Design of Adaptive I-PD Control Systems using Augmented Error Method

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Abstract: Proportional-Integral-Derivative (PID) controllers are undoubtedly the most employed controllers in industry. Also, since the so called exact model matching is impossible in the actual realities, the partial model matching from the lowest order term to as higher the term as possible is adopted by Kitamori. This controller has a structure which cousists of information about the controlled object and the reference model. PID can be obtained after some manipulation from the original solution. Thus a problem arises which is the superior, the original one or PID. In this paper, we propose adaptive I-PD controller using augmented error method for SISO systems. In addition, we show the effectiveness of the proposed method by the simulation result.

Keywords: Model reference adaptive control, PID control, SISO, Time lag, Transfer functions

1. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are undoubtedly the most employed controllers in industry. Despite their widespread use and considerable history, PID tuning is still an active area of research, both academic and industrial. During the past five decades, a comprehensive PID tuning literature has been developed. Roughly speaking, there are two different approaches to obtain PID and PID-like controller parameters.

First, tune the parameters of the PID structure by following one of several available tuning techniques: Ziegler-Nichol method[1], internal-model-control-based method[2], optimization method[3], and gain-phase margin method[4]. For single-input/single-output (SISO) plants, satisfactory control can be achieved by using established tuning rules.

Second, assume that the controller has a PID structure, and find the PID parameters by using some well-known optimization methods, e.g., H_∞[5], mixed H_2/H_∞[6], and semidefinite programming approaches[7]. These methods can be used to obtain the PID controller parameters such that the controllers have good time-domain performance and frequency-domain robustness. The main problem with this approach is that the resulting controllers are states-pace controllers of high-order rather than low-order controllers with a fixed structure. Although one can reduce or approximate it with a PID-like structure, it is not so far the reduced-order controller.

Also, since the so called exact model matching is impossible in the actual realities, the partial model matching from the lowest order term to as higher the term as possible is adopted by Kitamori[8]. This controller has a structure which cousists of information about the controlled object and the reference model. PID can be obtained after some manipulation from the original solution. Thus a problem arises which is the superior, the original one or PID. In this paper, we propose adaptive I-PD controller using augmented error method for SISO systems. In addition, we show the effectiveness of the proposed method by the simulation result.

2. PROBLEM STATEMENT

Consider the SISO, linear time-invariant systems in Fig.1 described by

$$y(t) = P(s)u(t) \tag{1}$$

$$\begin{aligned} P(s) &= \frac{b(s)}{a(s)} \\ &= \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + \cdots} \\ &= \frac{1}{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots} \\ &= \frac{1}{\beta(s)} \end{aligned} \tag{2}$$

where $u(t), y(t) \in \Re$ are the control input and the plant output respectively.

Then, we choose the following reference model.

$$y_M(t) = G_M(s)r(t) \tag{3}$$

$$\begin{aligned} G_M(s) &= \frac{1}{1 + \sigma s + \alpha_2(\sigma s)^2 + \alpha_3(\sigma s)^3 + \cdots} \\ &= \frac{1}{\alpha(s)} \end{aligned} \tag{4}$$

where $r(t), y_M(t) \in \Re$ are the reference input and the reference model output respectively.

The σ in the above model is a time-scaling parameter and, at the same time, a measure of response time because it is the first-order moment of the impulse response, that is, an average delay of the impulse response. The smaller the value of σ is, the higher the response speed is. The value of σ is left indeterminate in the model because the speed of designed system depends upon the speed of controlled object and ability of the compensator/controller used. The value is determined in the course of matching.

The α_i's are parameters to adjust the damping characteristics of designed system. Some recommendable sets of values for α_i's are known.A set is given as

$$\{\alpha_2, \alpha_3, \alpha_4, \alpha_5, \cdots\} = \{0.5, 0.15, 0.03, 0.003, \cdots\} \tag{5}$$

which gives rise to step responses of about 10 percent overshoot with good damping[9]. Some others are

$$\{\alpha_2, \alpha_3, \alpha_4, \cdots\} = \{0.425, 0.0975, 0.014344, \cdots\} \tag{6}$$
$$\{\alpha_2, \alpha_3, \alpha_4, \cdots\} = \{0.375, 0.0625, 0.003906, \cdots\} \tag{7}$$

The former, proposed by Shigemasa[10], gives rise to quicker step responses with negligible overshoot. The latter is the fourth order critical damping.

