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A nonlinear PI controller for optical tweezers system

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Abstract: Positioning an optical tweezer is a complex task due to the inherent nonlinear characteristic of the process and the multiple disturbances that affect its behavior. In this work, a nonlinear PID controller is proposed to address the challenges posed by this task. The control scheme is designed to minimize the effect of uncertainties and external forces that perturb the system. The stability of the closed loop system is analyzed by using standard Lypaunov theory. Some numerical simulations illustrate the robustness of the proposed control scheme under random thermal and constant perturbations.

Keywords: PI controllers, nonlinear control systems, Lyapunov stability, linearization, optical tweezers.

1. INTRODUCTION

The manipulation of microscopic or nanoscale particles is nowadays an important and challenging task in many biophysics applications where the optical trapping (OT) of single molecules, live cells, virus, bacterium, etc, plays a key role in the characterization of their mechanical and superficial structure Lee et. al. (2003), van Mameren (2002).

Aguilar et. al. (2010) have proposed a simple control scheme for manipulating the OT system. This approach uses a Lyapunov analysis to design a nonlinear controller with a saturation function to perform the stabilization of the system, but it does not consider the effect of constant disturbances. Constant perturbations are produced by physical conditions given by the molecule, medium or the tweezers themselves. We consider that the laser beam produce a potential field with a Gaussian distribution, Aguilar et. al. (2010), Lee et. al. (2003), Ibañez (2008).

In this paper, we propose a nonlinear control scheme based on a nonlinear PI controller for manipulating the position of the particle. The control structure considers an integral action in order to deal with constant disturbances and changes in the desired final positions. The stability of the closed loop is analyzed by using the standard Lyapunov theory, Khalil (2002).

The paper is organized as follows. In Section 2, the physical model based on the motion equations of the OT system is described. In Section 3, the control scheme is presented and the stability of the closed loop is analyzed. In Section 4, some numerical simulations of the open and closed loop systems, illustrate the effectiveness of the proposed control scheme. Finally, some conclusions and future work are summarized in section 5.

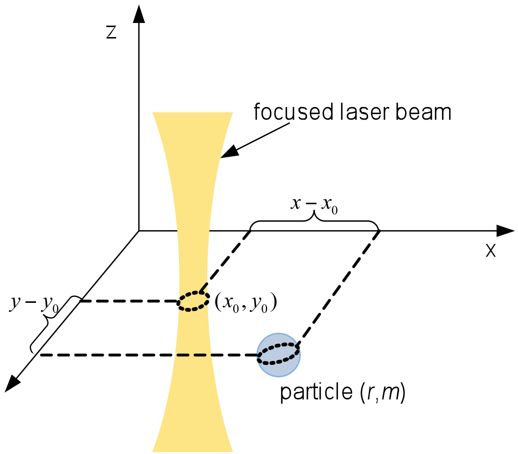

Fig.1. OT system with the coordinates system.

2. PHYSICAL MODEL OF AN OT SYSTEM

There are many models to describe an OT system. In this work, we consider the one where the trapping is produced by a potential field with a Gaussian distribution [1], [2], [4]. Let x and y be the horizontal and vertical position of the particle, we consider that there is not movement in the z axis. The control action can be carried out by focusing the laser beam in the position (x_0, y_0). Fig.1 shows a simple scheme of the OT system where a particle has a radius r and mass m.

The nonlinear model is then given by:

$$m\ddot{x} + [2\ln(2)p_0(x-x_0)/a^2] \cdot\exp\left[-\ln(2)\left((x-x_0)^2/a^2 + (y-y_0)^2/b^2\right)\right] = -\gamma\dot{x} + F_x(t), \quad (1)$$

$$m\ddot{y} + [2\ln(2)p_0(y-y_0)/b^2] \cdot\exp\left[-\ln(2)\left((x-x_0)^2/a^2 + (y-y_0)^2/b^2\right)\right] = -\gamma\dot{y} + F_y(t). \quad (2)$$

The left terms of these equations represent the inertial forces and the one produced by the potential field, where p_0 is related to the intensity of the beam and a and b are geometrical parameters of the beam waist. In the right side term, the first term represents the drag force produced by the medium and $\gamma > 0$ is the damping force coefficient; which can be estimated using the Stokes equation as $\gamma = 6\pi\rho r$ with ρ the viscosity of the medium and r the radius of the particle. The second terms are random thermal forces known as Langevin forces, they have a zero mean value and a constant power spectrum [1], and they can be modeled by

$$F_x(t) = \sqrt{4k_b T\gamma}\eta_x(t),\ \ F_y(t) = \sqrt{4k_b T\gamma}\eta_y(t)\,, \qquad (3)$$

with k_b the Boltzman constant, T the absolute temperature, $\eta_x(t)$ and $\eta_y(t)$ two independent white noise signals, this noise is modelated, as reported by Ranaweera [1], with a bandwidth of 10 kHz.

