IFAC Conference on Advances in PID Control
PID'12
Brescia (Italy), March 28-30, 2012 ThPS.11

A PI Controller with Dynamic Set-Point Weighting for Nonlinear Processes

R. J. Mantz

Laboratorio de Electrónica, Industrial Control e Instrumentación (LEICI).
FI. Universidad Nacional de La Plata UNLP. CC 91. La Plata (1900).
(Tel.:+54-221-4259306; e-mail mantz@ing.unlp.edu.ar)
Comisión de Investigaciones Científicas de la Provincia de Buenos Aires (CIC)

Abstract: The paper deals with the control of nonlinear processes with 2DOF-PI controllers. A new and practical tuning method, based on recent ideas of immersion and invariance, is proposed for the dynamic adjustment of the set-point weight. The main attractive feature of the proposal is the possibility of assigning a reduced-order linear dynamics for the tracking response beyond the nonlinear characteristics of the process and the amplitude of the set-point changes. From a practical point of view the adjustment is performed by a simple sliding mode regime that accepts a straightforward implementation.

Keywords: 2DOF-PI controller, nonlinear processes, invariance and immersion, sliding mode

1. INTRODUCTION

Even though new and more powerful tools have been developed, PI/PID control is still the most used control strategy in industrial applications. An attractive feature of these controllers is their relatively simple and intuitive design. Moreover, the fixed structure of PID controllers has made possible the development of ready-made hardware modules and software packages for a quick and easy implementation (Li et al., 2006). For these reasons, PI/PID controllers are commonly preferred even though more aggressive controllers can be obtained with more sophisticated techniques. Their popularity has encouraged the formulation of a large number of methods for tuning the controller gains. Over the years, several modifications to the standard algorithm have been proposed with the aim of improving the performance of PID controllers. One of these alternative structures is the so-called PID with set-point weighting or two degree of freedom PID (2DOF-PID). The advantage of the 2DOF-PID structure is that responses of the system to both disturbances and changes in the set-point can be adjusted separately. This characteristic results especially useful when the controller must accomplish several simultaneous specifications (Åström & Hagglund, 2005).

The effects of set-point weighting are rather intuitive in most of the simple processes. For this reason, empirical tuning methods are extensively used. Several methods with theoretical support have also been proposed for both SISO and MIMO processes (O'Dwyer, 2006), between them some approaches include dynamic weighting to improve the tracking behaviour (Dey, 2006; Hang & Cao, 1996), as well as to limit the coupling between variables in MIMO systems (Åström & Johansson, 2002; Bianchi et al., 2008). Up-to-date references of the most popular methods for tuning 2DOF-PID can be found in O'Dwyer (2006) and Mudi & Dey (2011).

This paper deals with the dynamic set-point weighting of 2DOF-PI controllers in nonlinear processes. It is worthy to point out that, in nonlinear processes, a suitable weight coefficient for a given set-point could be inappropriate for other reference values. Additionally, constant values for the weight coefficient in all nonlinear range of operation could drive to excessively conservative behaviours. Then, when the process is complex and/or highly nonlinear, variable weights should be considered. While in many works, this problem is addressed from a process of linearization (Bianchi et al., 2008), here the problem is focused using concepts from the theory of nonlinear systems. More precisely, with concepts of immersion of systems and invariance manifolds (Astolfi, A. & Ortega, R., 2003). In this framework a new methodology to the dynamic tuning of the set-point weight is presented. The proposal allows to assign a reduced-order linear dynamics for the tracking response independently of the set-point change. From a practical position the adjustment is performed by a simple sliding mode regime that accepts a straightforward implementation.

The paper is organized as follows. Section 2 briefly reviews the basic concepts of immersion of systems and invariant manifolds. In section 3, a new proposal for the dynamic adjustment of the set-point weight of 2DOF-PI controllers for nonlinear processes is presented. Then, the main features of the proposal are validated through an example. Finally, conclusions are summarized.

2. BASIC CONCEPTS OF IMMERSION AND INVARIANCE

Immersion and invariance concepts have always been linked to the control theory of nonlinear systems. The idea of immersion, is usually associated with the transformation of a system into another with specific properties. It has been used, for example, to the linearization of nonlinear systems by state feedback (Isidori, A., 1995), for robust regulation (Byrnes, C. et al, 1997), stabilization of infinite dimensional systems (Michel, A ., 2001), for interpretation of the Lyapunov method, etc.. In turn, the notion of invariant manifold has been extensively used to infer control actions in nonlinear systems.