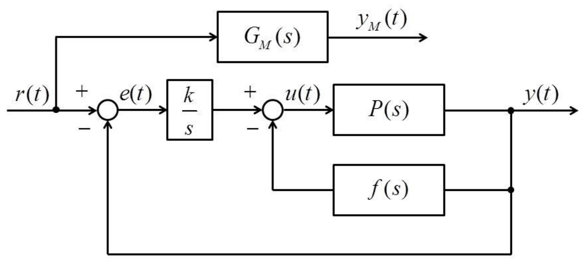

Fig. 1. I-PD type control system

3. ADAPTIVE I-PD CONTROL DESIGN

3.1 Choice of Control Law

The control objective is to design $u(t)$ such that asymptotically $y(t)$ tracks $y_M(t)$ with all generated signals remaining bounded.

We want to calculate the transfer function $W(s)$ from $r(t)$ to $y(t)$.

$$y(t) = P(s)\left[\frac{k}{s}\{r(t) - y(t)\} - f(s)y(t)\right] \tag{8}$$

Then

$$W(s) = \frac{1}{1 + \frac{s}{k}\{\beta(s) + f(s)\}} \tag{9}$$
$$f(s) = f_0 + f_1 s \tag{10}$$

If the following equations are satisfied, then $W(s) = G_M(s)$

Hence, the model matching condition is

$$\alpha(s) = 1 + \frac{s}{k}\{\beta(s) + f(s)\} \tag{11}$$

At this time, parameters σ, k, f_0, f_1 are

$$\sigma = \frac{\alpha_3\beta_3}{\alpha_4\beta_2} \tag{12}$$
$$k = \frac{\beta_2}{\alpha_3\sigma^3} \tag{13}$$
$$f_0 = k\sigma - \beta_0 \tag{14}$$
$$f_1 = k\alpha_2\sigma^2 - \beta_1 \tag{15}$$

We obtain the following error equation from the model matching condition.

$$\varepsilon(t) = \frac{s}{k\alpha(s)}\left[u(t) - \theta^{\mathrm{T}}\zeta(t)\right] \tag{16}$$

where

$$\theta = [k, f_0, f_1]^{\mathrm{T}} \tag{17}$$
$$\zeta(t) = \left[\frac{1}{s}e(t), -y(t), -\dot{y}(t)\right]^{\mathrm{T}} \tag{18}$$
$$e(t) = r(t) - y(t) \tag{19}$$
$$\varepsilon(t) = y(t) - y_M(t) \tag{20}$$

The control law when the parameters of plant are known.

$$u(t) = \theta^{\mathrm{T}}(t)\zeta(t) \tag{21}$$

3.2 Choice of Adaptation Law

When the plant parameters are unknown, true parameters θ are replaced by the adjustable parameters $\hat{\theta}(t)$. Controller parameters in $\hat{\theta}(t)$ provided by the adaptation law.

$$u(t) = \hat{\theta}^{\mathrm{T}}(t)\zeta(t) \tag{22}$$
$$\varepsilon(t) = \frac{s}{k\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] \tag{23}$$
$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta \tag{24}$$

However, the choice of adaptation law given by

$$\dot{\hat{\theta}}(t) = -\Gamma\zeta(t)\varepsilon(t), \Gamma = \Gamma^{\mathrm{T}} > 0 \tag{25}$$

cannot be used, because now the error transfer function $\frac{s}{\alpha(s)}$ is no longer strictly positive real. A famous technique called error augmentation can be used to avoid this difficulty in finding an adaptation law for this error model. The basic idea of the technique is to consider a so-called augmented error $\varepsilon'(t)$ which correlates to the parameter error in a more desirable way than the tracking error $\varepsilon(t)$.

First, let us define an auxiliary error $\eta(t)$ by

$$\eta(t) = \hat{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\hat{\theta}^{\mathrm{T}}(t)\zeta(t)\right) \tag{26}$$

It is useful to note two features about the auxiliary error. First, this error $\eta(t)$ can be computed on-line, since the estimated parameter vector $\hat{\theta}(t)$ and the signal vector $\zeta(t)$ are both available on-line manner. Secondly, this error $\eta(t)$ is caused by time-varying nature of the estimated parameters $\hat{\theta}(t)$, in the sense that when the estimated parameters $\hat{\theta}(t)$ is replaced by the constant parameter vector, then we have $\eta(t) = 0$.

