Design of PID Controller with Filter for Distributed Parameter Systems ⋆

J.D. Álvarez * **J.E. Normey-Rico** ** **M. Berenguel** *

* *Dpto. de Lenguajes y Computación, Universidad de Almería, Campus de Excelencia Internacional Agroalimentario, ceiA3, Ctra. Sacramento s/n, 04120, Almería, Spain (e-mail: jhervas,beren@ual.es)*
** *Depto. de Automação e Sistemas, Universidade Federal de Santa Catarina, 88040-900, Florianópolis, SC, Brazil (e-mail: julio@das.ufsc.br)*

Abstract: This paper presents a method for tuning simple controllers for distributed parameter systems in the process industry. These systems are usually described by partial differential equations (PDE), which are then simplified into lumped parameter systems represented by ordinary differential equations (ODE), providing low order transfer functions used for PID control design. The drawback of this approach is that the control system only performs well at low bandwidths and without disturbances, as relevant system dynamics, such as resonances lying at medium-high frequencies, are not taken into account, limiting the performance of the closed-loop system. This work proposes a method for SISO systems which uses a transfer function that models the resonances (obtained directly from the PDE) to tune a PID controller that incorporates a filter that is designed to fulfill control requirements.

Keywords: Tuneable filters, PID controllers, Distributed parameter systems, Resonance

1. INTRODUCTION

Distributed parameter systems (DPS) are well-known in process industry (e.g. heat exchangers, tubular and packed bed reactors, solar collector systems) and are usually modelled with partial differential equations (PDE) and have been studied since the middle of the last century (Bellman and Cooke (1963); Cohen and Johnston (1956)) until today (Curtain and Morris (2009)).

The most usual ways to deal with these systems and the associate drawbacks are: i) To simplify them to lumped-parameter systems described by ordinary differential equations (ODE). By this way, it is possible to obtain either a first or a second order transfer function to easily tune PID controllers (Camacho et al. (1997)), but in turn, this causes that relevant system dynamics are not taken into account when tuning the PID controller; ii) To find a high order transfer function which is subsequently used in a more complex (high order) control system, e.g. model predictive control (Camacho et al. (1997); Johansen et al. (2000)). Coefficients of this transfer function can be estimated using PRBS or multisine input signals in combination with a linear identification algorithm, but the physical meaning of the parameters is lost.

In this work an irrational transfer function is directly calculated from the PDE and split into two parts. One of them is a low order transfer function, from which it is easy to tune a PID controller (as mentioned before). The other one is an irrational term modelling resonance dynamics in the system response, previously used within an infinite order controller, e.g. repetitive control (Álvarez et al. (2007, 2009)) or a Smith's Predictor like control scheme (Meaburn and Hughes (1996); Ramanathan et al. (1989)). However, not all the resonance modes have the same influence in the closed-loop performance, as in real systems, the amplitude of these resonance modes declines with the frequency and the control requirements set a frequency limit from which the dynamics introduced by the resonant modes, are irrelevant. Therefore, the main objective and contribution of this work is a methodology, for SISO systems, to design a specific filter which is able to cancel those relevant resonance dynamics, reaching, at the same time, the same closed-loop performance achievable by an infinite order controller.

This work is organized as follows. A brief description of DPS model used for control design is described in Section 2. Section 3 is devoted to the controller filter design, while and illustrative example is shown in Section 4. Section 5 summarizes the main conclusions and future research.

2. CONTROL STRATEGY

2.1 System model

DPS, when linearized, can be represented by transfer functions as (Bellman and Cooke (1963); Ramanathan et al. (1989)):

⋆ This work has been funded by the following projects: DPI2010-21589-C05-04 and DPI2011-27818-C02-01 (financed by the Spanish Ministry of Science and Innovation and ERDF funds) and PHB2009-0008 (financed by the Spanish Ministry of Education; CNPq-BRASIL; CAPES-DGU 220/2010).

$$G(s) = P(s) \underbrace{\left(1 - e^{-\theta s} \underbrace{\left(\frac{-\beta s + 1}{\tau s + 1}\right)}_{Q(s)}\right)}_{R(s)} \quad (1)$$

where $P(s)$ and $Q(s)$ are low order rational transfer functions, being $P(s)$ the part of the system without resonance dynamics and $R(s)$ the irrational transfer function which models the resonance dynamics, where θ is usually related to the process residence time. For lack of space, the mathematical development which leads to $G(s)$ is not included here, but it can be found in (Álvarez et al. (2009)). Rational transfer functions can be obtained from $G(s)$ in Eq. (1), first order ones to reproduce the open loop step response of the system (that usually does not exhibit the effect of resonances lying at medium-high frequencies) or high order ones to account for resonances by a rational approximation of $R(s)$. Controllers based on first order models usually have to be detuned when high closed-loop bandwidth is desired, while high order models require more complex control methods.

