Practical Control of Surge Tanks Suffering from Frequent Inlet Flow Upsets

Peter Rosander, * **Alf J. Isaksson,** * **Johan Löfberg,** * **Krister Forsman** **

* *Department of Electrical Engineering, Linköping University, SE-581-83 Sweden (email: {rosander,alf,johanl}@isy.liu.se)*
** *Perstorp AB, (email: krister.forsman@perstorp.com)*

Abstract: In the presence of frequent inlet flow upsets, tuning of averaging level controllers is typically quite complicated since not only the size of the individual steps but also the time in between the subsequent steps need to considered. One structured way to achieve optimal filtering for such a case is to use Robust Model Predictive Control. The robust MPC controller is, however, quite computationally demanding and not easy to implement. In this paper two linear controllers, which mimic the behavior of the robust MPC, are proposed. Tuning guidelines to avoid violation of the tank level constraints as well as to achieve optimal filtering are presented.

Keywords: Averaging level control, PI controllers, Surge tanks

1. INTRODUCTION

The objective of averaging level controllers is to keep the outlet flow of the surge tank smooth by using the tank capacity to average out the effect of inlet flow upsets. Following an inlet flow upset, averaging is typically obtained by initially allowing the tank level to deviate from its nominal set-point while slowly adapting the outlet to the new throughput and ultimately bringing back the tank level to the set-point.

Early approaches to averaging control are the PL controller in (Luyben and Buckley, 1977) and the nonlinear approaches by Shunta and Fehervari (1976). The (de-)tuning of PI controllers has been addressed in (Cheung and Luyben, 1979), (Kelly, 1998) and (Shin et al., 2008) amongst others. A control structure permitting separate responses to set-point and load disturbances was proposed by Wu et al. (2001). The optimal non-linear continuous- and discrete-time controllers were derived in (McDonald et al., 1986) and (Campo and Morari, 1989) respectively.

We have an application in one of Perstorp's factories where the previously mentioned approaches struggle to obtain satisfactory flow filtering. The reason for this is that traditionally inlet flow upsets have been treated as separate occurrences, but, as seen in Figure 1, this clearly does not hold in this case. In (Rosander et al., 2011) optimal filtering in the presence of frequent inlet flow upsets was obtained using robust Model Predictive Control (MPC). The manner in which the robust MPC controller achieves flow filtering differs from previous approaches in that it does not return the tank level to a fixed set-point following an inlet flow step. Instead the steady state tank level depends on the current inlet flow level. The effect of this behavior was analyzed in (Rosander et al., 2012) where it was shown that the robust controller outperforms the optimal level controller by (McDonald et al., 1986) for frequent upsets while achieving comparable performance even for infrequent upsets.

In this paper, two linear controllers that mimic the behavior of the robust MPC controller are discussed. The first one is a special version of the proportional controller while the other one is a PI whose set-point is not fixed but given as an affine map of the inlet flow.

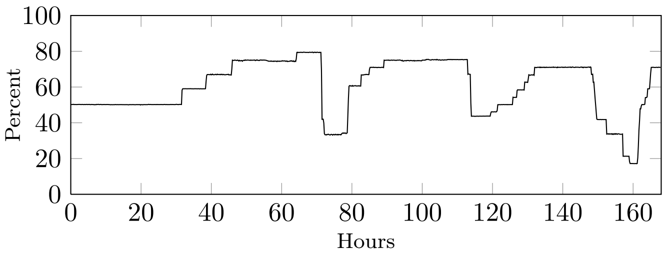

Fig. 1. Inlet flow data from Perstorp AB last week of March 2011.