Based on these two concepts, Astolfi et al. (2008) have formalized a theoretical framework called Immersion and Invariance (I&I) that reduces the design problem of nonlinear controllers to subproblems which might be substantially easier to solve and that do not require knowledge of Lyapunov functions. Although I&I ideas are applied to a wide range of problems, it is conceptually clearer to introduce them in the context of a stabilization issue. To this end, consider the nonlinear system,

$$\dot{x} = f(x) + g(x)u \tag{1}$$

with $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$, and where we are interested in getting a feedback control law $u = v(x)$ so that the controlled system presents an asymptotically stable equilibrium at the origin. Based on classical notions of system immersion and manifold invariance, the problem can be addressed by finding:

1- a target system with reduced-order dynamics

$$\dot{\zeta} = \alpha(\zeta) \qquad \zeta \in \mathbb{R}^{p<n}, \tag{2}$$

asymptotically stable at the origin;

2- a smooth mapping

$$x = \pi(\zeta), \tag{3}$$

3- a state feedback control $u = v(x)$ such that

$$\pi(\zeta(0)) = x(0), \tag{4}$$

$$\pi(0) = 0, \tag{5}$$

$$f(\pi(\zeta)) + g(\pi(\zeta))v(\pi(\zeta)) = \frac{\partial \pi}{\partial \zeta}\alpha(\zeta). \tag{6}$$

If the previous problem can be solved, any state trajectory x of the closed loop system can be seen as a mapping π of a trajectory ζ of the target system. As this target system is asymptotically stable at the equilibrium, $x(t)$ converges to the origin. From a geometric point of view all closed loop trajectories $x(t)$ live in the invariant manifold

$$M = \left\{ x \in \mathbb{R}^n \,\middle|\, x = \pi(\zeta),\ \zeta \in \mathbb{R}^{p<n} \right\} \tag{7}$$

with internal dynamics $\dot{\zeta} = \alpha(\zeta)$.

Although this formulation is theoretically correct, it is not always practical since both the mapping $x = \pi(\zeta)$ and the control $u = v(x)$ depend on the initial conditions, which complicates the calculus (indeed, in many applications, it could be impossible to be solved). From a practical standpoint, these limitations can be overcome by I&I ideas determining a solution for (5) and (6) (i.e. without requiring (4)), and modifying the control action $u = v(x)$ such that M is attractive, i.e., for any initial condition, the system trajectories $x(t)$ of the closed loop system

$$\dot{x} = f(x) + g(x)v(x) \tag{8}$$

converge to the manifold M. The attractiveness of M is defined in terms of a distance function ξ_1 whose absolute value

$$|\xi_1| = dist(x, M) \tag{9}$$

must be reduced to zero. This signal can be defined in different ways, which gives an additional degree of freedom to design.

3. DYNAMIC SET-POINT WEIGHTING BASED ON CONCEPTS OF I&I AND SM

Consider the nonlinear process

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{9}$$

where the state variables are measured online or can be estimated by a state observer, and the 2DOF-PI controller

$$u = k_p \left[(r' - y) + \frac{k_i}{k_p} \int (r - y) dt \right] \tag{10}$$

where $r' = r.b(t)$ is the weighted set-point for the proportional control action, and k_p and k_i are the proportional and integral gains, which have been tuned for the proper rejection of perturbations in the vicinity of $y=r$.

We are interested in adjusting the weight $b(t)$ in such a way that the tracking closed-loop response has a reduced-order linear dynamics. As it will be shown, both the order of the dominant tracking dynamics and the corresponding eigenvalues can be chosen without major difficulty from the basic I&I ideas. However, for ease of presentation, it is first introduced the case in which a first-order tracking dynamics is specified.