2.2 Proposed control structure

A methodology which has proved to be easy and effective, is to use the irrational transfer function given in Eq. (1) for control design purposes. A simple PID controller can be designed for the transfer function without resonance dynamics, $P(s)$, using for this aim any of the existing methods in literature (e.g. Rivera et al. (1986); Ziegler and Nichols (1942)). It is necessary to complement the control system with another controller designed to specifically counteract the resonance dynamics modelled by $R(s)$. In (Álvarez et al. (2007, 2009)), a repetitive control structure was used to cancel the resonance dynamics. The repetitive controller has infinity order in continuous time and a good performance in nominal situation, i.e, without model errors, requiring some modifications to account for model uncertainties (Álvarez et al. (2010)).

For the study at hand, an alternative is proposed based on replacing the repetitive controller by a filter specifically designed to counteract the resonance dynamics. Thus, the proposed control structure is shown in Fig. 1, where $Y(s)$, $Ref(s)$ and $D(s)$ are the closed-loop output, the reference signal and the disturbances. $G(s)$, $P(s)$ and $R(s)$ refer to the equivalent transfer functions in Eq. (1), $PID(s)$ is a PID controller and $F(s)$ is a filter specifically designed from the frequency response of $R(s)$. This filter has a high order in continuous time and only counteracts the most relevant resonance dynamics of the system but, it is able to reach the same performance than infinite order controllers such as repetitive ones.

3. PROPOSED FILTER DESIGN

Usually, to deal with the resonance dynamics of the system, a low-pass filter is designed. In closed-loop, this filter is able to smooth the resonance dynamics effects but, at the same time, deteriorates the performance. In this work, the filter is designed to counteract the resonance dynamic effects without reducing the closed-loop performance. The methodology to design the filter uses $R(s)$ and its frequency response analysis.

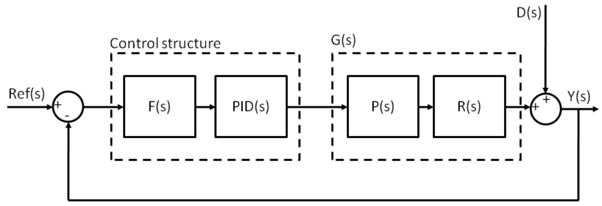

Fig. 1. Proposed control structure

First, the particular case with $Q(s) = 1$ is considered. This is the most simplest case. From this case a methodology to develop the filter is deduced. Later on, this methodology is extended to the general case, i.e. $Q(s) \neq 1$.

3.1 $Q(s) = 1$

Here, there is not any transfer function smoothing the resonance dynamics, since $Q(s)$ acts as a filter. Thus, the resulting transfer function which models the resonance dynamics can be expressed as:

$$R_s(s) = (1 - e^{-\theta s}) \quad (2)$$

The Bode magnitude plot of $R_s(s)$ for a particular case with $\theta = 160$, is shown in Fig. 2. It is possible to appreciate the resonance dynamics which produce that the magnitude suddenly tends to zero. An analysis of its frequency response considering $s = j\omega$ allows to find the frequencies in which these resonance dynamics are located.

$$R_s(j\omega) = \left[1 - e^{-\theta j\omega}\right] = \left[1 - \cos(-\theta\omega) + j\sin(-\theta\omega)\right] \quad (3)$$

The last equation has a real part which is equal to 0 when $\cos(\theta\omega) = 1$ and an imaginary part which is equal to 0 when $\sin(\theta\omega) = 0$. Thus, $\theta\omega = \pi \to \omega = \pi/\theta$ and, therefore, the zeros of the transfer function $R_s(s)$ are located in:

$$s = 0 \pm j\frac{(2\pi)n}{\theta} \;\; ; \forall n > 0 \quad (4)$$

That is, $R_s(s)$ has an infinity number of pairs of complex conjugate zeros. These zeros cause the resonance dynam-