2. SYSTEM DESCRIPTION

We consider a cylindrical buffer tank with level y and liquid constant density inlet and outlet flows q_{in} and q_{out} respectively. For the sake of notational brevity it is assumed that the outlet flow can be directly manipulated, $u = q_{out}$. Using mass balance we obtain the continuous time model

$$\dot{y}(t) = k_v \left(q_{in}(t) - u(t)\right), \tag{1}$$

where k_v is inversely proportional to the cross-sectional area. Furthermore we assume that all quantities are given in percent and that the outlet and inlet flow have equal range

$$y_{min} \leq y \leq y_{max}, \tag{2a}$$
$$q_{min} \leq q_{in},\, u \leq q_{max}. \tag{2b}$$

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Here $q_{min} = y_{min} = 0\%$ and $q_{max} = y_{max} = 100\%$ if no extra safety limitations are put on the tank level. The assumption (2b) guarantees that we do not risk violating the tank level constraints due to lack of outlet flow capacity. Consequently we could allow the outlet flow to have greater range than the inlet flow, but for the purpose of this paper that would mainly complicate the notation. For the same reason we assume that the inlet flow is directly measurable. In any case, the linear dynamics of the system allow for a straightforward estimation of the inlet flow using the Kalman filter as shown in (Khanbaghi et al., 2001).

Good averaging level control is characterized by a smooth outlet flow, i.e., that $\dot{u}$ is "small". Typically this is quantified by either the Euclidean norm

$$||\dot{u}||_2^2 = \int_0^\infty \dot{u}(t)^2 \, dt, \tag{3}$$

or the maximum change of the outlet flow

$$||\dot{u}||_\infty = \max_t |\dot{u}(t)|. \tag{4}$$

Which criterion that best captures "good flow filtering" depends of course on the nature of the downstream processes. In this paper we will display both to give as complete a picture as possible.

3. ROBUST AVERAGING LEVEL CONTROL

We will briefly describe the robust MPC approach to averaging level control taken in (Rosander et al., 2011).

The key idea behind the controller is that we want a controller that does not only focus on filtering the current upset but also is prepared to handle future ones. This is obtained by formulating the problem in the robust MPC framework. By treating future inlet flows as bounded and unknown, a controller that is robust towards future upsets and achieves optimal filtering is obtained. Mathematically this corresponds to, in every sample instant, solving the optimization problem

$$\min_{u(0:N)} \max_{q_{in}(1:N)} ||u(k) - u(k-1)||_\infty \tag{5a}$$

subject to

$$y(k+1) = y(k) + T_s k_v \left(q_{in}(k) - u(k)\right) \tag{5b}$$

$$y(1:N) \in [y_{min},\, y_{max}] \;\forall q_{in}(1:N) \in [q_{min},\, q_{max}] \tag{5c}$$

$$q_{in}(1:N),\, u(1:N) \in [q_{min},\, q_{max}] \tag{5d}$$

$$y(0),\, q_{in}(0),\, u(-1) \text{ known} \tag{5e}$$

where $y(0)$ and $q_{in}(0)$ are the current tank level and inlet flow respectively and $u(-1)$ is the actuated control signal from the previous sampling instant. The discretized dynamics and system constraints are represented by (5b-5d). Robustness towards future inlet flow upsets is obtained by taking the worst case situation (maximizing over q_{in}). Note that the only requirement put on the tank level is that it must stay within bounds (5c). The controller is thus allowed to "decide" the optimal tank level.

The behavior of the robust MPC controller differs from most previous approaches (apart from the P controller) as it does not return the tank level to a fixed set-point, but instead the steady state tank level depends on the actual level of the inlet flow. This is illustrated in Figure 2 for a sample feed where both the robust MPC controller as well as a standard PI controller is used to control the tank. The nature of the mapping from inlet flow to steady state tank level depends on the prediction horizon, but for prediction horizons that capture the transient response of the system, the mapping stays almost constant, see Figure 3. In Figure 3 also the approximation, the affine mapping, used in this paper is shown.