3. SYSTEM STABILIZATION

First the open loop stability is analyzed and then the controller is designed. The closed loop stability is analyzed by using standard Lyapunov arguments.

3.1 Open Loop Analysis

First we make a change of variables to obtain a standard representation in the form $\dot{\mathbf{w}} = \boldsymbol{f}(\mathbf{w})$ with $w_1 = x,\ \ w_2 = y,$ $w_3 = \dot{x},\ w_4 = \dot{y}$ and

$$\dot{\mathbf{w}} = \begin{pmatrix} w_3 \\ w_4 \\ -K_a(w_1 - x_0)f_{\exp} - K_d w_3 \\ -K_b(w_2 - y_0)f_{\exp} - K_d w_4 \end{pmatrix}, \qquad (4)$$

with $K_a = 2\ln(2)p_0/(ma^2), K_b = 2\ln(2)p_0/(mb^2)$, $K_d = \gamma/m$ and $f_{\exp}$ a function containing the exponential term. Now, it can clearly be seen that system (4) has three equilibrium points, $\mathbf{w}_{1,2} = (\pm\infty,\pm\infty,0,0)$ and $\mathbf{w}_3 = (x_0, y_0, 0, 0)$. Reference [2] demonstrates that the first two equilibrium points are unstable. The third equilibrium point is stable and [2] also provides information about the maximum distance between the particle and potential well so that the laser will trap the particle.

3.2 Control Design

In this work, we consider not only system (4) but also the perturbation forces (3) and a constant disturbance signal. We note this force $F_p(t) = |F_p| = const. \geq 0$. Before proposing the control scheme the following characteristics of this system must be taken into consideration:

- The nonlinear term in the original system is an exponential function bounded away from zero in a region of interest, and therefore it can be easily canceled using exact linearization [6].
- The magnitude of the constant perturbation is unknown and it can also appear in any moment; hence an integral action would be useful to suppress its effect.

The proposed controller has the following structure:

$$u_x = x - x_0 = \frac{K_{px}e_x + K_{ix}\sigma_x}{\hat{f}_{\exp}},\ u_y = y - y_0 = \frac{K_{py}e_y + K_{iy}\sigma_y}{\hat{f}_{\exp}}, \qquad (5)$$

where $e_x = x - x_f,\ e_y = y - y_f$ are the tracking errors between the actual and the desired positions, $\dot{\sigma}_x = e_x, \dot{\sigma}_y = e_y$ are the integral part and $K_{px}, K_{py},\ \ K_{ix}, K_{iy}$, are the gains of the controllers, and $\hat{f}_{\exp}$ is an estimation of $f_{\exp}$. Therefore replacing the control action into (1) and (2) we obtain:

$$\ddot{e}_x + K_a K e_x + K_a \sigma_x = -K_d \dot{e}_x + P_x(t)\,, \qquad (6)$$

$$\ddot{e}_y + K_a K e_y + K_a \sigma_y = -K_d \dot{e}_y + P_y(t)\,, \qquad (7)$$

where $P_x(t) = (F_x(t) + F_p(t))/m$ and $P_y(t) = (F_y(t) + F_p(t))/m$.

3.3 Closed loop Analysis

Under this control structure the closed loop dynamics of the coordinates are decoupled, and therefore the stability analysis will only consider just the x coordinate, since the same analysis can be carried out for y coordinate.
Rewriting equation (6) by using the auxiliary variables $x_1 = \sigma_x, x_2 = e_x,$ and $x_3 = \dot{e}_x,$ the closed loop equation becomes:

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_a K_{ix} & -K_a K_{px} & -K_d \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} P_x(t)\,, \qquad (8)$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}P_x(t), \qquad (9)$$

where $\mathbf{x} = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$. The matrix $\mathbf{A}$ can designed to be Hurwitz matrix by choosing suitable values for the controller gains. In order to carry out an analysis of the perturbed system (8) we propose the use of a quadratic Lyapunov function $V_{cl}(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$, where $\exists\mathbf{P}$ is a positive definite matrix that can be calculated from the Lyapunov equation $\mathbf{PA} + \mathbf{A}^T\mathbf{P} = -\mathbf{Q}$. The time derivative of the Lyapunov function along the trajectories of the system (8) is the form:

$$\dot{V}_{cl} = \tfrac{\partial V_{cl}}{\partial \mathbf{x}}\mathbf{A}\mathbf{x} + \tfrac{\partial V_{cl}}{\partial \mathbf{x}}P_x(t)\,, \qquad (10)$$

where the different terms can be bounded as:

$$\tfrac{\partial V_{cl}}{\partial \mathbf{x}}\mathbf{A}\mathbf{x} = -\mathbf{x}^T\mathbf{Q}\mathbf{x} \leq -\lambda_{\min}(\mathbf{Q})\,\|\,\mathbf{x}\,\|_2^2\,, \qquad (11)$$

$$\left\|\tfrac{\partial V_{cl}}{\partial \mathbf{x}}\right\|_2 = \|\,2\mathbf{x}^T\mathbf{P}\,\|_2 \leq 2\,\|\,\mathbf{P}\|_2\,\|\,\mathbf{x}\,\|_2 = 2\lambda_{\max}(\mathbf{P})\,\|\,\mathbf{x}\,\|_2\,, \qquad (12)$$

$$\|\,P_x(t)\,\| \leq \sqrt{4k_b T\gamma} + |\,F_p\,| = \delta_x\,, \qquad (13)$$

then (10) can be bounded as:

$$\dot{V}_{cl} \leq -\lambda_{\min}(\mathbf{Q})\,\|\,x\,\|_2^2 + 2\lambda_{\max}(\mathbf{P})\delta_x\,\|\,x\,\|_2\,, \qquad (14)$$

Thus $\dot{V}_{cl} \leq 0$ for all $\|\,x\,\|_2 \geq \mu = 2\lambda_{\max}(\mathbf{P})\delta_x/\lambda_{\min}(\mathbf{Q})$, and therefore the solution of the perturbed dynamic (8) is uniformly ultimately bounded with an ultimate bound

$$\|x\| \leq \frac{2\delta_x\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{Q})}\sqrt{\frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})}}, \qquad (15)$$

Since the above controller is designed considering perfect knowledge of the exponential term, it is necessary to carry out an analysis concerning the robustness of the proposed controller under uncertain nonlinear term. The analysis again considers the x coordinate. In this case, the closed loop system (6) is:

$$\ddot{e}_x + K_a K e_x + K_a \sigma_x + (\Psi - 1)(K_a K e_x + K_a \sigma_x) = -K_d \dot{e}_x + P_x(t), \quad (16)$$

where $\Psi = f_{\exp} / \hat{f}_{\exp}$. Writing (8) in matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \Delta\mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}P_x(t), \quad (17)$$

where,

$$\Delta\mathbf{A}(\mathbf{x}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -K_a K_{ix} & -K_a K_{px} & 0 \end{pmatrix} (\Psi - 1). \quad (18)$$

By performing the same analysis as in the nominal case, but assuming that this new perturbation term can be bounded as follows $\| \Delta\mathbf{A}(\mathbf{x}) \|_2 \leq \alpha$, $\alpha > 0$, and using the same Lyapunov function as before, the new condition for establishing the asymptotic stability of the error dynamic is:

$$\lambda_{\min}(\mathbf{Q}) - 2\lambda_{\max}(\mathbf{P})\alpha > 0. \quad (19)$$

Thus given that (19) is satisfied, we have that $\dot{V}_{cl} \leq 0$ for all $\| x \|_2 \geq \mu = 2\lambda_{\max}(\mathbf{P})\delta_x / (\lambda_{\min}(\mathbf{Q}) - 2\lambda_{\max}(\mathbf{P}))$, and therefore the solution of the perturbed dynamic (17) is uniformly ultimately bounded with an ultimate bound

$$\|x\| \leq \frac{2\delta_x \lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{Q}) - 2\alpha\lambda_{\max}(\mathbf{P})} \sqrt{\frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})}}. \quad (20)$$

The estimated ultimate bounds (15) and (20) can be quite conservative since the structure of the perturbation has not been considered.

4. SIMULATION RESULTS

Numerical simulations are carried out by considering the control scheme proposed (8), and using the original system (4). The simulations are dived in three parts. The first one considers the open loop simulation of a particle with a given initial condition (near enough to the beam for being trapped by the potential field). The second one considers both open and closed loop simulations being the particle in the focus of the laser beam under random-thermal perturbation.

