IFAC Conference on Advances in PID Control
PID'12
Brescia (Italy), March 28-30, 2012

Pseudo-PID Controller: Design, Tuning and Applications

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Abstract: In this paper, a pseudo PID (PPID) controller, including only one gain to be tuned, is proposed. The idea is to connect the I+PD control design with the Fertik and Ziegler-Nichols tuning rules in order to obtain not only a simple and efficient control algorithm but also to decrease the operator intervention time with respect to the calibration task and to obtain desired closed-loop dynamic. Three approaches for stable automatic tuning via, self-tuning, internal model control and small gain theorem, are investigated for adjusting the tuning parameter of the controller. Effectiveness and performance aspects of the proposed PPID controller are assessed in numerical and experimental plants.

Keywords: PID control, automatic tuning, nonlinear systems, stability, self-tuning control, robustness.

1. INTRODUCTION

The Proportional Integral Derivative (PID) controller is still the most popular in the industry of process control despite the advances in technology and control theory. The success of PID is due to its simple structure, efficient performance and applicability to a broad class of practical control systems. As real processes exhibit characteristics, such as high-order, time-delay and nonlinearity, sometimes it is necessary a retuning or a more elaborate algorithm in the PID controller design for servo and regulatory responses to provide good closed-loop dynamic in different operating points (Åström and Hägglund, 2000; Li et al., 2009).

To increase the efficiency of the PID controller in complex plants and to facilitate the control design, many calibration rules have been developed since the appearance of the first proposal of Ziegler-Nichols. In general, the PID controller projects are based on heuristic, analytical (parametric and non-parametric models), intelligent, optimization and advanced (minimum variance and predictive) methods (Ang et al., 2005; Visioli, 2006). Some features and difficulties of PID tuning methods are: i) the advanced and optimization techniques are time-consuming processing and can fail in plants with time-varying dynamic or large time-delay, ii) the adaptive control technique, called gain scheduling, requires prior knowledge of the operating condition of the plant at each operating range, to adjust the controller gains locally (non-trivial task), iii) the industry of process control shows interest in auto-tuning and self-tuning. The first has a lower computational complexity and understanding, while the second has a market barrier due to the greater complexity and feasibility in digital devices such as programmable logic controllers. Normally, it is due that this control topology has an excessive number of design parameters to be tuned (Kirecci et al., 2003; Bobal et al., 2005).

This paper proposes a digital design of a pseudo-PID controller characterized by the presence of only one parameter to be calibrated. The idea is to show not only the flexibility of using the proposed controller to reduce the commissioning time, but also to give a consistent performance for dynamic systems. The automatic calibration, based on simple tuning guidelines of the pseudo-PID controller, is linked to issues of stability, loop performance and is supported in the following advanced methods: i) self-tuning approach: the controller gain is directly estimated via recursive least-squares, at any operating point of the plant and with a reduced machine cycle for applications in microprocessors; ii) internal model control (IMC) approach: aims to ensure a consistent standard tuning of success in academia and industry to avoid the pursuit of gain by trial and error procedure (IMC technique uses the knowledge about the mathematical model of the controlled plant and the closed-loop dynamic must be specified); iii) small gain theorem approach: sufficient condition for stability, in the frequency domain, is employed to adjust the performance of the closed-loop system in the presence of additive uncertainty. Case studies and experiments are shown.

2. PSEUDO-PID CONTROLLER DESIGN

Most controllers used in industry are PID for many reasons: operational efficiency in closed-loop, programming and installation simplicity as a field device. The PID controller has different structures of implementation that range among manufacturers in terms of tuning, recursive equation, topology, filtering and scaling. The standard structure of the ideal discrete PID control law has the form

$$u(t) = K_c\left\{e(t) + \frac{T_s}{T_i}\sum_{i=1}^{t} e(i) + \frac{T_d}{T_s}\left[e(t) - e(t-1)\right]\right\} \qquad (1)$$

where $e(t) = y_r(t) - y(t)$ is the system error, K_c is the proportional gain, T_i is the integral time, T_d is the derivative time and T_s is the sampling period. The implementation of the incremental PID controller is given by

$$u(t) = u(t-1) + K_c\{e(t) - e(t-1) + \frac{T_s}{T_i}e(t) + \frac{T_d}{T_s}[e(t) - 2e(t-1) + e(t-2)]\} \tag{2}$$