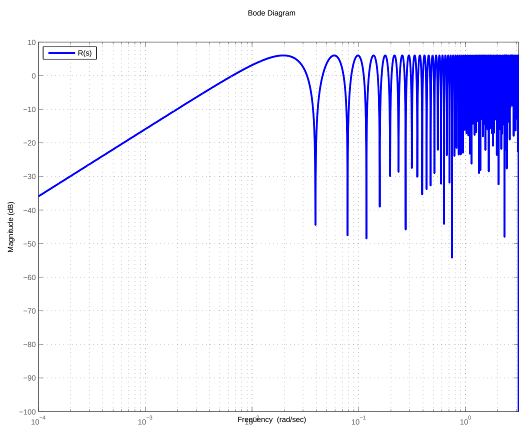

Fig. 2. Bode magnitude plot of $R_s(s)$

ics in the system and can be located as a function of the θ parameter. As commented previously, one way to counteract these resonance dynamics is to use a controller similar to a repetitive control with the form of the inverse of $R_s(s)$. It allows to cancel all the resonance modes but its performance strongly depend on a good estimation of θ parameter. A simple solution consist on defining a filter, $F(s)$, to counteract only the most relevant resonance modes (those lying within the desired closed-loop bandwidth).

The filter needs a pure integrator since $R_s(s)$ has a zero in the origin. On the other hand, Fig. 2 shows that, at high frequencies the magnitude response of the system oscillate around $1(0\ dB)$. Thus, a candidate transfer function base for the filter can be:

$$F_b(s) = \frac{(0.5s + k)}{s} \tag{5}$$

where k is the velocity constant of $R_s(s)$, i.e, $k = \theta$. In Fig. 3, the Bode magnitude plots of $F_b(s)$ and $1/R_s(s)$ are compared. Then, in order to counteract each resonance mode individually, a cosine function is used:

$$c_n(s) = \frac{k_n s}{s^2 + \alpha_n^2} \ \forall\ \alpha_n = (2n)\pi/\theta \quad n \in \mathbb{N}^+ \tag{6}$$

On the other hand, the k_n parameter is used to adjust the resonance mode width. Thus, it is necessary to add as cosine functions as resonance modes one wants to cancel. Figure 4 shows a comparison among the Bode magnitude plot of the inverse of $R_s(s)$, blue solid line, the cosine transfer function to counteract the first and second resonance mode, green and red solid line respectively, and the resulting filter, $F(s)$, composed of the base transfer function $F_b(s)$ plus the cosine functions, cyan solid line. In this picture, it is possible to appreciate as the designed filter, $F(s)$, reproduces accurately the inverse of $R_s(s)$ until the second resonance mode and, therefore, it is able to counteract the dynamics introduced in the system by the first and second resonance modes.

Thus, the methodology to design the filter can be summarized in the following sequence of points:

(1) Find a transfer function for the DPS with two differentiable parts. A low order transfer function without resonance dynamics, $P(s)$, and an irrational transfer function which models the resonance dynamics, $R(s)$.
(2) Design a PID controller for the free resonance dynamic part, $P(s)$. The chosen method to tune the PID can be any of the well-known methods in literature.
(3) Design a base transfer function for the filter, $F_b(s)$, as a function of the velocity constant of $R(s)$, see Eq. (5).
(4) For each resonance mode to be cancelled (depending on the closed loop bandwidth), it is necessary to choose a cosine transfer function tuned as function of θ parameter, see Eq. (6).
(5) Finally, in order to complete the filter, $F(s)$, it is necessary to add the cosine transfer functions to the base transfer function, $F_b(s)$.