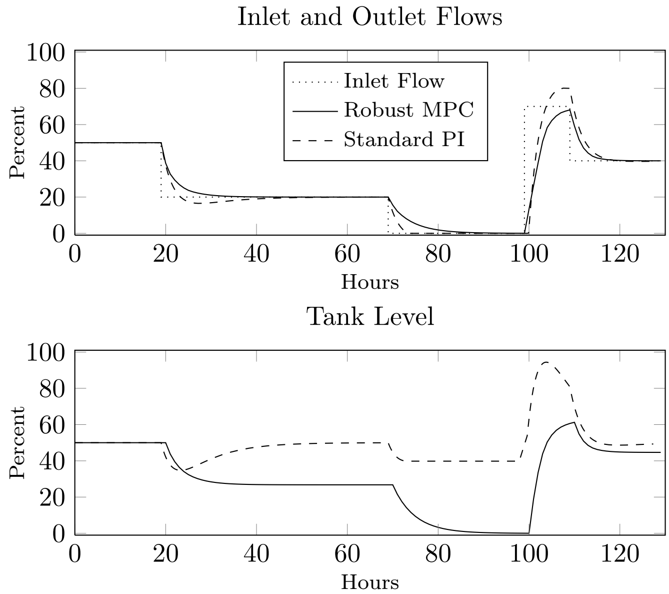

Fig. 2. Simulation of closed loop robust MPC with $N = 20$, shown in solid black. For comparison purposes a standard PI-controller with $K_c = -1.5$ and $T_I = 10$ h^{-1} is also shown dashed.

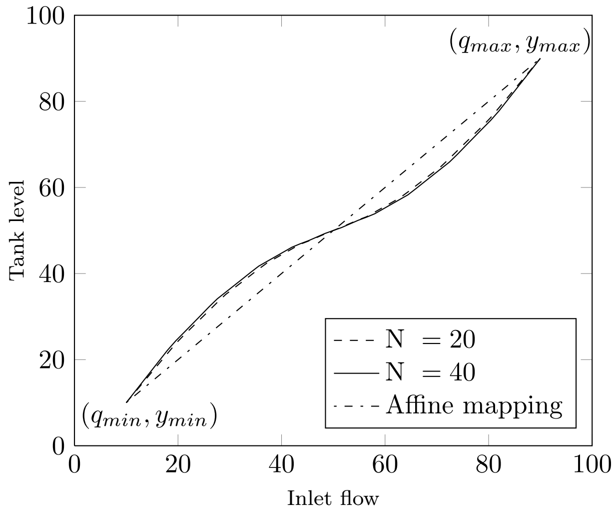

Fig. 3. The steady state tank level as a function of the inlet flow for different prediction horizons and $k_v = \frac{1}{3}$. Shown is also the affine mapping that will be used in this paper.

3.1 *Robust Optimal Averaging Level Control*

The robust MPC controller does not permit an analytical analysis, and to still obtain an explicit estimate of its performance we do the following. The filtering problem is formulated as an optimal control problem for which robustness towards future inlet flow upsets is obtained by letting the steady state tank level be given by the affine map shown in Figure 3. A robustly optimal control law can thus be obtained by solving the optimization problem

$$\min \|\dot{u}\|_\infty \text{ or } \|\dot{u}\|_2^2 \tag{6a}$$
subject to
$$\dot{y} = k_v(q_{in}(t) - u(t)) \tag{6b}$$
$$q_{in}(t) = \begin{cases} q_0, & t < 0 \\ q_0 + A, & t \geq 0 \end{cases} \tag{6c}$$
$$y(0) = K_{SP} q_0 + b_{SP} \tag{6d}$$
$$y(T) = K_{SP}(q_0 + A) + b_{SP},\ T \text{ free} \tag{6e}$$

where A is the size of the inlet flow upset. The constraints (6d) and (6e) ensure that the resulting controller will position the tank level analogous to the robust MPC controller. Using Figure 3 the parameters of this mapping can be derived to be

$$K_{SP} = \frac{y_{max} - y_{min}}{q_{max} - q_{min}}, \tag{7a}$$
$$b_{SP} = \frac{q_{max} y_{min} - q_{min} y_{max}}{q_{max} - q_{min}}. \tag{7b}$$

In (6) it is assumed that the system is in steady state (constraint (6d)) when the upset occurs but that can be relaxed.

The solution of (6) results in a nonlinear feedback controller, see (Rosander, 2011) and (Rosander et al., 2012), that we will not concern ourselves with in this paper. It does, however, provide a lower bound on the achievable performance for any controller using the affine mapping (7)

$$J_{OPT,2} = \frac{4 k_v A^2}{9 K_{SP}}, \tag{8a}$$
$$J_{OPT,\infty} = \frac{k_v |A|}{2 K_{SP}}. \tag{8b}$$

The performance attained by the linear controllers will be compared to these values.