Then, according to what was previously discussed in section 2, we can solve the problem finding a 'control action' $b(t)$ to achieve asymptotic immersion of the closed-loop tracking dynamics in a subspace of $\mathbb{R}$,

$$M=\left\{x\in\mathbb{R}^n \;\middle|\; x=\pi(\zeta),\ \zeta\in\mathbb{R}\right\} \tag{11}$$

being $\zeta(t)=y(t)$, and where the invariant manifold is parameterized by the solutions of

$$\dot{y}=-\lambda y+\lambda r \qquad \lambda>0\,. \tag{12}$$

A distance between the actual state trajectory $x(t)$ and M can be defined as

$$\left|\xi_1(x)\right|=dist(x,M)=\left|L_{f+gu}h(x)+\lambda h(x)-\lambda r\right|\cdot \tag{13}$$

Depending on the process nonlinear characteristics, model uncertainties and disturbances, the determination of the law of adjustment $b(t)=v(x)$ may present some difficulties. In this paper we propose to force the attractiveness of M via a simple sliding mode (SM) regime which adjusts $b(t)$. Fig. 1 illustrates the scheme of the 2DOF-PI control with a detail of the proposed tuning circuit for the weight $b(t)$.

Since the SM is implemented at the level of the set-point, chattering problems are completely negligible. On the other hand, the SM is easy to implement and gives greater robustness than other methods.

The resulting discontinuous signal $w(t)$ is employed to shape the weighted reference $r.b(t)$ through a first-order low-pass filter $F(s)$:

$$\begin{aligned}\dot{x}_f&=-\lambda_f x_f+\lambda_f(w+r)\\ r'&=x_f\end{aligned} \tag{14}$$

where $-\lambda_f$ is the filter eigenvalue. The choice of this eigenvalue is not critical since its effect is cancelled by the SM regime. Naturally, this eigenvalue must be chosen for the filter bandwidth to be much faster than the target dynamic system.

To provide the present proposal with theoretical support it is useful to reformulate the system model in the normal form considering the variable $\xi_1(x(t))$ as the first state, which will be zeroed by the action of the discontinuous signal $w(t)$. Then, if the relative degree of $\xi_1(x(t))$ with respect to $w(t)$ is ρ, we proceed to model the complete system ($n+2$ states: n states of the open-loop system, one of the PI controller and the remaining state due to the filter F) from the following state variables:

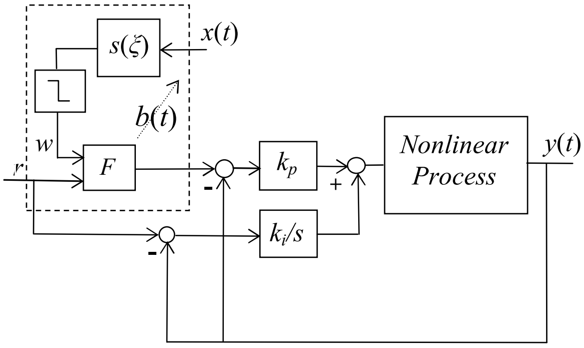

Fig.1: scheme of the 2DOF-PI control with the proposed tuning circuit for the weight $b(t)$.

$$x_e=\begin{bmatrix}\xi_{\rho x1}\\ \eta_{(n+2-\rho)x1}\end{bmatrix}, \tag{15}$$

with

$$\xi_{\rho x1}=\begin{bmatrix}\xi_1(x)\\ L_f\xi_1(x)\\ L_f^2\xi_1(x)\\ \vdots\\ L_f^{\rho-1}\xi_1(x)\end{bmatrix} \qquad \eta=\begin{bmatrix}\eta_1=y\\ \eta_2\\ \vdots\\ \eta_{n+2-\rho}\end{bmatrix}. \tag{16}$$

That is, the elements of the first subset of ρ states are the signal $\xi_1(x)$ and its ρ-1 successive derivatives. The surplus states η can be freely chosen. For simplicity, the first state (η_1) of the set η is chosen as the variable $y(t)$. Then, the new closed loop model results

$$\begin{bmatrix}\dot{\xi}_1=\xi_2\\ \dot{\xi}_2=\xi_3\\ \vdots\\ \dot{\xi}_{\rho-1}=\xi_\rho\\ \dot{\xi}_\rho=L_{f+gu}\xi_{\rho-1}(x)\big|_{x=\Phi(\xi)}=a(\xi)+b(\xi)w\\ \dot{\eta}_1=q_1(\xi,\eta)+p_1(\xi,\eta)w=\dot{y}=\xi_1(x)-\lambda y+\lambda r\\ \dot{\eta}_2=q_2(\xi,\eta)+p_2(\xi,\eta)w\\ \vdots\\ \dot{\eta}_{n+2-\rho}=q_{n+2-\rho}(\xi,\eta)+p_{n+2-\rho}(\xi,\eta)w\end{bmatrix}. \tag{17}$$

