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Extending the AMIGO PID tuning method to MIMO systems ⋆

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Abstract: In this paper a methodology for tuning decentralised PID is proposed, which is based on the AMIGO method for SISO systems. The study addresses MIMO systems with transfer matrix made up of first-order with time delay models that describe a large number of industrial processes. The proposed approach is evaluated by means of simulation studies that show the results of its application to several systems.

Keywords: PID, effective transfer function, MIMO systems, disturbances.

1. INTRODUCTION

The purpose of many control loops in industry is mainly to reject possible disturbances that tend to lead system outputs away from their reference values. The tuning of PID controllers in order to minimise the effect of disturbances in SISO systems has been widely addressed in the literature, Panagopoulos et al. (2002); Åström and Hägglund (2004); Åström et al. (1998); Sanchis et al. (2010); Romero et al. (2011). Many industrial processes, however, are of a multivariable nature in which the disturbances are transmitted to several outputs with the subsequent adverse affects from the point of view of their operation.

In this paper a new methodology for tuning decentralised PID controllers is proposed to minimise the effect of disturbances in MIMO systems. The methodology is based on extending the well known AMIGO method (Approximate M-constrained Integral Gain Optimisation) for tuning SISO PID control loops to the MIMO case. AMIGO method, Åström and Hägglund (2004), consists in applying a set of equations to calculate the parameters of the controller, thus their application is very simple. Furthermore, the method is applicable to systems whose behaviour can be approximated by a first-order plus time delay (FOPTD) model or integrator plus time delay, thereby covering a large number of applications in the process industry. Therefore the extension of AMIGO method to be applied in MIMO systems could be of interest in many industrial control applications.

The paper is structured as follows. In section 2 the problem of rejecting disturbances in TITO systems is stated formally. In section 3 the concept of Effective Transfer Function (ETF) is addressed, which is fundamental to be able to understand the methodology proposed here. In section 4 the general characteristics of the AMIGO method for minimising the effect of disturbances in SISO systems are discussed. The methodology for adjusting decentralised PID controllers is described in section 5. The results of applying the methodology in two multivariable systems are presented in section 6. Finally, in section 7 the conclusions from the study are discussed.

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2. STATEMENT OF THE PROBLEM

Let us consider the TITO system with decentralised control shown in Figure 1. $C_1(s)$ and $C_2(s)$ are PID controllers with transfer functions

$$C_n(s) = K_{p_n}\left(1 + \frac{T_{d_n}s}{1 + \frac{T_{d_n}s}{N_n}} + \frac{1}{T_{i_n}s}\right),\ n = 1, 2 \qquad (1)$$

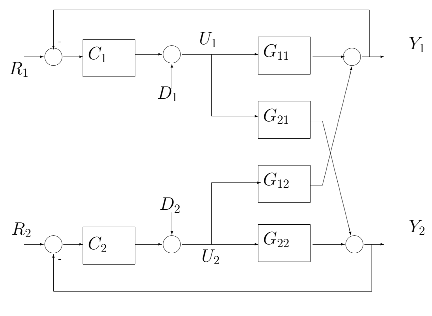

Fig. 1. TITO system with decentralised controllers

The aim is to adjust the controllers $C_1(s)$ and $C_2(s)$ in order to achieve a good degree of disturbance rejection. More specifically, the IAE index of the outputs $Y_1(s)$ and $Y_2(s)$ (IAE_1 and IAE_2) as a response to step-like inputs at $D_1(s)$ and $D_2(s)$, defined as

$$IAE_n = \int_0^t |r_n(\nu) - y_n(\nu)| d\nu,\ n = 1, 2 \qquad (2)$$

must be kept as small as possible. At the same time, the system must have a robust behaviour when faced with errors in models $G_{ij}(s)$.

3. EFFECTIVE TRANSFER FUNCTIONS

A key concept in the methodology proposed in this paper is that of effective transfer function, which is the transfer function between an output and an input in a MIMO system when the other input/output pairs are in a closed loop through their corresponding controllers.

