A Multi-objective Optimization Design Methodology for SISO PID Controllers ⋆

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Abstract: By today, PI-PID controllers remain as reliable control solutions in a wide variety of industrial applications. Several tuning techniques have been designed and proposed successfully over the years. However the difficulty for tuning procedures increase as multiple requirements and criteria to evaluate the closed loop performance are demanded to be fulfilled. In this work, a multi-objective optimization design methodology is applied to the SISO benchmark for PID control based on the Boiler Control Problem. The main advantage of a multi-objective design methodology, is the capacity to perform an analysis on the trade-off among objectives.

Keywords: Multiobjective optimization, Multiple-criterion optimization, PI controller, design.

1. INTRODUCTION

PID controller remains as a reliable and practical control solution for several industrial processes. Owing to this, research for new tuning techniques is an ongoing research topic (Ge et al. (2002); Toscano (2005); Goncalves et al. (2008); Astrom et al. (1998); Panagopoulos et al. (2002); Alfaro (2007); Reynoso-Meza et al. (2012b)). Current research points to develop tuning techniques to guarantee reasonable stability margins as well as a good overall performance for a wide variety of process. One of the main advantages of PI-PID controllers is their ease of implementation as well as their tuning, giving a good trade-off between simplicity and cost to implement (Tan et al. (2004); Stewart and Samad (2011)).

When multiple design objectives to fulfil are required, it is said to have a multi-objective problem (MOP). In a MOP, the designer (control engineer) has to deal with several requirements, and searches for a solution with a desired trade-off between objectives. As a Decision Maker (DM) she/he has to design or translates such desired trade-off in the optimization problem at hand. A traditional approach to solve a MOP is to translate it into a single-objective problem using weighting factors, indicating the relative importance among objectives. More elaborated methods to incorporate the preference handling have been developed (Marler and Arora (2004)) such as lexicographic methods, goal programming and physical programming. However, new mechanisms and techniques are still required to handle preferences in a flexible and meaningful way.

In this paper, we propose a tuning methodology based on multi-objective optimization (MOO) to adjust the parameters of a PID controller for the boiler control problem for the IFAC Conference on Advances in PID Controllers. The main advantage of this approach is to generate a set of potentials solutions, where the designer can analyze the trade-off among conflicting objectives. The rest of this paper remains as follows: in section 2, a brief benchmark introduction is given. In section 3, the multi-objective optimization design methodology is explained. In section 4 and 5 the results are discussed and evaluated in the benchmark framework. Finally, some concluding remarks are given.

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2. BENCHMARK DEFINITION

The process under consideration is the benchmark for PID control 2012 described by Morilla (2012). It is a benchmark which proposes a boiler control problem (Morilla (2010); Fernández et al. (2011)) based on the work of Pellegrinetti and Bentsman (1996). The original problem stated a 3x3 MIMO system with a measured load disturbance:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & 0 & g_{13}(s) \\ g_{21}(s) & g_{22}(s) & 0 \\ g_{31}(s) & 0 & g_{33}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix} + \begin{bmatrix} g_{1d}(s) \\ 0 \\ g_{3d}(s) \end{bmatrix} D(s) \quad (1)$$

Where the inputs are fuel flow, water flow and air flow whilst the outputs are steam pressure, oxygen level and water level. For this conference, a reduced MIMO and SISO version are available to evaluated different controller tuning procedures. In this work, we focus in the SISO PID controller tuning, where the fuel flow is manipulated to control the steam pressure.

The benchmark also defines an index to evaluate the performance for a given PID controller. It is an aggregate objective function, which combines ratios of IAE, ITAE and IAVU indexes. More details can be consulted in Morilla (2012).

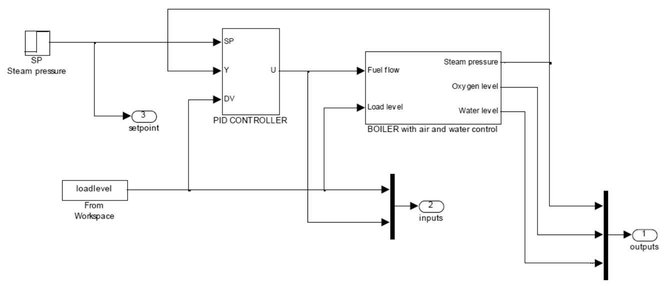


Fig. 1. SISO loop (taken from Morilla (2012)).

