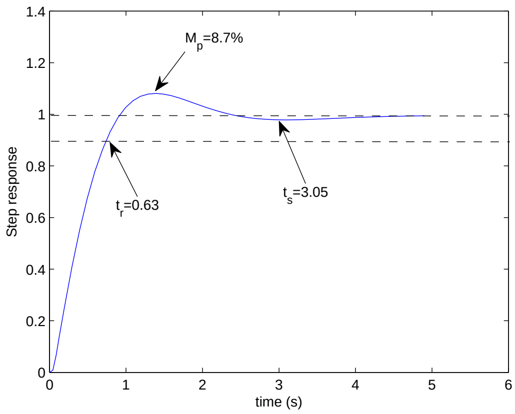
Fractional calculus is a generalization of the integration and differentiation to the non-integer (fractional) order fundamental operator ${}_bD_t^n$, where b and t are the limits and n is the order of the operation. The Riemann-Liouville definition of fractional derivative is (Podlubny, 1999)

$${}_bD_t^n f(t) = \frac{1}{\Gamma(m-n)} \frac{d^m}{dt^m} \int_b^t (t-\tau)^{m-n-1} f(\tau) d\tau, \tag{5}$$

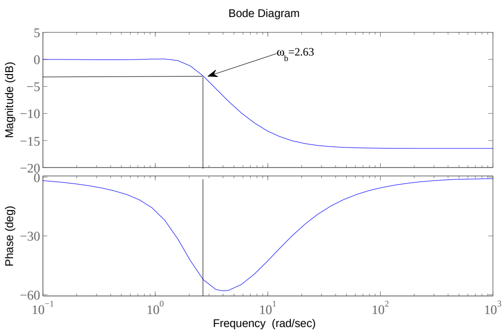
where m is the first integer larger than n, i.e., $m - 1 \leq n < m$ and Γ denotes the Gamma function defined as,

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}. \tag{6}$$

As can be observed, this generalized operator is local only when n is an integer. This nonlocal property and its ability



(a)



(b)

Fig. 2. PID closed loop behavior (a) Step response (b) Frequency response

to model memory phenomena has been extensively used in science and engineering, as well as in control theory to improve the robustness and the dynamic posibilities of the controlled systems (see (Monje et al., 2008), (Monje et al., 2010) and references therein).

3.2 Towards a more flexible control requirement space

Let us consider the following stable monovariable linear system

$$G(s) = \frac{(s+2)^2}{(s+1)^3} \tag{7}$$

As suggested in (Fliess and Join, 2009), the Broïda method is used to obtain the PID control gains, $K_P = 1.8181$ $K_I = 0.7754$ $K_D = 0.1766$. The time and frequency responses of the controlled system are depicted in Figure 2, where one can observe that the settling time is $t_s = 3.05$ s, the rising time is $t_r = 0.63$ s, the overshoot is $M_p = 8.7\%$, and the bandwidth is $\omega_b = 2.63$ rad·s^{-1}.

To evaluate the control requirement domain of both integer DD-PID and fractional DD-PID approaches, the PID controller above detailed has been used as a reference, and therefore its corresponding values of K_P, K_I and K_I were used in the all the subsequent tests. Hence, an iterative computation of the time response characteristics and the bandwidth from the frequency response of the linearized system have been performed to obtain an idea of the stable regions when varying $\alpha \in [1, 100]$ and $n \in [0, 2]$. Figure 3 shows two different representations of the requirement space for DD-PID when $n = 1$ (red points) and when n varies from 0 to 2 (blue points). In the first graph, the overshoot, settling time and rising time are plotted respectively in the X-Y-Z axes. In the second one, time response characteristics -overshoot and settling time- are confronted to a frequency domain attribute -bandwidth.

Note that in both cases the most attractive region (less overshoot, rising and settling time and more badwidth) is largely best covered by the fractional DD PID. As a consequence of this, not only the best closed loop behavior is provided by a fractional DD PID controller, but also a wider set of specifications can be reached with the latter. Specifically, the configuration providing a best dynamic response with the standard DD-PID ($n = 1$) is obtained for $\alpha = 1$ and the settling time, rising time, overshoot and bandwidth are, respectively, $t_s = 0.19$ s, $t_r = 0.06$ s, $M_p = 1.64\%$, $\omega_b = 2.98$ rad·s^{-1}. Note that the fractional DD-PID that provides the best dynamic behavior among all the plotted configurations in Fig. 3 is attained when $\alpha = 1$ and $n = 1.12$, guaranteing $t_s = 0.19$ s, $t_r = 0.04$ s, $M_p = 0.99\%$, $\omega_b = 100$ rad·s^{-1}, which is significantly better than the dynamic response of PID and even DD-PID.

