# Antiwindup-aware PI autotuning

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**Abstract:** This manuscript addresses the problem of "tuning the antiwindup mechanism" of PI(D) controllers, an issue seldom addressed in the literature but yielding improvement and reducing undesired large-transient behaviours if tackled correctly. A previously proposed framework is extended and related to both alternative antiwindup implementations and tuning methods, resulting in more methodological insight on the matter, and guidelines to set up the extension of a given method to include antiwindup. Simulation examples are reported to illustrate the achievable advantages.

*Keywords:* Antiwindup; autotuning; PI control.

## 1. INTRODUCTION

Virtually any PID (auto)tuning method available in the literature has to compute and – more or less explicitly – validate the obtained regulator parametrisation prior to its application to the controlled process, and to accomplish such a task, a linear dynamic framework is typically assumed.

There are a number of obvious reasons to do so, starting from the necessity of doing the overall job fast, and possibly with low-end hardware as an industrial single-loop stand-alone controller. However, this leaves out completely what happens when the control signal hits a saturation limit, delegating the matter entirely to the controller's antiwindup mechanism, see e.g. EnTech Control Engineeering Inc. (1993); Harrold (1999). For completeness it has to be noticed that loop nonlinearities are sometimes considered, but the focus is practically limited to those introduced by the process (Shinskey, 2002). As a result, autotuned PIDs may somehow exhibit undesired behaviour in the presence of large transients.

One may object that such transients are seldom managed via the feedback mechanism alone, resorting for example to *ad hoc* set point generation methods or similar solutions. This is true, but the fact that saturation may be hit remains, since from an industrial standpoint the fact that an actuator *never* saturates may even be considered a flaw in its sizing, and the idea that the same tuning method may produce different results when applied to industrial PIDs differing only for the antiwindup type, is unpleasant in any case.

In the past, the mainstream approach has been to avoid accounting for the antiwindup mechanism in the tuning method, and devise antiwindup schemes that provide "reasonably uniform results" whatever tuning method is used, and are easy to understand and set up. Alternatives were also attempted, but the typical result is quite cumbersome a mechanism, and sometimes even the idea of abandoning the PI(D) structure, see e.g. Tyan and Berstein (1994); Yamada and Funami (2000); Cao et al. (2002); Grimm et al. (2003); da Silva Jr. and Tarbouriech (2005), or very domain-specific solutions, as for example in Shin (1998); Charaabi et al. (2002). Although all the presented solutions are valid, however, the impression remains that not tackling the "antiwindup (auto)tuning" correctly results in quite significant a limitation – as far as large transients are concerned, of course – of an otherwise extremely flexible, powerful and general controller as the PI(D).

In fact, in a previous paper (Leva and Bascetta, 2006) it was already shown that neglecting the antiwindup functioning can lead to regulator parametrisations that appear very similar for small transients, but are very different for large ones: based on that, the same work proposed a new antiwindup scheme, and a *preliminary* method to tune its parameter. This manuscript continues the quoted work, introducing the following contributions. First, a common framework is established in which the presented antiwindup scheme (and conceptually any other) can be formally compared to existing ones. Second, further insight is provided on the parameters (now two instead of one) of the proposed antiwindup tuning, showing how they can be related not only to the closed-loop bandwidth anymore, but also to the prevalence of proportional or integral action (in the sense explained later on). Third, an application of the so refined idea is presented with reference to a widely accepted benchmark for PID control (Åström and Hägglund, 2000).

The manuscript is organised as follows. Section 2 provides a short sketch of the background, limiting the scope to the PI case as the PID is substantially analogous with just more computations to deal with. Then, section 3 reviews some widely used antiwindup schemes, and evidences their differences in a formal manner, that in section 4 are used to provide a stronger basis for the preliminary research quoted above, yielding to an improved antiwindup-aware autotuning proposal. Section 5 applies that proposal to a well assessed autotuning method so as to show how it can be plugged into a general one, and together with the following section 6 provides evidence of its effectiveness.

Finally, section 7 draws some conclusions and envisages future research.