Equation (2), which is appropriate to microcontrollers applications, is present in single-loops and is understandable for digital implementation from viewpoints of operators and engineers (Visioli, 2006). In addition, the proportional and derivative bands appear multiplied by the system error. This has an implication on the performance of the controller because abrupt changes in the reference, also in error, vary instantaneously, producing control actions with excessive magnitudes. This condition can degrade the implementation of the actuator and process dynamic. To avoid practical problems, including loop saturation, the following implementation can be chosen: i) keep the integral term with $e(t) = y_r(t) - y(t)$; ii) substitute to the proportional and derivative terms the error with $e(t) = -y(t)$. So, the ideal digital PID control (2), can be rewritten as

$$u(t) = u(t-1) + K_c\{-y(t) + y(t-1) + \frac{T_s}{T_i}e(t) + \frac{T_d}{T_s}[2y(t-1) - y(t) - y(t-2)]\} \tag{3}$$

that represents the control structure called I+PD (Ang et al., 2005; Bobál et al., 2005; Moudgalya, 2007). Selection of the PID control gains to adequately interfere on the closed-loop dynamic of the controlled plant is a hard task (PiMira et al., 2000; Åström and Hägglund, 2000). The necessity of simple and efficient control algorithms to code in platforms like CLP, DSP, FPGA, and microcontroller applications (that are highlight in the PID controller industry) are evident nowadays (Visioli, 2006).

In order to have a simple practical calibration that not only ensures the stability and the closed-loop performance, but also facilitates tuning task by the operator, a pseudo-PID (PPID) controller is proposed with a single parameter. First, based on the relationship established by H. A. Fertik (Seborg et al., 1989) and J. G. Ziegler and N. B. Nichols (Visioli, 2006) is possible to set

$$\frac{T_s}{T_i} > \frac{1}{100} \quad ; \quad T_i = [2 \ldots 5]T_d \tag{4}$$

Second, from (4) and (3) it is possible to obtain the following normalized expressions:

$$\frac{T_s}{T_d} = 0.4 \quad ; \quad \frac{T_i}{T_d} = 4 \quad ; \quad \frac{T_s}{T_i} = 0.1 \tag{5}$$

Finally, the digital equation of the PPID controller takes the form

$$u(t) = u(t-1) + K_c\{0.1y_r(t) - 3.6y(t) + 6y(t-1) - 2.5y(t-2)\} \tag{6}$$

Some characteristics for the pseudo-PID controller design are: i) there is only one parameter, K_c, to be tuned and, classical (Jury, root locus), optimal or advanced (adaptive, robust, fuzzy, neural) techniques can be applied; ii) this type of control law provides good performance in simple and complex plants (nonlinear); iii) the structure of the PPID equation is appropriate from the viewpoint of implementation in digital technologies (hardware and software) and understanding by plant operators.

2.1 Numerical Results with Plant Models

Next, the PPID controller is evaluated and the parameter K_c, of (6), is adjusted by trial and error in different standard plants proposed in the process control literature (Åström and Hägglund, 2000). These models represent dynamic with simple and complex behaviors found in the industry. The gain K_c, with the mathematical model of each plant, is listed in Table 1.

