Fuzzy Control of a Microalgae Growth Process in Photobioreactors

Laurențiu Luca, Marian Barbu, George Ifrim, Emil Ceanga, Mihaela Miron, Sergiu Caraman
Faculty of Automation, Computers, Electrical Engineering and Electronics
"Dunărea de Jos" University of Galați
Galați, Romania
(Laurentiu.Luca, Marian.Barbu, George.Ifrim, Emil.Ceanga, Mihaela.Miron, Sergiu.Caraman)@ugal.ro

***Abstract*— The paper deals with the fuzzy control of a photoautotrophic microalgae growth process in photobioreactors. The non-linear mathematical model of the process is presented and a fuzzy control algorithm is implemented in order to control simultaneously the algal biomass concentration, the pH and the average irradiance inside the photobioreactor. The results of the fuzzy control are obtained in numerical simulation and experimentally. The experiments were developed using the photobioreactor designed in our laboratory, from "Dunarea de Jos" University of Galati. The microalga *Desmodesmus Quadricauda* supplied by the Biological Research Institute from Cluj-Napoca, Romania, was used within the experiments.**

Keywords—microalgae culture, photobioreactor, non-linear process, modelling, fuzzy control.

I. INTRODUCTION

Microalgae represent an interesting research domain in order to obtain different products such as long chain polyunsaturated fatty acids, pigments, poly-carbohydrates, vitamins or other biologically active components. Many applications in which microalgae are used were developed: obtaining biofuels, CO_2 biomitigation [1], wastewater treatment etc. Microalgae grow in photobioreactors (PBRs). They need a carbon source, macro-nutrients (nitrogen, phosphorus etc.) At the same time a light source is necessary. Depending on the light source, the PBRs can be classified in PBRs with artificial light (this type being approached in this paper) and solar PBRs (using the natural light). The light can be also considered as a substrate for microalgae.

The photoautotrophic growth process of microalgae is complex, strong non-linear and affected by uncertainties (hidden dynamics and variable parameters). It can be also mentioned the multivariable character of this process. In this respect, it is quite a challenge for the specialists in control to increase the efficiency of these processes. This is the reason why aspects regarding modeling and control issues are frequently approached in the literature.

A series of mathematical models of the PBRs can be found in [7], [14], [15], [16] and [17]. Usually they contain nonlinear ordinary differential and algebraic equations for the liquid phase and for describing the light driven biological phenomena. The distribution of the light inside the reactor is described through the radiative-transfer models [18]. At the same time, different control solutions can be found in [2] – [9]. In the present paper a fuzzy control algorithm in order to control the algal biomass concentration, the pH and the average irradiance inside the PBR is implemented in numerical simulation. Fuzzy control of the biomass concentration is also implemented within experiments provided in our laboratory.

The paper contains the following sections: the second section presents the non-linear model of the microalgae growth process; the third section describes the fuzzy control scheme, the fourth section shows the results regarding the PBR's fuzzy control obtaining experimentally and in numerical simulation. Finally the last section presents the conclusions.

II. THE MATHEMATICAL MODEL OF MICROALGAE PHOTOSYNTHETIC GROWTH PROCESS

The mathematical model of the microalgae growth process in a photobioreactor is described in detail in [7], [8] and [10]. The intracellular mechanism that takes place at the microalgae level is the following: a photoautotroph microorganism is capable to assimilate inorganic carbon forms (i.e. CO_2, HCO_3^-) and to convert it in substances necessary for their own cellular metabolism. At the same time, it produces O_2 by the light-induced water oxidation reaction. In addition to its substrate role, CO_2 is used to lower the pH of the microalgae culture because it is a weak acid. When dissolved CO_2 in water, a series of chemical species is formed in the liquid medium (i.e. $CO_{2,aq}$, HCO_3^-, CO_3^{2-}) through reactions such as hydration, dehydration and protonation. The ions of HCO_3^- interfere with the ions of NH_3 (nitrogen source) leading to the formation of a multi-solute system ($NH_3 - CO_2 - H_2O$) which consists in 9 chemical species: NH_3, NH_4^+, NH_2COO^-, CO_2, HCO_3^-, CO_3^{2-}, H_2O, OH^- and H^+. Inorganic carbon and nitrogen sources are consumed in the growth process with reaction rates governed by the availability of light inside the PBR.

The mathematical model contains three main components, as follows: 1. biological model, 2. radiative model and 3. thermo-dynamic model.

