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Spectral Abscissa Minimization when Algebraic Control of Unstable LTI-TDS

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Abstract: Optimal pole assignment minimizing the spectral abscissa when algebraic control of linear time-invariant time delay systems (LTI-TDS) is focused in this paper. We concentrate on algebraic controller design approach in the R_{MS} ring resulting in delayed controllers as well. In the case of unstable delayed plants, the use a simple feedback loop results in a characteristic quasipolynomial instead of polynomial is obtained which means that the closed loop has an infinite spectrum. Thus, it is not possible to place all feedback poles to the prescribed positions exactly by a finite number of free controller parameters. The pole placement problem is translated to the minimization of the spectral abscissa which is a nonsmooth nonconvex function of free parameters in many cases. We initially solve the problem via standard quasi-continuous shifting algorithm followed by a comparative utilization of three iterative optimization algorithms; namely, Nelder-Mead algorithm, Extended Gradient Sampling Algorithm and Self-Organizing Migration Algorithm. Simulation control of an unstable LTI-TDS - the roller skater on the swaying bow - serves as an illustrative example for the algebraic control with the spectral abscissa minimization.

Keywords: Algebraic approaches, Time delay, Pole assignment, Parameters optimization, Minimization, Numerical methods.

1. INTRODUCTION

Expedient control of linear time-invariant time delay systems (LTI-TDS) has been a challenging task in control theory for decades, which is apparent from several books, conferences and journal publications dedicated to this topic, see e.g. Byrnes et al. (1984), Górecki et al. (1989), Loiseau (2000), Richard (2003), Partington and Bonet (2004), Michiels and Vyhlídal (2005), Michiels et al. (2010), Vyhlídal et al. (2010), Pekař and Prokop (2011a), etc. A modern way how to cope with the problem, among many others, is based on algebraic parlance and tools like rings and linear equations. Concerning an input-output description of single-input single-output (SISO) systems, a transfer function representation in the form of quasipolynomial fractions is not suitable for controller design.

A possibility is to introduce so called pseudopolynomials, Loiseau (2000), or a quasipolynomial fraction (meromorphic) function representation can be extended to any type of the fractional description, Kučera (1993). In order to meet natural requirements of asymptotical stability and controller properness (realizability), one may introduce the ring of stable and proper quasipolynomial (RQ) meromorphic functions ($\mathbf{R}_{MS}$), see Zítek and Kučera (2003), Pekař and Prokop (2011b). Originally, the ring was developed for retarded systems only; however, the conception can be easily extended to neutral ones, Hale and Verduyn Lunel (1993).

Control design in this ring employing the Bézout identity to along with the Youla-Kučera parameterization yields an infinite-dimensional feedback system, in the case of an unstable controlled plant in a simple feedback loop. In other words, a characteristic quasipolynomial instead of a polynomial is obtained which decides about the stability (in most cases) as usual for finite-dimensional systems.

A natural task regarding controller parameters tuning, assignment of all feedback poles to the prescribed positions exactly by a finite number of free controller parameters, is hence unsolvable. We can place exactly as many poles (including their multiplicities) as free controller parameters are only, see e.g. Zítek and Vyhlídal (2008). Alternatively, it is possible to place some dominant poles and move the rest of the spectrum to the left, Michiels et al. (2010), Pekař and Prokop (2011c). Or, which is the aim of this paper, to translate the pole placement problem into the minimization of the spectral abscissa which is a nonsmooth nonconvex function of free parameters in many cases, Michiels et al. (2002), Michiels and Vyhlídal (2005), Vanbiervliet et al. (2008).

In this contribution, we originally combine the algebraic controller design for an unstable LTI-TDS determining the controller structure with the (quasi)optimal poles shifting. We initially solve the tuning problem via standard quasi-continuous shifting algorithm (QCSA), see Michiels et al. (2002), followed by a comparative utilization of three iterative optimization algorithms; namely, Gradient Sampling Algorithm (GSA), Burke et al. (2005), Nelder-Mead algorithm (NM), Nelder and Mead (1965), and Self-Organizing Migration Algorithm (SOMA), Zelinka (2004). The last two enumerated methods have not been used while solving the task yet.