being our objective to force the fast convergence of the output $\xi_1(t)$ to zero. To this end, i.e. to warrantee the attractiveness of M and, as a consequence, the practical asymptotic immersion of the tracking dynamic response in the target system, it is proposed a sliding mode regime on the control surface

$$s(\xi)=k_1\xi_1+k_2\xi_2+k_3\xi_3+.....+k_\rho\xi_\rho=0 \tag{18}$$

where the coefficients k_i define the convergence dynamics of ξ_1 to zero (Utkin et al., 1999; Garelli et al., 2011).

The selected surface $s(\xi)$ has relative degree $\rho=1$ with respect to the signal $w(t)$ fulfilling the condition of existence of the sliding motion. Then it is always possible to choose w^+ and w^- high enough for guaranteeing the SM. These extreme values can be calculated from necessary and sufficient condition for the sliding mode regime:

$$w^- \le w_{eq}(t) \le w^+ \tag{19}$$

where $w_{eq}(t)$ is the equivalent control (i.e. the fictitious continuous signal that produce the same effect than the actual discontinuous signal $w(t)$ on the sliding surface) calculated from the invariance SM conditions

$$s(\xi)=0 \quad \dot{s}(\xi)=0\,. \tag{20}$$

However, as the discontinuous control action $w(t)$ is used here to shape the changes of the reference, then it is reasonable to choose values of the order of the set-point changes for $w^{\pm}$ (taking into account bounds on possible disturbances). It should be also pointed out that the selection of $w^{\pm}$ can be made in a conservative manner because the SM is restricted to the low-power side of the system.

Once achieved the sliding regime results,

$$\xi_\rho = -\frac{1}{k_\rho}(k_1\xi_1 + k_2\xi_2 + k_3\xi_3 + \ldots\ldots + k_{\rho-1}\xi_{\rho-1}) \tag{21}$$

then substituting (21) in (17), the reduced-order state model (order $n+1$) is obtained

$$\begin{bmatrix} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \xi_3 \\ \vdots \\ \dot{\xi}_{\rho-1} = \xi_\rho = -\frac{1}{k_\rho}(k_1\xi_1 + k_2\xi_2 + k_3\xi_3 + \ldots\ldots + k_{\rho-1}\xi_{\rho-1}) \\ \dot{\eta}_1 = \dot{y} = \underbrace{L_{f+gu}h(x) + \lambda y - \lambda r}_{\xi_1(x)} - \lambda y + \lambda r \\ \dot{\eta}_2 = q_2(\xi,\eta) + p_2(\xi,\eta)w \\ \vdots \\ \dot{\eta}_{n+2-\rho} = q_{n+2-\rho}(\xi,\eta) + p_{n+2-\rho}(\xi,\eta)w \end{bmatrix}. \tag{22}$$

where the extinction speed of the new controlled output ξ_1 is defined by a linear dynamics whose eigenvalues are assigned by the proper selection of the k_i coefficients. Then, if these coefficients are chosen so that ξ states present a fast dynamics compared to the ones corresponding to the target system (defined by the eigenvalue λ), the dynamic equation of state η_1, approaches to:

$$\begin{aligned} \dot{\eta}_1 &= \dot{y} = \underbrace{L_{f+gu}h(x) + \lambda y - \lambda r}_{\xi_1(x)\to 0} - \lambda y + \lambda r \\ \dot{\eta}_1 &= \dot{y} \to -\lambda y + \lambda r \end{aligned} \tag{23}$$

that is, the target dynamics. Obviously, all the remaining states dynamics η_i must meet the stability requirements (i.e. minimum phase zero dynamics).

Observation 1. Note that the tracking objective is achieved in an indirect form conditioning the signal $r.b(t)$ for zeroing the distance ξ_1 between the trajectory $x(t)$ and the manifold M. Then, the proposed adjusting action could be interpreted as a special case of SM reference conditioning (Mantz and De Battista, 2002; Garelli et al., 2011). Note also that (23) is one of the zero dynamics.