Let us consider the TITO system with decentralised controllers shown in Figure 1. The ETF between output $Y_1(s)$ and input $U_1(s)$ is:

$$G_{11}^e(s) = \frac{Y_1(s)}{U_1(s)} = G_{11}(s) - \frac{C_2(s)G_{12}(s)G_{21}(s)}{1 + C_2(s)G_{22}(s)} \quad (3)$$

where, for the sake of simplicity, the complex variable s has been omitted. Likewise, the ETF between output Y_2 and input U_2 is:

$$G_{22}^e(s) = \frac{Y_2(s)}{U_2(s)} = G_{22}(s) - \frac{C_1(s)G_{12}(s)G_{21}(s)}{1 + C_1(s)G_{11}(s)} \quad (4)$$

As can be seen in equations (3) and (4), the ETF between an input/output pair depends on the controllers of the other input/output pairs.

3.1 Simplification of the ETF

The dependence of ETF on the controllers of the other control loops in a MIMO system, which are initially unknown, restricts their use for designing controllers. Under some considerations, however, this dependence can be removed. Following on with the TITO case in the previous section, if it is supposed that controllers $C_1(s)$ and $C_2(s)$ are ideal, that is to say, that the closed loop transfer functions satisfy the following condition:

$$\frac{Y_1(s)}{R_1(s)} = \frac{C_1(s)G_{11}(s)}{1 + C_1(s)G_{11}(s)} = 1 \quad (5)$$

$$\frac{Y_2(s)}{R_2(s)} = \frac{C_2(s)G_{22}(s)}{1 + C_2(s)G_{22}(s)} = 1 \quad (6)$$

then the ETF can be simplified as follows:

$$G_{11}^{er}(s) = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{22}(s)} \quad (7)$$

$$G_{22}^{er}(s) = G_{22}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{11}(s)} \quad (8)$$

depending only on the transfer functions of the system. Equations (7) and (8) are known as reduced effective transfer functions (RETF).

4. AMIGO METHOD FOR TUNING PID CONTROLLERS

The problem of adjusting PID controllers in order to minimise the effect of disturbances in SISO systems has been addressed in a number of different studies. In Åström and Hägglund (2004) an approximate method is proposed that accomplishes this goal in a simple way. The method, which is known as AMIGO (Approximate M-constrained Integral Gain Optimisation), consists in applying a set of equations to calculate the parameters of the controller, in a similar way to the procedure used in the Ziegler-Nichols method. The robustness of the design can be specified by means of the maximum value of the sensitivity function (M_{s_d}) within a range between 1.1 and 2. The method is applicable to systems whose behaviour can be approximated by a FOPTD model or integrator plus time delay, thereby covering a large number of applications in the process industry. For a system with a transfer function $G(s) = Ke^T/(\tau s + 1)$, the tuning rules are:

$$K_p = \frac{\alpha_1 T + \alpha_2 \tau}{KT},\ T_i = \frac{\alpha_3 T + \alpha_4 \tau}{T + \alpha_5 \tau},\ T_d = \frac{\alpha_6 T \tau}{T + \alpha_7 \tau} \quad (9)$$

where the parameters α_i depend on the value of M_{s_d} that is sought for the design.

5. EFFECT OF THE DISTURBANCES

The problem of tuning the controllers in order to minimise the effect of the disturbances present in a SISO system can be approached as one of maximising the integral gain ($K_i = K_p/T_i$) of the PID controller, thereby satisfying certain conditions regarding robustness that are considered to be restraints to the problem of maximising K_i, Åström et al. (1998); Panagopoulos et al. (2002). This approach is based on the fact that under a set of conditions offering an acceptable level of robustness (that is to say, with a low-oscillation response), the IAE can be approximated by the error integral (EI), which, as is well known, is inversely proportional to K_i. More specifically, for a SISO system, $IE = 1/K_i$ is satisfied. This section looks at whether it is possible to apply a similar strategy to minimise disturbances in MIMO systems.

By applying the ETF concept, the decentralised control system in Figure 1 can be broken down into two systems like those shown in Figure 2. G_{12}^e and G_{21}^e are the ETF between output Y_1 and disturbance D_2 and output Y_2 and disturbance D_1, respectively, which are given by equations (10) and (11).