Table 1. PPID gains for the plants

Plant Characteristic	Plant Model	K_c
multiple equal poles	$G(s)=\frac{1}{(s+1)^4}$	1
fourth order	$G(s)=\frac{1}{(s+1)(0.3s+1)(0.09s+1)(0.027s+1)}$	2
inverse unstable	$G(s)=\frac{(1-5s)}{(s+1)^3}$	0.2
FOPDT	$G(s)=\frac{e^{-s}}{(10s+1)}$	0.5
SOPDT	$G(s)=\frac{e^{-s}}{(5s+1)^2}$	1
heat conduction problem	$G(s)=\frac{0.57s^4+13.59s^3+5.59s^2+0.29s+0.001}{s^5+9.57s^4+24.23s^3+7.61s^2+0.32s+0.001}$	2.5
fast and slow modes	$G(s)=\frac{100}{(s+10)^2}\left(\frac{1}{s+1}+\frac{0.5}{s+0.005}\right)$	6
conditionally stable	$G(s)=\frac{(s+6)^2}{s(s+1)^2(s+36)}$	0.8
oscillatory	$G(s)=\frac{25}{(s+1)(s^2+s+25)}$	0.2
unstable	$G(s)=\frac{1}{(s^2-1)}$	4
integrating	$G(s)=\frac{e^{-s}}{s(10s+1)}$	0.04

In PiMira et al. (2000) these benchmark models were also evaluated for a proposed LS-3000 digital PID controller, with self-tuning and fuzzy properties. Differently from the results presented by the PID controller of PiMira, which did not control the unstable plant and did not test in the heat conduction problem, the PPID controller stabilized all plant models from Table 1 (simulation results are not shown).

2.2 Numerical Results with a Reactor Model

Another case study of numerical simulation, in order to illustrate the implementation feasibility of the PPID controller, is performed in a continuous stirring reactor (CSTR). The following discrete nonlinear equations describe the dynamic of the reactor (Chen and Peng, 1997):

$$x_1(t+1) = x_1(t) + \\ +T_s\left[-x_1(t) + D_a(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma}}\right] \quad (7)$$

$$x_2(t+1) = x_2(t) - T_s x_2(t)(1+\beta) + \\ +T_s\left[BD_a(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma}} + \beta u(t)\right] \quad (8)$$

where $x_1(t)$ and $x_2(t)$ represent the concentration of reactants (dimensionless) and reactor temperature, respectively. The control input $u(t)$ is the dimensionless cooling jacket temperature. Physical parameters of the reactor model are given by: $D_a = 0.072$ (Damköhler number), $\gamma = 0.072$ (activation energy), $B = 8$ (heat of reaction), $\beta = 0.3$ (coefficient of heat transfer), $T_s = 0.2$ s. Figure 1 shows the phase plane of the CSTR, where there are two stable points and an unstable central point. Thus, this type of dynamic behavior is a good challenge for evaluating the performance and efficiency of the proposed PPID control strategy.

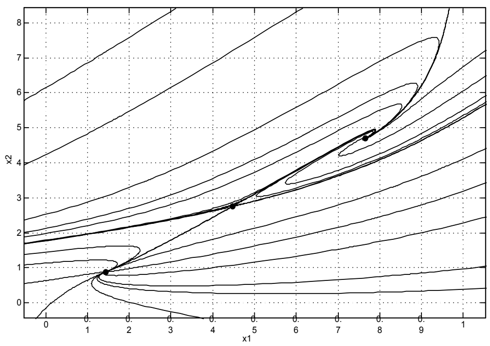

Fig. 1. Phase plane of the reactor.

To analyze the servo dynamic three reference changes are used: $y_r(t) = 1$ (sample 1 to 200), $y_r(t) = 3$ (sample 201 to 400) and $y_r(t) = 6$ (sample 401 to 600). Figure 2 illustrates the output and control of the CSTR system with the PPID controller, with the calibration being $K_c = 3.5$ (adjusted by trial and error). The closed-loop response shows a good servo dynamic behavior in three different operating points with a small control variance.

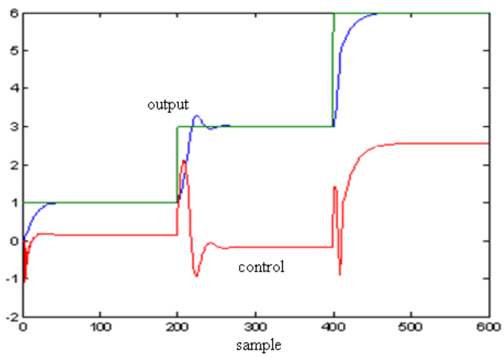

Fig. 2. Servo response of the reactor with PPID controller.