4. ROBUSTLY OPTIMAL P CONTROL

The obvious choice of a linear controller which gives a steady state offset is the P controller

$$u(t) = K_P(r - y(t)) + b_P, \tag{9}$$

where K_P is the gain, r the set-point and b_P a bias term. The set-point is actually also a bias term which can be seen accordingly

$$u(t) = K_P(r - y(t)) + b_P = -K_P y(t) + \underbrace{K_P r + b_P}_{\text{bias}}.$$

To mimic the affine mapping in Figure 3, maximum tank level, y_{max}, should be mapped to maximum outlet flow, q_{max}, and vice verse for the minimum values. Solving that system of equations yields the tuning

$$\begin{aligned} K_P &= -\frac{q_{max} - q_{min}}{y_{max} - y_{min}}, \\ \text{bias} &= \frac{q_{min} y_{max} - q_{max} y_{min}}{y_{max} - y_{min}}. \end{aligned} \tag{10}$$

This P controller tuning is admittedly not new but is also advocated by Taylor and la Grange (2002) among others. That the tuning does result in robustly optimal filtering has, however, not previously been presented.

The performance of the P controller, for an upset of size A, can be calculated by first noting that it gives the outlet flow

$$u(t) = A\left(1 - e^{K_P k_v t}\right) + \text{bias}. \tag{11}$$

Using this, straightforward, but tedious, calculations give that the performance evaluates to

$$\begin{aligned} J_{P,2} &= \|\dot{u}\|_2^2 = -\frac{A^2 k_v K_P}{2}, \\ J_{P,\infty} &= \|\dot{u}\|_\infty = -|A| k_v K_P, \end{aligned} \tag{12}$$

where the minus signs come from the fact that $K_P < 0$ to obtain stability. From (10) and (7) it follows that $K_{SP} = -\frac{1}{K_P}$ and we can thus conclude that in terms of the Euclidean norm the performance degrades by approximately 12% while the infinity norm actually is twice as bad.

Apart from bad flow smoothing in terms of the infinity norm criterion there is also an issue with how the controller is perceived by the operators. The term b_P is typically not available since it is used to handle bump less transfers between operating modes. To then obtain the bias as in (10) a rather unintuitive set-point must be used, and furthermore one that is never attained by the controller. The risk is that the controller is perceived by the operator as not functioning correctly and hence disconnected.

A PI controller, put in cascade mode, whose set-point is given by a calculation block resolves these two drawbacks of the P controller. The operators can easily see what set-point the system is trying to achieve and the set-point is also obtained. The PI controller also achieves better smoothing, especially in terms of the infinity norm.

5. PI WITH VARIABLE SET-POINT

One easy way to mimic the robust MPC with a PI controller is to let its set-point be given by the affine mapping

$$r(t) = K_{SP} q_{in}(t) + b_{SP}, \tag{13}$$

where K_{SP} and b_{SP} are given by (7a) and (7b) respectively. A block diagram of the proposed control structure is shown in Figure 4 where the block MAP is the affine map (13). In the subsequent analysis we assume that the effect b_{SP} has on u and y has reached its steady state value and therefore only focus on q_{in}.

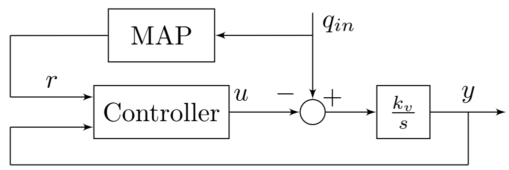

Fig. 4. Block diagram of the proposed PI controller.

5.1 Transfer functions

In addition to changing the set-point according to (13) a β-factor in the controller will be used

$$U(s) = K_c\left(\beta R(s) - Y(s) + \frac{1}{T_I s}(R(s) - Y(s))\right). \tag{14}$$

The resulting closed loop dynamics from Q_{in} can be derived to be

$$Y(s) = \frac{k_v(T_I s - K_c\beta K_{SP} T_I s - K_c K_{SP})}{T_I s^2 - k_v K_c T_I s - k_v K_c} Q_{in}(s), \quad (15a)$$

$$U(s) = \frac{K_c\big(\beta K_{SP} T_I s^2 + (K_{SP} - k_v T_I)s + 1\big)}{T_I s^2 - k_v K_c T_I s - k_v K_c} Q_{in}(s). \quad (15b)$$

