

TKP4140 Process Control  
Department of Chemical Engineering NTNU  
Autumn 2020 - Midterm Exam

8 October 2020

- Write your answers on a separate sheet of paper.
- Time: **95** minutes (**14:10-15:45**) + **5** extra minutes for scanning.
- Upload your answers via Blackboard not later than **15:50**.

**Problem 1** (25 points)

Consider the following transfer function

$$g(s) = \frac{(-4s + 1)e^{-s}}{(2s + 1)^2(6s + 1)}$$

- (a) Approximate by a first-order model using the half-rule.
- (b) Draw the response of the approximated model in Figure 1. Sketch the responses on a separate sheet of paper.
- (c) Tune a PI controller using the SIMC rules (using  $\tau_c = \theta$ ).
- (d) After a first inspection, you realized that the above controller is too fast. Should you increase or decrease  $\tau_c$  to slow down the response?

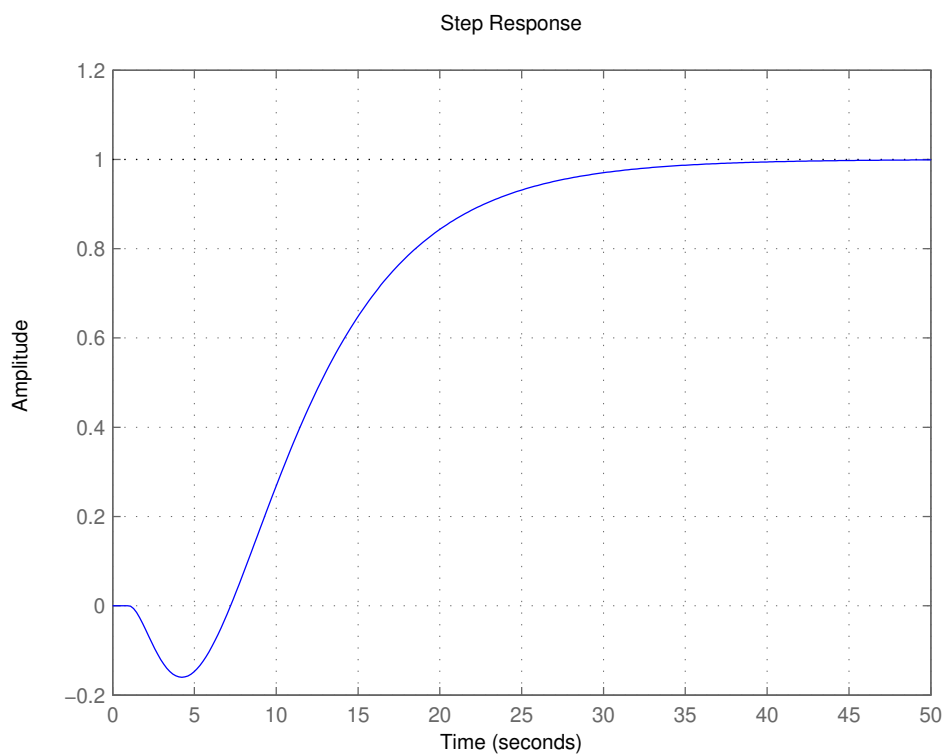


Figure 1: Response of  $g(s)$  to a unit step at time 0.

**Problem 2** (20 points)

Given

$$g_1 = \frac{1}{6s + 1} - \frac{1.2}{2s + 1}$$

$$g_2 = \frac{1}{2s + 1} - \frac{1.2}{6s + 1}$$

$$g_3 = \frac{0.8}{10s + 1}$$

$$g_4 = \frac{0.8e^{-4s}}{10s + 1}$$

$$g_5 = \frac{(10s + 1)}{(8s + 1)(4s^2 + 4s + 1)}$$

$$g_6 = \frac{1}{s^2 + 0.4s + 1}$$

Fill in the missing values (poles, zeros, steady state gain, initial gain and initial slope) in Table 1. As conclusion, identify the step responses in Figure 2.