The correctness and usability of the designed controller and parameters optimization via the spectral abscissa minimization are tested on a simulation control of the unstable LTI-TDS - the roller skater on the controller swaying bow, Zítek et al. (2008).

2. CONTROLLER DESIGN IN RMS RING

As we are focused on Laplace transform models, they are subjected below. A general case of non-commensurate or rationally unapproximated delays results in a fraction of quasipolynomials $a(s)$, $b(s)$ as follows

$$G(s)=\frac{b(s)}{a(s)} \tag{1}$$

where a general quasipolynomial has the form

$$x(s)=s^n+\sum_{i=0}^{n}\sum_{j=1}^{h_i}x_{ij}s^i\exp(-s\eta_{ij}),\eta_{ij}\geq 0 \tag{2}$$

If $\sum_{j=1}^{h_n}x_{nj}\exp(-\eta_{nj}s)\neq$ constant holds for the denominator quasipolynomial, a neutral system is given; otherwise, the system is retarded. Note that retarded systems only are considered in this paper; however, ring definition and controller design are applicable for a neutral case as well.

However, a meromorphic (i.e. quasipolynomial fraction) transfer function representations are not suitable in order to satisfy some basic control requirements, e.g. controller feasibility, feedback H_∞ stability, etc., see Loiseau (2000). Rather more general approaches, Vidyasagar (1985), Kučera (1993) utilize a field of fractions where a transfer function is expressed as a ratio of two coprime elements of a suitable ring.

The ring of stable and proper RQ-meromorphic functions ($\mathbf{R}_{MS}$) is one of those powerful algebraic tools. Since the original definition of $\mathbf{R}_{MS}$ in Zítek and Kučera (2003) does not constitute a ring, some minor changes in the definition were made in Pekař and Prokop (2009), Pekař (2011c). A term $T(s)\in\mathbf{R}_{MS}$ ring is represented by a proper ratio of two quasipolynomials $y(s)/x(s)$ where $x(s)$ is of degree n and $y(s)$ can be factorized as

$$y(s)=\tilde{y}(s)\exp(-\tau s) \tag{3}$$

where $\tilde{y}(s)$ is a quasipolynomial of degree l and $\tau\geq 0$. Note that the degree of a quasipolynomial means its highest s-power. $T(s)$ is analytic and bounded in $\mathbb{C}^+$, particularly, there is no pole s_0 such that $\operatorname{Re}s_0\geq 0$. Thus, it lies in the space $H_\infty(\mathbb{C}^+)$ providing the finite norm defined as

$$\|T\|_\infty:=\sup\{|T(s)|:\operatorname{Re}s\geq 0\} \tag{4}$$

It is said that $T(s)$ is H_∞ stable, Partington and Bonet (2004). Notice, for instance, that $T(s)$ having no pole in the right-half complex plane but with a sequence of poles with real part converging to zero can be H_∞ unstable due to an unbounded gain at the imaginary axis. Moreover, $T(s)$ is also stable in the strong sense, see details in Hale and Verduyn Lunel (1993), which means that

$$\sum_{j=1}^{h_n}|x_{nj}|<1 \tag{5}$$

holds for $x(s)$. In addition, the ratio is proper, i.e. $l\leq n$. More precisely, there exists a real number $R>0$ for which holds that

$$\sup_{\operatorname{Re}s>0,|s|\geq R}|T(s)|<\infty \tag{6}$$

Hence, factorize (1) as

$$G(s)=\frac{B(s)}{A(s)},A(s),B(s)\in\mathbf{R}_{MS} \tag{7}$$

The ratio is Bézout coprime, i.e. there is non-trivial (non-unit) common factor of both elements, and moreover it holds that

$$\inf_{\operatorname{Re}s\geq 0}(|A(s)|,|B(s)|)>0 \tag{8}$$

A plant must be formally stable, see details in Loiseau et al. (2002).