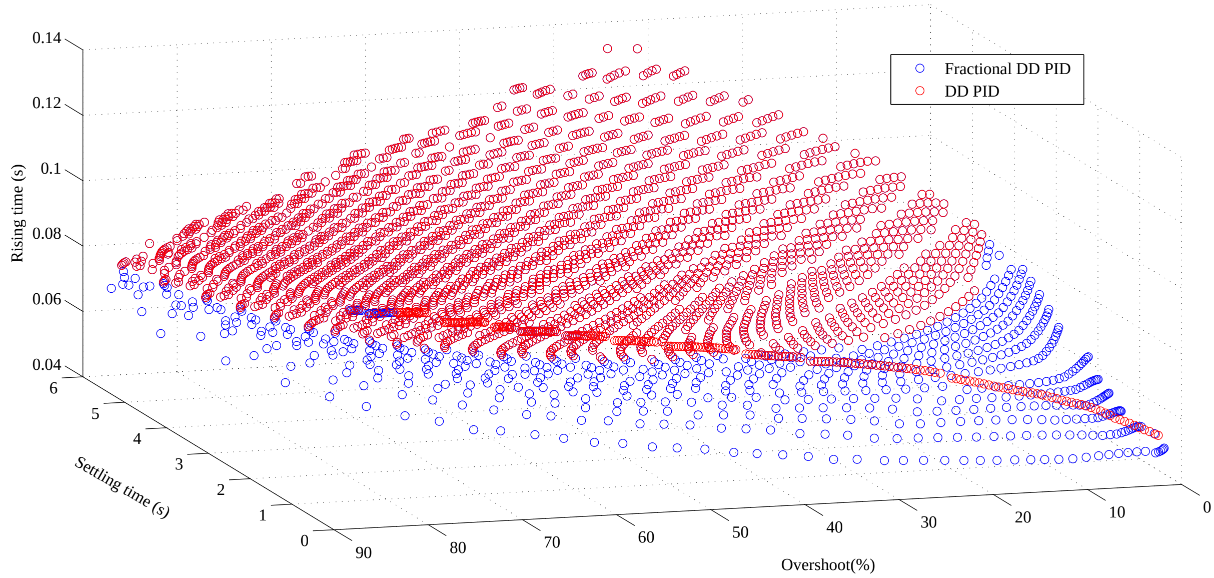
4. APPLICATION TO A DC MOTOR CONTROL IN A ROBOT FLEXIBLE JOINT

4.1 Joint motion control under uncertainty

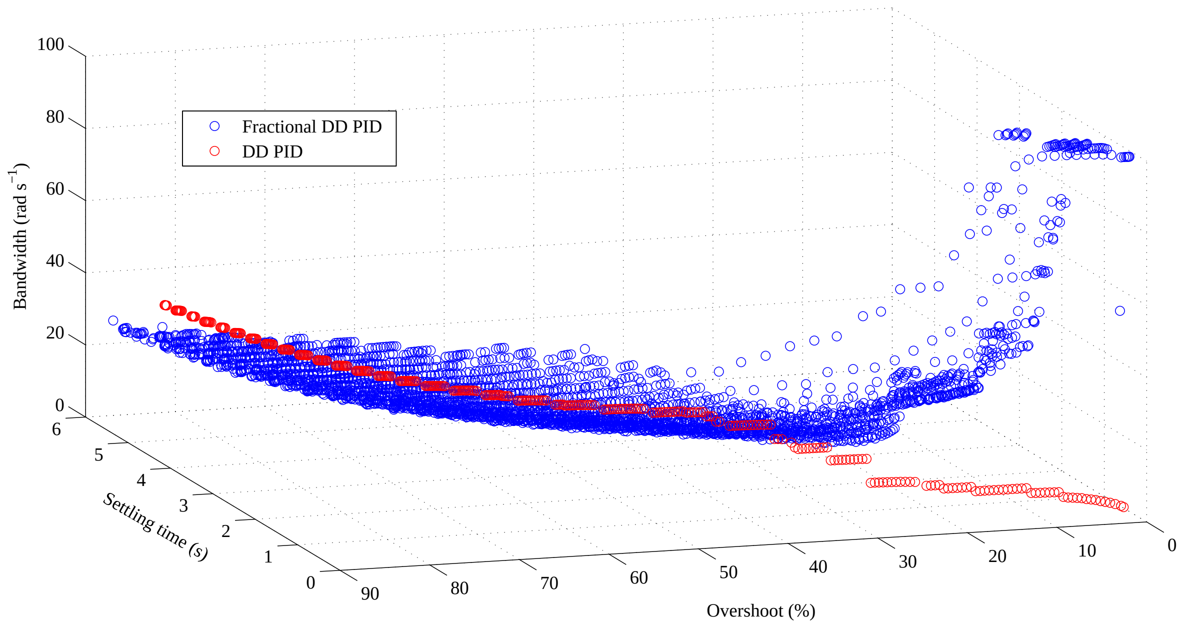
The accurate position control of robot joints has been extensively studied in the last decades. Thus, many different feedback techniques have achieved good position tracking when considering electrically driven rigid robots (cf. e.g. (Tarn et al., 1991)). However, the transmission systems usually introduce nonlinear dynamics between the motor output and the real joint. Moreover, it is not easy to obtain a precise model of these effects, because of the great number of parts that intervene in the transmission. Some authors (Tjahjowidodo et al., 2007) have tried to obtain, through careful modelling, an accurate feedforward control which, in turn, allows to follow a desired trajectory with low-feedback gains. The main problem of this sort of approaches is that they have to deal with highly cross-coupled non-linear models, where the parameters are not always identifiable. Thus, several robust/intelligent control approaches (Oya et al., 2004), (Kwan et al., 1998) and adaptive control techniques (Chang, 2002), (Ishii et al., 2001) have been proposed to tackle with an appropriate feedback law the transmission uncertainty problem in rigid joint tracking. However, not so many efforts (Chien and Huang, 2007) have been addressed to control electrically driven flexible-joints under uncertainty. This is because the presence of joint flexibility greatly increases the complexity of the system dynamics.