6. TUNING OF PI

When adapting the level, additional requirements (apart from closed loop stability) have to be put on the controller: an inlet flow step response cannot yield an under or overshoot. Why this is the case can be understood by viewing an example: Assume that the system is in steady state at $t = t'$. Then q_{in} performs a step from q_{min} to q_{max} and eventually let us say at $t = t''$ the system will again be in steady state. We then have that $u(t') = q_{min}$, $y(t') = y_{min}$, $u(t'') = q_{max}$ and $y(t'') = y_{max}$. An overshoot in either $y(t)$ or $u(t)$ [1] would then lead to tank overflow while an undershoot would give an empty tank.

Before turning to investigate the under and overshoot of the system we note that the controller has a direct term from the inlet to the outlet flow

$$\beta K_c K_{SP}. \quad (16)$$

A step in the inlet flow will thus give a discontinuous outlet flow which obviously contradicts the main idea of using surge tanks. The obvious choice is thus $\beta = 0$ to decouple the outlet from the inlet flow. This yields the transfer functions

$$Y(s) = \frac{k_v\left(K_{SP} T_I s - K_c K_{SP}\right)}{T_I s^2 - k_v K_c T_I s - k_v K_c} Q_{in}(s), \quad (17a)$$

$$U(s) = \frac{K_c\left((K_{SP} - k_v T_I)s + k_v)\right)}{T_I s^2 - k_v K_c T_I s - k_v K_c} Q_{in}(s). \quad (17b)$$

Both systems are thus of the form $K\frac{s+z}{(s+p_1)(s+p_2)}$ for which there exist necessary and sufficient conditions to give a non under or overshooting step response. The requirements, as derived in (Kwon et al., 2002), are real stable poles and that the zero is farther away from the origin than the rightmost pole

$$-\Re\{p_1,\, p_2\} < 0,\ \Im\{p_1,\, p_2\} = 0,\ z \geq \max \Re\{p_1,\, p_2\}. \quad (18)$$

The resulting step response will actually be monotone and henceforth we will refer to monotonicity as the sought after behavior.

As derived in Appendix A the system given by (17a) and (17b) has a monotone step response if and only if

$$T_I > \frac{K_{SP}}{k_v}, \quad (19a)$$

$$K_c \leq -\frac{4}{k_v T_I},\ T_I \in \left(\frac{K_{SP}}{k_v}, \frac{2K_{SP}}{k_v}\right], \quad (19b)$$

$$K_c \leq \frac{k_v T_I}{K_{SP}\left(K_{SP} - k_v T_I\right)},\ T_I > \frac{2K_{SP}}{k_v}. \quad (19c)$$

The permitted area, defined by (19), is shown in Figure 5 for some numerical values of k_v and K_{SP}. In general k_v is, of course, not known exactly but replacing it by appropriate estimates (for example a lower bound in (19a) and an upper one in (19b)) it is still possible to ensure a monotone step response in y and u.

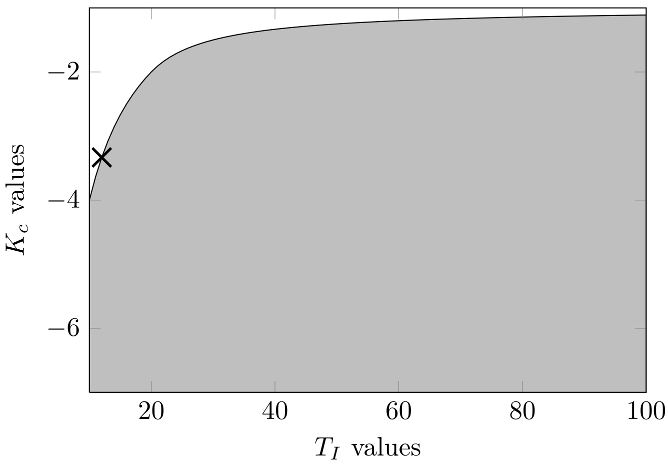

Fig. 5. Shaded area is the permitted region for $k_v = \frac{1}{10}$, $K_{SP} = 1$ and $b_{SP} = 0$. The optimal tuning for these system parameters is also shown as a cross.