$$G_{12}^e = \frac{G_{12}}{1 + C_2 G_{22}} \quad (10)$$

$$G_{21}^e = \frac{G_{21}}{1 + C_1 G_{11}} \quad (11)$$

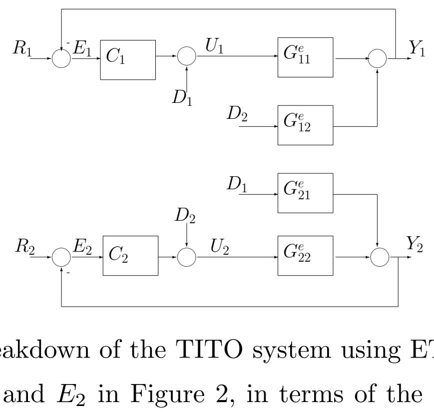

Fig. 2. Breakdown of the TITO system using ETF

Errors E_1 and E_2 in Figure 2, in terms of the ETF, can be expressed by the following equations:

$$E_1 = D_1 \frac{G_{11}^e}{1 + C_1 G_{11}^e} + D_2 \frac{G_{12}^e}{1 + C_1 G_{11}^e} \quad (12)$$

$$E_2 = D_2 \frac{G_{22}^e}{1 + C_2 G_{22}^e} + D_1 \frac{G_{21}^e}{1 + C_2 G_{22}^e} \quad (13)$$

If $E_{n,m}$ is defined as the error in loop n due to disturbance D_m, then $E_n(s) = E_{n,1}(s) + E_{n,2}(s)$ and consequently:

$$\begin{aligned} IE_n &= \lim_{t\to\infty} \int_0^t e_n(\tau)d\tau = \lim_{s\to 0} E_n(s) \\ &= \lim_{s\to 0} E_{n,1}(s) + \lim_{s\to 0} E_{n,2}(s) \\ &= IE_{n,1} + IE_{n,2} \end{aligned} \quad (14)$$

By applying this equation to the errors in equations (12) and (13), after substituting G^e_{11}, G^e_{22}, G^e_{12}, G^e_{21} and C_n with their corresponding expressions, we have that:

$$IE_{1,1} = \frac{T_{i_1}}{K_{p_1}} = \frac{1}{K_{i_1}} \quad (15a)$$

$$IE_{1,2} = 0 \quad (15b)$$

$$IE_{2,2} = \frac{T_{i_2}}{K_{p_2}} = \frac{1}{K_{i_2}} \quad (15c)$$

$$IE_{2,1} = 0 \quad (15d)$$

and therefore

$$IE_1 = IE_{1,1} = \frac{1}{K_{i_1}}, \; IE_2 = IE_{2,2} = \frac{1}{K_{i_2}} \quad (16)$$

As an example, Figure 3 shows the graphs of $E_{n,m}$ and $IE_{n,m}$ for a TITO system with decentralised PID controllers. The behaviour of $E_{1,2}$ and $E_{2,1}$ is such that the value of their integral in the steady state is null ($IE_{1,2} = 0$ and $IE_{2,1} = 0$) and therefore in the steady state $IE_1 = IE_{1,1}$ and $IE_2 = IE_{2,2}$. Yet, during the transient, these equalities are not fulfilled because the integrals of errors $E_{1,2}$ and $E_{2,1}$ are not null.

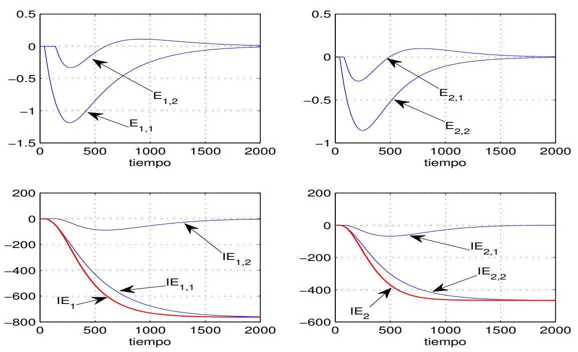

Fig. 3. Effect of disturbances on a TITO system with decentralised PID controllers

Remark 1. From equations (15a) and (15c) it is obvious that minimising $IE_{n,n}$ is equivalent to maximising the integral gain (K_i) of the controller C_n. If, in addition, the response to disturbance D_n has a low degree of oscillation, which is accomplished by a sufficiently robust adjustment of C_n, then $IE_{n,n} = 1/K_{i_n} \approx IAE_{n,n}$, which would mean that maximising K_{i_n} would be equivalent to minimising the index $IAE_{n,n}$.

Remark 2. Equations (15b) and (15d) do not imply that the disturbance D_n has no effect on the error $E_m, m \neq n$. As shown in Figure 3, errors $E_{1,2}$ and $E_{2,1}$ are not null, but their integrals in the steady state are: $IE_{1,2} = 0$ and $IE_{2,1} = 0$.