Controller design in $\mathbf{R}_{MS}$ has been presented many times, e.g. in Pekař and Prokop (2011a, b, c), therefore, and with regard to the limited space here, a brief overview of the procedure is given only.

Consider the well-known simple feedback loop. If a controller has the following transfer function

$$G_R(s)=\frac{Q(s)}{P(s)},\ Q(s),P(s)\in R_{MS} \tag{9}$$

the closed-loop asymptotic stability is given by the solution of the Bézout identity

$$A(s)P(s)+B(s)Q(s)=1 \tag{10}$$

The solution exists since whenever the plant is formally stable, the ring constitutes a Bézout domain. A particular stabilizing solution of (10), say $P_0(s),Q_0(s)$, can be then parameterized as

$$P(s)=P_0(s)\pm B(s)Z(s)\neq 0,\ Q(s)=Q_0(s)\mp A(s)Z(s) \tag{11}$$

where $Z(s)\in\mathbf{R}_{MS}$.

Given reference and load disturbance signals expressed by

$$W(s)=\frac{H_W(s)}{F_W(s)},\ H_W(s),F_W(s)\in R_{MS}$$
$$D(s)=\frac{H_D(s)}{F_D(s)},\ H_D(s),F_D(s)\in R_{MS} \tag{12}$$

asymptotic reference tracking and load disturbance attenuation are solved by the suitable choice of $Z(s)$ in (11) so that both $F_W(s)$ and $F_D(s)$ divide $P(s)$. Some details about the divisibility in $\mathbf{R}_{MS}$ can be found e.g. in Pekař and Prokop (2009, 2011c).

3. SPECTRAL ABSCISSA MINIMIZATION FOR RETARDED LTI-TDS

The presented controller design in $\mathbf{R}_{MS}$, using a simple feedback loop for an unstable LTI-TDS controlled plant, yields a characteristic quasipolynomial instead of polynomial, i.e. the closed loop system is infinite-dimensional having the infinite spectrum. Similarly as for delayless systems, the characteristic quasipolynomial decides about asymptotic stability, except cases of distributed delays where zeros of the quasipolynomial do not coincide with system poles. Pole assignment philosophy, which places closed-loop poles to the prescribed positions, can not be adopted here as for finite-dimensional selectable (free) parameters. A possibility is to optimize the whole spectrum so that the right-most pole is moved to the left as much as possible.

Define the spectral abscissa which agrees with the objective function for retarded LTI-TDS. For neutral systems, some additional conditions due to strong stability must be added, the reader is referred to Vyhlídal et al. (2010) for details.

Let the controller obtained by the algebraic approach in $\mathbf{R}_{MS}$ be with k selectable parameters $\mathbf{\Gamma} = \{\gamma_1, \gamma_2, ..., \gamma_k\}$. The spectral abscissa function, $\alpha(\mathbf{\Gamma})$, is defined as follows

$$\alpha(\mathbf{\Gamma}) = \max \operatorname{Re} s_i \tag{13}$$

where s_i are system poles and $\alpha(\mathbf{\Gamma})$ is strictly negative, see e.g. Vanderbliert et al. (2008), Vyhlídal et al. (2010). The objective is to solve the optimization problem

$$\min_{\mathbf{\Gamma}} \Phi(\mathbf{\Gamma}) = \min_{\mathbf{\Gamma}} \alpha(\mathbf{\Gamma}) \tag{14}$$

The question is why a complex optimization algorithm ought to be used instead of a standard one, say, the well known steepest descent (gradient) algorithm. Reason lie in some spectral abscissa function properties, namely $\alpha(\mathbf{\Gamma})$ is non-convex, i.e. it may have multiple local minima, it is non-smooth w.r.t. parameter changes in points where are more the one real poles or conjugate pairs with the maximum real part Michiels et al. (2002), Vanderbliert et al. (2008) – and thus not differentiable at these points, and the function is non-Lipschitz, for example, at points where the maximum real part has multiplicity greater than one, Burke et al. (2005). It is clear that with such behaviour the global minimum is hard to find, and many optimization algorithms will converge to a local minimum of the objective function $\Phi(\mathbf{\Gamma})$. However, it is assumed that the spectral abscissa is differentiable almost everywhere.