## 2. BACKGROUND

This section provides a minimum background review, see the previous paper Leva and Bascetta (2006) for a more detailed explanation. A two-degree-of-freedom (2-dof) PI in the ISA form, i.e., the control law

$$U(s) = K\left[ (bY^\circ(s) - Y(s)) + \frac{1}{sT_i}(Y^\circ(s) - Y(s)) \right] \quad (1)$$

is considered, where $Y^\circ(s)$, $Y(s)$ and $U(s)$ are the Laplace transforms of the set point, the controlled variable and the control signal, respectively, and the other symbols have the standard ISA meanings, see the famous work Åström and Hägglund (1995). As anticipated, extending the ideas proposed there to the PID or even more complex regulators is not difficult, but would unduly complicate their explanation.

Denoting by $I$ the integral term and by $u_{min}$ and $u_{max}$ the lower and upper control bounds, saturation does not occur when

$$u_{min} \leq K(by^\circ - y) + I \leq u_{max} \quad (2)$$

holds; (2) can also be written as

$$\frac{I - u_{max}}{K} + by^\circ \leq y \leq \frac{I - u_{min}}{K} + by^\circ \quad (3)$$

which means that, at any given time, there is no saturation if the output does not exceed the limits

$$PB_{lo} = \frac{I - u_{max}}{K} + by^\circ, \quad PB_{hi} = \frac{I - u_{min}}{K} + by^\circ \quad (4)$$

and the difference $PB_{hi} - PB_{lo}$, equalling $(u_{max} - u_{min})/K$, is called the proportional band (amplitude).

As will be shown, antiwindup schemes can be characterised by observing the signal

$$PBC = \frac{PB_{hi} + PB_{lo}}{2} = by^\circ - \frac{I - u_c}{K}, \ u_c = \frac{u_{min} + u_{max}}{2}, \quad (5)$$

i.e., the proportional band centre. Moreover, focusing the *synthesis* of the PI antiwindup part on the idea of suitably managing $PBC$ allows to devise tuning methods capable of complementing existing (linear) ones and improving interpretability with respect to other approaches. This is the subject of the subsequent section 4, but preliminary to that, it can be shown how the behaviour of $PBC$ can be used to characterise an existing antiwindup strategy, to formally evidence its difference with respect to others, and thus the generality of the proposed one.

## 3. ANTIWINDUP SCHEMES

The purpose of this section is to show that different antiwindup schemes produce different large-transient behaviours – which is obvious – and to illustrate that this can be explained by studying the behaviour of $PBC$ induced by them. For space reasons, only the positional PI realisation and the most frequently used schemes are here considered; possible extensions are straightforward, and left as an exercise to the interested reader.

The three addressed schemes are the so-called "integral term recomputation", "actuation error feedback"

and "feedback-based integration" ones. Since in the literature there is no uniformity on the corresponding nomenclature, said schemes are here shown as discrete-time block diagrams obtained from (1) via the backward difference method with time-step $T_s$ (which causes no generality loss), and then analysed to determine the induced $PBC$ behaviour.

The integral term recomputation scheme is shown in figure 1. Its *rationale* is to compute the unconstrained control, and then in the presence of saturation reset the integral term to match $u$ with the current proportional term. It is a very simple scheme, not involving any additional parameter apart from the linear PI ones, and thus widely used.
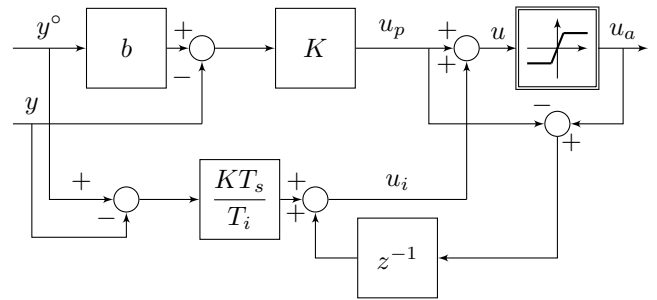


Fig. 1. Antiwindup by integral term recomputation.

Recalling (5), the behaviour of $PBC$ is given by

$$PBC(k) = by^\circ(k) + \frac{I(k) - u_c}{K} \quad (6)$$

with

$$I(k) = \text{sat}\left[ K\left( by^\circ(k-1) - y(k-1) \right) + I(k-1) + \right.$$
$$\left. + \frac{KT_s}{T_i}\left( y^\circ(k-1) - y(k-1) \right) \right] - K\left( by^\circ(k-1) - y(k-1) \right) \quad (7)$$

Another well known scheme is the actuation error feedback one of figure 2, based on the idea of feeding the difference of the unconstrained and constrained controls back to the integrator. A merit of this scheme is the possibility of reading $u_a$ back from the real actuator instead of simply having its saturation characteristic as minimal model, which for example naturally accounts for actuation dynamics or possible modifications of the saturation values. On the other hand, parameter $T_t$ is known as "difficult to tune" (Åström and Hägglund, 1995).