The biological model is of 6th order and it is characterized by the state vector $\xi = [X\ c_{TIN}\ c_{TIC}\ c_{O2}\ y_{out}^{CO2}\ y_{out}^{O2}]^T$, where X is the biomass concentration, c_{TIN} – total inorganic nitrogen

concentration, c_{TIC} - total inorganic carbon concentration, c_{O2} – dissolved oxygen concentration, y_{out}^{CO2} – molar fraction of carbon dioxide in the output gas and y_{out}^{O2} represents the molar fraction of oxygen in the output gas. The model equations are the following:

$$\dot{X} = r_x - D \cdot X \tag{1}$$

$$\dot{c}_{TIN} = -\frac{Y_{N/X}}{M_x} r_x + D\left(c_{TIN,in} - c_{TIN}\right) \tag{2}$$

$$\dot{c}_{TIC} = -\frac{1}{M_x} r_x + d\left(c_{TIC,in} - c_{TIC}\right) + (K_{La})_{O2} \frac{D_{CO2}}{D_{O2}} \left(\frac{\left(y_{in}^{CO2} + y_{out}^{CO2}\right)/2 \cdot P}{\gamma_{CO2} \cdot H_{CO2}} - c_{CO2} \right) \tag{3}$$

$$\dot{c}_{O2} = \frac{Q_p}{M_x} - D \cdot c_{O2} + (K_{La})_{O2} \left(\frac{\left(y_{in}^{O2} + y_{out}^{O2}\right)/2 \cdot P}{\gamma_{CO2} \cdot H_{O2}} - c_{O2} \right) \tag{4}$$

$$\dot{y}_{out}^{CO2} = \frac{R \cdot T}{P \cdot V_g} \left(y_{in}^{CO2} G_{in} - y_{out}^{CO2} G_{out} - V_L (K_{La})_{O2} \cdot \cdot \frac{D_{CO2}}{D_{O2}} \left(\frac{\left(y_{in}^{CO2} + y_{out}^{CO2}\right)/2 \cdot P}{\gamma_{CO2} \cdot H_{CO2}} - c_{CO2} \right) \right. \tag{5}$$

$$\dot{y}_{out}^{O2} = \frac{R \cdot T}{P \cdot V_g} \left(y_{in}^{O2} G_{in} - y_{out}^{O2} G_{out} - V_L (K_{La})_{O2} \cdot \cdot \frac{D_{CO2}}{D_{O2}} \left(\frac{\left(y_{in}^{O2} + y_{out}^{O2}\right)/2 \cdot P}{\gamma_{CO2} \cdot H_{O2}} - c_{O2} \right) \right. \tag{6}$$

The notations used in eq. (1) – (6) are: D – the dilution rate, r_x – photosynthetic growth rate, $y_{in}^{CO2}, y_{in}^{O2}$ – the molar fraction of CO_2 and O_2, respectively, in the inflow, $Y_{N/X}$ – yield coefficient of total inorganic nitrogen (*TIN*) conversion, Q_p – photosynthetic quotient, $(K_La)_{O2}$ – the overall volumetric mass-transfer coefficient of the oxygen, M_x – the C-mole mass, D_{CO2} and D_{O2} – the molecular diffusivity of CO_2 and O_2 respectively, P – total pressure, R – the universal gas constant, T – temperature, H_{CO2} and H_{O2} – Henry's constant of CO_2 and O_2 respectively, V_L and V_g – volumes of liquid and gas respectively, G_{in} and G_{out} – gas inflow and gas outflow.

All the molar fractions of the gas components at the inlet are known and they are equal to: $y_{in}^{CO_2} = G_{in}^{CO_2}/G_{in}$; $y_{in}^{O_2} = G_{in}^{O_2}/G_{in}$; $y_{in}^{N_2} = G_{in}^{N_2}/G_{in}$, where $G_{in}^{CO_2}$, $G_{in}^{O_2}$ şi $G_{in}^{N_2}$ are suppling flows with CO_2, O_2 and N_2. G_{in} is the total gas flow at the inlet and, therefore, the sum of all gases at the inlet. The same relations can be also written for the gases at the outlet and G_{out} can be calculated on the basis of molar fractions of N_2 in the gas at the inlet and the outlet, respectively:

$$G_{out} = \frac{y_{in}^{N_2}}{y_{out}^{N_2}} G_{in} \tag{7}$$

The molar fractions of N_2 in the gas at the inlet and the outlet can be determined as follows:

$$y_{in}^{N_2} = 1 - y_{in}^{CO_2} - y_{in}^{O_2} \tag{8}$$

$$y_{out}^{N_2} = 1 - y_{out}^{CO_2} - y_{out}^{O_2} \tag{9}$$

The radiative model gives the light attenuation along the depth of the culture of microalgae which, being heterogeneous in terms of the availability of light, is treated as a uniformly distributed system. The radiative model, coupled with a kinetic law of the biomass growth, provides the local photosynthetic responses, $\mu\big(G(z)\big)$, where G(z) is the local irradiance. However, the microalgae culture is considered homogeneous in terms of the components from the liquid phase and, therefore, the photosynthetic responses are integrated in order to obtain an average photosynthetic response. In order to determine the light attenuation within the microalgae culture, a radiative-transfer model dedicated to the single-side rectangular reactors has been adopted. The local irradiance, $G(z)$, is given by:

$$G(z) = q_0 \cdot exp\left(-\frac{1+\alpha}{2\alpha} E_a \cdot X \cdot z\right) \tag{10}$$

where $\alpha = \sqrt{(E_a)/(E_a + 2bE_s)}$ is the coefficient of the linear light dispersion. E_a and E_s are the coefficients of mass absorption and mass dispersion of the light, and b is the light dispersion fraction by reflection (non-dimensional). The variable q_0 represent the incident light flux. Returning to the biological model, the volumetric growth rate of microalgae is given by the eq. (11):

$$r_x = (\mu_G - \mu_s)X = \frac{\mu_0}{L} \int_0^L \mu\big(G(z)\big) dz \cdot f(pH) \cdot X - \mu_s X \tag{11}$$

where L is the depth of the PBR, μ is a function of local irradiance:

$$\mu = \frac{G(z)}{K_I + G(z)} \tag{12}$$

with $G(z)$ given by (10), K_I – semi-saturation constant, μ_0 – the maximum specific growth rate, μ_s – kinetic parameter associated to the the breathing process (considered most of the time constant). The average photosynthetic response, $\langle \mu_G \rangle$, calculated for the entire volume of PBR is determined by the integration of the average photosynthetic responses along the culture depth, z:

$$\langle \mu_G \rangle = \frac{1}{L} \int_0^L \mu\big(G(z)\big) dz \tag{13}$$

The function $f(pH)$ from (11) is:

$$f(pH) = \frac{pH_{max} - pH}{pH - pH_{min}} exp\left(1 - \frac{pH_{max} - pH}{pH - pH_{min}}\right) \tag{14}$$

The specific growth rate of microalgae is optimal when *pH* is maintained in an optimal domain depending on chemical species. In the case of the photoautotrophic processes *pH* is controlled by adding CO_2 in PBR which means that the specific growth rate of microalgae is a function of total inorganic carbon (*TIC*), present in the culture medium. For the mathematical description of this dependence, it was preferred to use a term of Monod type which multiplies the relation (14), $c_{\mathrm{TIC}}/(K_{\mathrm{TIC}} + c_{\mathrm{TIC}})$. Thus, if c_{TIC} has very low values, the *pH* will limit the microalgae growth rate.

The thermodynamic model: the multi-solute system $(NH_3 - CO_2 - H_2O)$ is composed of 9 distinct chemical species, as mentioned before: NH_3 (molecular), NH_4^+,

NH_2COO^-, CO_2 (molecular), HCO_3^-, CO_3^{2-}, H_2O (molecular), OH^- and H^+. It results 17 equations, as follows:

- 5 chemical equilibria:

$$K_1 = \frac{a_{H^+} a_{HCO_3^-}}{a_{CO_2} a_w}, K_2 = \frac{a_{H^+} a_{CO_3^{2-}}}{a_{HCO_3^-}}, K_3 = \frac{a_{NH_4^+} a_{OH^-}}{a_{NH_3} a_w}$$

$$K_4 = \frac{a_{NH_3} a_{HCO_3^-}}{a_{NH_2COO^-} a_w}, K_w = \frac{a_{H^+} a_{OH^-}}{a_w} \quad (15)$$

where $a_i = \gamma_i c_i$ represens the activity of the component i, γ_i – activity coefficient of the component i and c_i – the concentration of the component i. The activity constants K_1, K_2, K_3, K_4 and K_w can be found in [12] for different temperatures.

- 2 mass-balance equations:

$$c_{TIC} = c_{CO_2} + c_{HCO_3^-} + c_{CO_3^{2-}} + c_{NH_2COO^-} \quad (16)$$

$$c_{TIN} = c_{NH_3} + c_{NH_4^+} + c_{NH_2COO^-} \quad (17)$$

- 1 electro-neutrality equation written for the main chemical species in the culture medium:

$$c_{NH_4^+} + c_{H^+} + c_{Na^+} =$$

$$= c_{HCO_3^-} + 2c_{CO_3^{2-}} + c_{NH_2COO^-} + c_{OH^-} + c_{Cl^-} \quad (18)$$

- 8 equations for the calculus of the activity coefficients (one for each species except for the water) [12].