*Hint: Initial slope of response to unit step input:  $\lim_{t \rightarrow 0} y'(t) = \lim_{s \rightarrow \infty} sg(s)$ .*

Table 1: Table for Problem 2; SS: steady state; TF: transfer function

TF	Poles	Zeros	SS gain	Initial gain	Initial slope	Conclusion
$g_1$						
$g_2$						
$g_3$						
$g_4$						
$g_5$						
$g_6$						

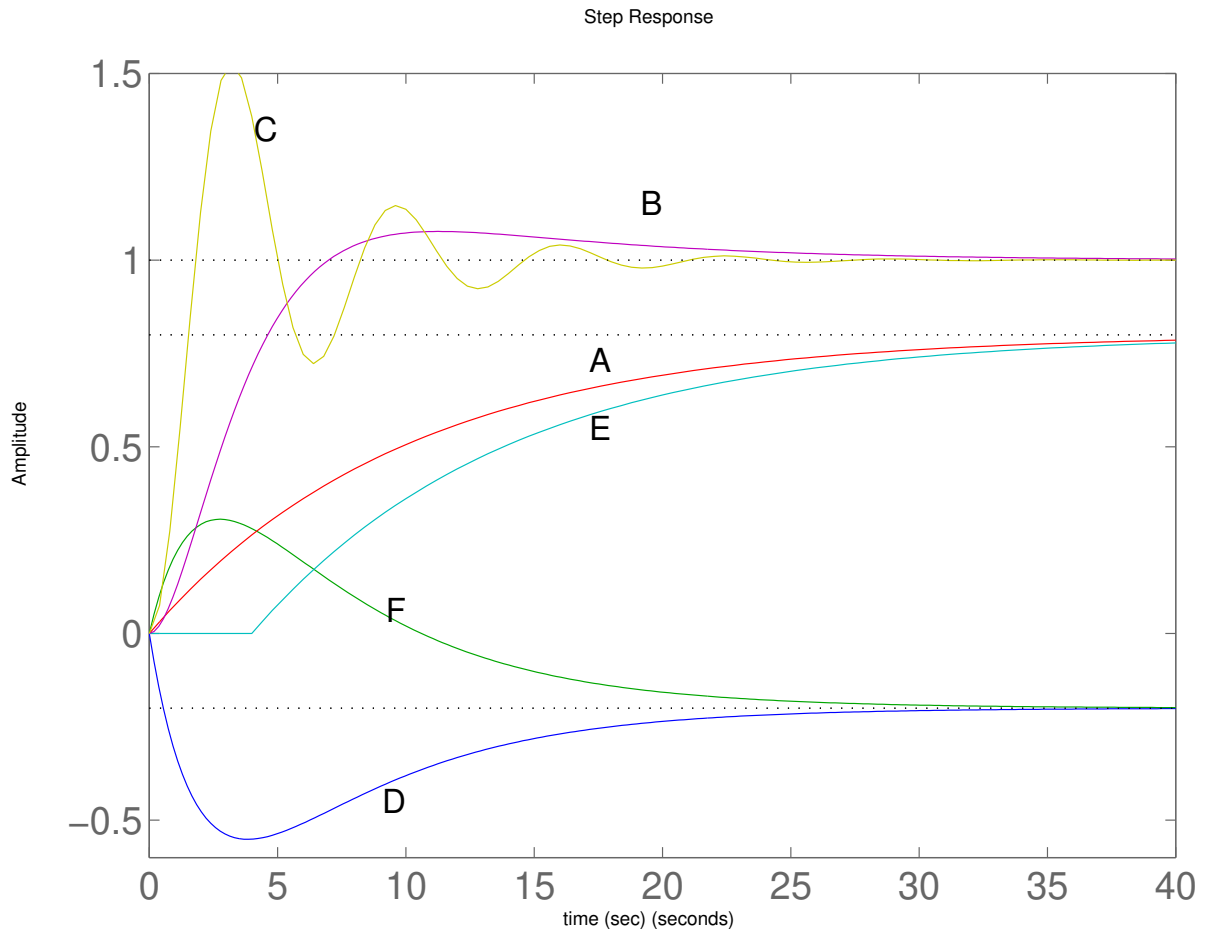


Figure 2: Step responses.