With the scheme of figure 2, $PBC$ is ruled by (6), but with

$$I(k) = I(k-1) + \frac{KT_s}{T_i}\left( y^\circ(k-1) - y(k-1) \right)$$
$$- \frac{T_s}{T_t}\left( u(k-1) - \text{sat}\left[ u(k-1) \right] \right) \quad (8)$$

The third scheme considered is that of figure 3, here called "feedback based integration" despite it has been given more than one name in the literature, to evidence that the existence of the integral term corresponds to the linear behaviour of the saturation block. When saturation is conversely hit, the positive-feedback loop opens and the
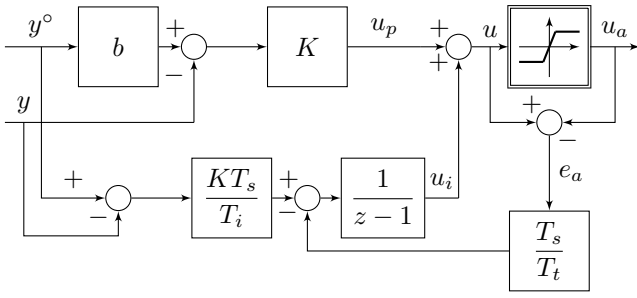
Fig. 2. Antiwindup by actuation error feedback.

integrator ceases to exist. This scheme shares with that of figure 1 the absence of antiwindup-specific parameters, but despite it is frequently used to explain antiwindup in the continuous time domain, it is less frequently used than the integral recomputation one.
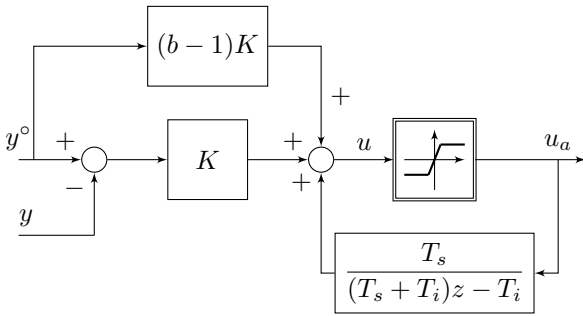


Fig. 3. Antiwindup by feedback-based integration.

In this last case, $PBC$ obeys again to (6), but, denoting with

$$u_a(k) = \text{sat}\left[K\left(y^\circ(k) - y(k)\right) + K(b-1)y^\circ(k) + x(k)\right]$$

$$x(k) = \frac{1}{T_s + T_i}\left(T_i x(k-1) + T_s u_a(k-1)\right)$$

(9)

The integral action $I(k)$ can be computed as

$$I(k) = u_a(k) - K\left(y^\circ(k) - y(k)\right) + K(b-1)y^\circ(k) \quad (10)$$

Comparing the aspect of (6) when one of the alternatives (7,8,10) – or any other – is plugged in, one immediately sees how many hard-to-predict results can be obtained. The reported analysis thus backs up the original idea of Leva and Bascetta (2006) to achieve antiwindup by "governing PBC", and provides a formal means to compare the so induced behaviour to that of classical policies.

## 4. THE PROPOSED ANTIWINDUP SCHEME

The mentioned preliminary proposal already stated, as a means to handle the problem, that the integral term should be recomputed in such a way that the proportional band centre $PBC$ follow a specified trajectory, in turn obtained by filtering the set point through a conveniently chosen transfer function.