3. MULTI-OBJECTIVE OPTIMIZATION DESIGN METHODOLOGY

MOO can handle with MOP owing to its simultaneous optimization approach. The MOO approach offers to the designer a set of solutions, the Pareto set, where all the solutions are Pareto-optimal (Marler and Arora (2004)). This set of solutions will offer to the DM a higher degree of flexibility at the decision making stage. The role of the designer is to select the most preferable solution according to his needs and preferences for a particular situation. A MOP, without loss of generality (since a maximization problem can be converted to a minimization problem), can be stated as follows:

$$\min_{\boldsymbol{\theta}\in\Re^n} \boldsymbol{J}(\boldsymbol{\theta}) = [J_1(\boldsymbol{\theta}), \dots, J_m(\boldsymbol{\theta})] \in \Re^m \tag{2}$$

In general, it does not exist an unique solution because there is not a solution better than other in all the objectives. Therefore a set of Pareto-optimal solutions, the Pareto set $\boldsymbol{\Theta_P}$ is defined and its projection into the objective space is known as the Pareto front $\boldsymbol{J_P}$. Each point in the Pareto front is said to be a non-dominated solution. MOO techniques search for a discrete approximation $\boldsymbol{\Theta_P^*}$ of the Pareto set $\boldsymbol{\Theta_P}$ capable of generate a good quality description of the Pareto front $\boldsymbol{J_P^*}$. In this way, the DM has a set of solutions for a given problem and a higher degree of flexibility to choose a particular or desired solution.
A general framework is required to successfully incorporate the MOO approach into any engineering process. A multi-objective optimisation engineering design (MOOD) methodology is shown in Figure 2 (Reynoso-Meza et al. (2012a)). It consists in three main steps: the MOP definition, the MOO process and the decision-making stage.

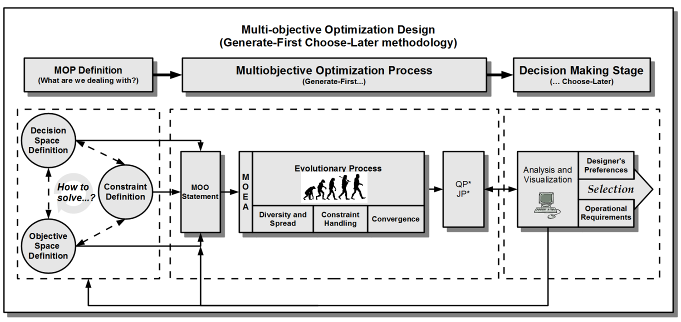


Fig. 2. A multiobjective optimisation engineering methodology.

3.1 MOP definition

A PI controller is selected, mainly for comparison purposes with the benchmark proposals:

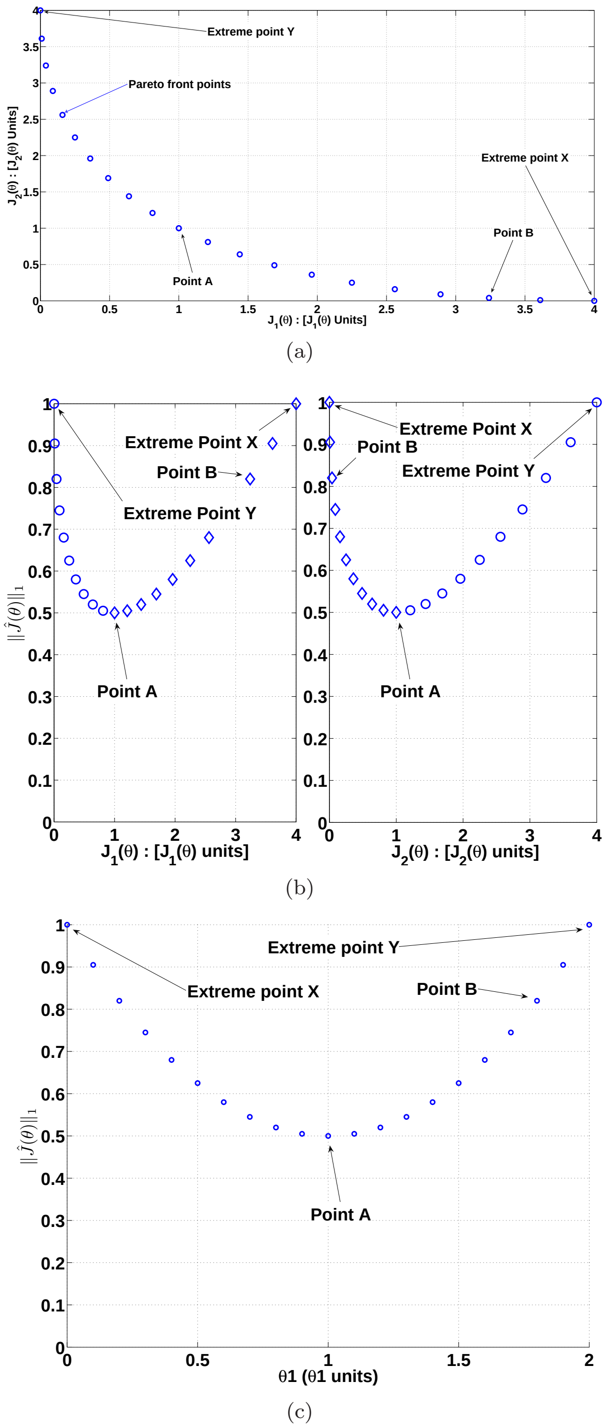


Fig. 3. Typical representation of the Pareto front (a) and the Pareto set (b). Points at the same level in the LD have correspondence on each graph.

$$G_c(s) = k_c \left(1 + \frac{1}{T_i s}\right) E(s) \tag{3}$$

where k_c is the proportional gain, T_i the integral time (sec.) and $E(s)$ the error signal. A feedforward compensator is integrated with the PI controller, according to Guzmán and Hägglund (2011)
A multi-objective problem with three objectives is proposed:

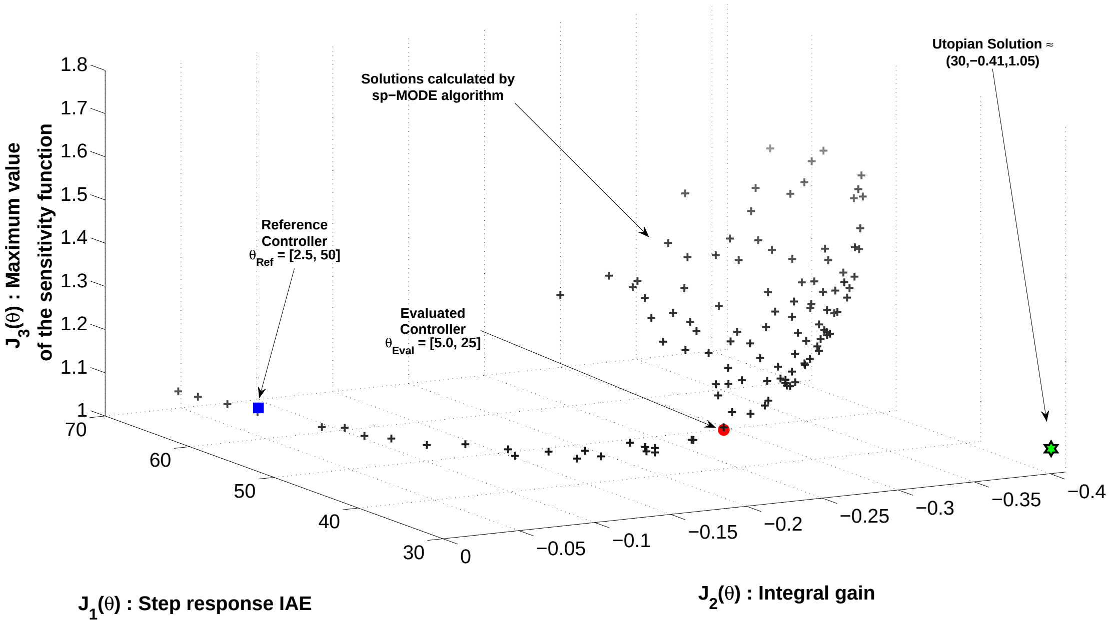


Fig. 4. 3D visualization. The darker the solution, the most preferable according to the benchmark criteria (test 2).

$$\min_{\boldsymbol{\theta} \in \Re^2} \boldsymbol{J}(\boldsymbol{\theta}) = [f^{IAE}(\boldsymbol{\theta}), -k_i, M_s] \tag{4}$$

where $\boldsymbol{\theta} = [k_c, T_i]$, $f^{IAE}(\boldsymbol{\theta})$ is the integral of the absolute error for a step reference change in simulation test (with a previously identified model), $k_i = k_c/T_i$ the integral gain of the controller and M_s the maximum value of the sensitivity function $M_s = \max|S(\mathrm{j}\omega)| = \max\left|\frac{1}{1+G_p(\mathrm{j}\omega)G_c(\mathrm{j}\omega)}\right|$. It has been notice in Astrom et al. (1998) that the integral gain is related with the integral of error in the load response ($\int e \cdot dt \approx 1/k_i$). Therefore, $\boldsymbol{J}(\boldsymbol{\theta}) = [f^{IAE}(\boldsymbol{\theta}), -k_i, M_s]$ describes a trade-off between set-point response, load rejection performance and robustness.

3.2 Multi-objective optimization

Several algorithms used to calculate $\boldsymbol{J}_P^*$ have been widely proposed, for example the Normal Boundary Intersection method of Das and Dennis (1998) or the Normal constraint method from Messac et al. (2003), but lately multi-objective evolutionary algorithms (MOEA's) have been used due to its flexibility to deal with non-convex and highly constrained functions (Coello and Lamont (2004)). In this work, the sp-MODE algorithm from Reynoso-Meza et al. (2010); Reynoso-Meza (2009) is used to generate the Pareto front approximation $\boldsymbol{J}_P^*$.

3.3 Decision making

The selection of a trade-off solution, according with the designer preferences, takes place in a posteriori analysis on the Pareto front $\boldsymbol{J}_P^*$. In this work, to visualize the calculated front, the Level Diagram (LD) visualization of Blasco et al. (2008) is used. The LD helps the DM to perform an analysis on the obtained Pareto front $\boldsymbol{\Theta}_P^*$, which is not a trivial task when the number of objectives is bigger than three. The LD visualization is a useful technique to analyze m-objective Pareto fronts (see for example Reynoso-Meza et al. (2009, 2012a)); it is based on the classification of the $\boldsymbol{\Theta}_P^*$ calculated. Each objective $J_q(\boldsymbol{\theta})$ is normalized with respect to its minimum and maximum values. Afterwards, to each normalized objective vector $\hat{\boldsymbol{J}}(\boldsymbol{\theta})$ a p-norm $\|\hat{\boldsymbol{J}}(\boldsymbol{\theta})\|_p := \left(\sum_{q=1}^{m} \|\hat{J}(\boldsymbol{\theta})_q\|^p\right)^{1/p}$ is applied to evaluate the distance to an ideal solution $\boldsymbol{J}^{ideal} = \boldsymbol{J}^{min}$.