Observation 2. An analytic expression of $b(t)$ can be obtained from

$$b(t) = \frac{r + w_{eq}\left(\xi(t)\right)}{r} \tag{24}$$

where $w_{eq}\left(\xi(t)\right)$ is the equivalent control. However (24) has only theoretical value, because it is not required for the implementation of the present proposal. The adjustment of $b(t)$ is forced by the SM of the basic circuit of Fig. 1 without need of calculating it.

Observation 3. Previously, it was considered the asymptotic immersion of the tracking response in a target manifold parameterized by the trajectories of a first order dynamics. The extension of the previous ideas to the general case with higher order dynamics is straightforward. Indeed, it is enough:

1) to choose a new manifold parameterized by the solutions of the target tracking dynamics

$$\overset{m}{y} = -(a_1 y + \cdots + a_{m-1}\overset{m-2}{y} + a_m \overset{m-1}{y}) + a_1 r \tag{25}$$

with $m<n+2-\rho$, and a signal ξ_1 defining a distance between the actual state trajectory $x(t)$ and M for example

$$\left|\xi_1(x)\right| = dist(x,M) = \left|L^m_{f+gu}h(x) + a_1 y + \cdots + a_m \overset{m-1}{y} - a_1 r\right|; \tag{26}$$

2) to modify (22) in such a way that the first m states of η are the variable $y(t)$ and its successive $m-1$ derivatives, i.e.

$$\begin{aligned} \eta_1 &= y \\ \eta_2 &= \dot{y} \\ &\vdots \\ \eta_m &= \overset{m}{y} = \underbrace{L^m_{f+gu}h(x) + a_1 y + \cdots + a_m \overset{m-1}{y} - a_1 r}_{\xi_1(x)\to 0} - (a_1 y + \cdots + a_m \overset{m-1}{y}) + a_1 r \end{aligned}$$

3) and to force the trajectories to converge asymptotically to M by a SM regime as (18).

Then, the zero dynamics verifies the target tracking dynamics (25).

$$\begin{aligned}
\dot\eta_1 &= \eta_2 \\
\dot\eta_2 &= \eta_3 \\
&\vdots \\
\dot\eta_m &= a_1\eta_1 + \cdots + a_m\eta_m
\end{aligned} \tag{28}$$

4. EXAMPLE

Consider the nonlinear system

$$\dot x = \begin{bmatrix} 0{,}1\; x_2^2\; sig(x_2) - x_1 \\ -x_2 + u \end{bmatrix}, \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{29}$$

and a PI controller with variable set-point weighting (10), where the gains k_i=0,657 and k_p=8,9 have been tuned from the linearized model (in the proximity of the steady state point of regulation) for the proper rejection of perturbations. In this case, the disturbance rejection has a characteristic close to that known as "quarter decay" which is considered adequate for many chemical processes (it is important to keep in mind that the present proposal is independent of the PI gains tuning). Obviously, due to the nonlinear characteristics of the system, this type of response is not obtained in other operation points without the suitable readjustment of the controller gains. Fig. 2 shows, in thin lines, the dynamic behaviour of the closed loop system considering constant values of b (1; .9; .8; .7; .6; .5 and .1). It is observed a clear trade off between overshoot and large settling time. These poor nonlinear tracking responses contrast with the proper disturbance rejection. This result is not surprising since the PI controller was tuned based on a linearized model that is only valid on the surrounding of the steady state point.

The dynamics of the closed loop system, including the filter F, is given by the next differential equations

$$\begin{bmatrix} \dot x_1 \\ \dot x_2 \\ \dot x_i \\ \dot x_F \end{bmatrix} = \begin{bmatrix} 0{,}1\; x_2^2\; sig(x_2) - x_1 \\ -x_2 + k_p\left[(x_F - x_1) + \dfrac{k_i}{k_p} x_i \right] \\ r - x_1 \\ \lambda_F (x_F + w + r) \end{bmatrix}. \tag{30}$$

We propose as target tracking dynamics

$$\dot\zeta = -\lambda\zeta + \lambda r = -.2\zeta + .2r \tag{31}$$

with $\zeta = y$ and a signal

$$\xi_1(x) = 0{,}1\; x_2^2\; sig(x_2) - x_1 + \lambda y - \lambda r \tag{32}$$

as a measure between the actual trajectories and the manifold defined from the solutions of the target dynamics (31). This signal ξ_1 has relative degree $\rho = 2$ with respect to the discontinuous signal $w(t)$. Then its absolute value can be reduced in a controlled way forcing a sliding mode on the surface

$$s(\xi) = k_1\xi_1(x) + k_2\xi_2(x) = 0 \tag{33}$$

with $k_1/k_2 >> \lambda$ to guarantee a convergence speed faster than corresponding to the selected for the tracking response. In the present case we choose k_1/k_2=2 (i.e. the time constant of the extinction speed of ξ_1 ten times less than the corresponding to the target dynamics λ^{-1} = 5sec).