Prior to the work presented here, the idea was simply to lowpass-filter the set point through a first-order transfer

function in the form $1/(1 + sT_{bc})$, where the single parameter $T_{bc}$ was chosen based on a convenient estimate of the closed-loop bandwidth, that most tuning methods are capable of providing. However, as experience gathered after that work has shown, simply low-passing the set point signal may in some cases be not enough. In suggestive but somehow rigorous terms, one could state the actual desire as follows:

> *The proportional band centre should move from its position before a "large" transient to that after the same transient (assuming for simplicity and quite generally that this means transferring the system from one steady-state condition to another) so that the control signal stay in saturation as long as possible, to minimise the transition duration, but exits saturation early enough to prevent the controlled variable from overshooting.*

The problem is then (heuristically) cast into that of finding a sensible way to give the statement above some quantitative meaning, and quite intuitively, having $PBC$ follow $y^\circ$ with a lagging behaviour is not always adequate. In some cases, for example one may want that signal itself to overshoot, in a view precisely to achieve the speed-smoothness trade-off just mentioned.

It is therefore better, in general, to obtain the desired $PBC$ (termed $PBC^\circ$ from now on with obvious meaning) by passing $y^\circ$ through a zero-pole transfer function, i.e.,

$$F(s) := \frac{PBC^\circ(s)}{Y^\circ(s)} = \frac{1 + s\tau_{bc}}{1 + sT_{bc}}, \quad (11)$$

where $\tau_{bc}$ and $T_{bc}$ are the two design parameters to be brought into play.

Carrying on, a non-ambiguous measure of the obtained results needs introducing, and some systematic procedure needs devising to select the parameters that will be brought into play. To this end, the proposal formulated here can be summarised as follows.

(1) Start from the PI tuning method of choice; in the example reported later on, the contextual IMC-PI with prescribed high-frequency control sensitivity Leva et al. (2010) is used. In principle any method is suitable, provided it is capable of providing a reasonable estimate of the obtained (nominal) closed-loop cutoff frequency $\omega_{cn}$.

(2) Decide a cost function to evaluate the quality of the "large" transient of interest; in the example, the ISE for a set point step variation is chosen, but apparently here too the idea is general.

(3) Identify a sufficiently wide class of processes to which the antiwindup-aware autotuner is to be targeted; in the example the benchmark set proposed in Åström and Hägglund (2000) is used, although only a part of the results are reported for space reasons.

(4) Perform an off-line simulation campaign by sweeping the process classes, their possible characteristic parameters and the tuning method's design variables, so as to determine for each of the so defined *scenarii* the best $(T_{bc}, \tau_{bc})$ couple for the chosen cost function.

(5) Seek convenient interpolating function to chose $T_{bc}$ and $\tau_{bc}$ based on data available at tuning time.

Once this is done, set up the antiwindup scheme so that the PI behave normally when not in saturation – of course – while in the opposite case the integral term is recomputed so that the proportional band centre be equal to the so obtained $PBC°$.

For such a scheme and tuning procedure to make sense, it is necessary that the last step of the list above can be carried out successfully, and producing meaningful results. To justify the conjecture that this may be taken as true in many relevant cases, one can observe that the necessity of exiting saturation sooner or later when moving toward the set point, depends to quite significant an extent on whether the linear PI tuning privileges proportional or integral action.

In the case of a "weak" integral action, in fact, when the controlled variable is approaching the set point and the proportional term is vanishing – think for simplicity of the one-degree-of-freedom case, the presence of a non unitary $b$ in (1) does not alter the conceptual *panorama* – the integral term will not be keen to cause controlled variable's overshoots, and as a limit condition $F(s) \approx 1$ is adequate; the opposite is true in the case of a "strong" integral action, where an overshooting $F$ (i.e., $\tau_{bc} > T_{bc}$) is generally advised.

A good – albeit heuristic – way to discriminate one case from the other, finally, is to observe the quantity $\omega_{cn}T_i$, associating the idea of "strong integral action" to $\omega_{cn}T_i < 1$. The following section shows how so simple an idea can yield quite interesting results.

## 5. APPLICATION TO A TUNING METHOD

The tuning method used here is the "contextual IMC-PI with prescribed high-frequency control sensitivity", applied based on a relay test yielding the process frequency response point with phase $-90°$ and causing the corresponding measured frequency to become $\omega_{cn}$. With that method, a number of simulation experiments were conducted on a set of processes included in the well known benchmark devised in Åström and Hägglund (2000). In particular, here we present the results for the benchmark process class

$$G(s) = \frac{1 - \alpha s}{(1 + s)^3}, \quad \alpha = 0.1, 0.2, 0.5, 1 \qquad (12)$$

Experiments on the benchmark process (12) show a significant regularity for the dependence of the parameters of $F$ producing the best set point step response ISE on the product $\omega_{cn}T_i$. Figure 4, shows the obtained optimal values for $T_{bc}$ and $\tau_{bc}$, while the dashed line shows the fit of the curves with exponential functions that have been obtained numerically.