The LD visualization proposes a graphical array for every objective $q \in [1, \ldots, m]$ and every decision variable $l \in [1, \ldots, n]$. The ordered pairs $\left(J_q(\boldsymbol{\theta}), \|\hat{\boldsymbol{J}}(\boldsymbol{\theta})\|_p\right)$ in each objective sub-graphic and $\left(\theta_l, \|\hat{\boldsymbol{J}}(\boldsymbol{\theta})\|_p\right)$ in each decision variable sub-graphic are plotted. Therefore, a given solution will have the same y-value in all graphics (see figure 3). This correspondence will help to evaluate general tendencies along the Pareto front and to compare solutions according with the selected norm. In all cases, the lower the norm, the closer to the ideal solution; to plot the LD visualization, the LD-Tool [1] is used. For the remaining of this work the $\|\cdot\|_2$ norm will be used for the sake of simplicity.

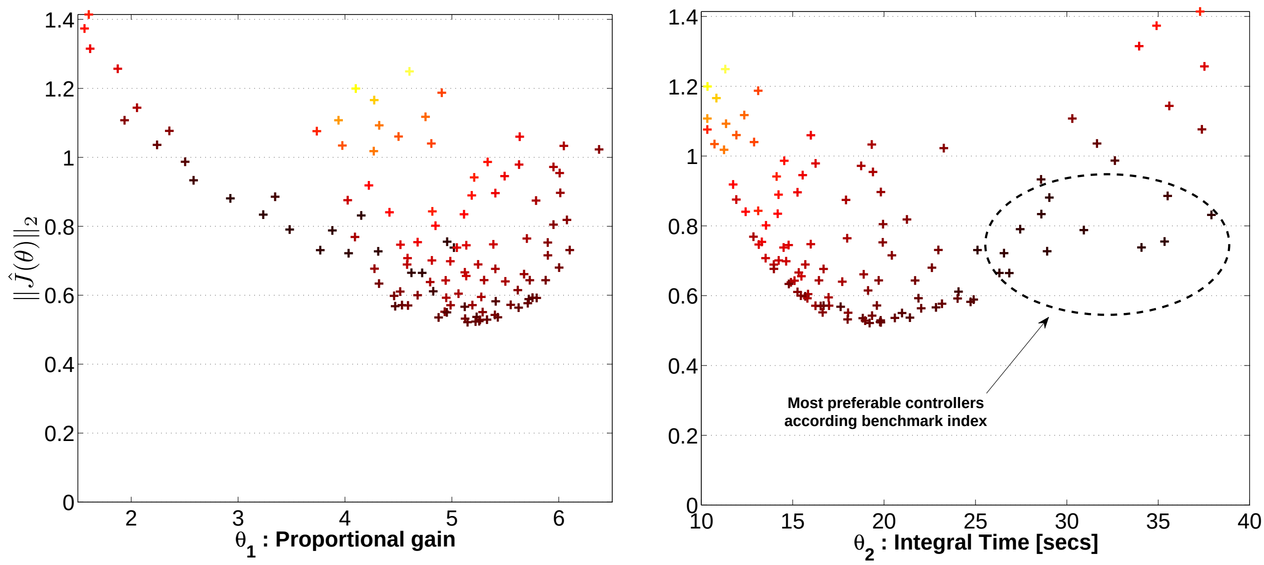
4. DEVELOPMENT

As we are looking for a controller with better performance than the reference controller ($\boldsymbol{\theta}_{Ref} = [2.5, 50]$), we evaluate it in the identified model [2]. In this case, $J_1(\boldsymbol{\theta}_{Ref}) \approx 60$. Therefore, we constraint the solutions in the Pareto front approximation to have a $J_1(\boldsymbol{\theta}) \leq 70$. The sp-MODE algorithm executed 2020 function evaluations to approximate a Pareto front. In figure 4 and 5 the 3D and LD visualization are shown, respectively.