Fig. 2 (thick line) and 3a show the response of the non linear process with the 2DOF-PI controller with the proposed dynamic weighting. From a practical point of view, this tracking response presents the target dynamics (31) with a much better performance than any of the corresponding to constant weights. This fact can also be verified from Fig. 3c, which shows how ξ_1 converges to zero with the dynamic assigned through the choice of k_1/k_2 (time constant 0.5sec). As a consequence, it results

$$\dot y = \underbrace{0{,}1\; x_2^2\; sig(x_2) - x_1 + \lambda y - \lambda r}_{\xi_1(x)\to 0} - \lambda y + \lambda r \quad \Rightarrow \quad -\lambda y + \lambda r. \tag{34}$$

Part b of the Fig. 3 shows the weight coefficient $b(t)$ that guarantees the desired behaviour. In terms of I&I, this is the control that assures the trajectories convergence to M where the dynamics (31) is verified.

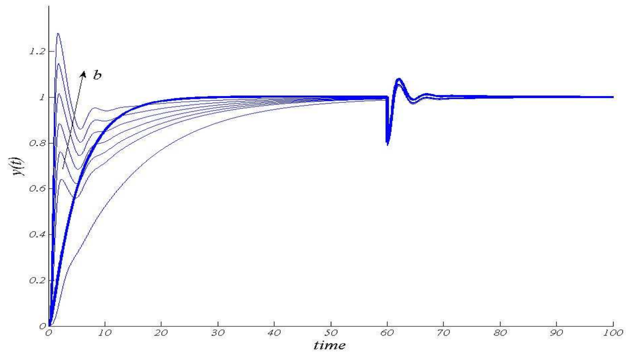

Fig. 2: Process output. Thin lines: 2DOF-PI with constant b values .1,.5, .6, .7, .8, .9 and 1. Thick line: 2DOF-PI with $b(t)$ based on I&I ideas.

Fig. 4 shows the trajectory corresponding to the thick curve in Fig. 2 in the $(y, \dot y)$ plane. Three sections can be distinguished. The first section A-B corresponds to the fast dynamics of ξ_1. Subsequently, the B-C straight with slope -0.2 is consistent with the dynamic assigned for the tracking response (first-order linear dynamics), and the third part of the trajectory shows the underdamped dynamics corresponding to the disturbance rejection (third-order nonlinear dynamics) which is exclusively defined by the PI gains.

Note that the sliding surface $s(\xi) = k_1\xi_1(x) + k_2\xi_2(x) = 0$ is the same regardless of the tuning of the PI gains.

Therefore, if the nonlinear system operates with different set-points, which could require different k_i and k_p settings to the proper disturbances rejection, no changes would be required for the implementation of the proposed tracking method.

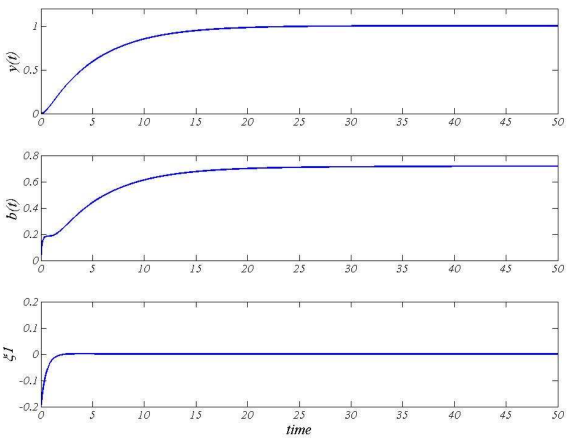

Fig. 3: a) set-point tracking response, b) set-point weight $b(t)$, c) distance ξ_1 between $x(t)$ trajectories and M.