Finally, the precise calculus of the chemical species concentrations allows to determine the pH:

$$pH = -\lg(a_{H^+}) \quad (19)$$

The numerical values of the model parameters are given in [10]. The photobioreactor was modeled as a non-linear system of 16th order. In order to tune the control loops of PBR the dynamics characterization was preferred in the neighborhood of a nominal point of the system. The system linearization has been achieved through the identification of each channel of the non-linear system. It resulted a 14th order model of the linearized system presented in appendix 1. Further on, the following *I/O* representation of PBR is considered (Fig. 1).

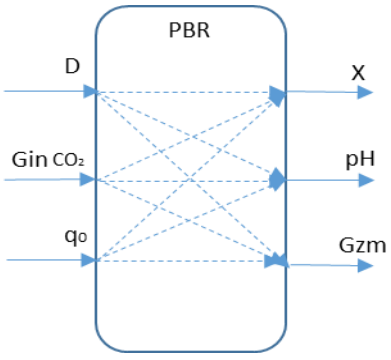

Fig. 1. *I/O* representation of PBR

The input variables are: the dilution rate, D [l/h], the light incident flux q_0 [µmol photon/m^2/s] and the flow of CO_2 at the inlet, G_{in}^{CO2}. The output variables are: the algal biomass concentration, the pH and the average irradiance inside the PBR. It has to be mentioned the interactions between the input and output variables. Practically, the three channels disturb each other, creating difficulties in controlling the whole process.

III. FUZZY CONTROL OF THE PHOTOBIOREACTOR

Three requirements are imposed to the microalgae growth process:

1. the control of the algal biomass concentration to a desired setpoint (through the dilution rate, D);
2. providing an average light flux in the photobioreactor section, G_{zm}, which corresponds to an efficient regime, from the point of view of the use of this energy resource in the growth process of the algal biomass. The biomass control to the desired setpoint for the average light flux, G_{zm}, is done by controlling the incident light flow, q_0;
3. setting a pH value for the culture medium to ensure the maximum biomass growth rate under the given culture conditions. This requirement is provided by the flow control of CO_2 at the inlet, G_{in}^{CO2}.

Fuzzy logic is based on the expertise of the specialist in solving various problems, including control issues. In the present control application FLCs of Mamdani type were used. Fig. 2 presents the architecture of a Fuzzy Logic Controller (FLC) [13]:

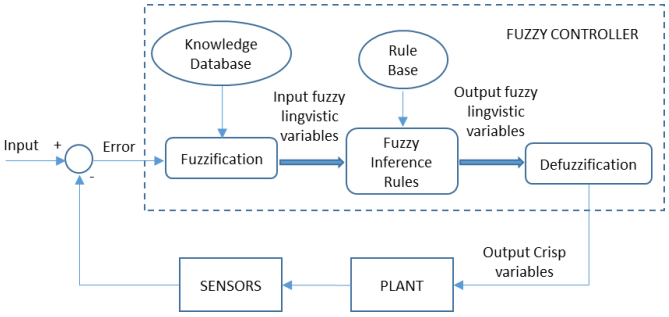

Fig. 2. The architecture of a fuzzy control system

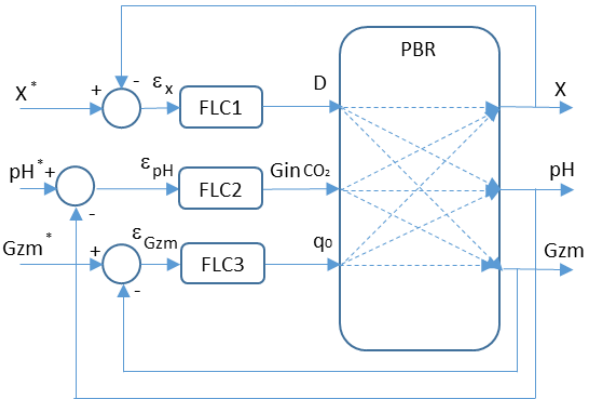

Fig. 3. The control scheme of PBR

Generally, fuzzy control is applied successfully when the process model is not known accurately, as is the case of the

photosynthetic growth process of microalgae. As mentioned before it contains model and parameters uncertainties and is affected by a high level of noise. According to [13] fuzzy control has the following advantages: fuzzy control is cheaper, robust, customizable, emulates human deductive thinking, reliable and efficient. In this respect, the following control scheme has been implemented in Matlab - Simulink (Fig. 3). In Fig. 3, the first FLC controls the algal biomass concentration, the second FLC controls the pH and the third FLC controls the average irradiance, G_{zm}.