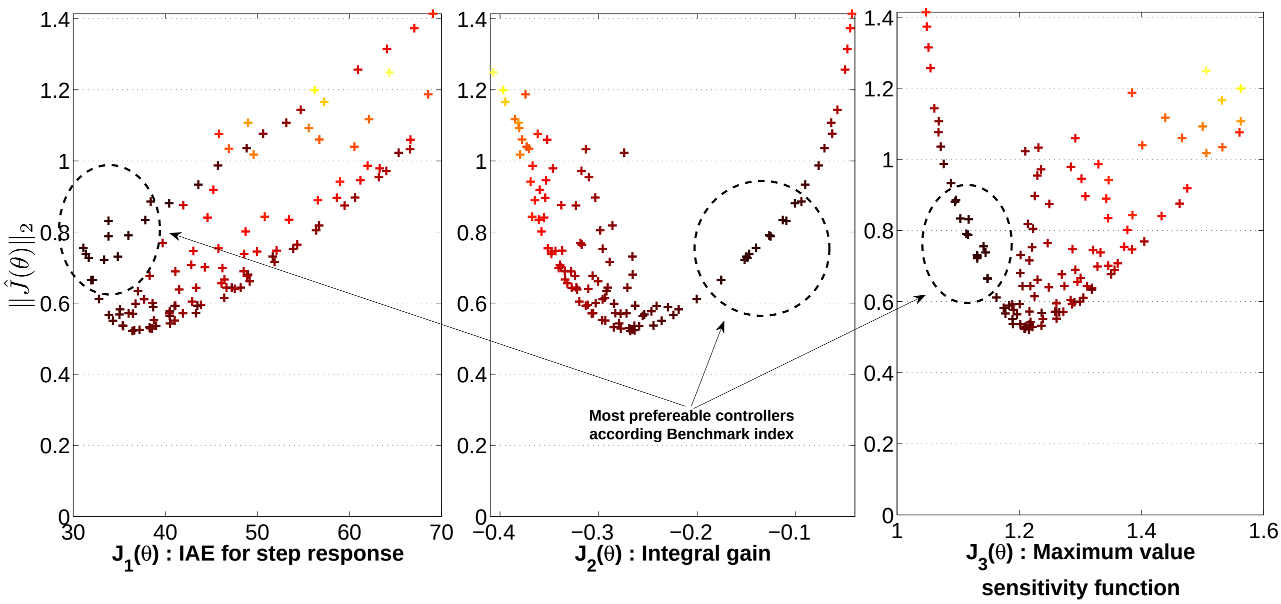
In figure 4 the reference controller, as well as the evaluated controller ($\boldsymbol{\theta}_{Eval} = [5.0, 25]$) of the benchmark are shown.

[1] Free GUI available at `http://www.mathworks.com/matlabcentral/fileexchange/24042`

[2] $G(s) = \frac{0.3934}{45.6794s+1}e^{-3.42s}$; This model was obtained with a step response experiment using the identification toolbox of matlab.



(a) Pareto set



(b) Pareto front

Fig. 5. LD visualization. The darker the solution, the most preferable according to the benchmark criteria (test 2).

Notice how both controllers converge into the Pareto front approximation. It means that both of them are Pareto optimal (according to the objective vector statement using in the optimization). The evaluated controller is not better than the reference controller, but it is a most desirable solution, according with the benchmark function cost. All controllers were evaluated in the benchmark model with the second test (which is similar to the one used in the identified model in the optimization step) to use their index value as a new indicator for selection. This information can be included in the LD visualization using a colour coding. In this case, the darker the solution, the most preferable according with the benchmark function cost. It is possible to appreciate in figure 5 the preferable zone according with such indicator.

5. TEST

The most preferable controller from the Pareto front approximation is selected ($\boldsymbol{\theta}_{Pref} = [3.48, 27.45]$), and compared with the evaluated controller from the benchmark. This controller has a small k_i value when compared with others controller along the Pareto front. This means it will have a worst load rejection performance. Nevertheless, as the main load is measured and tackle by the feedforward compensator, we can rely in this controller. A new experiment is defined, with a lineal load disturbance (from 46 to 50 %) from time 100 to 800 sec. followed by a setpoint change (60 to 50 %) at time = 1000 secs. Results from this experiment are shown in figures 6 to 13. The most appreciable change is in the fuel flow, where the manipulated variable saturates (same anti-windup mechanism is incorporated in both controllers).

6. CONCLUSIONS

In this work, a multi-objective optimization design methodology (MOOD) for PID controller tuning has been used. As illustrative example, a PI controller was adjusted for the SISO benchmark control problem of the IFAC Conference on Advances in PID Controllers. The MOOD methodology consist on three main steps: multi-objective problem definition, multi-objective optimization process and multi-criteria decision making.