Due to the limited space, utilized minimization techniques are described very briefly and the reader is referred to the literature for more details.

3.1 Quasi-Continuous Shifting Algorithm (QCSA)

QCSA for retarded systems was introduced in Michiels et al. (2002) and it was extended e.g. in Michiels and Vyhlídal (2005), Michiels et al. (2010). The algorithm can be described as follows.

Algorithm 1 (QCSA).
Input: Objective function $\Phi(\mathbf{\Gamma})$.
Step 1: Set termination parameters and the number of move (controlled) poles $m = 1$.
Step 2: Compute the right-most poles, e.g. using quasipolynomial mapping based rootfinder (QPMR), Vyhlídal and Zítek (2003).
Step 3: Compute the sensitivity of m right-most poles w.r.t. changes in $\mathbf{\Gamma}$. (Sensitivity matrix)
Step 4: Move m right-most poles to the left-half plane by applying small changes in $\mathbf{\Gamma}$ using the sensitivity matrix.
Step 5: If necessary, increase m. Stop when the available degrees of freedom in the controller do not allow to further reduce $\alpha(\mathbf{\Gamma})$; otherwise, go to Step 2.
Output: Values of $\mathbf{\Gamma}$.

3.2 Gradient Sampling Algorithm (GSA)

The original algorithm, Burke et al. (2005), and its modifications, Vanbliervliet et al. (2008), Vyhlídal et al. (2010), are essentially extensions of the well-known steepest descent method. The basic difference lies in the computation of the non-smooth direction where EGSA requires a numerical estimation of the gradient, even in points where the objective function is not differentiable. It is however expected that $\Phi(\mathbf{\Gamma})$ is differentiable almost everywhere. The basic steps of EGSA can be given as follows.

Algorithm 2 (GSA).
Input: Objective function $\Phi(\mathbf{\Gamma})$.
Step 1: Initialize a starting point $\mathbf{\Gamma}_0$ arbitrarily and set control and termination parameters.
Step 2: Choose $k+1$ points near by $\mathbf{\Gamma}_0$. Compute the Clarke subdifferential and the (non-smooth) steepest descent direction. If the norm of the direction is very small, then terminate the algorithm.
Step 3: Calculate the step length along the direction from Step 2. If it fails choose a substitute direction. If all possible directions fail, stop.
Step 4: Update the current position $\mathbf{\Gamma}_i$ to $\mathbf{\Gamma}_{i+1}$ and go to Step 2.
Output: The best position and its function value.

3.3 Nelder-Mead Algorithm (NM)

The NM algorithm belonging to the class of comparative (direct search) algorithms was originally published in Nelder and Mead (1965). The method does not require derivatives of the objective function and thus it is suitable for non-smooth functions. The method typically requires only one or two function evaluations per iteration, which is useful especially in applications where each function evaluation is time-consuming; however, it can require an enormous amount of iterations to obtain a significant improvement in Φ.

Algorithm 3 (NM).
Input: Objective function $\Phi(\mathbf{\Gamma})$.

Step 1: Construct the initial working simplex *S*, set transformation and termination parameters.
Step 2: Calculate the termination test information. If the test is satisfied, stop the algorithm.
Step 3: Order simplex vertices as the worst, second worst and the best one.
Step 4: Calculate the central point and reflex the worst vertex. If the reflection is successful, accept the reflected point in the new working simplex and go to Step 3.
Step 5: Try to use contraction or expansion. If this succeeds, the accepted point becomes the new vertex; otherwise, shrink the simplex towards the best vertex. Go to Step 3.
Output: The best vertex and its function value.