4.2 System model

The system to be controlled consists of a DC motor and an Harmonic Drive Transmission system. Furthermore, flexion torques due to the robot structure compliance have been taken into account (see (Villagra and Balaguer, 2011)



(a)



(b)

Fig. 3. Requirement space comparison between standard DD PID control and fractional DD PID control. (a) Settling time-overshoot-rising time requirement space (b) Settling time-overshoot-bandwidth requirement space

for more details). The classical direct current (DC) motor model is considered:

$$J_m\ddot{\theta}_m(t) = K_t i(t) - \tau_{fr}(t) - \tau_{fl}(t)$$
$$L_m\frac{di(t)}{dt} = -R_m i(t) + Eu_m(t) - K_b\dot{\theta}_m(t) \tag{8}$$

where i denotes the armature current, θ_m the angular position of the motor shaft, J_m the rotor inertia, L_m the terminal inductance, R_m the motor terminal resistance, K_b the back electromotive force constant and K_t the torque constant. E represents the maximum available voltage, in absolute value, which excites the machine, while u_m is an input voltage modulation signal, acting as the ultimate control input, with values restricted to the closed real set $[-1, 1]$. Flexible torque τ_{fl} due to the structure compliance will be above detailed.

The model of the reducer can be written as follows (cf. (Abba and Sardain, 2005) for more details):

$$\dot{\theta}_t = \frac{\dot{\theta}_m}{N}, \ \tau_t = \alpha_m N \tau_m \tag{9}$$

where θ_t is the transmission angular position, τ_t is the transmission at the Harmonic Drive output, N is the gear-box ratio and α_m represents the torque transfer coefficient. Consider the dynamics of the flexible system as follows (cf. (Ghorbel et al., 1989)):

$$J_l\ddot{\theta}_l + \tau_l - \tau_{fl} = 0,\ \tau_{fl} = c(\theta_t - \theta_l) \qquad (10)$$

where J_l is the joint inertia, c the joint stiffness constant, τ_l the load torque, and the τ_{fl} the flexible torque. Remark that the torque τ_{fl}, generated by joint flexible dynamics, is here the same than in equation (8), but with opposite sign.

The system to be controlled is therefore the one conformed by equations (8)-(9)-(10), where the input signal is the motor voltage $u = Eu_m$ and the output is the link tip position $y = \theta_l$.

4.3 Controllers comparative simulation

The fractional DD-PID is compared with the standard DD-PID in a scenario, where the position reference is a sinus of amplitude equal to 1 rad and frequency of 1 Hz, and a varying load τ_l is applied to the flexible link (a sinusoidal of 1 Nm and $15Hz$, see Villagra and Balaguer (2011) for more details)

Figure 4 shows the response of the closed loop system to the sinusoidal excitation with 4 different configurations: the best PID, the best DD-PID ($n = 1$), the best fractional DD-PID and the best configuration (K_P, K_I, K_D, α) for two controllers with fixed fractional derivative order ($n = 0.7$ and $n = 1.2$). Note that the optimal configuration in each case is obtained by minimizing the Integral Absolute Error (IAE) via a sequential combination of two nonlinear optimizers: Sequential Quadratic Programming and Nelder Mead. Note that even though the varying disturbances imposes a quite demanding control challenge (that it is not easy to handle nor by PID neither by DD-PID) the fractional DD-PID achieves a remarkable tracking quality, and therefore a very satisfactory disturbance rejection.