In particular, the obtained functions are

$$T_{bc}(\omega_{cn}, T_i) = 8.667 e^{-1.188\omega_{cn}T_i}$$
$$\tau_{bc}(\omega_{cn}, T_i) = 49.97 e^{-2.207\omega_{cn}T_i} \qquad (13)$$

Observing the behaviour of said interpolants, reported in figure 5, one can notice that the proposed idea does actually make sense. For low values of $\omega_{cn}T_i$ the resulting $F(s)$ has a sometimes quite significantly overshooting step response, while for large values of $\omega_{cn}T_i$ its pole and
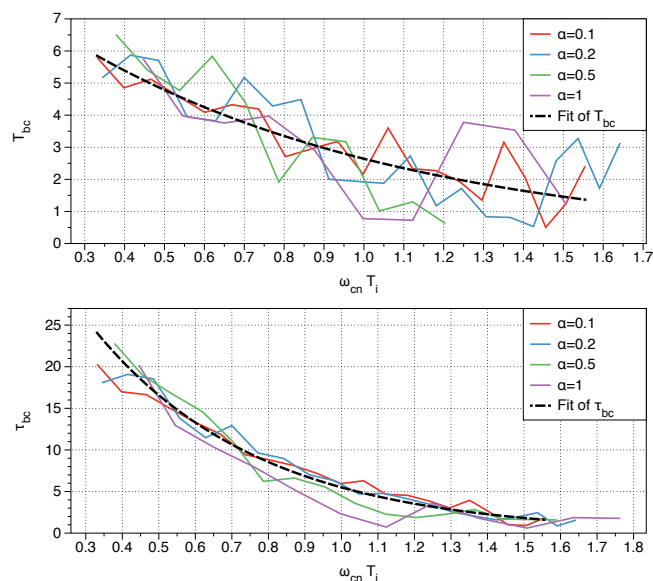


Fig. 4. Relationship between $\omega_{cn}T_i$ and the parameters of $F(s)$ for process class (12) in the reported application example.
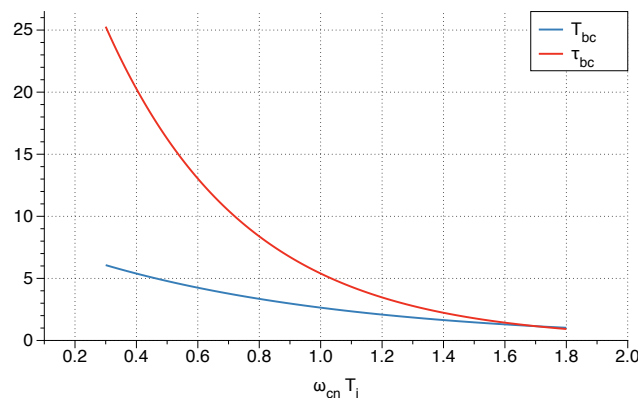


Fig. 5. Interpolated $(T_{bc}, \tau_{bc})$ as function of $\omega_{cn}T_i$ in the reported application example.

zero tend to essentially cancel one another, meaning that the proportional band centre should follow the set point algebraically. Note that in this case, which is definitely one of the least suited for PI control in the benchmark owing to the nonminimum-phase nature of the process, the proposed idea still provides meaningful results, while it is never advisable to use as $F(s)$ a low-pass filter. The proposal formulated here is therefore a generalisation of the previous one, as intended.