3.4 Self-Organizing Migration Algorithm (SOMA)

SOMA is ranked among genetic algorithms, dealing with populations, see e.g. Zelinka (2004). Population specimens cooperate while searching the best solution and, simultaneously, each of them tries to be a leader. They move to each other and the searching is finished when all specimens are localized on a small area. The method converges very fast; however, the number of function evaluations in every iteration can be very high. The main steps of the basic algorithm strategy called "All to One" can be formulated as follows.

Algorithm 4 (SOMA).
Input: Objective function.
Step 1: Set control and termination parameters. Generate a population based on a selected prototypal specimen.
Step 2: Find the best specimen (leader), i.e. that with the minimal function value.
Step 3: Move all other specimens towards the leader and evaluate their function values in each step.
Step 4: Select the new population and test the minimal divergence of the population. If it succeeds, stop; otherwise, go to Step 2.
Output: The leader and its function value.

4. UNSTABLE LTI-TDS PLANT – A STUDY CASE

The aim of this section is to demonstrate the controller design in $\mathbf{R}_{MS}$ followed by the spectral abscissa minimization using a real-life unstable LTI-TDS, namely a model of the roller skater on a controlled swaying bow, Zítek et al. (2008), see Fig. 1.

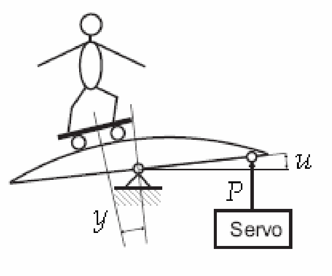

Fig. 1. Roller skater on a controlled swaying bow.

In Zítek et al. (2008) it has been stated that the transfer function of the plant reads

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b\exp(-(\tau+\vartheta)s)}{s^2\left(s^2-a\exp(-\vartheta s)\right)} \tag{15}$$

where $y(t)$ is the skater's deviation from the desired position, $u(t)$ expresses the slope angle of a bow caused by force *P*, delays τ, ϑ means the skater's and servo latencies, respectively, and *b*, *a* are real parameters. Skater controls the servo driving by remote signals into servo electronics. As presented in the literature, $b = 0.2$, $a = 1$, $\tau = 0.3$ s, $\vartheta = 0.1$ s.

4.1 Controller design

Let the reference signal and load disturbance be step-wise functions with Laplace forms

$$W(s)=\frac{H_W(s)}{F_W(s)}=\frac{\frac{w_0}{m_W(s)}}{\frac{s}{m_W(s)}},\; D(s)=\frac{H_D(s)}{F_D(s)}=\frac{\frac{d_0}{m_D(s)}}{\frac{s}{m_D(s)}} \tag{16}$$

respectively, where w_0, $d_0 \in \mathbb{R}$, $m_W(s)$ and $m_W(s)$ are arbitrary stable (retarded) (quasi)polynomials of degree one and $H_W(s), H_D(s), F_W(s), F_D(s) \in \mathbf{R}_{MS}$.

Moreover, factorize (15) as

$$G(s)=\frac{Y(s)}{U(s)}=\frac{B(s)}{A(s)}=\frac{\frac{b\exp(-(\tau+\vartheta)s)}{(s+m_0)^4}}{\frac{s^2\left(s^2-a\exp(-\vartheta s)\right)}{(s+m_0)^4}} \tag{17}$$

$A(s), B(s) \in \mathbf{R}_{MS}$

where $m_0 > 0$ is a selectable real parameter. Using e.g. the extended Euclidean algorithm, a stabilizing particular solution of (10) reads

$$Q_0(s)=\frac{\left(q_3s^3+q_2s^2+q_1s+q_0\right)(s+m_0)^4}{s^2\left(s^2-a\exp(-\vartheta s)\right)\left(s^3+p_2s^2+p_1s+p_0\right)+b\exp(-(\tau+\vartheta)s)\left(q_3s^3+q_2s^2+q_1s+q_0\right)}$$
$$P_0(s)=\frac{\left(s^3+p_2s^2+p_1s+p_0\right)(s+m_0)^4}{s^2\left(s^2-a\exp(-\vartheta s)\right)\left(s^3+p_2s^2+p_1s+p_0\right)+b\exp(-(\tau+\vartheta)s)\left(q_3s^3+q_2s^2+q_1s+q_0\right)} \tag{18}$$

where p_2, p_1, p_0, q_3, q_2, q_1, $q_0 \in \mathbb{R}$ are free controller parameters. It has been shown by simulations (not demonstrated here due to the limited space) that a fewer number of free parameters introduced by the Euclidean algorithm in (18) do not give a stabilizable feedback by numerical algorithms described above, i.e. the spectral abscissa can not be further reduced so that $\alpha(\mathbf{\Gamma}) < 0$.