IV. Results regarding fuzzy control of PBR

A. Simulation results

Figs. 4 - 6 present the step responses of the three loops of the microalgae growth process: the biomass control loop, the pH control loop and the average irradiance control loop. Thus a step succession has been applied to the setpoints, as follows: $X^{ref} = 0.85$ to 0.95 [g/l], $pH^{ref} = 6.5$ to 7 and $G_{zm}^{ref} = 80$ to 85 [µmol photon/m^2/s].

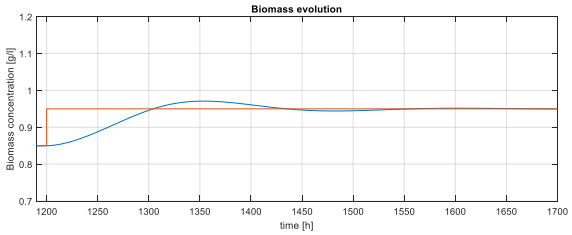

Fig. 4. The biomass concentration evolution, $X_{ref} = 0.85$ [g/l] (red), blue – the biomass concentration

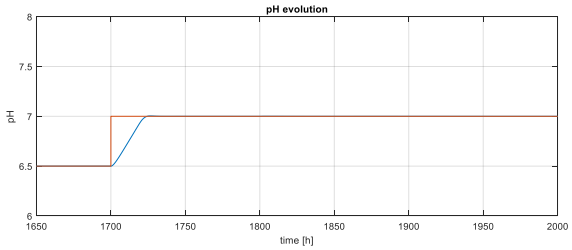

Fig. 5. The pH evolution, $\mathrm{pH_{ref}} = 7$ (red), blue – the pH

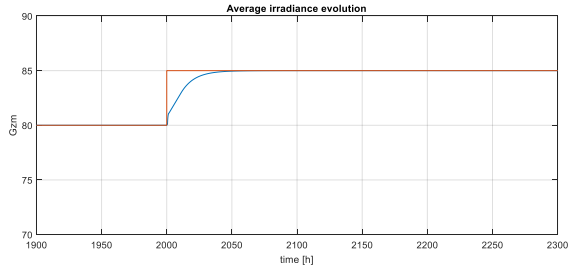

Fig. 6. The average irradiance evolution, $G_{zm} = 80$ [µmol photon/m^2/s] (red), blue – the average irradiance

Fig. 7 presents the dilution rate which has a smooth evolution comparing to the case when conventional PI controllers are used [11]. Figs. 4 – 6 show that all the three variables track accurately the setpoints. Figs. 8 – 10 present the influences between channels.

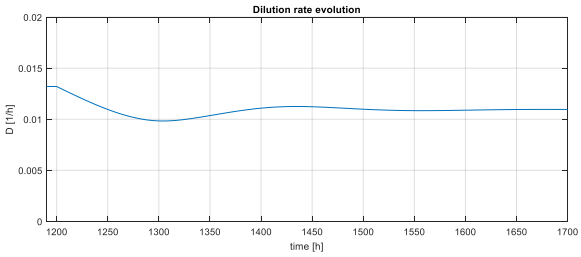

Fig. 7. The dilution rate evolution, D [h^{-1}]

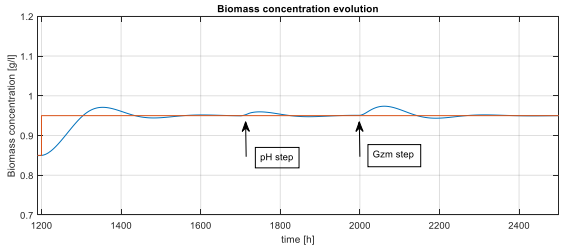

Fig. 8. The biomass concentration evolution, $X_{ref} = 0.85 \rightarrow 0.95$ [g/l] (red), blue – the biomass concentration

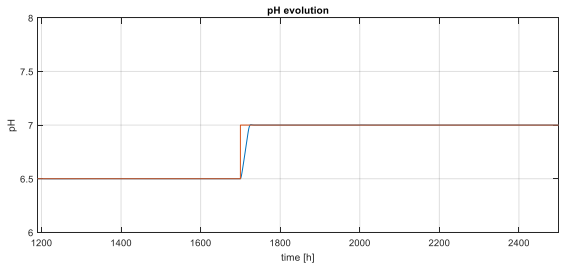

Fig. 9. The pH evolution, $\mathrm{pH_{ref}} = 6.5 \rightarrow 7.0$ (red), blue – the pH