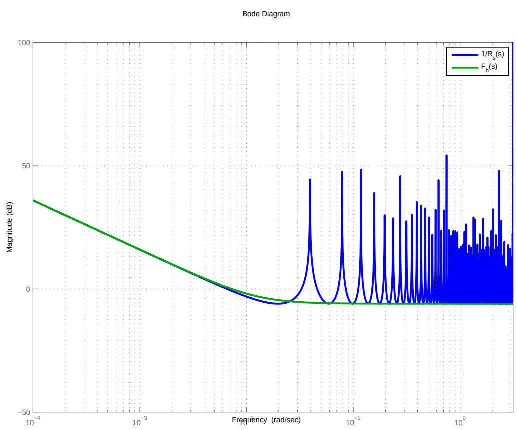

Fig. 3. Bode magnitude plot of $1/R_s(s)$ and $F_b(s)$

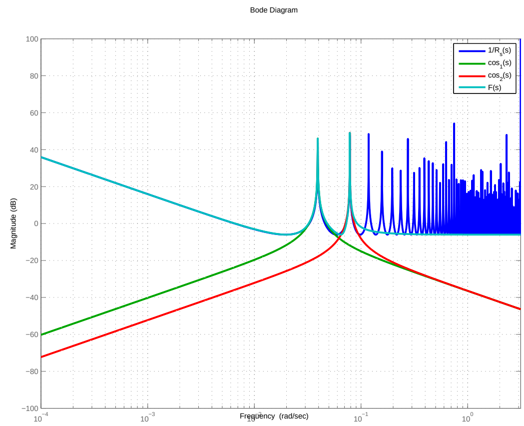

Fig. 4. Bode magnitude plot of $1/R_s(s)$, cosine functions for the first $(c_1(s))$ and second $(c_2(s))$ resonance modes and calculated filter $F(s)$

3.2 Second case. $Q(s) \neq 1$

In this subsection, the methodology previously proposed is extended to the general case, where $Q(s) \neq 1$. It is important to highlight that in this case, the resonance modes are not as abrupt as in the previous case shown in Fig. 2, since the transfer function $Q(s)$ acts as a filter. This is illustrated in Fig. 5, where the Bode magnitude plot of $R(s)$ for a particular case of $\theta = 160$, $\beta = -10$ and $\tau = 50$ is shown. Moreover, these resonance modes are less pronounced in the medium and high frequencies. Unlike the particular case $Q(s) = 1$ where all the resonance modes have the same magnitude, in this case, it is only necessary to counteract the first two or three resonance modes, since those are the most meaningful and that would lie within the closed-loop bandwidth.

The first three steps of the proposed methodology, in the previous section, can be applied in this case in the same way, with the only difference that k, the velocity constant of $R(s)$ and parameter of the base transfer function of the third step, is in this case equal to $k = \theta + \tau + \beta$ (Álvarez et al. (2009)). However, the fact that $Q(s) \neq 1$ complicates the frequency analysis. For this reason, the fourth step has to be modified. The frequency response can be calculated considering $s = j\omega$ in $R(s)$ in Eq. (1):

$$|R(j\omega)| = M(\omega) = \left| \left(1 - e^{-\theta j\omega} \left(\frac{-\beta j\omega + 1}{\tau j\omega + 1}\right)\right) \right| \tag{7}$$

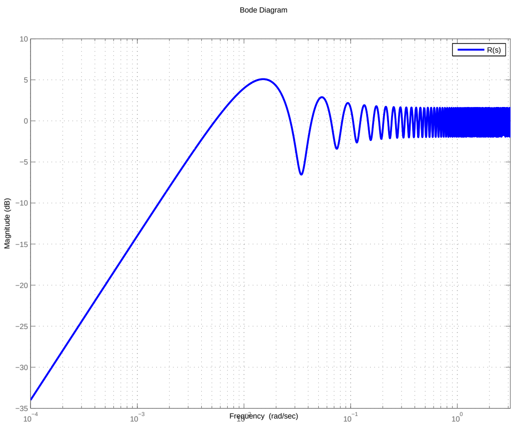

Fig. 5. Bode magnitude plot of $R(s)$

$$M^2(\omega) = 1 + \frac{(1+\beta^2\omega^2)}{(1+\tau^2\omega^2)} + \cos(\theta\omega)2\frac{(\beta\tau\omega^2-1)}{(1+\tau^2\omega^2)} + \\ + \quad \sin(\theta\omega)2\frac{(\omega\tau+\omega\beta)}{(1+\tau^2\omega^2)} \tag{8}$$