6.1 Optimal performance for a step disturbance

A tuning fulfilling (19) guarantees that the system will stay within its bounds. The question then arises how to select K_c and T_I to achieve optimal filtering? We derive the optimal parameters for the Euclidean norm since it yields an analytical expression. For a flow upset of size A this corresponds to solving

$$\begin{aligned} &\min_{K_c,\, T_I} J_{PI,2} = \min_{K_c,\, T_I} \ ||\dot{u}||_2^2 \\ &\text{subject to} \\ &(17b), (19) \end{aligned} \quad (20)$$

The calculation of $J_{PI,2}$ has to be split into three parts depending on the pole-zero configuration of the system. For $K_c = -\frac{4}{k_v T_I}$ the system has a double pole and obtains

$$J_{PI,2}^{double} = \frac{5k_v^2 T_I^2 + 4K_{SP}^2 - 8k_v T_I K_{SP}}{2k_v^2 T_I^3}, \quad (21)$$

where the superscript *double* is used to denote a double pole. For $K_c = \frac{k_v T_I}{K_{SP}(K_{SP} - k_v T_I)}$ pole zero cancellation occurs (regardless of T_I value) and the PI controller essentially [2] becomes a P controller and consequently the criterion evaluates to

$$J_{PI,2}^{single} = J_{P,2} = \frac{k_v}{2K_{SP}}. \quad (22)$$

For K_c not on the boundary we have an over damped second order system with performance criterion

$$J_{PI,2}^2 = \frac{(2k_v T_I K_{SP} - K_{SP}^2 - k_v^2 T_I^2)K_c + k_v T_I}{2k_v T_I^2}. \quad (23)$$

Investigating $J_{PI,2}^2$ closer we note that

$$\begin{aligned} \frac{\partial J_{PI,2}^2}{\partial K_c} =& 2k_v T_I K_{SP} - K_{SP}^2 - k_v^2 T_I^2 = \\ & -\left(k_v T_I - K_{SP}\right)^2 < 0 \end{aligned} \quad (24)$$

which means that the objective function decreases with increasing K_c. It is thus optimal to choose K_c on the

[1] Strictly speaking this is only required close to the boundary and might thus be obtained for a non-linear gain. Relaxing (2b) would also permit over or undershoot in $u(t)$ even in the linear case.

[2] The integrating part of the controller is of course still present.

boundary. Consequently either $J_{PI,2}^{double}$ or $J_{PI,2}^{single}$ is optimal. The optimal T_I-value for $J_{PI,2}^{double}$ is either on the boundary (19a) or the solution of

$$\frac{d\,J_{PI,2}^{double}}{dT_I} = 0 \quad \frac{d^2\,J_{PI,2}^{double}}{dT_I^2} > 0. \tag{25}$$

Carrying out the calculations, we obtain the optimal value

$$T_I = \frac{6K_{SP}}{5k_v}. \tag{26}$$

Inserting this into (21) the objective evaluates to

$$J_{PI,2}^{double} = \frac{25k_v}{54K_{SP}}, \tag{27}$$

which clearly is smaller than $J_{PI,2}^{single}$. Concluding, the optimal tuning is

$$\begin{aligned} K_c &= -\frac{4}{k_v T_I}, \\ T_I &= \frac{6K_{SP}}{5k_v}. \end{aligned} \tag{28}$$

Turning to the maximum criterion we obtain that with the proposed tuning it evaluates to

$$J_{PI,\infty} = \frac{10|A|k_v e^{-\frac{1}{2}}}{9K_{SP}}. \tag{29}$$

Comparing the performance values obtained for the proposed PI controller with that of the robustly optimal controllers we see that it is 4 % and 35 %[3] worse in terms of Euclidean and infinity norm respectively. Furthermore, the PI is approximately 7% and 33 % better than the P controller in terms of the two and infinity norm criterion respectively.