3. TUNING OF PPID CONTROLLER

To avoid the trial and error tuning procedure for K_c and to make the PPID design flexible and automatic, in terms of calibration, operator intervention and dynamic performance, three effective and in evidence control approaches in products already industrially manufactured are derived.

3.1 Tuning for K_c with Self-Tuning Approach

In this proposal, the PPID controller is implemented using a direct adaptive control algorithm (self-tuning strategy). To calibrate K_c the recursive least-squares estimator is used (Kirecci et al., 2003). In this way, (2) can be rewritten as

$$\Delta u(t) = \varphi^T(t)\theta(t) \quad (9)$$

$$\varphi^T(t) = \begin{bmatrix}\Delta e(t) & e(t) & \Delta^2 e(t)\end{bmatrix} \quad (10)$$

$$\theta^T(t) = \begin{bmatrix}K_c & K_c T_s / T_i & K_c T_d / T_s\end{bmatrix} \quad (11)$$

$$\Delta e(t) = e(t) - e(t-1) \quad (12)$$

$$\Delta^2 e(t) = \Delta e(t) - \Delta e(t-1) \quad (13)$$

that represents the recursive tuning. Measurement and estimated parameters vectors, for the scalar case, are given by the following equations:

$$\varphi(t) = 0.1y_r(t) - 3.6y(t) + 6y(t-1) - 2.5y(t-2) \quad (14)$$

$$\theta(t) = K_c \quad (15)$$

The recursive least-squares algorithm with forgetting factor can be directly used when the measurements $\Delta u(t)$ and $\varphi(t)$ are available at time t. Thus, the update of $\theta(t)$, which is the estimative of K_c, can be expressed as

$$\theta(t) = \theta(t-1) + K(t)\left\{\Delta u_r(t) - \varphi(t)\theta(t-1)\right\} \quad (16)$$

$$\Delta u_r(t) = \mathrm{sign}(e(t))\left\{\lambda_e e^2(t) + \lambda_u \Delta u^2(t-1)\right\} \quad (17)$$

$$K(t) = \frac{P(t-1)\varphi(t)}{\lambda + P(t-1)\varphi^2(t)} \quad (18)$$

$$P(t) = \frac{P(t-1)}{\lambda + P(t-1)\varphi^2(t)} \quad (19)$$

The constant λ is called forgetting factor ($0 < \lambda < 1$). For the initialization of $P(0)$ and $\theta(0)$ is useful to consider, in the absence of prior knowledge about the plant dynamic, the following values: $P(0) = \alpha$, with α of magnitude $[10 \ldots 10^6]$ and $\theta(0)$ calibrated with $[0.1 \ldots 0.001]$ (Kirecci et al., 2003; Bobal et al., 2005). The factor λ_e weights the dynamic behavior of the closed-loop system in terms of reference tracking and λ_u regulates the control energy.

3.2 Tuning for K_c with IMC Approach

To evaluate the calibration of PPID controller, which reflects the performance of the closed-loop system, a standard tuning and of interest to the industry is used to derive the fixed gain for the PPID controller. In Morari and Zafiriou (1989) a design methodology for the internal model control has the PID gains based on the following typical models of industrial plants: FOPDT (First-Order Plus Dead-Time), SOPDT (Second-Order Plus Dead-Time) and IPDT (Integral Plus Dead-Time), as shown in Table 2. The design parameter τ_{MF} adjusts the response speed of the closed-loop system (Ravichandran and Karray, 2001; Li et al., 2009). These models can be used to represent a variety of real situations. Using equations of Table 2, the respective IMC for PPID tuning is obtained and the PPID controller gain, for each model, is adjusted in Table 3.