From Eq. (8), it is difficult to find the frequencies where the resonance modes are located, i.e. the minima of Eq. (8). Although, these minima must be close to the frequencies where the minima are located when $Q(s) = 1$, see Eq. (4). This can be seen by doing $dM^2(\omega)/d\omega = 0$

$$\frac{dM^2(\omega)}{d\omega} = 1 + 2\frac{\beta^2\omega(1+\tau^2\omega^2) - (1+\tau^2\beta^2)\tau^2\omega}{(1+\tau^2\omega^2)^2} + \\ +\cos(\theta\omega)2\frac{(2\beta\tau\omega)(1+\tau^2\omega^2) - (\beta\tau\omega^2-1)(2\tau^2\omega)}{(1+\tau^2\omega^2)^2} + \\ +\theta\cos(\theta\omega)2\frac{(\omega\tau+\omega\beta)}{(1+\tau^2\omega^2)} - \theta\sin(\theta\omega)2\frac{(\beta\tau\omega^2-1)}{(1+\tau^2\omega^2)} + \\ +\sin(\theta\omega)2\frac{(\tau+\beta)\left((1+\tau^2\omega^2) - \omega(2\tau^2\omega)\right)}{(1+\tau^2\omega^2)^2} = 0 \tag{9}$$

However, the trigonometric expression which appears in Eq. (9) cannot be solved as an algebraical equation. A good approximated solution can be obtained replacing the trigonometric expressions cos and sin for algebraical approximations around the point where the minimum should be found.

$$\sin(\theta\omega) \approx (\theta\omega - (2\pi)n) \ \forall n > 0 \tag{10a}$$

$$\cos(\theta\omega) \approx 1 - \frac{(\theta\omega - (2\pi)n)^2}{2} \ \forall n > 0 \tag{10b}$$

Let n be the resonance mode to be approximated. Thus, replacing sin and cos expressions in Eq. (9) for Eqs. (10a) and (10b) respectively, after simplifications, a polynomial equation can be obtained:

$$\frac{dM^2(\omega)}{d\omega} \approx \sum_{x=0}^{5} a_{n_x}\omega^x = 0 \tag{11}$$

where the a_{n_x} terms ($x = 0..5$) are function of n, β, τ and θ parameters. Then, solving Eq. (11) and choosing the closest root to $(2\pi)n/\theta$, it is possible to find the frequency where the resonance mode is located. As an example, Fig. 6 shows the Bode magnitude plot of $R(s)$, blue solid line, its derivative with respect to ω, green solid line, and two derivative approximations, one for the resonance mode close to $\omega = 2\pi/\theta$, red solid line and with a circle with the same colour marking the frequency where the resonance mode is located, and other for the resonance mode close to $\omega = 4\pi/\theta$, magenta solid line and with other circle with the same colour marking the frequency where the resonance mode is located.

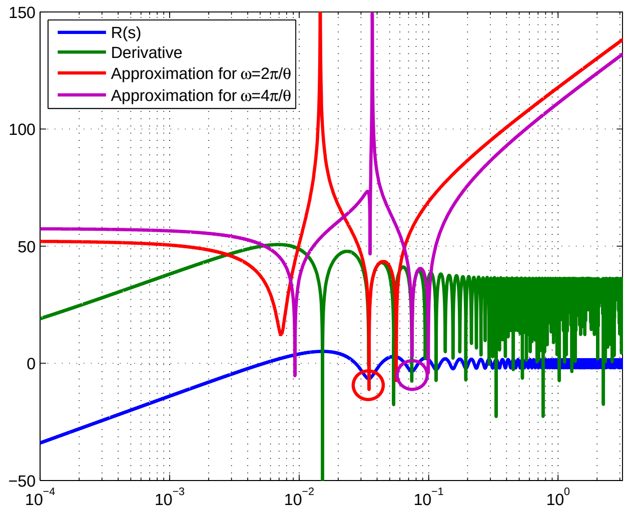

Fig. 6. Bode magnitude plot of $R(s)$, its derivative and the derivative approximations for $\omega = 2\pi/\theta$ and $\omega = 4\pi/\theta$

Comparing Figs. 2 and 5, in the last case the resonance modes are less pronounced, thus a pure cosine transfer function is not necessary to individually counteract each resonance mode, but the following approximation can be used:

$$c_n(s) = \frac{\delta_n s}{s^2 + k_n s + \alpha_n^2} \tag{12}$$

Contrarily to what happens in Eq. (6), three parameters must be estimated in Eq. (12), α_n, k_n and δ_n. As in the previous case, the α_n parameter is directly linked to the frequency where the resonance mode is located (that depends on θ, τ and β), that can be found finding the roots of Eq. (11) as has been commented before. The k_n parameter is used to adjust the width of the resonance mode and, at last, the δ_n parameter is used to set the maximum amplitude of the transfer function in Eq. (12). To obtain its value in absolute terms, just replace in Eq. (8) the value obtained for ω and later, the magnitude value of Eq. (5) in absolute terms and in the same frequency than ω, must be subtracted to the previous obtained result.

