TKP4140 Process Control

Department of Chemical Engineering NTNU

Autumn 2019 - Midterm Exam

11. October 2019

Student number: Signed ______________________

- Write your student number on **every** page in the indicated space.
- Write your answers on the enclosed pages.
- Use the last page for details if you have too little space.
- Do not separate the enclosed pages.
- Time: **90** minutes

Problem 1: System analysis (16 points)

(a) Calculate the poles and zeros for $g_1(s)$ and $g_2(s)$.

i. $g_1(s) = 3 - \frac{1}{5s+1}$

$$g_1 = \frac{3(5s+1)-1}{5s+1} = \frac{15s+2}{5s+1} = \frac{2(7.5s+1)}{5s+1} =$$

Pole: $p = -1/5 = -0.2$

Zero: $z = -1/7.5$

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ii. $g_2(s) = 3 - \frac{4}{5s+1}$

$$g_2 = \frac{3(5s+1)-4}{5s+1} = \frac{15s-1}{5s+1}$$

pole: $p = -1/5$

zero: $z = +1/15$ (RHP-zero!)

2 (+1)

+1 for noting that RHP-zero gives somewhat inverse response.

(b) Sketch the step responses for $g_1(s)$ and $g_2(s)$ for a unit step ($u(t) = 1$) given at time $t = 2$ in Fig. 1

8
5+5
(4 for correct steady states)

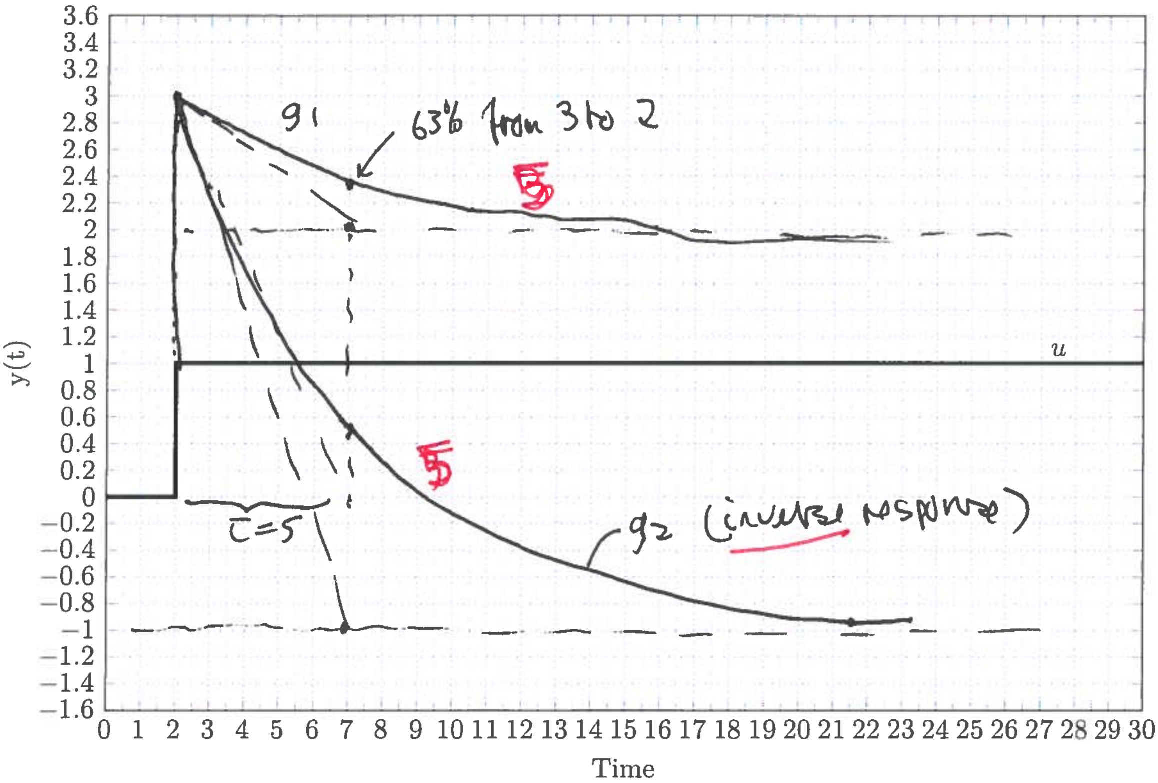

Figure 1: Step responses for $g_1(s)$ and $g_2(s)$

Problem 2: Transfer function responses (16 points)