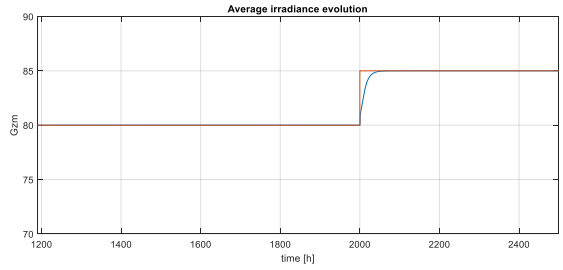

Fig. 10. The average irradiance setpoint, $G_{zm} = 80 \rightarrow 85$ [µmol photon/m^2/s] (red), blue – the average irradiance

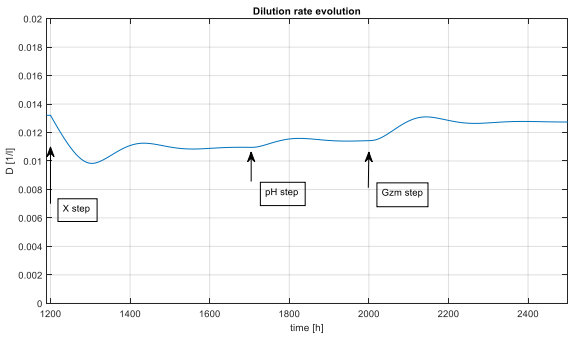

Fig. 11. The dilution rate evolution, D [h^{-1}]

The same step succession is considered at the following moments: biomass step → t = 1200 h, pH step → t = 1700 h

and G_{zm} step $\rightarrow t$ = 2000 h. One can notice that the pH and G_{zm} steps influences significantly the biomass control loop which is very slow. On the other hand, the biomass control loop does influence the pH and G_{zm} loops but, as these loops are much faster, the effect on them is practically insignificant. Fig. 11 presents the dilution rate evolution.

B. *Experimental results*

The fuzzy control of algal biomass concentration has been implemented in a plant designed and made entirely in our laboratory, totally controlled by a computer through a dSpace system.

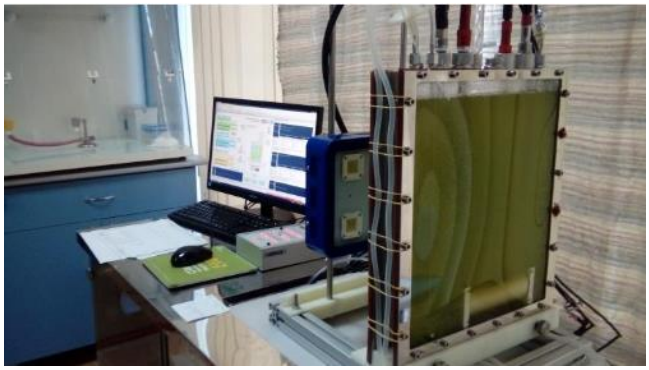

Fig. 12. The PBR system used in the experiments

Fig. 13. The PBR system used in the experiments

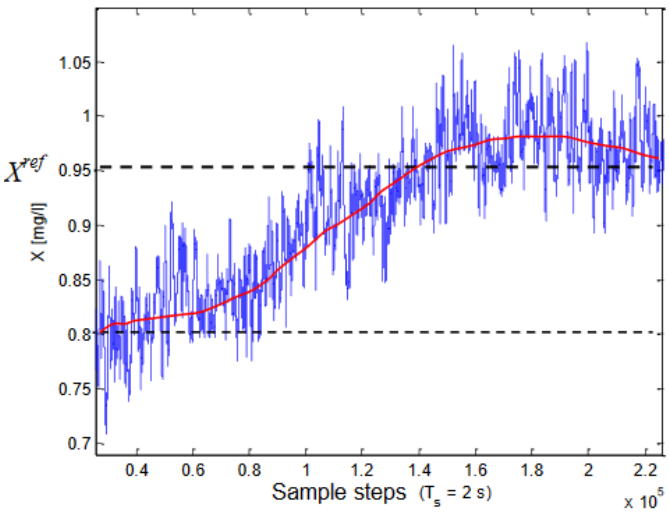

Fig. 14. The biomass concentration evolution, X[g/l]; red – filtered biomass, blue – real biomass, dotted line – the setpoint (step from 0.8 to 0.95 [g/l])