Since one of the key features of this data-driven approach is its ability to adapt to unmodelled dynamics or parameter uncertainty, a second experiment has been simulated, where three of the most influent parameters (J_m, K_t and c) in the model are modified. The results on Figure 5 shows the dynamic behavior of the best DD-PID and fractional DD-PID when an increment of 20% is applied to each parameter with respect the experimental plotted in Figure 4.

Both in Figure 5 and 4 the behavior of each controller is quantified with an arrowed text where the corresponding IAE is plotted -in Figure 4 the value associated to the arrows corresponds to the worst case among the 3 parameter variations. Looking at these, it is clear that the introduction of a fractional derivative order significantly increases the control performance to track the desired reference while rejecting disturbances and under an important degree of uncertainty.

5. CONCLUDING REMARKS

A first attempt to introduce fractional derivative orders in the local model on which the presented data-driven PID controllers rely has been explored. The results obtained firstly in an academic example and thereafter in a more complex application highlight the interest of fractional calculus in this new control framework.

Since the main advantage of this new approach is the enlargement of control requirements accessibility, an important breakthrough would be to obtain a systematic procedure that allows firstly to analytically characterize the stable regions of the closed loop system, and thereafter to assign a set of specifications to a particular controller configuration. Our current research work is looking for some answers to this challenging problem.

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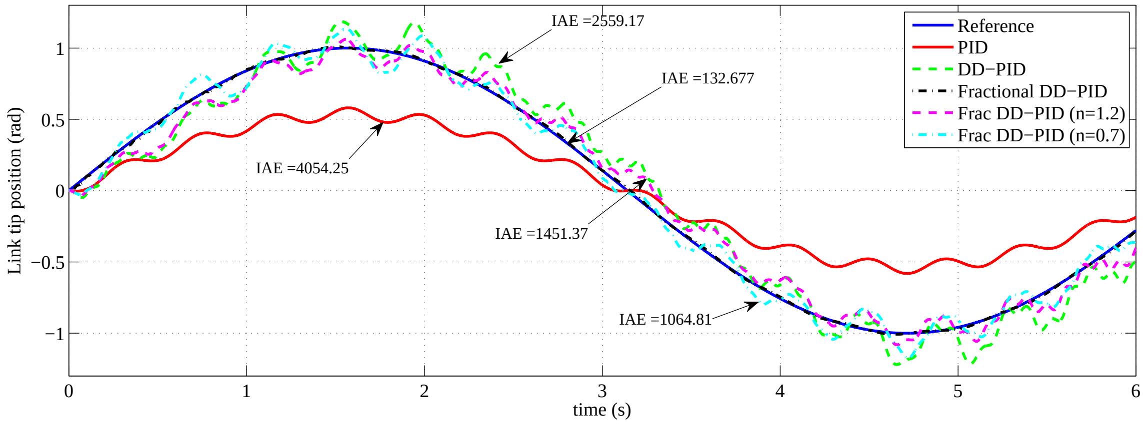

Fig. 4. Time response with varying load disturbances.

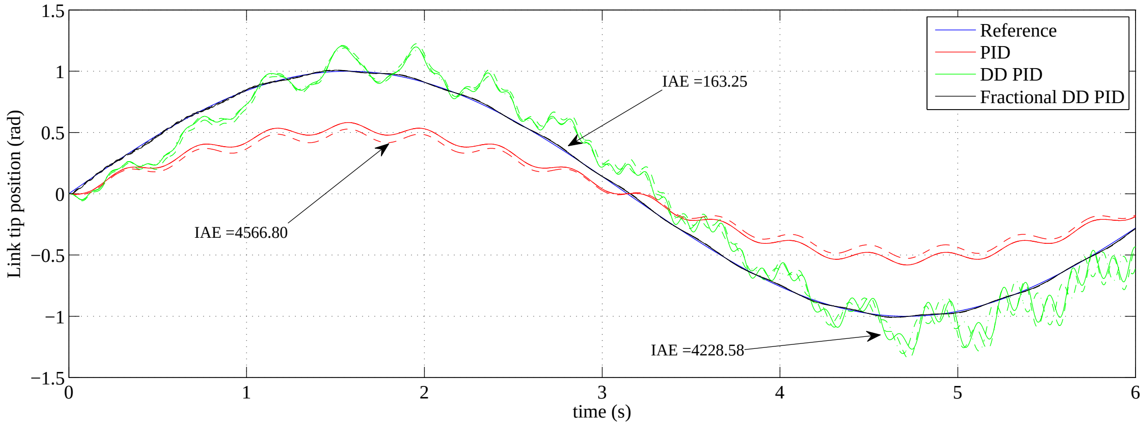

Fig. 5. Time response with varying load disturbances and parameter uncertainty.

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