Given the transfer functions

$$g_1 = k_1$$
$$g_2 = k_2 e^{-\theta s}$$
$$g_3 = \frac{k_3}{\tau_3 s + 1}$$
$$g_4 = \frac{T_4 s + 1}{\tau_4 s + 1}$$

And given the responses for a unit step ($u(t) = 1$) given at time $t = 2$ shown in Fig. 2

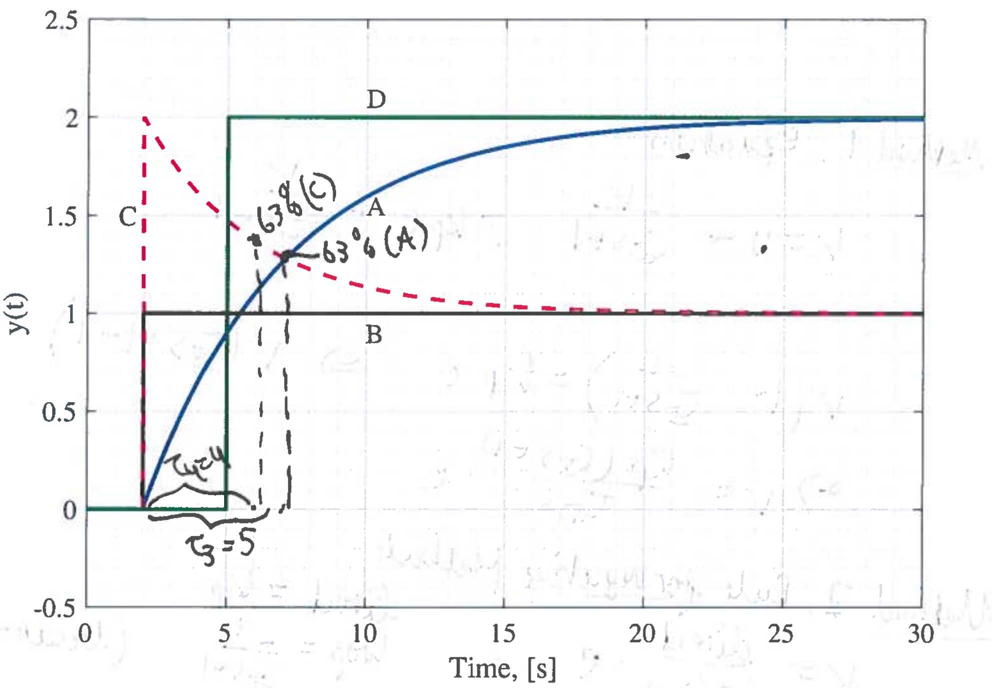

Figure 2: Step responses

(a) Match function g_1, g_2, g_3, g_4 with functions A, B, C, D.

2 each
2·4=8

A = g_3 B = g_1 C = g_4 D = g_2

(b) Find the missing parameter for g_1, g_2, g_3, g_4. Comment your choice.

~~3 each~~

$g_1 = k_1 = 1$ $g_2 = k_2 e^{-\theta s} = 2e^{-3s}$ $g_3 = \frac{k_3}{\tau_3 s+1} = \frac{2}{5s+1}$ $g_4 = \frac{8s+1}{4s+1}$

$\frac{T_4}{\tau_4} = 2$

$\tau_4 = 4$ (63% from 2 to 1)

1 2 2 3

8 total

(-1 if they forget that step starts at t=2)

Problem 3: Block Diagrams (16 points)

Given the block diagram from Fig. 3

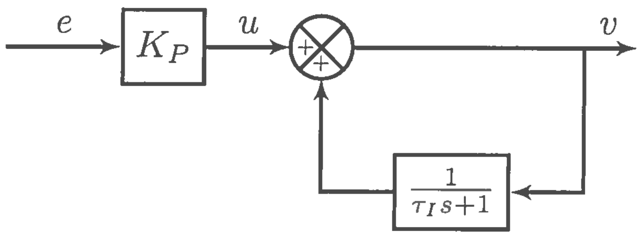

Figure 3: Block diagram

(a) Find the closed loop transfer function $K(s)$ from e to v. (Note that this is positive feedback.)