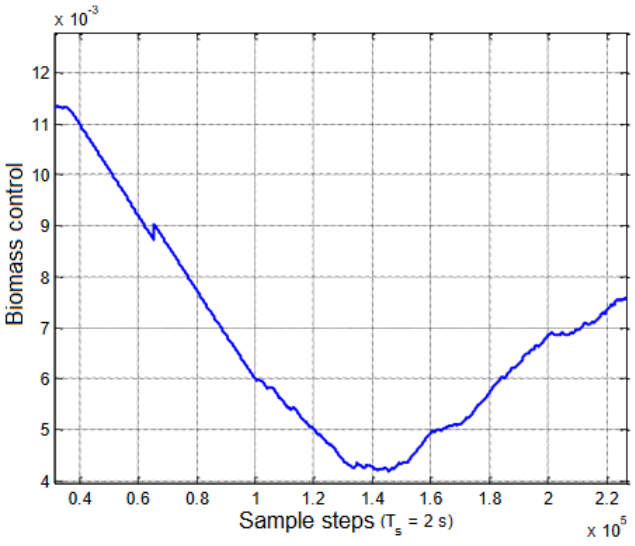

Fig. 15. The dilution rate evolution, D [h^{-1}]

PBR is of rectangular shape and of air-lift type, having a 5 liters useful volume. It operates in continuous regime and is equipped with the following sensors: turbidity, carbon dioxide, dissolved oxygen, pH and temperature. The PBR is stirred by introducing nitrogen and the pH control is done with CO_2 introduced inside the PBR. Both nitrogen and CO_2 are introduced inside PBR using two peristaltic pumps. For illumination a special panel which provides the light in the spectrum 400 – 700 nm (it contains white and red LEDs) was used. The experimental PBR is presented in the two figures, 12 and 13. The process control was implemented in Matlab-Simulink and a friendly interface facilitates the operating of the system.

It should be mentioned that in the experiment, only the biomass concentration has been controlled by a fuzzy controller. The biomass was measured with the turbidity sensor being necessary a conversion relationship between turbidity and biomass. For the rest of the two variables (pH and average irradiance) conventional PI controllers were used. Figs. 14 and 15 present the experimental results. As it can be noticed in Fig. 14 the noise overlaid over the turbidity signal is significant being necessary a powerful filtering.

V. CONCLUSIONS

The paper objective is focused on the fuzzy control of photoautotrophic microalgae growth process in photobioreactors. The microalgae growth process was treated as a multivariable one. It has as control inputs the dilution rate, the flow of CO_2 and the influent light flux. The outputs are the algal biomass concentration, the pH and the average irradiance inside the PBR. The fuzzy control of the biomass has been validated in numerical simulation and experimentally. Both the simulation results and the experimental ones are promising and show that the fuzzy control can be a viable alternative to other types of control. Two research steps are followed in the future: 1. to implement fuzzy control for the pH and average irradiance loops on the experimental plant and 2. to implement a multivariable fuzzy controller for the entire plant.

REFERENCES

[1] Lam, M.K., Lee, K.T. and Mohamed, A.R. (2012). Current status and challenges on microalgae-based carbon capture. *International Journal of Greenhouse Gas Control*, 10, pp. 456–469.

[2] S. Tebbani, F. Lopes, R. Filali, D. Dumur and D. Pareau, ”Nonlinear predictive control for maximization of CO2 bio-fixation by microalgae in a photobioreactor,” Bioprocess Biosyst Eng., vol. 37(1), pp. 83-97, 2014.

[3] Bastin, Georges; Nesic, Dragan; Tan, Ying; et al., ”On Extremum Seeking in Bioprocesses with Multivalued Cost Functions”, BIOTECHNOLOGY PROGRESS, Volume: 25, Issue: 3, Pages: 683-689, 2009.

[4] Mairet, R. Munoz-Tamayo and O. Bernard, ”Adaptive control of light attenuation for optimizing microalgae production,” J. Process Contr., vol. 30, pp. 117-124, 2015.

[5] Chen, Xue; Goh, Qianru Yvonne; Tan, Weifeng; et al., ”Lumostatic strategy for microalgae cultivation utilizing image analysis and chlorophyll a content as design parameters”, BIORESOURCE TECHNOLOGY, Vol. 102, Issue 10, pp. 6005 - 6012, 2011.

[6] S.J. Yoo, D.H. Jeong, J.H. Kim and J.M. Lee, ”Optimization of microalgal photobioreactor system using model predictive control with experimental validation,” Bioprocess Biosyst. Eng., vol. 39(8), pp. 1235-1246, 2016.

[7] G.A. Ifrim, M. Titica, M. Barbu, L. Boillereaux, G. Cogne, S. Caraman and J. Legrand, ”Multivariable feedback linearizing control of Chlamydomonas reinhardtii photoautotrophic growth process in a torus photobioreactor,” Chem. Eng. J., vol. 218, pp. 191-203, 2013.