## 6. SIMULATION EXAMPLES

This section reports a couple of simulation examples. The purpose is to show how the use of the proposed antiwindup scheme and tuning procedure actually reflects into the obtained closed-loop transients in the presence of control saturation, since the interpolation results shown in a view to minimise the ISE may not be informative enough as far as the actual signals' behaviour is of concern.
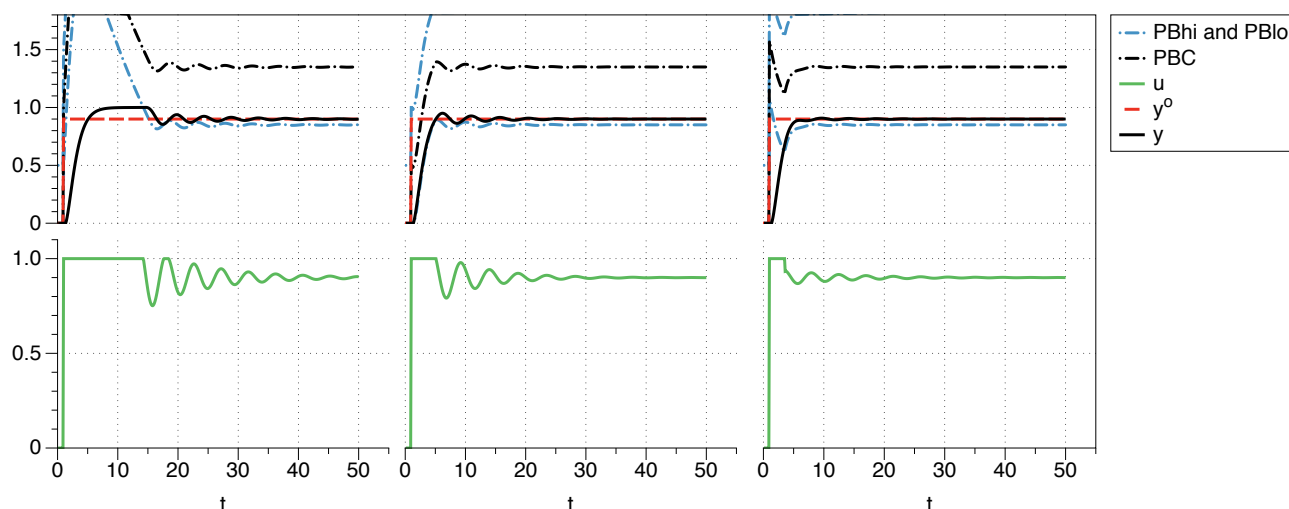
Fig. 6. Responses of the controlled variable (top) and the control signal (bottom) to a set point step causing saturation in example 1.

### 6.1 Example 1

The process considered in the first example (incidentally, not contained in the benchmark) is described by the transfer function

$$P(s) = \frac{1 - 0.2s}{(1 + s)^2} \qquad (14)$$

and the obtained simulation results are depicted in figure 6. The PI was tuned prescribing a unitary high-frequency control sensitivity magnitude.

Each column of figure 6 shows the response of the controlled variable and the proportional band limits (top row) and the response of the control signal (bottom row) to a set point step large enough to induce a relevant saturation. The top row also shows the behaviour of the proportional band centre (PBC) and of its high and low limits (PBhi and PBlo, respectively).

The leftmost column of figure 6 presents the transients obtained when the autotuner has no awareness of anti-windup, and the used scheme for that functionality is that of figure 2. The center column shows the same case, where however the antiwindup scheme is that of figure 3. The rightmost column, conversely, reports the outcome of the proposed scheme with the tuning method used in section 5.

The parameters obtained with the various schemes and methods are shown below (the antiwindup scheme of figure 3 has no additional parameters other than those of the PI):

| | |
|---|---|
| PI parameters | $K = 2$, $T_i = 0.89$ |
| AW scheme of figure 3 | $T_t = 20$ |
| Proposed scheme and method | $T_{bc} = 2.16$, $\tau_{bc} = 3.79$ |

As can be seen, the antiwindup-unaware scheme based on actuation error feedback experiences the well known difficulty of tuning $T_t$: here quite large a value was used to evidence possible problems: smaller values make the results resemble those of the central column, but not

those of the proposed technique. The method relying on the feedback-based integral term, conversely, is in general better than the former, but evidences some oscillations induced by an "incorrect" saturation exit. The proposed scheme, finally, does achieve the envisaged "correct exit", with evident advantages.