**Problem 3** (25 points)

Consider the pressure tank shown in Figure 3. The inflow  $F_1$  is taken from a reservoir with pressure  $P_r$  (and  $P_r$  does not change with  $F_1$ ).

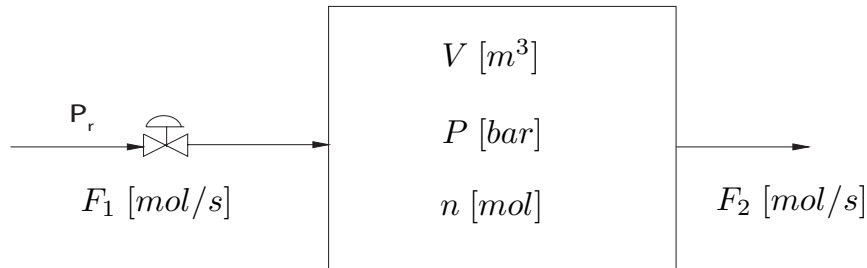


Figure 3: Pressure tank scheme

- Assume ideal gas with constant temperature  $T$  and volume  $V$ .
- Valve equation:  $F_1 = C_v z \sqrt{P_r^2 - P^2}$  [mol/s] where  $z$  is the opening of the valve.
- MV:  $z$ .
- CV:  $P$ .
- DV:  $F_2$ .

Table 2: Parameters

$P_r$	=	12	[bar]
$T$	=	300	[K]
$V$	=	100	[m <sup>3</sup> ]
$C_v$	=	2	[mol/s/bar]
$R$	=	8.314	[J/K/mol]
$F_2^*$	=	2.0	[mol/s]
$z^*$	=	0.5	[-]

Please do the following:

- Formulate a dynamic model for the system.
- Determine the steady state value  $P^*$ .
- Write the model in the form  $\frac{dP}{dt} = \dots$  and linearize the dynamics in terms of deviation variables ( $\Delta P = P - P^*$ ,  $\Delta z = z - z^*$ ,  $\Delta F_2 = F_2 - F_2^*$ ).
- Compute the transfer function  $g(s)$ :

$$P(s) = g(s)z(s)$$

**Problem 4** (5 points)

Indicate whether the sentences are true or false .

- (a) The PID controller  $c(s) = K_c \left( \frac{\tau_I s + 1}{\tau_I s} \right) (\tau_D s + 1)$  is in the ideal (parallel) form.
- (b) The PID-SIMC rules gives tunings for PID in cascade (series) form.
- (c) For the system  $y(s) = \frac{k}{\tau s + 1} u(s)$ , the response for a step in  $u$  is  $y(t) = k e^{-t/\tau} \Delta u$ .
- (d) The PID in the ideal (parallel) form is more general than PID-series (cascade) form since it can have complex zeros, whereas the series form can only have real zeros.
- (e) The derivative action is necessary to eliminate the steady state off-set (error).

**Problem 5** (25 points)

Derive a modified SIMC PID tuning rule for a first order with delay process

$$g(s) = \frac{ke^{-\theta s}}{(\tau s + 1)} \quad (1)$$

Do the following:

- (a) Write the transfer function for a PID controller  $c(s)$  in cascade form.
- (b) Specify a desired first-order with delay response  $T(s) = y/y_s$  and introduce the tuning parameter  $\tau_c$ .
- (c) Draw the block diagram showing  $g, c, y, u$  and  $y_s$  and write the transfer function  $T(s)$  in terms of  $c(s)$  and  $g(s)$ .
- (d) Solve this to find  $c(s)$  as a function of  $g(s)$  and  $T(s)$ .
- (e) Put in the desired  $T(s)$  from (b) and given  $g(s)$  in (1) and derive the controller  $c(s)$  (as a function of  $\tau, \theta$  and  $\tau_c$ ).
- (f) What is the first-order Padé approximation (with a single pole and a RHP-zero) of a time delay?
- (g) Approximate the time delay using the Padé approximation and derive a PID controller.
- (h) Give the resulting PID tunings  $(K_c, \tau_I, \tau_D)$  and a derivative filter  $\tau_F$  for the model (1).
- (i) Comment on the difference with respect to the original SIMC-rule.