[8] G.A. Ifrim, M. Titica, M. Barbu, E. Ceanga and S. Caraman, Optimization of a microalgae growth process in photobioreactors”, 11th IFAC Symposium on Dynamics and Control of Process Systems, including Biosystems, Trondheim, Norway, 218-223, 2016.

[9] M. Sbarciog, I. Saraiva and A. Vande Wouwer, ”Accelerating animal cell growth in perfusion mode by multivariable control: Simulation Studies”, Bioprocess and Biosystems Engineering 36(5), 517-530, 2013.

[10] Sbarciog, M., Ifrim, G., A., Caraman, S., Vandewouwer, A., 2017, Multivariable Predictive Control of a Photobioreactor System”, Proc. of 18th International Carpathian Control Conference, ICCC 2017, Eds.: D. Popescu, D. Şendrescu, M. Roman, E. Popescu, L. Bărbulescu, ISBN: 978 – 1 – 50905825 - 9, May 28 – 31, 2017, Palace Hotel, Sinaia, Romania.

[11] http://www.biocon.ugal.ro.

[12] Edwards TJ, Maurer G, Newman J, Prausnitz JM. Vapour - liquid equilibria in multicomponent aqueous solutions of volatile weak electrolytes. AIChE Journal. 1978; 24:966 – 975.

[13] https://www.tutorialspoint.com/fuzzy_logic/fuzzy_logic_control_system.htm

[14] Fouchard, S., Pruvost, J., Degrenne, B., Titica, M. and Legrand, J. (2009). Kinetic modeling of light limitation and sulfur deprivation effects in the induction of hydrogen production with *Chlamydomonas reinhardtii*: part I. Model development and parameter identification. *Biotechnology and Bioengineering*, 102(1), 232–245.

[15] Takache, H., Christophe, G., Cornet, J.F. and Pruvost, J. (2010). Experimental and theoretical assessment of maximum productivities for BIORESOURCE TECHNOLOGYof photobioreactors. *Biotechnology Progress*, 26(2), 431–440.

[16] Pruvost, J., Cornet, J. F., Goetz, V. and Legrand, J. (2011). Modeling Dynamic Functioning of Rectangular Photobioreactors in Solar Conditions. *AIChE Journal*, 57(7), 1947–1960.

[17] Bernard, O. (2011). Hurdles and challenges for modelling and control of microalgae for CO_2 mitigation and biofuel production. *Journal of Process Control*, 21, 1378–1389.

[18] Pottier, L., Pruvost, J., Deremetz, J., Cornet, J.F., Legrand, J. and Dussap, C.G. (2005) A fully predictive model for one-dimensional light attenuation by *Chlamydomonas reinhardtii* in a torus photobioreactor. *Biotechnology and Bioengineering*. 91(5), 569–582.

APPENDIX

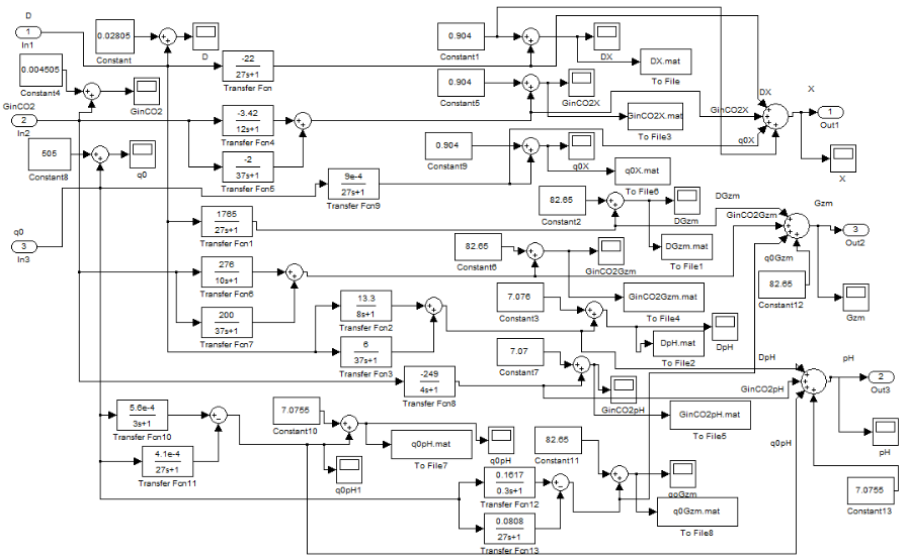

Fig. 16. The linearized model of photoautotrophic microalgae growth process in PBR