7. SIMULATION RESULTS

To illustrate the benefits with the proposed controllers they are compared with a standard PI with a fixed set-point (hereafter denoted Fix-PI) on data for the first 21 weeks (5 months) of 2011. The system is assumed to have $k_v = \frac{1}{3}$ and

$$\begin{aligned} q_{min} &= y_{min} = 0, \\ q_{max} &= y_{max} = 100. \end{aligned}$$

The Fix-PI is augmented with the tracking anti-windup (tracking gain $\frac{1}{T_a} = \frac{1}{T_I}$) in (Tharayil and Alleyne, 2002). Furthermore the minimum and maximum outlet flows are perturbed an additional 5 % to improve the Fix-PI's handling of flows on or close to the boundary, hence an outlet flow of -5 % and 105 % is permitted for the Fix-PI controller. The tuning of the Fix-PI is a delicate business as a too detuned controller will violate the level constraints and too tight tuning will give bad disturbance attenuation. We will do what is typically done in industry: use historical data to find parameters that are optimal for that data and somewhat robust towards the uncertain future. We use four months of data from 2010 and find a tuning which is optimal for that data. Robustness towards future uncertainty is obtained by requiring that the control parameters must keep the tank level within 10 % and 90 % for the 2010 data. This yields the parameters

$$\begin{aligned} K_c^{2010} &= -1.1, \\ T_I^{2010} &= 3.5. \end{aligned} \tag{30}$$

To obtain a lower bound for what is possible to achieve with a Fix-PI we use the actual data from 2011 and by an extensive search the optimal K_c and T_I values are found. The parameters which obtained the least average Euclidean norm value were

$$\begin{aligned} K_c^{oracle} &= -1.3, \\ T_I^{oracle} &= 9.5. \end{aligned} \tag{31}$$

The two tunings of Fix-PI as well as the proposed P and PI controllers were simulated for the 2011 data and their normalized weekly performance is showed in Figure 6. The P controller performs, as expected, almost as well as the variable set-point PI controller in terms of the Euclidean norm, but much worse in terms of the infinity norm. Furthermore we see that the performance of the oracle tuning of the Fix-PI (which is a lower bound for what a PI can achieve) is still ≈ 50 % worse than the performance of the variable set-point PI controller. The tuning using data from 2010 performs almost as well as the oracle solution.

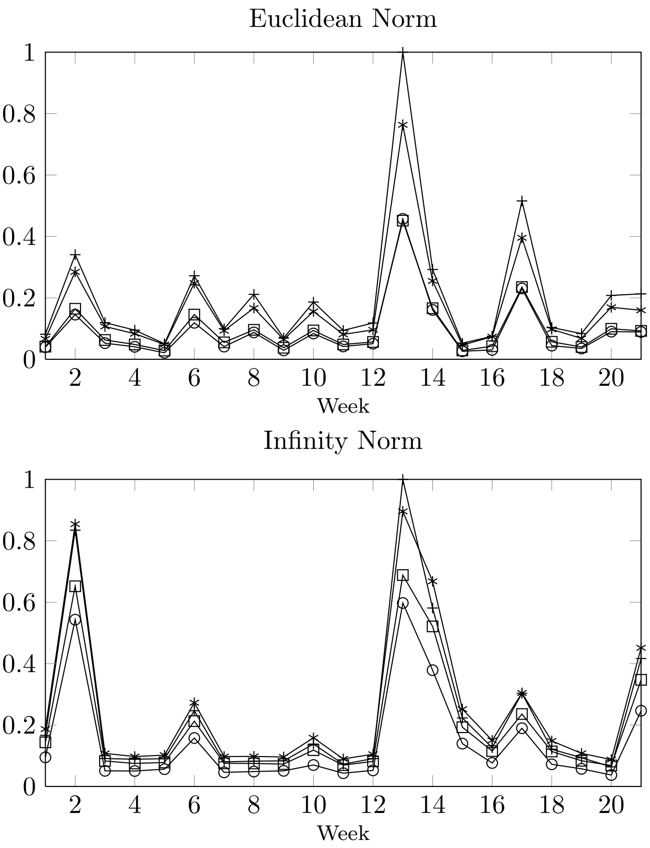

Fig. 6. Normalized Euclidean and infinity norm for the controllers: Fix-PI controller with the tuning using data from 2010 (+), the Fix-PI controller with the oracle tuning using the actual data from 2011 (∗), the optimal filtering P controller (□) and the variable set-point PI controller (○).