With this approach, a set of potential solutions is calculated. This allows to analyse the trade-off among solutions and select the most preferable solution, according to a given problem. Although the index defined in the benchmark is used to detect a desirable zone in the Pareto front, the MOOD allows to embed the DM in the final selection of the solution. This is a more flexible approach, since it is possible to select another Pareto-optimal solution (but sub-optimal according with a given index) to improve one desirable objective, in exchange of another(s). Furthermore, new indexes or measurements not used in the optimization stage could be included in the decision making step (control effort, raising time, overshoot, for example). This would be helpful to the DM to perform and justify his/her final selection.

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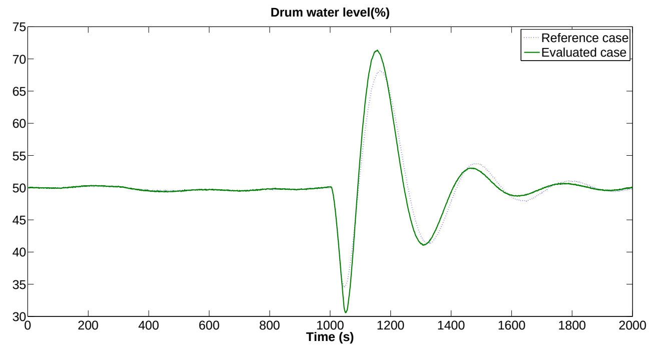

Fig. 6. Drum water level performance for evaluated controller $\boldsymbol{\theta}_{Eval} = [5.0, 25]$.

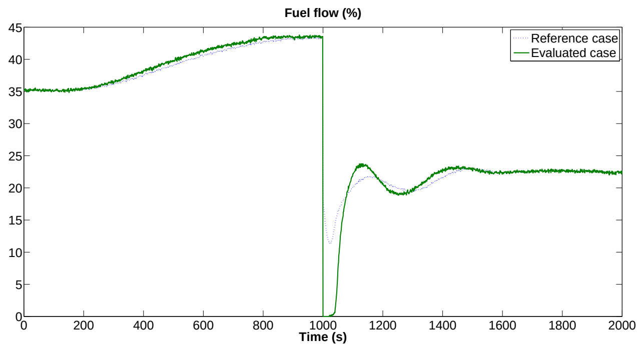

Fig. 7. Fuel flow performance for evaluated controller $\boldsymbol{\theta}_{Eval} = [5.0, 25]$.

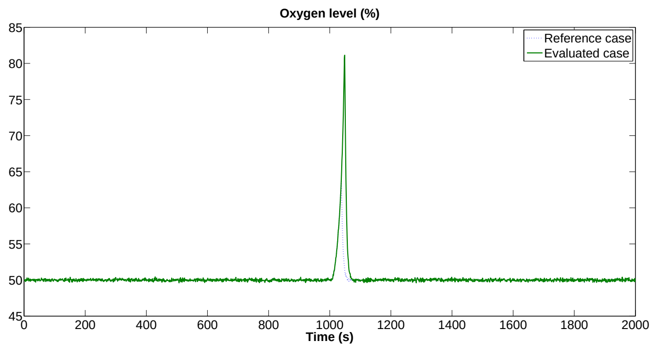

Fig. 8. Oxygen level performance for evaluated controller $\boldsymbol{\theta}_{Eval} = [5.0, 25]$.

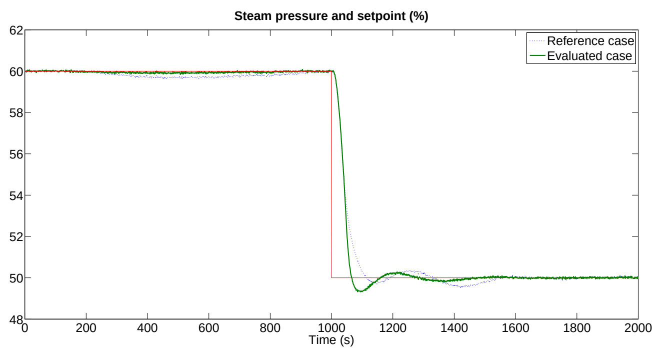

Fig. 9. Steam pressure performance for evaluated controller $\boldsymbol{\theta}_{Eval} = [5.0, 25]$.