Taking remark 1 into account, the design of controllers C_1 and C_2 will focus on minimising $IE_{1,1}$ and $IE_{2,2}$ in the SISO systems shown in Figure 2, which result from applying the concept of ETF to the TITO system.

6. DESIGNING DECENTRALISED CONTROLLERS

The tuning of decentralised PID controllers using the ETF concept has recently been addressed in Nguyen and Lee (2010), more particularly for the case of IMC-PID Lee et al. (1998). The proposal is based on obtaining an approximate model of the RETF and applying the design methodology from the IMC-PID to that model. The drawback of this method is that the *ideal controllers* hypothesis that forms the basis of the process of obtaining the RETF may not be true, thus giving rise to discrepancies between the real ETF and the RETF used in the design. Moreover, in Vázquez et al. (1999) the authors proposed the iterative design of decentralised controllers for MIMO systems using ETF. The method basically consists in alternating the calculation of the ETF and the tuning of controllers, taking arbitrary controllers as the starting point for the design. In that work, it is also suggested that non-parametric models should be used, given the complexity of the ETF.

The iterative design methodology that is proposed in this work uses RETF as the initial model to carry out the tuning of decentralised controllers by means of the AMIGO method. Moreover, since this method of tuning requires FOPTD models, the ETF must be adjusted by this type of models. The method can be summarised in the following steps:

(1) Define the robustness specifications sought for each loop: M_{sd_1} and M_{sd_2}.
(2) Calculate a preliminary approximation to the RETF using equations (7) and (8).
(3) Approximate the RETF ($G^{er}_{1,1}$ and $G^{er}_{2,2}$) by means of FOPTD models ($G^{ea}_{1,1}$ and $G^{ea}_{2,2}$) and calculate controllers C_1 and C_2 using equation (9).
(4) Calculate the ETF using equations (3) and (4), taking into account the controllers designed in the previous step.
(5) Approximate the ETF ($G^e_{1,1}$ and $G^e_{2,2}$) by means of FOPTD models ($G^{ea}_{1,1}$ and $G^{ea}_{2,2}$). Recalculate the controllers C_1 and C_2 using those models and equations (9).
(6) Repeat steps 4 and 5 until the values of the parameters of the controllers converge to the final values, which indicates that no improvements are produced in the identification of the ETF used in the designs.

The previous methodology can be applied to systems in which the ETF can be adjusted by FOPTD models. This is generally possible in systems in which the inputs of the transfer matrix are models of this type, as illustrated in the examples given in the next section.

7. EXAMPLES

The algorithm above was applied to two models of TITO systems with the aim of evaluating its behaviour. The systems have the following transfer matrices:

$$G_1(s) = \begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix} \quad (17a)$$

$$G_2(s) = \begin{bmatrix} \dfrac{4.3e^{-40s}}{383s+1} & \dfrac{1.8e^{-140s}}{383s+1} \\ \dfrac{1.2e^{-80s}}{281s+1} & \dfrac{2.5e^{-40s}}{281s+1} \end{bmatrix} \quad (17b)$$

Model G_1 corresponds to a distillation column Vinante and Luyben (1972) and model G_2 describes the behaviour of a hydraulic system made up of four interconnected tanks Ho et al. (1996).

In designing the controllers it was considered that $M_{sd_1} = M_{sd_2} = M_{sd} = 1.4$. Figures 4 and 5 show the results obtained over 10 iterations. They show the values of the IAE when faced with unit step inputs in disturbances D_1 and D_2 and the real value of M_s that is achieved in each loop, which is calculated by using the ETF instead of the FOPTD models used in the design. In the two cases, it can be seen how convergence of the iterative design is accomplished after four or five iterations.

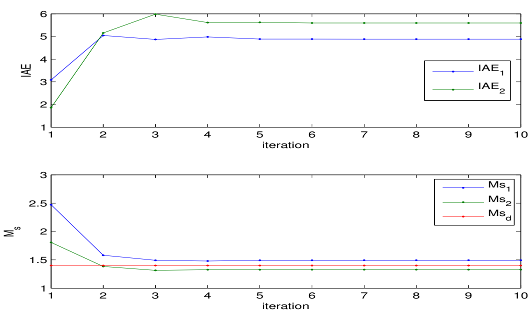

Fig. 4. Results of the iterative design for model G_1: IAE and the real M_s of each loop