Finally, the proposed control strategy is tested by considering a step change in the desired positions. In addition, a constant disturbance is also considered. Open and closed loop simulations are depicted in the same figures to gain insight about the effectiveness of the control strategy. Also, simulations are compared to the closed loop strategy proposed by Aguilar et. al. (2010), more details and stability analysis of this strategy can be found in the reference.

In order to test the robustness of the controller some simulations considering changes in the viscosity of the medium and also mismatch in the nonlinear terms are included.

The parameters used in the simulations are $m = 1\times10^{-16}$(kg), $a = b = 1\times10^{-5}$(m), $r = 2\times10^{-7}$(m), $p_0 = 1\times10^{-15}$(J), γ was estimated using the Stokes equation and considering that the medium is water at ambient temperature (20(°C)) we have that $\rho = 10^{-3}$(kg / ms), so $\gamma = 3.7699\times10^{-9}$(kg / s).

The controller gains were set $K_{px} = K_{py} = 50$ and $K_{ix} = K_{iy} = 50$, and $\mathbf{Q} = \mathbf{I}$ since this choice maximizes the ratio $\lambda_{\min}(\mathbf{Q}) / \lambda_{\max}(\mathbf{P})$ [6]. These values give the following **P** matrix

$$\mathbf{P} = \begin{pmatrix} 62 & 1839 & 3.6067\times10^{-12} \\ 1839 & 9193,2 & 2.6533\times10^{-10} \\ 3.6067\times10^{-12} & 2.6533\times10^{-10} & 1.3263\times10^{-8} \end{pmatrix}, (15)$$

having eigenvalues $\lambda_s(\mathbf{P}) = [1.3263\times10^{-8} \quad 25 \quad 91968]$.

In the first simulation example, the initial condition was $\mathbf{w}(0) = (2.0\times10^{-5}, -2.0\times10^{-5}, 0, 0)$ and $(x_0, y_0) = (0,0)$.

Fig.2 shows the particle trajectory as it is attracted to the origin where the center beam is.

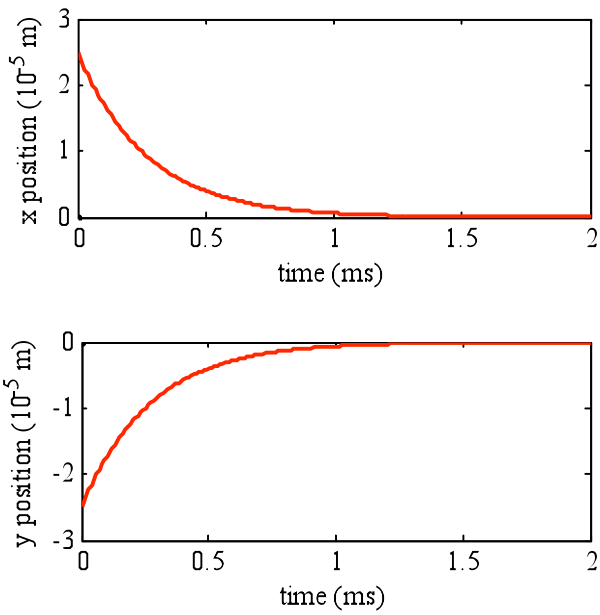

Fig.2. Open loop response of the OT system.

The random thermal noise is simulated using (3) and the constant perturbation with amplitude proportional to the amplitude of the Langevin forces. Fig. 3 shows how the control strategy minimizes the effects of the disturbances acting over the system.

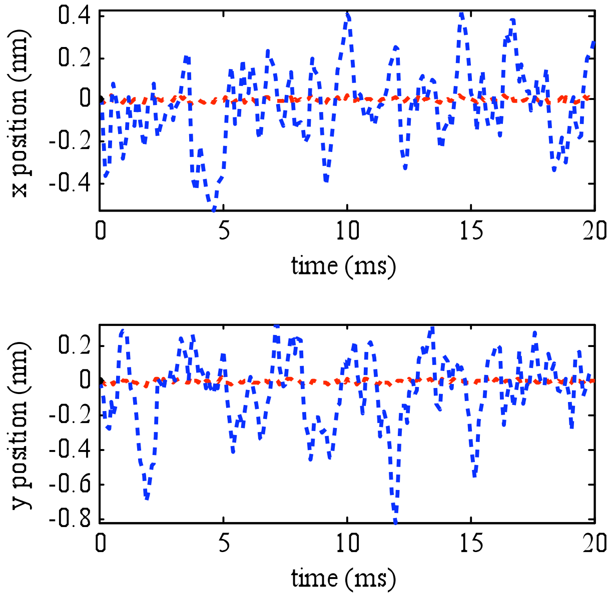

Fig.3. Open and closed loop simulation without constant perturbation and without position displacement. Closed loop (red) and open loop (blue).