Table 2. IMC Tuning for PID

Model	Tuning
FOPDT $\dfrac{K_p e^{-\theta s}}{\tau s+1}$	$K_c = \dfrac{(2\tau+\theta)}{2K_p(\theta+\tau_{MF})}$ $K_i = \dfrac{K_c 2T_s}{(2\tau+\theta)}$ $K_d = \dfrac{K_c \tau\theta}{T_s 2(\theta+\tau_{MF})}$
SOPDT $\dfrac{K_p e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$K_c = \dfrac{2\zeta\tau}{K_p(\theta+\tau_{MF})}$ $K_i = \dfrac{K_c T_s}{2\zeta\tau}$ $K_d = \dfrac{\tau K_c}{2\zeta T_s}$
IPDT $\dfrac{K_p e^{-\theta s}}{s}$	$K_c = \dfrac{\theta+2\tau_{MF}}{K_p(\theta+\tau_{MF})^2}$ $K_i = \dfrac{K_c T_s}{(\theta+2\tau_{MF})}$ $K_d = \dfrac{K_c^2}{4K_i T_s}$

Table 3. IMC Tuning for PPID

Model	Tuning
FOPDT	$K_c = \dfrac{\tau\theta(2\tau+\theta)}{10K_p(\theta+\tau_{MF})^2 T_s}$
SOPDT	$K_c = \dfrac{(\tau^2+2\zeta\tau T_s+T_s^2)}{3.6K_p(\theta+\tau_{MF})T_s}$
IPDT	$K_c = \dfrac{(\theta+2\tau_{MF}+T_s)}{3.6K_p(\theta+\tau_{MF})^2}$

In this way, this tuning set avoids a calibration for K_c, in practical applications, by the trial and error procedure. Additionally, the settings of Table 3 can give a pre-tuning in self-tuning implementations or start-up commissioning of other industrial loops.

3.3 Tuning for K_c with Small Gain Theorem Approach

In order to ensure stability for the closed-loop system, it is possible to analyze the effect of the tuning parameter K_c in the frequency domain. In this way, the robust stability under the presence of model plant mismatch with the small gain theorem can be analyzed (Banerjee and Shah, 1992). Using the digital equation of the pseudo-PID control, then (6) can be rewritten in the RST canonic structure as follows:

$$R(z^{-1})u(t) = T(z^{-1})y_r(t) - S(z^{-1})y(t) \quad (20)$$

$$R(z^{-1}) = \Delta = 1 - z^{-1}$$

$$S(z^{-1}) = K_c\{3.6 - 6z^{-1} + 2.5z^{-2}\} \quad (21)$$

$$T(z^{-1}) = 0.1K_c$$

where K_c is the parameter to be tuned that not only penalizes the control effort but also adjusts the closed-loop system performance. To evaluate the stability and robustness of the pseudo-PID controller, the RST loop structure, with additive uncertainty, is utilized as shown in Figure 3.

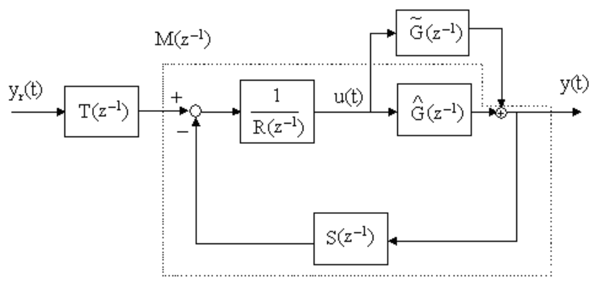

Fig. 3. RST control system with additive uncertainty.

The transfer function $\hat{G}(z^{-1})$ is the plant model and $\tilde{G}(z^{-1})$ is the model uncertainty. The small gain theorem applied to Figure (3) leads to the following sufficient condition for stability:

$$\left|\tilde{G}(e^{-j\omega})\right| < \left|\frac{1}{M(e^{-j\omega})}\right| = \left|\frac{R(e^{-j\omega}) + \hat{G}(e^{-j\omega})S(e^{-j\omega})}{S(e^{-j\omega})}\right| \quad (22)$$

where $M(z^{-1})$ includes the plant and controller models $(\forall\omega \in [0, \pi])$. By using the criterion of (22), the system stability can be evaluated by observing if the curve that represents the uncertainty (MPM – Model Plant Mismatch) is below from the curve that represents $1/M(z^{-1})$. Therefore, the robustness of the system increases as the spectrum of $1/M(z^{-1})$ moves away and up from the spectrum of uncertainty, and consequently, a stable control action for the controlled plant is obtained (detuned behavior).