Thus, the resulting filter $F(s)$ is composed of the base transfer function $F_b(s)$ in Eq. (5) plus the transfer functions $c_n(s)$ in Eq. (12). Figure 7 shows a comparative among the Bode magnitude plot of the inverse of $R(s)$, blue solid line, the transfer function to counteract the first and second resonance mode, green and red solid line respectively, and the resulting filter, $F(s)$, composed of the base transfer function $F_b(s)$ and the necessary transfer functions to counteract until the third resonance mode,

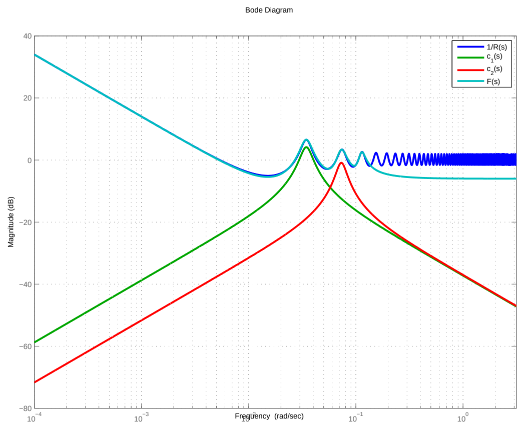

Fig. 7. Bode magnitude plot of $1/R(s)$, transfer function to counteract the first resonance mode, $c_1(s)$, transfer function to counteract the second resonance mode, $c_2(s)$ and calculated filter $F(s)$

cyan solid line. As for the previous case, it is possible to appreciate how the design filter, $F(s)$, reproduces accurately the inverse of $R_s(s)$ until the third resonance mode.

4. CASE STUDY

In this section, an example is presented in order to test the performance of the proposed methodology. The case study is a solar plant based on parabolic through collector technology. This plant can be modelled by the following non-linear model, that describe the energy balance in the fluid, Eq. (13), and in the pipe (subscript w), Eq. (14):

$$A_i \rho C \frac{\partial T}{\partial t} + q \rho C \frac{\partial T}{\partial x} = \pi D_i h_i (T_w - T) \tag{13}$$

$$\rho_w C_w A_o \frac{\partial T_w}{\partial t} = I n_o G - \pi D_o h_o (T_w - T_g) - \\ - \pi D_i h_i (T_w - T) \tag{14}$$

For lack of space, as the meaning of the physical parameters, that appear in the previous equations, as the complete description of the plant cannot be included here but, they can be found in (Camacho et al. (2007)). In a certain operating point (defined by solar irradiance $I = 900\ W/m^2$, ambient temperature $T_g = 25^o C$, fluid flow $q = 0.0005\ m^3/s$, inlet fluid temperature $T_{in} = 200^o C$ and outlet fluid temperature $T = 252^o C$), the relation between the controlled variable T and the control variable q, can be approximated by the transfer function:

$$G(s) = \underbrace{\underbrace{\frac{-430(57.44s+1)}{s(27.44s+1)}}_{P(s)} \left(1 - e^{-105s} \underbrace{\left(\frac{55s+1}{134s+1} \right)}_{Q(s)} \right)}_{R(s)} \tag{15}$$

Three different control systems are compared. The first one is a PI controller tuned through a first order transfer function approximation to the model in Eq. (15) obtained from the frequency response in Fig. 8 (without accounting for resonances). The second one is a PD controller tuned using only $P(s)$ together with the inverse of $R(s)$, similar to that presented in (Álvarez et al. (2007)) using an infinite order controller. Finally, the third one is the same PD controller used in the second case, together with a filter $F(s)$ developed following the methodology presented in the previous section.