If $Z(s)$ in (11) is chosen as

$$Z(s)=\frac{z_0(s+m_0)^4}{s^2\left(s^2-a\exp(-\vartheta s)\right)\left(s^3+p_2s^2+p_1s+p_0\right)+b\exp(-(\tau+\vartheta)s)\left(q_3s^3+q_2s^2+q_1s+q_0\right)}$$

where $z_0 \in \mathbb{R}$ is selected as

$$z_0 = \frac{-p_0 m_0^4}{b} \tag{19}$$

then $P(s)$ is in a simple form and, in particular, both $F_W(s)$ and $F_D(s)$ divide $P(s)$. Then the controller transfer function and the characteristic quasipolynomial, respectively, are

$$G_R(s) = \frac{Q(s)}{P(s)} = \frac{b\left(q_3s^3+q_2s^2+q_1s+q_0\right)\left(s+m_0\right)^4 + p_0m_0^4s^2\left(s^2-a\exp(-\vartheta s)\right)}{b\left[\left(s^3+p_2s^2+p_1s+p_0\right)\left(s+m_0\right)^4 - p_0m_0^4\exp\left(-(\tau+\vartheta)s\right)\right]} \tag{20}$$

$$m(s) = (s+m_0)^4 \left[s^2\left(s^2-a\exp(-\vartheta s)\right)\left(s^3+p_2s^2+p_1s+p_0\right)+b\exp\left(-(\tau+\vartheta)s\right)\left(q_3s^3+q_2s^2+q_1s+q_0\right)\right] \tag{21}$$

To cancel the impact of the quadruple real pole $s_1 = -m_0$ in (21), the assignment of which is trivial, it must hold that $m_0 >> \alpha(\mathbf{\Gamma})$, and hence the aim is to minimize $\alpha(\mathbf{\Gamma})$ of the quasipolynomial factor with seven unknown parameters.

4.2 Spectral abscissa minimization

Let the minimization be started from $\mathbf{\Gamma} = [1,1,1,1,1,1,1]^T$ with QCSA as the initial global optimization algorithm. Contrary to the original paper, Michiels et al. (2002), whenever a bunch of dominant roots secedes from the rest of the spectrum and the number of currently controlled roots is higher then the number of seceded ones, the number of controlled roots decreases so that only of seceded roots are controlled. The evolution of dominant poles is displayed in Fig. 2 (controlled ones are in bold).

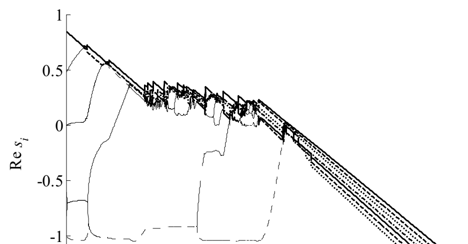

Fig. 2. Evolution of real parts of dominant roots of (21) using QCSA.

It is obvious from Fig. 2 that QCSA can improve $\alpha(\mathbf{\Gamma})$ and the trend is seemed to be continued. Thus, try to test the other optimization methods from the point $\mathbf{\Gamma}_{2400}$. The results are compared in Fig. 3 where the time range instead of iterations is chosen since, in Matlab, NM is approximately 8x faster then GSA and 70x faster then SOMA and comparable with QCSA (measured by the duration of an iteration step). A high time consumption of SOMA is given by an enormous number of pole locations evaluations via QPMR.