Method 1. Equations

$$v = u + \frac{v}{\tau_I s+1} = K_p e + \frac{v}{\tau_I s+1}$$

$$v\left(1 - \frac{1}{\tau_I s+1}\right) = K_p e \Rightarrow v(\tau_I s + 1 - 1) = K_p(\tau_I s+1)e$$

$$\Rightarrow v = \frac{K_p(\tau_I s+1)}{\tau_I s} \cdot e$$

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(well ✓ if they use rule for negative feedback)

Method 2. Rule for negative feedback

$$v = \frac{\text{direct}}{1+\text{loop}} \cdot e \qquad \text{direct} = K_p \quad \text{loop} = \frac{-1}{\tau_I s+1}$$

(because of positive feedback)

$$= \frac{K_p}{1 - \frac{1}{\tau_I s+1}} \cdot e = \frac{K_p(\tau_I s+1)}{\tau_I s} \cdot e$$

(b) What can you say about $K(s)$?

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$$K(s) = K_p \frac{\tau_I s+1}{\tau_I s} = K_p\left(1 + \frac{1}{\tau_I s}\right).$$

This is the transfer function of a PI-controller

Problem 4: Controller design (16 points)

(a) What is the transfer function for a PI-controller?

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$$C(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right) = K_c\left(1+\frac{1}{\tau_I s}\right) = K_c + \frac{K_I}{s}$$

when $K_I = K_c/\tau_I$

(b) Design a SIMC-controller for

$$g(s) = k\frac{-\theta s+1}{\tau s+1}$$

We first do the "half-rule" to get the effective delay

that is, assume $-\theta s + 1 \approx e^{-\theta s}$

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$$g(s) \approx k\frac{e^{-\theta s}}{\tau s+1}$$

SIMC-rule $\quad K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} \quad \tau_I = \min(\tau_1, 4(\tau_c+\theta))$

(c) What SIMC-controller do you get for $g(s)$ if $\tau = 0$?

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– When $\tau = 0$ we get $K_c = 0$ and $\tau_I = 0$

– This is an I-controller with

$$K_I = \frac{K_c}{\tau_I} = \lim_{\tau\to 0}\frac{K_c}{\tau_I} = \frac{1}{k(\tau_c+\theta)}$$

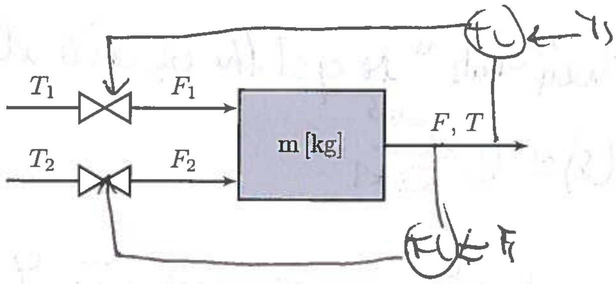
Problem 5: Modelling and linearization. (36 points)

Consider the mixing process shown in Fig. 4, where stream F_1 with temperature T_1 is mixed with stream F_2 with temperature T_2 to produce stream F [kg s^{-1}] with temperature T [°C]. We assume constant mass m, $c_p \approx c_V$ (liquid) and constant and equal c_p.

The nominal operating conditions are:

$F_1^* = 0.5\,\text{kg}\,\text{s}^{-1}$ $\quad F_2^* = 1.5\,\text{kg}\,\text{s}^{-1}$ $\quad T_1^* = 80\,°\text{C}$ $\quad T_2^* = 20\,°\text{C}$ $\quad m = 1\,\text{kg}$

The control objective is to keep the outlet flow at setpoint ($F = F^{sp}$) and the outlet temperature at setpoint $T = T^{sp}$.



Variables

- **CVs =** (F, T)
- **MVs =** (F_1, F_2)
- **DVs =** (T_1, T_2)

Figure 4: Mixing process (shower)

(a) Derive the mass balance (note that m is constant).