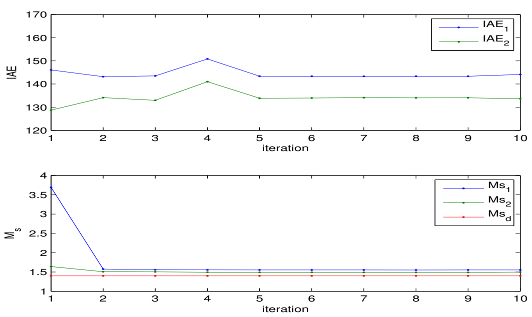

Fig. 5. Results of the iterative design for model G_2: IAE and the real M_s of each loop

The difference that is observed in the figures between the design and the real M_s is due to the error that is made on approximating the ETF by means of an FOPTD model, as can be observed in Figures 6 and 7. This difference, however, is not significant and the values of M_s that are obtained with it in the design are good enough to ensure the robustness of the control loops.

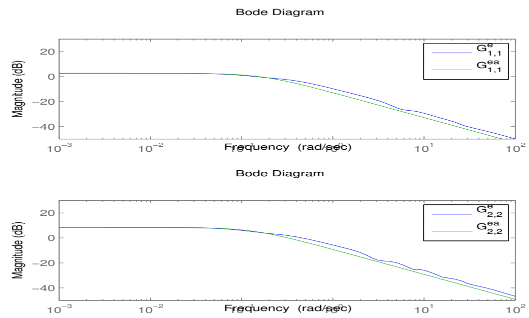

Fig. 6. Magnitude diagram of the ETF and their approximation by means of FOPTD models for the model G_1 and PID designed with $M_{s_d} = 1.4$

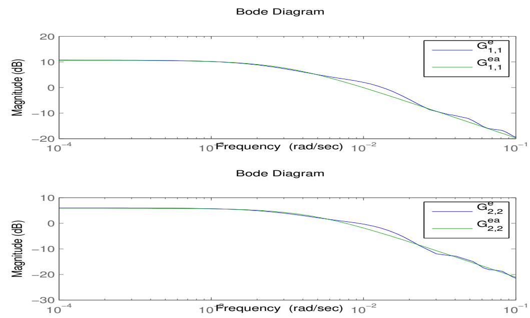

Fig. 7. Magnitude diagram of the ETF and their approximation by means of FOPTD models for the model G_2 and PID designed with $M_{s_d} = 1.4$

The time response to changes in the references and in the disturbances for the two systems are shown in Figures 8 and 9. In both cases, responses with low degrees of oscillation were obtained, which is a result that was to be expected owing to the robustness achieved in the designs.

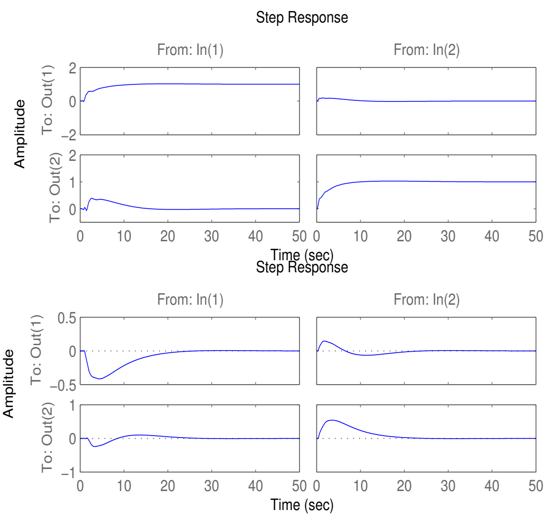

Fig. 8. Response to unit step-like inputs in the references (upper) and in the disturbances (lower) for the model G_1

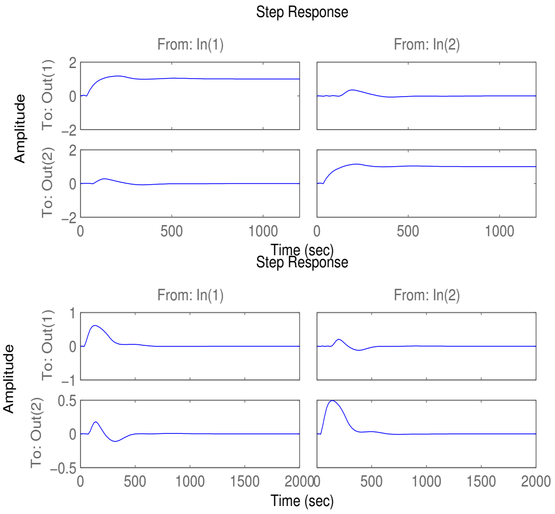

Fig. 9. Response to unit step-like inputs in the references (upper) and in the disturbances (lower) for the model G_2

Figures 10 and 11 show the control actions of controllers C_1 and C_2 to unit step-like input in the disturbances for the model G_1 and G_2 respectively. As can be noted, the PIDs produce smooth control actions which are enough to fix the disturbances effect.