4. PRACTICAL AND NUMERICAL APPLICATIONS

4.1 Air Flow Control of a Small Wind Tunnel

The first experimental essay covers a process with overdamped behavior and varying loop gain, called wind tunnel (WT), as shown in Figure 4.

Fig. 4. Air flow experimental plant: WT.

Figure 5 shows the input and output responses when the plant is subjected to two reference changes and a load disturbance at *60 s*. Using IMC-PPID tuning of Table 3, for $K_p = 1.21$, $\tau = 1.07$, $\zeta = 1.006$, $\theta = 0$, $T_s = 0.1$ *s*, $\tau_{MF} = 3.5$ *s*, the gain of PPID is $K_c = 0.9$ (SOPDT model of Table 2 is obtained by the reaction curve at the operating point of *3 V*). It can be observed that the PPID control system can stabilize the nonlinear loop in different points with good dynamic for setpoint tracking and disturbance rejection.

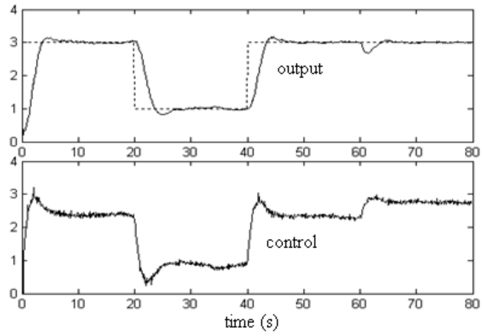

Fig. 5. WT behavior with PPID controller.

4.2 Position Control of a Damped Pendulum

The second experiment uses a nonlinear underdamped plant, called damped pendulum, as shown in Figure 6.

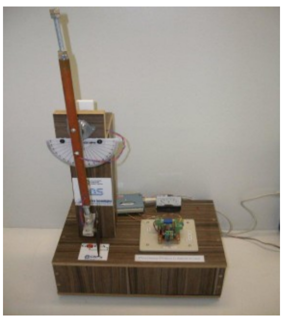

Fig. 6. Position experimental plant: PAM.

The process contains a vertical bar where there is a potentiometer at the pivot point for measuring the angular position. In the extreme of the bar there is a propulsion system consisting of a DC motor and a propeller. When an input voltage is applied, the angular position of the bar is changed. The goal is to position the bar to a specified angle with a desired dynamic. Figure 7 shows the simulation results for the self-tuning PPID controller. The automatic tuning structure uses a conservative initial value for the PPID gain, which varies smoothly, ensuring stability and smooth loop response. The adaptation of the PPID gain can be observed and also the small variance control signal.

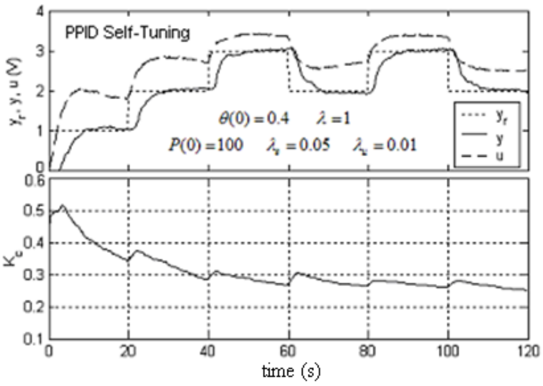

Fig. 7. PAM behavior with PPID controller.

4.3 Linear Plant Control with Robustness Approach

The third simulation considers a continuous stable process given by

$$G(s) = \frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)} \qquad (23)$$

with $K_p = 1$, $\tau_1 = 1$ *s*, $\tau_2 = 3$ *s*, $\tau_3 = 5$ *s* and $T_s = 1$ *s*. A discrete first-order model is employed as

$$\hat{G}(z^{-1}) = \frac{0.0854z^{-1}}{1-0.9163z^{-1}} \qquad (24)$$

in order to assess the robust stability of the PPID controller in the presence of a model uncertainty.