[3] This could have been decreased even further by instead minimizing $||\dot{u}||_\infty$.

8. CONCLUSIONS

Two linear controllers that mimic the behavior of the robust MPC have been presented. Tuning guidelines to guarantee that the level does not violate any constraints were derived. The PI with a variable set-point was shown to perform considerable better than the P controller in terms of worst case performance.

Future work includes investigating if the inlet flow characteristics can be used to design alternatives to the affine mapping and if gain-scheduling can be used to just enforce monotonic step responses close to the boundary. A low pass filter to the mapping should probably also be added to improve the controller's noise sensitivity.

Another extension is to apply the PI controller to a cascade of surge tanks. The inlet flow to the second tank is then not well described as a step as it is the outlet from the first tank. The optimal control parameters might then differ and whether the tuning can be done sequentially or not needs to be investigated.

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Appendix A. MONOTONE STEP RESPONSE

With $\beta = 0$ the system has poles

$$\{-p_1, -p_2\} = \frac{k_v K_c T_I \pm \sqrt{k_v^2 K_c^2 T_I^2 + 4 k_v K_c T_I}}{2T_I}, \quad \text{(A.1)}$$

where $-p_1$ is the rightmost one. The zeros are placed in

$$-z_{q_{in},y} = \frac{K_c K_{SP}}{T_I}, \; -z_{q_{in},u} = \frac{k_v}{K_{SP} - k_v T_I}. \quad \text{(A.2)}$$

Stable poles are obtained if $K_c < 0$ and real poles iff

$$k_v^2 K_c^2 T_I^2 + 4 k_v K_c T_I \geq 0 \Leftrightarrow K_c \leq -\frac{4}{k_v T_I}. \quad \text{(A.3)}$$

Straightforward calculations gives that $-z_{q_{in},y} \leq -p_1$ is fulfilled iff

$$\begin{aligned} K_c &\leq \frac{k_v T_I}{K_{SP}\left(K_{SP} - k_v T_I\right)}, \; T_I > \frac{2K_{SP}}{k_v} \\ K_c &\leq -\frac{4}{k_v T_I}, \; T_I \leq \frac{2K_{SP}}{k_v}. \end{aligned} \quad \text{(A.4)}$$

The condition $-z_{q_{in},u} \leq -p_1$ requires more attention. A necessary condition is that the zero lies in the left half plane

$$-z_{q_{in},u} < 0 \Leftrightarrow T_I > \frac{K_{SP}}{k_v}. \quad \text{(A.5)}$$

That fulfilled it follows that

$$\begin{aligned} &\frac{k_v}{K_{SP} - k_v T_I} \leq \frac{k_v K_c T_I + \sqrt{k_v^2 T_I^2 K_c^2 + 4 k_v T_I K_c}}{2T_I} \Leftrightarrow \\ &\frac{2k_v T_I - \alpha(k_v K_c T_I + \sqrt{k_v^2 T_I^2 K_c^2 + 4 k_v T_I K_c})}{2\alpha T_I} \leq 0, \end{aligned} \quad \text{(A.6)}$$

where $\alpha \triangleq K_{SP} - k_v T_I$. Since $\alpha < 0$ it follows that

$$\underbrace{-\alpha\sqrt{k_v^2 T_I^2 K_c^2 + 4 k_v T_I K_c}}_{\geq 0} \geq k_v T_I(\alpha K_c - 2). \quad \text{(A.7)}$$

We note that for a negative right-hand side

$$K_c \geq \frac{2}{K_{SP} - k_v T_I} \quad \text{(A.8)}$$

the inequality is trivially fulfilled and if not we can square both to obtain the requirement

$$K_c \leq \frac{k_v T_I}{K_{SP}\left(K_{SP} - k_v T_I\right)}. \quad \text{(A.9)}$$

It turns out that (A.8) and (A.9) do not impose any additional restrictions on K_c since

$$\frac{-4}{k_v T_I} \leq \frac{k_v T_I}{K_{SP}\left(K_{SP} - k_v T_I\right)} \Leftrightarrow (k_v T_I - 2K_{SP})^2 \geq 0. \quad \text{(A.10)}$$

Combining (A.4) and (A.5) concludes the derivation.