TKP4140 Process Control

Department of Chemical Engineering NTNU

Autumn 2020 - Solution Midterm Exam

8 October 2020

- Write your answers on a separate sheet of paper.
- Time: **95** minutes (**14:10-15:45**) + **5** extra minutes for scanning.
- Upload your answers via Blackboard not later than **15:50**.

Problem 1 (25 points)

Given the transfer function

$$g(s) = \frac{(-4s+1)e^{-s}}{(2s+1)^2(6s+1)}$$

the time constants are $\tau_1 = 6, \tau_2 = 2$ and $\tau_3 = 2$. The delay is $\theta_0 = 1$ and the gain $k = 1$.

(a) The first-order model approximation is

$$\frac{ke^{-\theta s}}{\tau s + 1}$$

where $\tau = \tau_1 + \tau_2/2 = 7$, $\theta = \theta_0 + \tau_2/2 + \tau_3 = 8$ and $k = 1$.

(b) Figure 1 shows the response of the original transfer function (in blue) and the approximation (in red).

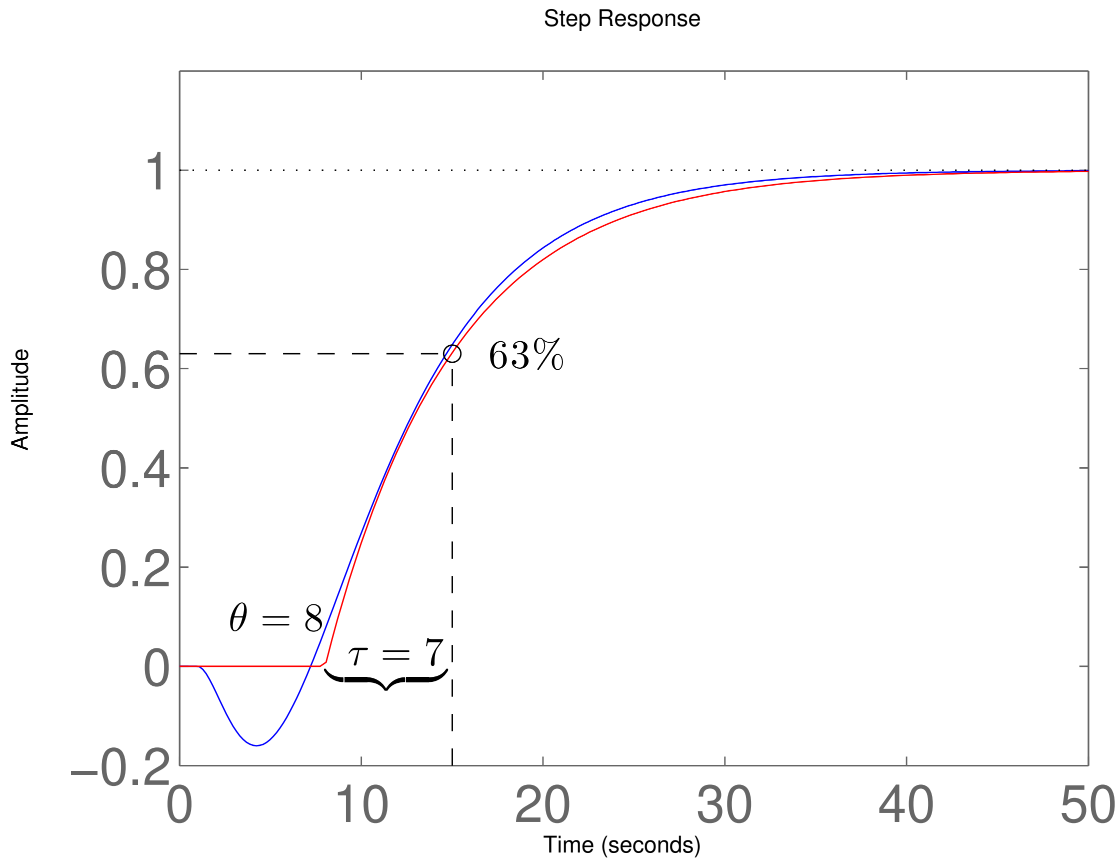


Figure 1: Response of $g(s)$ to a unitary step.