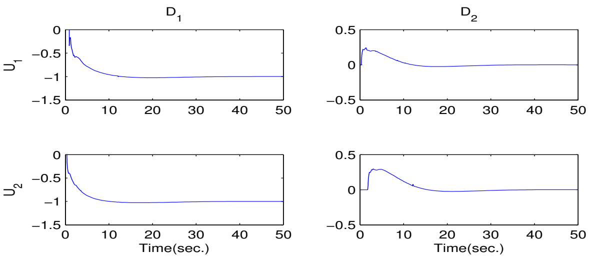

Fig. 10. Control actions to unit step-like input in the disturbances for the model G_1

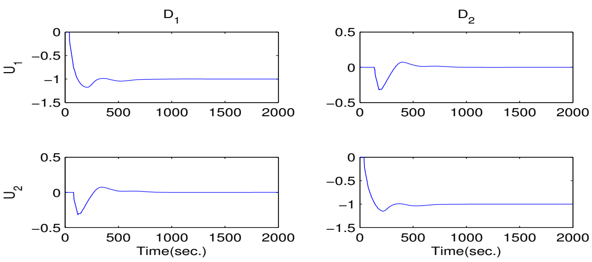

Fig. 11. Control actions to unit step-like input in the disturbances for the model G_2

7.1 Effect of M_{s_d} on the design

As discussed in section 4, the AMIGO method makes it possible to adjust PID controllers for values of $M_s \in [1, 2]$, thereby giving rise to different degrees of robustness in the

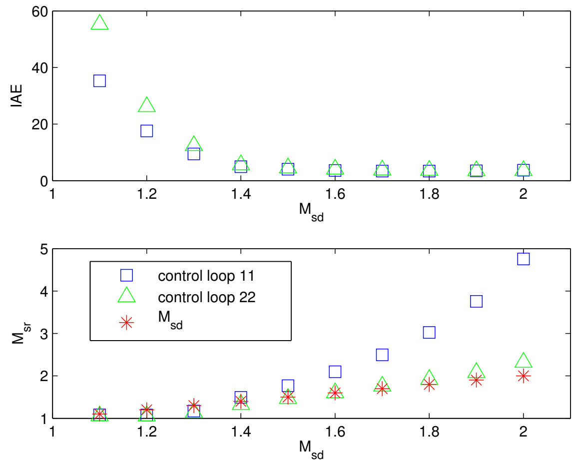

Fig. 12. Values of the IAE of the disturbances and Ms depending on M_{s_d} for the process $G_1(s)$

design. The greater the value of M_s is, the less robust the design and the more active the controller will be.

To be able to study the effect of M_s on the methodology proposed in section 6, designs for controllers were carried out for different values of $M_s \in [1, 2]$. The results can be seen in Figures 12 and 13 for systems (17a) and (17b) respectively. They show the behaviour of the IAE of the disturbance for each output and the values of M_s that are achieved for each loop.

In the IAE graphs it can be observed that for small values of M_{s_d} there is an increase in IAE. This is due to the fact that the designs thus obtained have a high degree of robustness, therefore controllers are not very active and take time to correct the effect of the disturbances.

The figures also offer the values of M_{s_r} , which is the real M_s that is obtained with the design and is calculated using the ETF rather than its approximation by means of an FOPTD model. It can be seen that when the value of M_{s_d} is increased, the difference between this value and that of M_{s_r} also increases, that is, the results that are obtained diverge away from the design specifications.

The difference between M_{s_d} and M_{s_r} is due to the error that is made in approximating the ETF, equations (3) and (4), by means of an FOPTD model. The approximations of the ETF by FOPTD models for designs with $M_{s_d} = 2$ can be seen in Figures 14 and 15. It can be observed that the discrepancy between the models is much greater in this case than for designs with $M_{s_d} = 1.4$, which are represented in Figures 6 and 7. This gives rise to considerable errors in the value of M_s that is really achieved with the design (M_{s_r}).