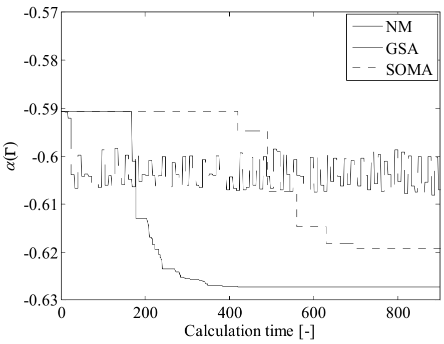

Fig. 3. Comparison of evolutions of $\alpha(\mathbf{\Gamma})$ using NM, GSA and SOMA in time range.

Control responses for the result from NM, i.e.

$$\mathbf{\Gamma} = [5.44288, 9.83811, 27.59276, 148.61548, 161.17249, 5.24605, 0.51339]^T \tag{22}$$

when $d(t) = -0.1$ enters at $t = 100$ are displayed in Fig. 4.

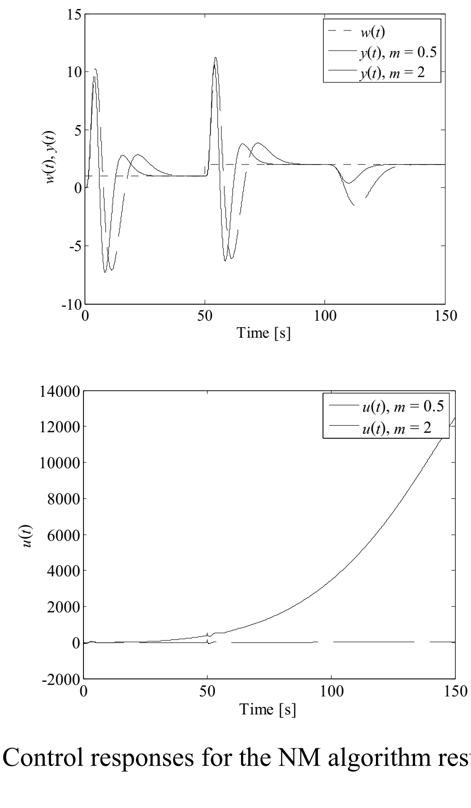

Fig. 4. Control responses for the NM algorithm result.

6. CONCLUSIONS

An original combination of algebraic controller design for unstable LTI-TDS in $\mathbf{R}_{MS}$ ring with the spectral abscissa minimization, as a controller tuning task, using four iteration algorithms has been presented in this paper.

First, the ring definition followed by basic controller design steps and principles has been introduced. Then, we have provided a brief overview of utilized minimization techniques. Finally, the whole methodology with the numerical algorithms comparison has been tested on an attractive model of the roller skater on a controlled swaying bow.

Simulation experiments have proved the usability of a standard QCSA for global $\alpha(\mathbf{\Gamma})$ optimization. With regard to other algorithms, the best result together with a short calculation time due to the small number of cost function evaluations have been given by NM algorithm. SOMA has provided good comparable results, yet with rather long calculation time. On the other hand, our test has not verified the usability of GSA which has though provided a fast spectral abscissa improvement, yet not followed by any significantly better evolution. The three last mentioned algorithms can be used rather for a local minimization.

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REFERENCES

Burke, J., Lewis, A., and Overton, M. (2005). A robust gradient sampling algorithm for nonsmooth, nonconvex optimization. *SIAM Journal of Optimization*, 15 (3), 751-779.

Byrnes, C.I., Spong, M.W., and Tarn, T.J. (1984). A several complex variables approach to feedback stabilization of neutral delay-differential systems. *Mathematical Systems Theory*, 17 (1), 97-133.

Górecki, H., Fuksa, S., Grabowski, P., and Korytowski, A. (1989). *Analysis and Synthesis of Time Delay Systems*. John Wiley & Sons, New York.

Hale, J.K. and Verduyn Lunel, S.M. (1993). Introduction to Functional Differential Equations. In *Applied Mathematical Sciences*, 99, Springer-Verlag, New York.