Finally the trajectories of the particle from $(x_0, y_0) = (0,0)$ to a final position $(x_f, y_f) = (2.0\times10^{-10}, -2.0\times10^{-10}, 0, 0)$ are simulated. The simulation of the control strategy considers a step change of magnitude equal to the desired rest positions.

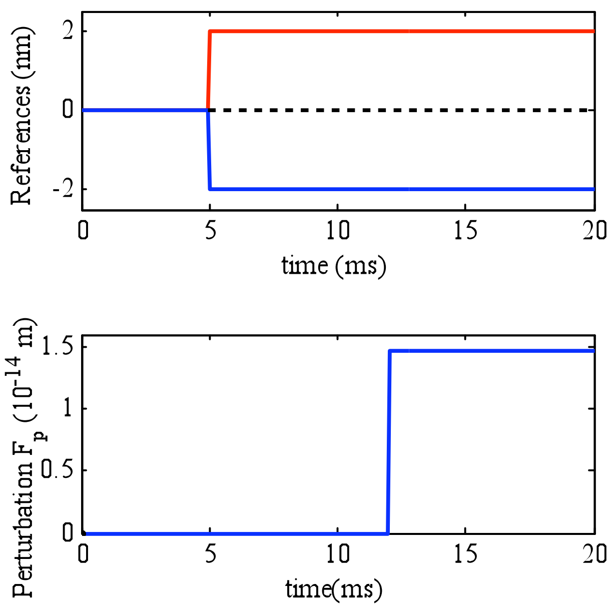

Fig.4. Reference and constant disturbance signal.

Fig.4 shows the step changes in both the reference and perturbation.

In Fig.5, the proposed controller stays near the final position whereas the open loop does not follow the reference when the constant perturbation appears.

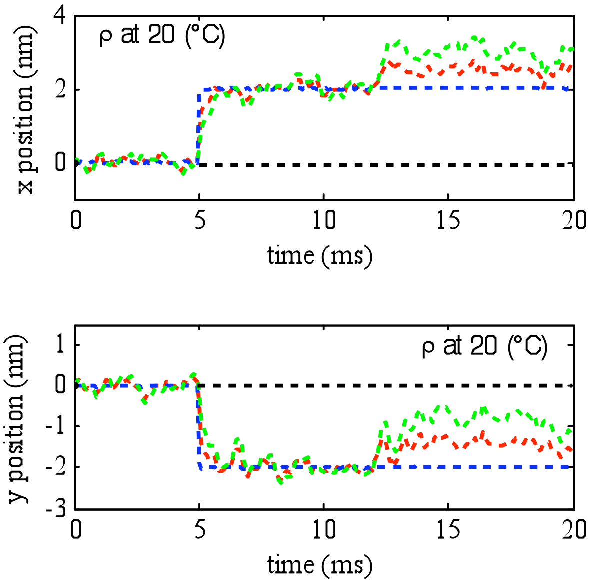

Fig.5. Open and closed loop simulation with constant disturbance and position displacement. Closed loop (blue), closed Aguilar et. al. (2010) (red), and open loop (green).

Some simulations considering changes in the viscosity of water, due to e.g. some temperature perturbation are also included. These changes are associated to changes in the value of γ ,