(c) The PI settings (using $\tau_c = \theta$) are

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c + \theta} = \frac{1}{1}\frac{7}{8+8} = 0.44$$

and

$$\tau_I = \min(\tau, 4(\tau_c + \theta)) = \min(7, 4(8+8)) = 7$$

(d) τ_c should be increased to make the response slower.

Problem 2 (20 points)

TF	Poles	Zeros	SS gain	Initial gain	Initial slope	Conclusion
g_1	$p_1 = -1/6; \quad p_2 = -1/2$	$z_1 = -1/26$	-0.2	0	-0.43	D
g_2	$p_1 = -1/6; \quad p_2 = -1/2$	$z_1 = 1/18$	-0.2	0	0.3	F
g_3	$p_1 = -1/10$	none	0.8	0	0.08	A
g_4	$p_1 = -1/10$	none	0.8	0	0	E
g_5	$p_1 = -1/8; p_2 = -1/2; p_3 = -1/2$	$z_1 = -1/10$	1	0	0	B
g_6	$p_1 = -0.2 + j0.98; p_2 = -0.2 - j0.98$	none	1	0	0	C

Table 1: Table for Problem 2; SS: steady state; TF: transfer function

where $j = \sqrt{-1}$.

Problem 3 (25 points)

(a) Mass balance:

$$\frac{dm}{dt} = w_1 - w_2 \qquad [kg/s]$$

since there is no reaction, we can write it on molar basis:

$$\frac{dn}{dt} = F_1 - F_2$$

where

$$F_1 = C_v z\sqrt{P_r^2 - P^2}$$

and

$$n = \frac{PV}{RT}; \quad \frac{dn}{dt} = \frac{V}{RT}\frac{dP}{dt}$$

(b) Steady-state:

$$\frac{dn}{dt} = 0; \quad F_1 = F_2$$

$$C_v z^*\sqrt{P_r^2 - P^{*2}} = F_2$$

$$2 \cdot 0.5\sqrt{12^2 - P^{*2}} = 2$$

$$P^* = \sqrt{12^2 - 2^2} = 11.83 \quad [bar]$$

(c) The nonlinear model is given by

$$\frac{dP}{dt} = \frac{RT}{V}(F_1 - F_2) = \frac{RT}{V}\left(C_v z\sqrt{P_r^2 - P^2} - F_2\right)$$

Using the notation $f = \frac{dP}{dt}$ we have the following linear approximation:

$$\frac{d\Delta P}{dt} = \left.\frac{\partial f}{\partial z}\right|_* \Delta z + \left.\frac{\partial f}{\partial P}\right|_* \Delta P + \left.\frac{\partial f}{\partial F_2}\right|_* \Delta F_2$$

where

$$\left.\frac{\partial f}{\partial z}\right|_* = \frac{RT}{V}C_v\sqrt{P_r^2 - P^{*2}} = 99.77$$

$$\left.\frac{\partial f}{\partial P}\right|_* = \frac{RT}{V}C_v z^*\frac{-2P^*}{2\sqrt{P_r^2 - P^{*2}}} = -147.56$$

and

$$\left.\frac{\partial f}{\partial F_2}\right|_* = -\frac{RT}{V} = -24.94$$

The linearized model becomes

$$\frac{d\Delta P}{dt} = 99.77\Delta z - 147.56\Delta P - 24.94\Delta F_2$$

(d) Using the notation $P(s) = \mathcal{L}\{\Delta P\}$, $F_2(s) = \mathcal{L}\{\Delta F_2\}$, and $z(s) = \mathcal{L}\{\Delta z\}$ we may write

$$s \cdot P(s) = 99.77z(s) - 147.56P(s) - 24.94F_2(s)$$

Since we are only interested in the effect of $z(s)$ in $P(s)$ we can set $F_2(s) = 0$ (this is valid due to the superposition property of linear systems). Solving the equation for $P(s)$ gives

$$P(s) = \frac{99.77}{s + 147.56}z(s) = \frac{0.68}{0.0068s + 1}z(s)$$

Problem 4 (5 points)