Therefore, the proposed tuning methodology is valid for the range of values of M_{s_d} in which the ETF can be properly approximated by FOPTD models. The selection of M_{s_d} for the design can be performed from graphs like those shown in Figures 12 and 13. A suitable value for examples $G_1(s)$ and $G_2(s)$ is $M_{s_d} = 1.4$, since $M_{s_d} = M_{s_r}$ while at the same time the IAE is kept to a minimum.

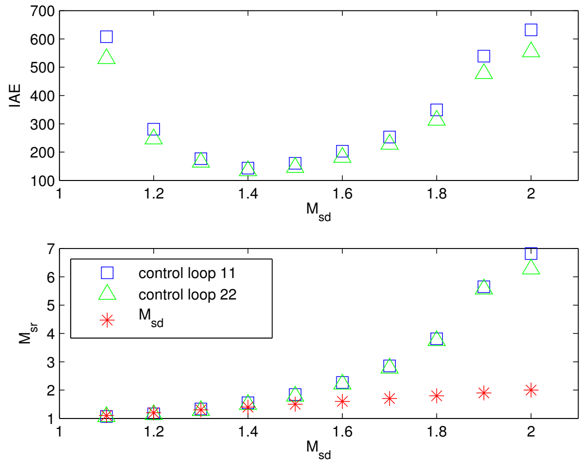

Fig. 13. Values of the IAE of the disturbances and Ms depending on M_{s_d} for the process $G_2(s)$

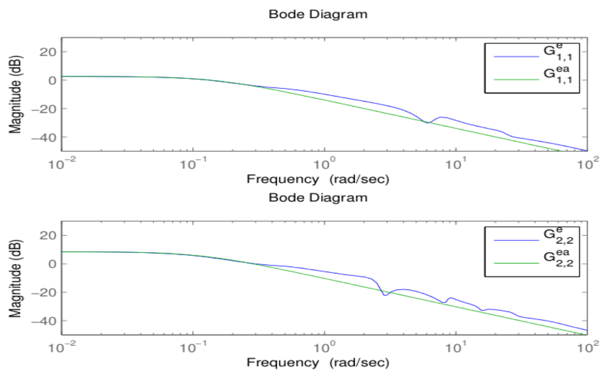

Fig. 14. Magnitude diagram of the ETF and their approximation by FOPTD models for the model G_1 and PID designed with $M_{s_d} = 2$

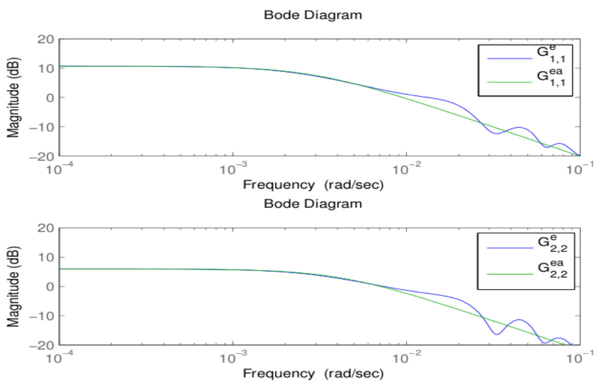

Fig. 15. Magnitude diagram of the ETF and their approximation by FOPTD models for the model G_2 and PID designed with $M_{s_d} = 2$

8. CONCLUSIONS

In this paper a methodology for tuning decentralised PID controllers has been presented, which is based on extending the well known AMIGO method to MIMO systems. The proposal consists in an iterative identification and design approach, where the estimation of FOPTD models to approximate the effective transfer functions and the PID tuning by AMIGO method are combined.

The feasibility of the methodology has been demonstrated by simulation study. It has been probed that the robustness specification, given by the maximum magnitude of the sensitivity function (M_s), has an important effect on the design results because of the errors introduced when approximating the effective transfer function by FOPTD models. These errors increased as M_{s_d} increased and consequently the design specifications are not fulfilled for large values of M_{s_d} . On the other hand, for small values of M_{s_d} the FOPTD models properly fit the effective transfer functions and the design requirement are accomplished.

Future work should be aimed at analysing the error made in the approximation of the effective transfer function by means of FOPTD models in order to improve the degree to which the design specifications are accomplished.

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