### 6.2 Example 2

The second example refers to a process in the benchmark, but not of the same class as (12), namely that with transfer function

$$P(s) = \frac{1}{(1 + s)(1 + 0.2s)(1 + 0.2^2 s)(1 + 0.2^3 s)} \qquad (15)$$

and figure 7, organised in the same way as figure 6, shows the results. Here too, the controller parameters obtained with the various schemes and methods are shown below:

| | |
|---|---|
| PI parameters | $K = 1$, $T_i = 0.19$ |
| AW scheme of figure 3 | $T_t = 0.1$ |
| Proposed scheme and method | $T_{bc} = 4.92$, $\tau_{bc} = 12.26$ |

In this case the antiwindup scheme of figure 2 is applied with a "small" $T_t$, to show that the remarks made above still apply. Needless to say, for $T_t$ it is not as straightforward to find interpolating functions based on the tuning and nominal parameters as for $(T_{bc}, \tau_{bc})$, however.

As can be seen, also in this case the proposed scheme and method achieve better results than the others.

## 7. CONCLUSIONS AND FUTURE WORK

The purpose of the presented research is to complement PI(D) autotuning with the capability of suitably accounting for control saturation, and the consequently necessary antiwindup. The scope was here limited to the PI controller to ease the presentation, but extensions are quite straightforward.
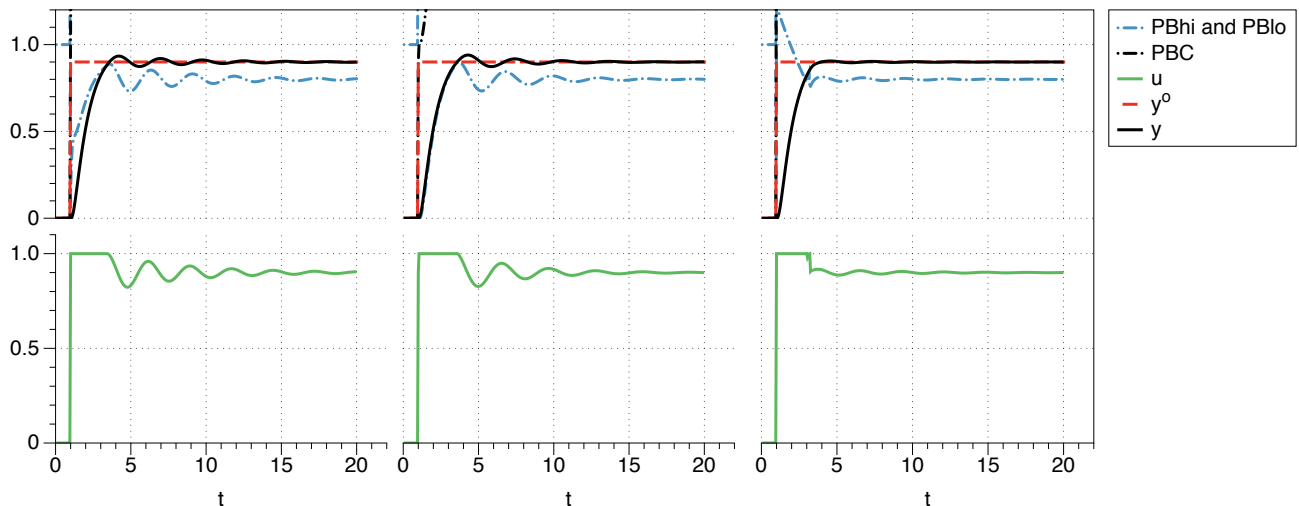
Fig. 7. Responses of the controlled variable (top) and the control signal (bottom) to a set point step causing saturation in example 2.

After showing – by means of a brief review of previous results – that the matter has an inherent relevance and difficulty, a general antiwindup scheme was presented by extending a previous one based on the idea of controlling the behaviour of the proportional band centre in the presence of control saturation. A tuning procedure was also devised for the presented scheme, illustrating that some previous assumptions on the type of filter to be used needed extending, and defining a systematic procedure by means of which any tuning method can be in principle endowed with the presented antiwindup awareness.

The devised solution quite intuitively contains some heuristics, but has some theoretically grounded justification and generality. The resulting scheme is simple and the same is true for the tuning procedure, that is based on a potentially long *off line* simulation phase to gather the necessary data, but has a minimum impact on the tuning procedure running on the plant. Some simulation results were presented – only a sample of the available ones for space reasons – that back up the proposal.

Future research will apply the presented idea to the PID controller, and also to more complex ones such as those emerging from the synthesis of control structures such as the cascade one, where the need for a systematic way to handle antiwindup is even stronger than in the single-loop case addressed herein.

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