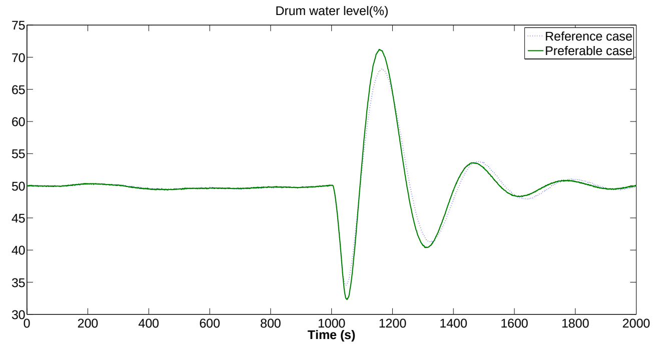

Fig. 10. Drum water level performance for preferable controller $\boldsymbol{\theta}_{Pref} = [3.48, 27.45]$.

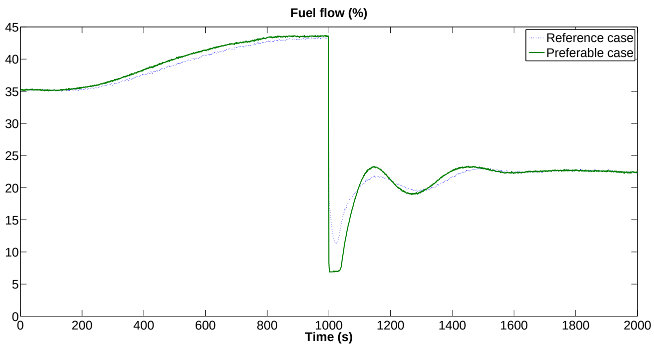

Fig. 11. Fuel flow performance for preferable controller $\boldsymbol{\theta}_{Pref} = [3.48, 27.45]$.

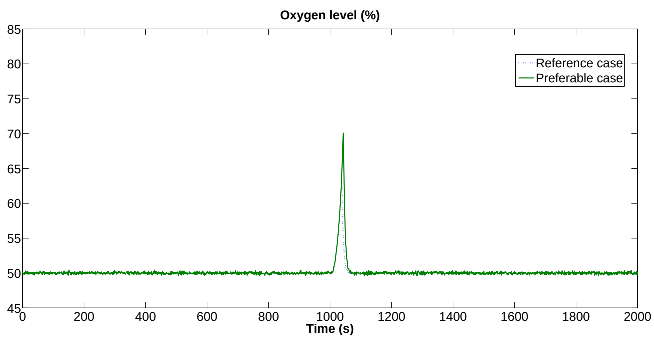

Fig. 12. Oxygen level performance for preferable controller $\boldsymbol{\theta}_{Pref} = [3.48, 27.45]$.

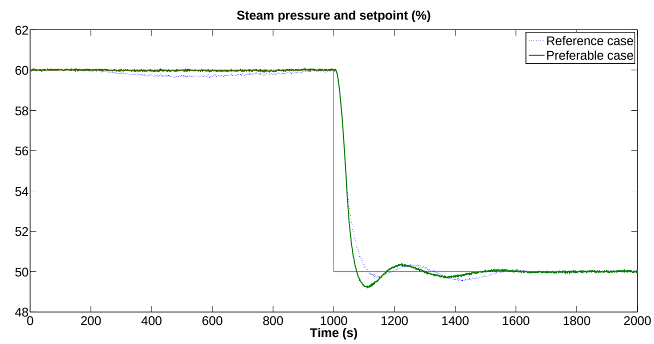

Fig. 13. Steam pressure performance for preferable controller $\boldsymbol{\theta}_{Pref} = [3.48, 27.45]$.