Alternative	True or False
(a)	False
(b)	True
(c)	False
(d)	True
(e)	False

Problem 5 (25 points)

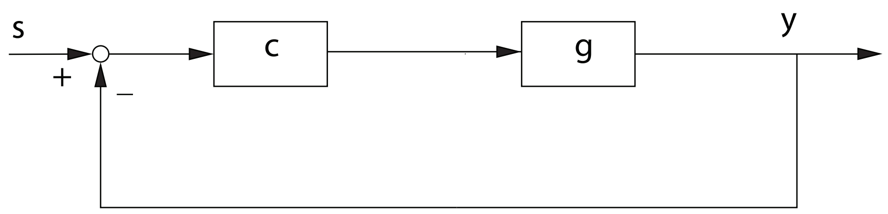
(a) The PID controller in cascade form is

$$c(s) = K_c \left(\frac{\tau_I s + 1}{\tau_I s} \right) (\tau_D s + 1)$$

(b) The desired response is

$$T(s) = y/y_s = \frac{e^{-\theta s}}{\tau_c s + 1}$$

(c) The block diagram of the closed-loop system is



The closed-loop transfer function is given by

$$T(s) = \frac{\text{direct}}{1 + loop} = \frac{gc}{1 + gc}$$

(d) Solving for $c(s)$ we get

$$c(s) = \frac{1}{g} \frac{1}{\left(\frac{1}{T} - 1\right)}$$

(e) Putting in the transfer functions we get

$$c(s) = \frac{(\tau_1 s + 1)}{k e^{-\theta s}} \frac{1}{\frac{\tau_c s + 1}{e^{-\theta s}} - 1}$$

$$c(s) = \frac{(\tau_1 s + 1)}{k(\tau_c s + 1 - e^{-\theta s})}$$

which can be seen as a Smith Predictor controller.

(f) The first-order Padé approximation (with a single pole and a RHP-zero) of a time delay is

$$e^{-\theta s} \approx \frac{1 - \frac{\theta}{2} s}{1 + \frac{\theta}{2} s}$$

(g) Inserting a first-order Padé approximation into the Smith Predictor controller we get

$$\begin{aligned} c(s) &= \frac{(\tau_1 s + 1)}{k(\tau_c s + 1 - \frac{1 - \frac{\theta}{2} s}{1 + \frac{\theta}{2} s})} \\ &= \frac{(\tau s + 1)(1 + \frac{\theta}{2} s)}{k(\tau_c + \frac{\theta}{2} \tau_c s + \theta) s} \\ &= \frac{(\tau s + 1)(1 + \frac{\theta}{2} s)}{k(\tau_c + \theta)(\frac{\theta \tau_c}{2(\tau_c + \theta)} s + 1) s} \end{aligned}$$

(h) Comparing with a series (cascade) PID controller with a derivative filter

$$c(s) = K_c \left(\frac{\tau_I s + 1}{\tau_I s} \right) \left(\frac{\tau_D s + 1}{\tau_F s + 1} \right)$$

we obtain the following relations:

$$\tau_I = \tau$$

$$\tau_D = \frac{\theta}{2}$$

$$K_c = \frac{\tau}{k(\tau_c + \theta)}$$

$$\tau_F = \frac{\theta \tau_c}{2(\tau_c + \theta)}$$

(i) The original SIMC-rule gives a PI controller for this process (with $\tau_D = 0$) instead of a PID controller with a derivative filter.

Note the following:

- $\tau_F = 0$ (no filter) for $\tau_c = 0$
- $\tau_F = \frac{\theta}{4}$ for $\tau_c = \theta$
- $\tau_F = \frac{\theta}{2}$ (filter cancels D-action) for $\tau_c = \infty$