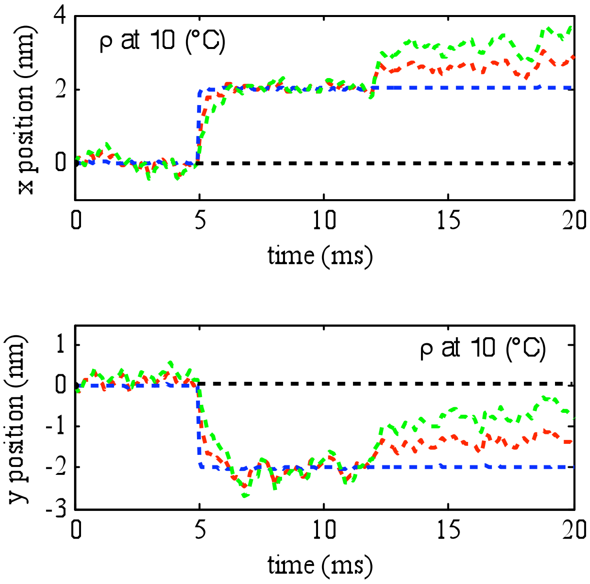

Fig.6. Open and closed loop simulation with changes in the viscosity of water at 10 °C. Closed loop (blue), closed loop Aguilar et. al. (2010) (red), and open loop (green).

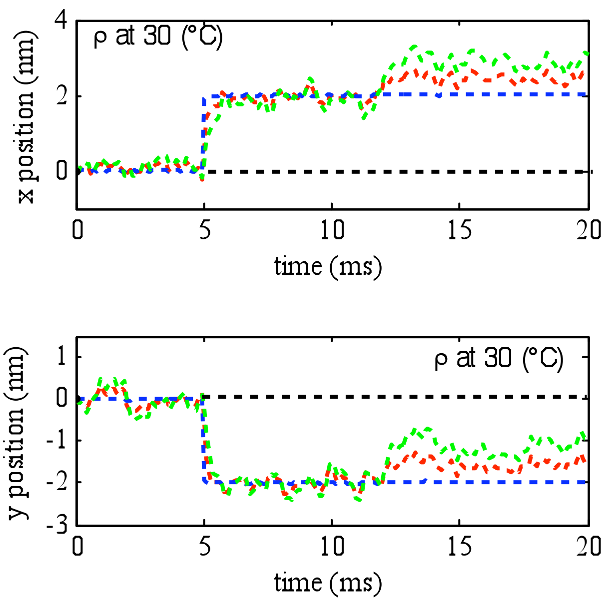

Fig.7. Open and closed loop simulation with changes in the viscosity of water at 30 °C. Closed loop (blue), closed loop Aguilar et. al. (2010) (red), and open loop (green).

Fig.6 and 7 show that there is not a big difference between the nominal closed loop response and the one considering a different viscosity. Future work will address the problem tuning the controllers considering uncertainties in this parameter.

Finally, the robustness of the system is tested when the exponential terms is partially known. In this example, the parameter a and b of the exponential function in the controller law are considered to have the following values $\hat{a} = \hat{b} = 1.2\times10^{-5}\,\mathrm{m}$. The same control strategy is used this time. The results, Fig. 8, show that the stability is preserved.

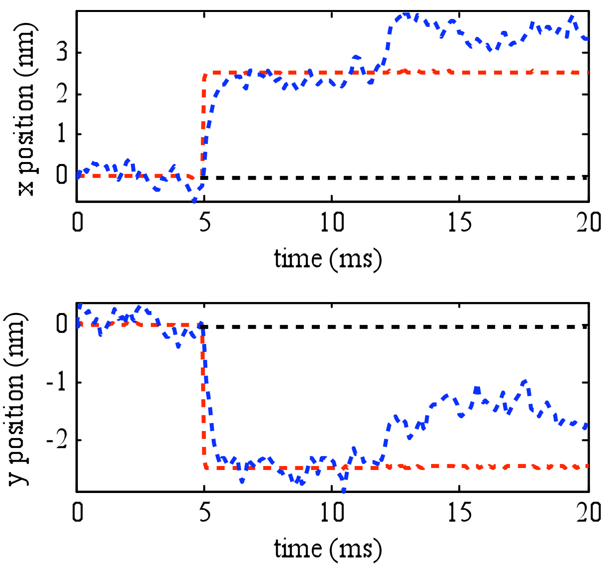

Fig.8. Open and closed loop simulation with no exact linearization performed by the controller. Closed loop (red) and open loop (blue).

6. CONCLUSIONS

In this work, a simple nonlinear PI control scheme for OT systems has been proposed to deal with the problem of constant disturbances, stationary reference tracking and changes in parameters. Standard Lyapunov theory shows that this approach provides stable closed loop behaviour. Simulation results illustrate its capabilities to deal with random thermal and constants perturbations produced by the medium or the particle *per se*. These results also show the big difference between the open and closed loop responses when a nonlinear PI controller is used. Further work is underway to establish tighter ultimate bounds and develop control design procedures taking into account uncertainties and bounded control signals.

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