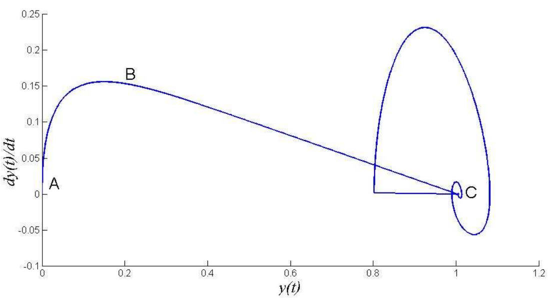

Fig. 4: Trajectory in the $(y, \dot{y})$ plane in correspondence with thick line in Fig. 2.

5. CONCLUSION

PI with set-point weighting or two degree of freedom PI controllers (2DOF-PI) are widely used in industrial environments. Although lots of heuristic rules and analytic methods have been proposed for the tuning of these controllers, little has been written that explicitly deals with nonlinear processes. In this case, suitable weight coefficients for the tracking of a given set-point can be inappropriate for other set-points values. This fact encourages the formulation of 2DOF-PI with variable weights. In this paper, a new method for dynamic tuning of the weight $b(t)$ is proposed for the tracking control of nonlinear processes. The technique proposed is supported by recent concepts of system immersion and invariant manifolds, which have shown to be useful for reducing the complexity of various problems of analysis and design in nonlinear systems. The distinctive characteristic of the tuning method proposed in this paper is the possibility of assigning a reduced-order linear dynamics for the tracking response. From a practical position, the proposal is easily implemented by a simple circuit operating in a sliding mode regime, making it suitable for industrial application. Other attractive characteristic of the proposal is its independence with respect to the set-point amplitude.

Acknowledgements. This work was supported by ANPCyT, CICpBA, and UNLP.

REFERENCES

Astolfi A. and Ortega R. (2003). Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems. *IEEE Trans. on Automatic Control.* 4, 590–606.

Astolfi, A., Karagiannis, D. and Ortega, R. (2008) *Nonlinear and Adaptive Control with Applications.* Springer-Verlag. London. UK.

Åström, K., Johansson, K. y Wang, Q. (2002). Design of decoupled PID controllers for two-by-two systems. *IEE Proc. on Control Theory and Applications*. 49, 74–81.

Åström, K. and Hagglund, T. (2006). *Advanced PID control.* ISA. Research Triangle Park, USA.

Bianchi, F., Mantz, R. and Christiansen, C. (2008). Multivariable PID control with set-point weighting via BMI optimisation. *Automatica*. 44, 472– 478.

Byrnes, C., Delli Priscoli, F. y Isidori, A. (1997). Output Regulation of Uncertain Nonlinear Systems. Birkhauser, Boston.

Dey, C., Mudi, R. and Lee, T. (2006). A PID controller with dynamic set-point weighting. *In Proceedings of the IEEE Int. Conf. on Industrial Technology*, 1071–1076.

Garelli F., Mantz R.J., De Battista H. (2011). *Advanced Control for Constrained Processes and Systems. A unified and practical approach.* Chapter 3. IET, Control Engineering Series. London, UK.

Hang, C. and Cao, L. (1996). Improvement of transient response by means of variable set point weighting. *IEEE Trans. on Industrial Electronics.* 4, 477–484.

Isidori, A. (1995). *Nonlinear Control Systems.* Springer-Verlag, Berlin.

Mantz R.J. and De Battista H. (2002). Sliding mode compensation for windup and direction of control problems in two-input-two-output PI controllers. *Ind. and Engineering Chemistry Research.* 41, 3179-3185.

Mudi R.K. and Dey Ch.(2011). Performance Improvement of PI Controllers through Dynamic Set-point Weighting. *ISA Transactions.* 50, 220-230.

Michel A., Wang K., and Hu, B. (2001). *Qualitative Theory of Dynamical Systems: The Role of Stability Preserving Mappings.* Marcel Dekker, NY.

O'Dwyer, A. (2006). *Handbook of PI and PID controller tuning rules.* Imperial College Press. London, UK.

Michel A., Wang K., and Hu, B. (2001). *Qualitative Theory of Dynamical Systems: The Role of Stability Preserving Mappings.* Marcel Dekker, NY.

Utkin V., Guldner J. and Shi. J (1999). *Sliding Mode Control in Electromechanical Systems.* Taylor and Francis. London.