This also implies that the auxiliary error can be written

$$\eta(t) = \tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right) \tag{27}$$

Now let us define an augmented error $\varepsilon'(t)$, by combining the tracking error $\varepsilon(t)$ with the auxiliary error $\eta(t)$ as

$$\varepsilon'(t) = \varepsilon(t) + \hat{h}\eta(t) \tag{28}$$

$$h = \frac{1}{k} \tag{29}$$

where $\hat{h}(t)$ is a time-varying parameter to be determined by adaptation. Note that $\hat{h}(t)$ is not a controller parameter, but only a parameter used in forming the new error $\varepsilon'(t)$.

$$\begin{aligned}\varepsilon'(t) &= \varepsilon(t) + \hat{h}\eta(t)\\ &= \frac{sh}{\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] + \hat{h}\eta(t) + h\eta(t) - h\eta(t)\\ &= \frac{sh}{\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] + \hat{h}\eta(t)\\ &\quad + h\left(\tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right)\right) - h\eta(t)\\ &= h\tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] + \tilde{h}\eta(t)\end{aligned} \tag{30}$$

We obtain

$$\varepsilon'(t) = h\tilde{\theta}^{\mathrm{T}}(t)\xi(t) + \tilde{h}\eta(t) \tag{31}$$

$$\xi(t) = \frac{s}{\alpha(s)}\zeta(t) \tag{32}$$

This implies that the augmented error can be linearly parameterized by the parameter errors $\tilde{\theta}(t)$ and $\tilde{h}(t)$. Then a number of standard techniques such that the gradient method or the least-squares method can be used to update the parameters.

Using the gradient method with normalization, the controller parameters $\hat{\theta}(t)$ and the parameter $\hat{h}(t)$ for forming the augmented error are updated by

$$\dot{\hat{\theta}}(t) = -\frac{\Gamma\xi(t)\varepsilon'(t)}{\rho + \xi^{\mathrm{T}}(t)\xi(t)} \tag{33}$$

$$\dot{\hat{h}}(t) = -\frac{\gamma\eta(t)\varepsilon'(t)}{\rho + \xi^{\mathrm{T}}(t)\xi(t)} \tag{34}$$

4. STABILITY OF CLOSED-LOOP SYSTEM

Consider the Lyapunov function candidate

$$V = \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\theta}(t) \tag{35}$$

where Γ is symmetric positive definite constant matrix

The derivative of $\dot{V}$ can be computed as,

$$\begin{aligned}\dot{V} &= \tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\dot{\hat{\theta}}(t)\\ &= -\frac{\tilde{\theta}^{\mathrm{T}}(t)\xi(t)\varepsilon'(t)}{\rho + \xi^{\mathrm{T}}(t)\xi(t)}\\ &= -\frac{\varepsilon'^2(t)}{\rho + \xi^{\mathrm{T}}(t)\xi(t)} < 0\end{aligned} \tag{36}$$

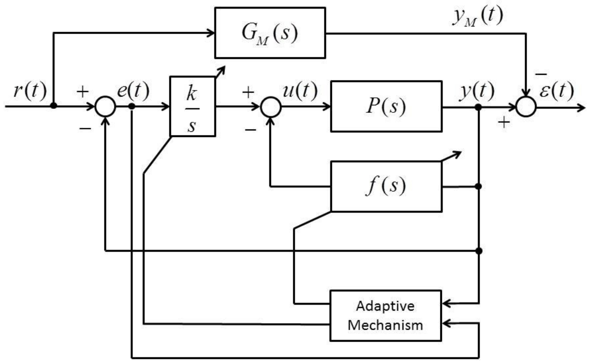

Fig. 2. Model reference adaptive control

5. SIMULATION EXAMPLE

The system description is discussed next. Most overdamped processes can be sufficiently well-approximated by a first-order system with a time-lag element as follows

$$P(s) = \frac{K}{1+Ts}e^{-Ls} \tag{37}$$

where K, T and L denote the system gain, the time-constant and the time-lag respectively.

The controlled object is given by

$$P(s) = \begin{cases}\dfrac{1}{1+s}e^{-Ls} & 0 \le t < 5\\ \dfrac{2}{1+s}e^{-Ls} & 5 \le t\end{cases} \tag{38}$$

Now we choose the following parameters α_i

$$\{\alpha_2, \alpha_3, \alpha_4\} = \{0.5, 0.15, 0.03\} \tag{39}$$

The proposed control scheme was employed, and the control result is shown in Fig.3. Fig.4 shows the trajectories of PID parameters $\hat{\theta}(t)$ corresponding to Fig.3.