Kučera, V. (1993). Diophantine equations in control - A survey, *Automatica*, 29, 1361-75.

Loiseau, J.J. (2000). Algebraic tools for the control and stabilization of time-delay systems. *Annual Reviews in Control*, 24, 135-149.

Loiseau, J.J., Cardelli, M., and Dusser, X. (2002). Neutral-type time-delay systems that are not formally stable are not BIBO stabilizable. *IMA Journal of Mathematical Control and Information*, 19 (1-2), 217-227.

Michiels, W., Engelborghs, K., Vansevenant, P., and Roose, D. (2002). Continuous pole placement for delay equations. *Automatica*, 38 (6), 747-761.

Michiels, W. and Vyhlídal, T. (2005). An eigenvalue based approach for the stabilization of linear time-delay systems of neutral type. *Automatica*, 41 (6), 991-998.

Michiels, W., Vyhlídal, T., and Zítek, P. (2010). Control design for time-delay systems based on quasi-direct pole placement. *Journal of Process Control*, 20 (3), 337-343.

Nelder, J.A. and Mead, R. (1965). A simplex method for function minimization. *The Computer Journal*, 7 (4), 308-313.

Partington, J.R. and Bonnet, C. (2004). H_∞ and BIBO stabilization of delay systems of neutral type. *Systems & Control Letters*, 52 (3-4), 283-288.

Pekař, L. and Prokop, R. (2009). Some observations about the RMS ring for delayed systems. In *Proceedings of the 17th International Conference on Process Control '09*, Štrbské Pleso, Slovakia, 28-36.

Pekař, L. and Prokop, R. (2011a). Non-stepwise reference tracking for time delay systems. In *Proceedings of the 12th International Carpathian Conference (ICCC)*, Velke Karlovice, Czech Republic, 292-297.

Pekař, L. and Prokop, R. (2011b). On the reference tracking and disturbance rejection for time delay systems. In *Proceedings of the 31st IASTED International Conference on Modelling, Identification and Control*, Innsbruck, Austria, track 718-049.

Pekař, L. and Prokop R. (2011c). Implementation of a new quasi-optimal controller tuning algorithm for time-delay systems. In Perůtka, K. (ed.), *Matlab for Engineers - Applications in Control, Electrical Engineering, IT and Robotics*, 1-24. InTech, Rijeka, Croatia.

Richard, J.P. (2003). Time-delay systems: An overview of some recent advances and open problems. *Automatica*, 39 (10), 1667-1694.

Vanbiervliet, T., Verheyden, K., Michiels, W., and Vandewalle, S. (2008). A nonsmooth optimization approach for the stabilization of time-delay systems. *ESIAM Control, Optimisation and Calculus of Variations*, 14 (3), 478-493.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, M. A.

Vyhlídal, T. and Zítek, P. (2003). Quasipolynomial mapping based rootfinder for analysis of time delay systems. In *Proceedings IFAC Workshop on Time-Delay Systems* (*TDS '03*), Rocquencourt, France, 227-232.

Vyhlídal, T., Michiels W., and McGahan P. (2010). Synthesis of a strongly stable state-derivative controller for a time delay system using constrained nonsmooth optimization. *IMA Journal of Mathematical Control and Information*, 27 (4), 437-455.

Zelinka I. (2004). SOMA-self organizing migrating algorithm. In Onwobolu G.C. and Babu B.V. (eds.), *New Optimization Techniques in Engineering*. Springer, Berlin.

Zítek, P. and Kučera, V. (2003). Algebraic design of anisochronic controllers for time delay systems, *International Journal of Control*, 76 (16), 905-921.

Zítek, P. and Vyhlídal, T. (2008). Argument-increment based stability criterion for neutral time delay systems. In *Proceedings of the 16th Mediterranean Conference on Control and Automation*, Ajaccio, France, 824-829.

Zítek, P., Kučera, V., and Vyhlídal, T. (2008). Meromorphic observer-based pole assignment in time delay systems. *Kybernetika*, 44 (5), 633-648.