Figures 8 and 9 illustrate the frequency response and closed-loop dynamic behavior of the plant with the PPID controller tuned with the following parameters: $K_c = 3$ and $K_c = 13$. As shown in Figure 8, the spectrum of $1/M(z^{-1})$ does not touches the MPM spectrum and a good control performance is obtained. It is possible to observe that the stability criterion is violated for $K_c = 13$ (plotted in Figure 9), given an instability for the closed-loop plant.

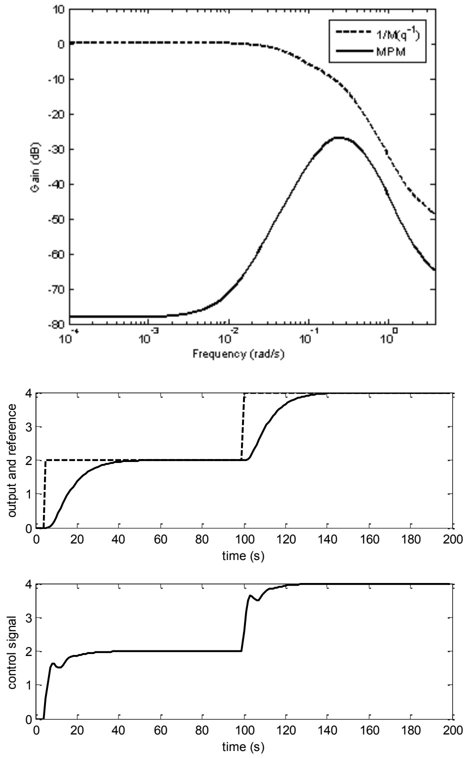

Fig. 8. Stable behavior for PPID with $K_c = 3$.

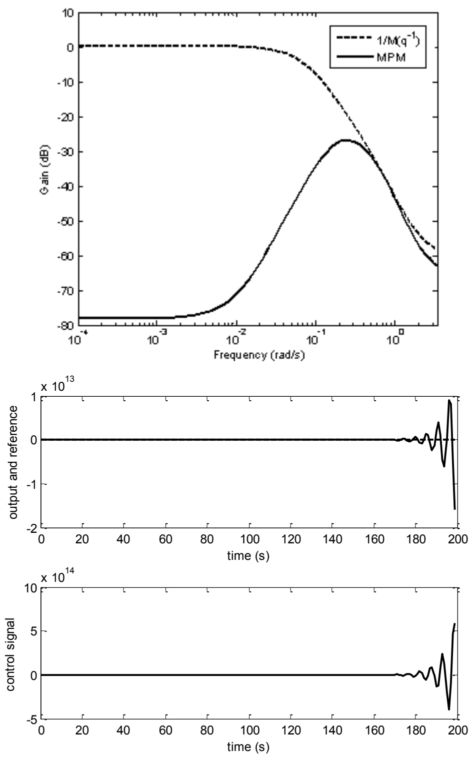

Fig. 9. Unstable behavior for PPID with $K_c = 13$.

5. CONCLUSIONS

The tuning and digital structure to implement PID controllers is a challenge for process control engineers, since there is a dependence on the complexity of the plant and control goals.

In this paper we have proposed a pseudo PID controller design that can be interesting from several viewpoints: as a general purpose device, it provides a good dynamic loop performance, presents one calibration parameter, is simple to implement, easy to use and maintain, is applicable in a variety of plant classes. So, the programming code of the PPID control law, in digital technologies, is easy to perform.

To avoid the trial and error task, automatic tuning procedures, including self-tuning, IMC and frequency criterion, were used for adjusting a single design parameter, in other words, tuning the digital controller and ensuring closed-loop stability.

Future work will include the PPID implementation in multivariable applications in order to verify its suitability in coupled and decoupled systems as a good field device in process control scenarios.

ACKNOWLEDGMENT

This research was supported by the DAS of the Federal University of Santa Catarina, GPqTCA, WEG and CNPq.

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