In the first case (PI control), the parameters of the first order approximation are given by a static gain $k = -79120\ ^\circ Cs/m^3$ and a time constant $\tau = 90\ s$. From this approximation, a PI controller can be tuned based on the Internal Model Control (IMC) approach (Rivera et al. (1986)). To use the IMC methodology, the parameter ε must be tuned. This parameter, that can be adjusted by the user and determines the speed of the response, is tuned searching a tradeoff between robustness and performance. Thus, if ε is set to $\varepsilon = 50$ and following the tune rules, row A, given in (Rivera et al. (1986)), the values of the proportional gain and the integral time of the PI controller are $k_p = -2.275e^{-5}\ m^3/^\circ Cs$ and $\tau_i = 90\ s$, respectively.

On the other hand, the PD controller is tuned using $P(s)$ in Eq. (15) and using the IMC methodology as well. In this case, row P of the tune rules given in (Rivera et al. (1986)) are used. With the same value for ε parameter than in the PI controller designed previously, the values of the proportional gain and the derivative time are set to $k_p = -4.65e^{-5}\ m^3/^\circ Cs$ and $\tau_d = 27.44\ s$. Moreover, a filter is added to the derivative part of the PD controller in order to: i) obtain a causal transfer function and ii) cancel the effects of the zero in $P(s)$. Thus, the parameter of this derivative filter is $\tau_{df} = 57.44\ s$. This PD controller is complemented in two different ways: i) a transfer function which is the inverse of $R(s)$ in Eq. (15) and, ii) with a filter $F(s)$, which is designed with the aim to counteract the dynamics of the first three resonance modes, following the methodology presented in the previous section.

The closed-loop results of these three control systems, the PI controller, the PD controller together with the inverse of $R(s)$ and the PD controller together the designed filter $F(s)$, at the time to control the non-linear model presented in Eqs. (13) and (14) are shown in Fig. 9. The PI controller,

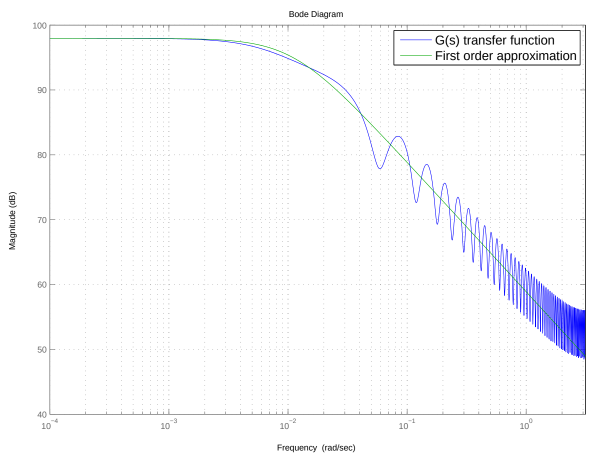

Fig. 8. Bode magnitude plot of $G(s)$, blue solid line, and its first order approximation, green solid line

magenta solid line in the graph, shows oscillations both in tracking and disturbance rejection coming from unmodelled dynamics, not fulfilling control requirements imposed for the ϵ parameter. The PD controller together with the inverse of $R(s)$, red solid line in the graph, is able to almost counteract the resonance dynamics and fulfill the control requirements. Notice that Eq. (15) is an approximation to the nonlinear model in Eqs. (13) and (14) and thus, unmodelled dynamics prevent full compensation of the resonance modes, but they are compensated to a large extend.

The third control system composed by the same PD controller than the previous case together with the designed filter $F(s)$, obtains almost the same results than the previous case (even smoother ones), what was the main objective of this work. Notice that in both cases, the control signals provided by the PD controller with both the inverse of $R(s)$ and the filter $F(s)$, have an oscillatory form in order to obtain an overdamped closed-loop response cancelling the resonances.

5. CONCLUSIONS AND FUTURE WORKS

In this work, a new methodology to design a filter to counteract the effects of resonance dynamics in distributed parameter systems is described. It has been probed that, with a control system composed of only a PID controller, the effects produced by the resonance modes cannot be cancelled, since the PID controller has not enough degrees of freedom to do it. The best solution is to complement the PID controller with a filter, but not with a low-pass filter, since it deteriorates the closed-loop performance. In this work, the filter is specifically designed to counteract the