Mass Balance: $\frac{dm}{dt} = F_1 + F_2 - F$ [kg/s]

Since m is constant we get

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$0 = F_1 + F_2 - F \Rightarrow F = F_1 + F_2$ [kg/s] (1)

(b) Derive the energy balance in temperature form ($\frac{dT}{dt} = ...$).

$H_{in} \quad H$

$= \overbrace{H_{in}} - \overbrace{H_{out}}$

Energy balance: $\frac{dE}{dt} = E_{in} - E_{out} + \underbrace{W_s + Q}_{=0} + \underbrace{W_{pv}}_{\text{Flow work (in-out)}}$

For a thermodynamic system ~~E=U and~~ with constant C_p

$E = mC_V(T - T_{ref})$ Assume $C_V \approx C_p$ (Liquid)

$H = FC_p(T - T_{ref})$. Set $T_{ref} = 0$ for simplicity.

Get $\frac{d(m\cancel{C_p}T)}{dt} = F_1\cancel{C_p}T_1 + F_2\cancel{C_p}T_2 - F\cancel{C_p}T$ [J/s]

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$\frac{dT}{dt} = \frac{1}{m}(F_1T_1 + F_2T_2 - FT)$ (2)

(c) Find the steady-state values for F and T.

6 { 3, 3

From (1): $F^* = F_1^* + F_2^* = 0.5 + 1.5 = 2$ kg/s

From (2): $T^* = \frac{F_1T_1 + F_2T_2}{F} = \frac{0.5 \cdot 80 + 1.5 \cdot 20}{2}$ °C

$= 35\,°C$

(d) Introduce deviation variables and linearize the two balances.

(1) gives $\Delta F^{(t)} = \Delta F_1 + \Delta F_2$ (it's already linear)

$\Rightarrow \Delta F(s) = \Delta F_1(s) + F_2(s)$

(2) gives

$$\frac{d\Delta T}{dt} = \frac{1}{m}\left[F_1^* \Delta T_1 + T_1^* \Delta F_1 + F_2^* \Delta T_2 + T_2^* \Delta F_2 - F^* \Delta T - T^* (\Delta F_1 + \Delta F_2)\right]$$

6

Laplace

$$msT(s) = F_1^* T_1(s) + T^* F_1(s) + F_2^* T_2(s) + T_2^* F_2(s) - F^* T(s) - T^* F_1(s) - T^* F_2(s)$$

$$\Rightarrow (\tau s + 1) T(s) = \frac{F_1^*}{F^*} T_1(s) + \frac{(T_1^* - T^*)}{F^*} F_1(s) + \frac{F_2^*}{F^*} T_2(s) + \frac{(T_2^* - T^*)}{F^*} F_2(s)$$

$\frac{m}{F^*}$

(e) Let

$$F(s) = g_{11}(s)F_1(s) + g_{12}(s)F_2(s)$$
$$T(s) = g_{21}(s)F_1(s) + g_{22}(s)F_2(s) + g_{d1}(s)T_1(s) + g_{d2}(s)T_2(s)$$

What are g_{11}, g_{12}, g_{21}, g_{22}, g_{d1} and g_{d2}?

6 2

Sol: $g_{11} = 1$, $g_{12} = 1$

Note that $\tau = \frac{m}{F^*} = \frac{1}{2} = 0.5s$, $\frac{F_1^*}{F^*} = 0.25$ etc., $\frac{T_1^* - T^*}{F^*} = \frac{80-35}{2} = 22.5$

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$\frac{T_2^* - T^*}{F^*} = \frac{20-35}{2} = -7.5$

$g_{21} = \frac{22.5}{0.5s+1}$, $g_{22} = \frac{-7.5}{0.5s+1}$, $g_{d1} = \frac{0.25}{0.5s+1}$, $g_{d2} = \frac{0.75}{0.5s+1}$

(f) Suggest a control structure based on single loop controllers, that is suggest where to place (TC) and (FC) in Fig. 4. Comment on why you made this choice.

We use the largest flow (F_2) to control flow (F) and the smallest flow (F_1) to control T.

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We also note that the gain $|g_{21}|$ (from F_1 to T) is larger than $|g_{22}|$ (from F_2 to T) so this agrees with "pair on large gain".

(5 points if they have correct with no argument, 4 if wrong with no argument)

Extra space

Please indicate clearly to which problem the solution belongs.