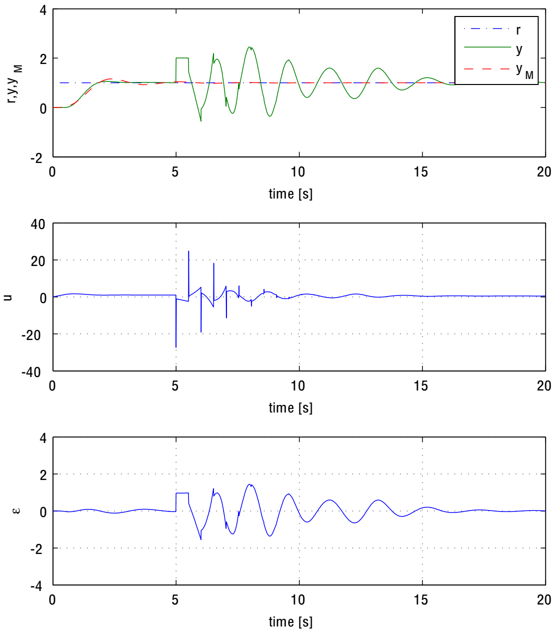

Fig. 3. Control result using the proposed method

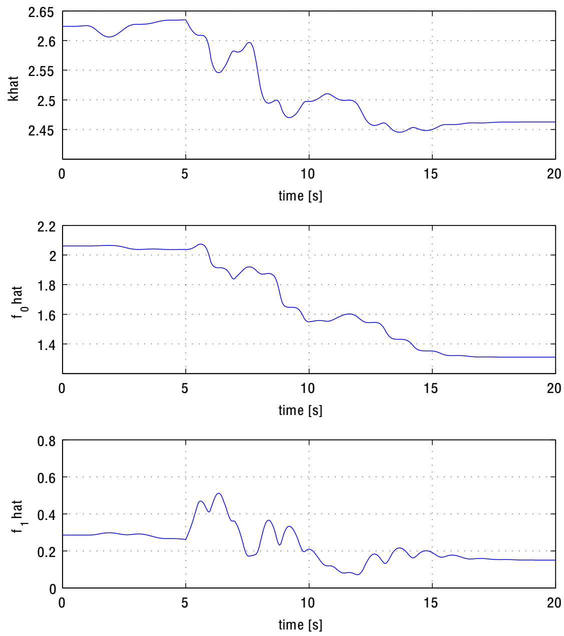

Fig. 4. Estimated value of parameters

Finally, for the purpose of comparison, the control behavior of the conventional scheme was also examined. The control result is shown in Fig.5. Fig.5 indicate that the control performance could not hold, because θ was fixed. In order to go on to obtain the desired control performance, θ has to be adequately chosen corresponding to the system property. From these viewpoints, it is clear that the proposed adaptive I-PD controller works well.

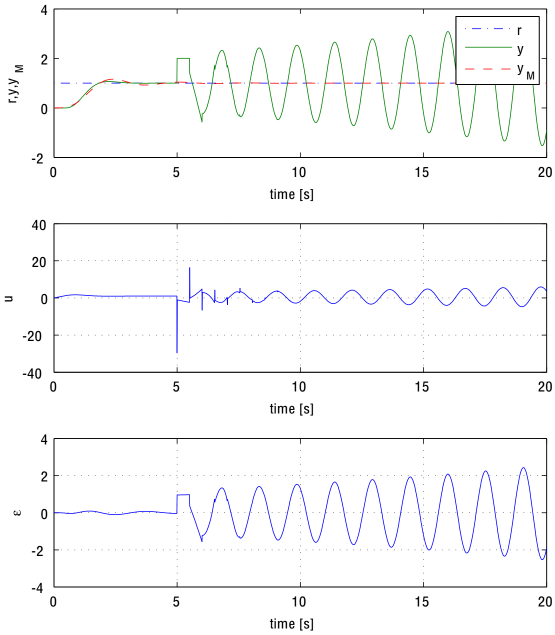

Fig. 5. Control result using the conventional method

6. CONCLUSION

In this paper, an adaptive I-PD control scheme has been proposed, in which PID parameters are computed and updated the model quality changes.In addition, the superiority of the proposed method to the conventional method is confirmed by the simulation.

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