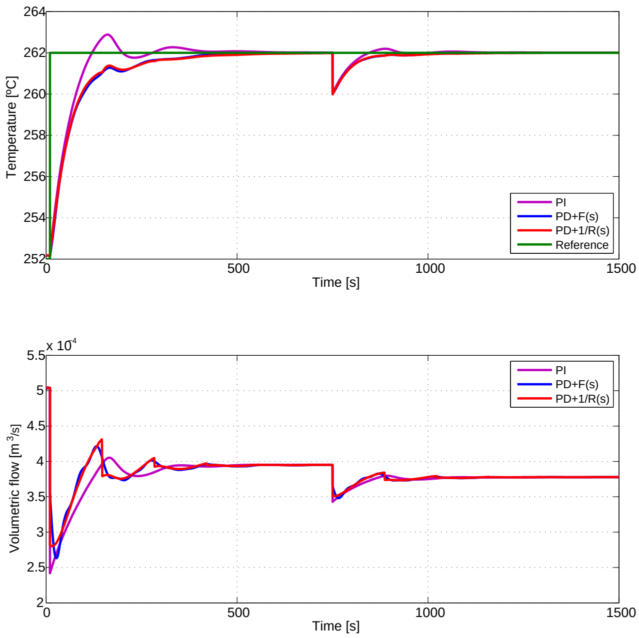

Fig. 9. Closed loop results. System control composed by PI controller, cyan solid line. System control composed by PD controller plus the inverse of $R(s)$, red solid line. System control composed by PD controller plus the filter $F(s)$, blue solid line

of the closed-loop performance. The filter can be tuned with a few parameters, all of them related with the frequencies where the resonance modes to be counteracted are located. Notice that it is not necessary to compensate for all the resonance modes, but the most significant ones. Moreover, the filter has produced almost the same performance than an infinity order controller that cancels all the resonance modes (Álvarez et al. (2007)). Future works are aimed to do the filter more robust in the presence of uncertainty in the frequencies in which the resonance dynamics are located, to include similar filters in disturbance compensation controllers (as done in (Álvarez et al. (2009))), aimed at avoiding the excitation of the resonance modes by the disturbances.

REFERENCES

Álvarez, J.D., Costa-Castelló, R., Berenguel, M., and Yebra, L.J. (2010). A repetitive control scheme for distributed solar collector field. *International Journal of Control*, 83(5), 970–982.

Álvarez, J.D., Yebra, L.J., and Berenguel, M. (2007). Repetitive control of tubular heat exchangers. *Journal of Process Control*, 17(9), 689–701.

Álvarez, J.D., Yebra, L.J., and Berenguel, M. (2009). Adaptive repetitive control for resonance cancellation of a distributed solar collector field. *International Journal of Adaptive Control and Signal Processing*, 23, 331–352.

Bellman, R.E. and Cooke, K.L. (1963). *Differential-difference equations.* Academic Press, New York, USA.

Camacho, E.F., Berenguel, M., and Rubio, F.R. (1997). *Advanced Control of Solar Plants (1st edn).* Springer, London.

Camacho, E.F., Rubio, F., Berenguel, M., and Valenzuela, L. (2007). A survey on control schemes for distributed solar collector fields. part i: Modeling and basic control approaches. *Solar Energy*, 81, 1240–1251.

Cohen, W.C. and Johnston, E.F. (1956). Dynamic characteristics of double-pipe heat exchangers. *Industrial and Engineering Chemistry*, 48, 1031–1034.

Curtain, R. and Morris, K. (2009). Transfer functions of distributed parameter systems: A tutorial. *Automatica*, 45(5), 1101–1116.

Johansen, T.A., Hunt, K.J., and Petersen, I. (2000). Gain-scheduled control of a solar power plant. *Control Engineering Practice*, 17(9), 689–701.

Meaburn, A. and Hughes, F.M. (1996). A simple predictive controller for use on large scale arrays of parabolic through collectors. *Solar Energy*, 56(6), 583–595.

Ramanathan, S., Curl, R.L., and Kravaris, C. (1989). Dynamics and control of quasirational systems. *AIChE Journal*, 35(6), 1017–1028.

Rivera, D.E., Morari, M., and Skogestad, S. (1986). Internal model control. 4. PID controller design. *Industrial and Engineering Chemistry Process Design and Development*, 25, 252–265.

Ziegler, J.G. and Nichols, N.B. (1942). Optimum settings for automatic controllers. *Transactions of the ASME*, 64, 759–768.