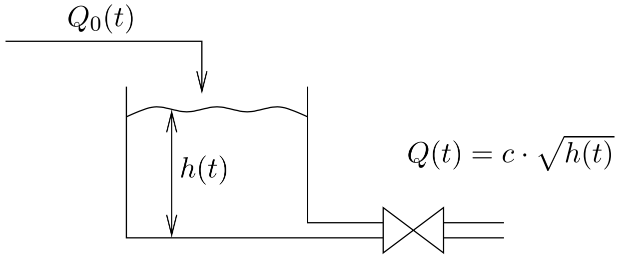
TKP4140 Process control — Midterm Exam

12. October 2017

Student number:SOLUTION

- Write your student number on **every** page in the indicated space.
- Write your student answers on the enclosed pages.
- Use the last two pages for details if you have too little space.
- Do not separate the enclosed pages.

1. (30 points) Consider a tank with one inflow and one outflow, as given in Figure 1. Assume constant density, $\rho = const$.



Parameters:

$$A = 10\text{m}^2$$

$$c = 10\frac{\text{m}^3}{\text{min}\sqrt{\text{m}}}$$

$$Q_0^* = 10\frac{\text{m}^3}{\text{min}} \text{ (nominal value)}$$

Figure 1: Tank system with volume $V = Ah$

(a) (7 points) Formulate the dynamic mass balance and write it in the form $\frac{dh}{dt} = \ldots$

The dynamic mass balance is:

$$\begin{aligned}
\frac{dm}{dt} &= m_{in} - m_{out} \\
\frac{d(\rho V)}{dt} &= \rho(Q_0(t) - Q(t)) \\
\frac{d(\rho A h)}{dt} &= \rho(Q_0(t) - Q(t)) \\
\rho\frac{dAh}{dt} &= \rho(Q_0(t) - Q(t)) \\
\frac{dAh}{dt} &= (Q_0(t) - Q(t)) \text{ after dividing by constant } \rho \\
\frac{dh}{dt} &= \frac{1}{A}(Q_0(t) - Q(t)) \\
\frac{dh}{dt} &= \frac{1}{A}(Q_0(t) - c\sqrt{h(t)})
\end{aligned}$$

(b) (3 points) What are the steady state values of $h(t)$ and $Q(t)$?

At steady state we have

$$0 = \frac{1}{A}(Q_0(t) - c\sqrt{h(t)}).$$

So

$$Q_0^* = c\sqrt{h(t)},$$

and

$$h^* = \left(\frac{Q_0^*}{c}\right)^2 = 1\text{m}$$

and

$$Q^* = Q_0^* = 10\frac{\text{m}^3}{\text{min}}$$

(c) (6 points) Linearize the model and introduce deviation variables.

The model is

$$\frac{dh}{dt} = f(h, Q_0)$$

where

$$f(h, Q_0) = \frac{1}{A}(Q_0(t) - c\sqrt{h(t)})$$

We linearize around the stationary point. The deviation variables are:

$$\Delta h = h - h^* \quad \Delta Q_0 = Q - Q_0$$

The linearized model takes the form

$$\frac{d\Delta h}{dt} = \mathcal{A}\Delta h + \mathcal{B}\Delta Q_0.$$

The constants $\mathcal{A}$ and $\mathcal{B}$ are found as

$$\mathcal{A} = \left.\frac{\partial f}{\partial h}\right|_* = \frac{-c}{2A\sqrt{h_0}} = -\frac{1}{2\text{min}}$$

$$\mathcal{B} = \left.\frac{\partial f}{\partial Q}\right|_* = \frac{1}{A} = \frac{1}{10\text{m}^2}$$

Omitting the units, the linearized model is

$$\frac{d\Delta h}{dt} = -0.5\Delta h + 0.1\Delta Q_0.$$

(d) (6 points) Take the Laplace transform and derive the transfer function $g(s)$:

$$h(s) = g(s)Q_0(s)$$

Taking the Laplace transform of the linearized model, we have

$$sh(s) = -0.5h(s) + 0.1Q_0(s).$$

Solving for $h(s)$

$$h(s) = \frac{0.1}{s+0.5}Q_0(s) = \frac{0.2}{2s+1}Q_0(s),$$

and comparing $h(s) = g(s)Q_0(s)$ with the above gives the transfer function $g(s)$ as

$$g(s) = \frac{0.2}{2s+1}$$

This is a first order transfer function of form

$$g(s) = \frac{k}{\tau s+1}$$

(e) (2 points) What is the value of the steady state gain and the time constant in $g(s)$?

The gain is $k = 0.2$ and the time constant $\tau = 2$.

(f) (4 points) Fill in the corresponding transfer functions in the block diagram from Figure 2.

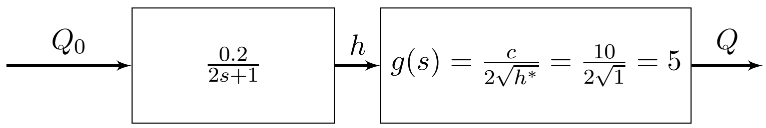


Figure 2: Block Diagram Tank System

(g) (2 points) Does the block diagram from Figure 2 correspond to an open loop or a closed loop system?

Open loop system.

2. (10 points) Given the first-order transfer function

$$g(s) = \frac{2e^{-7s}}{5s+1} \tag{1}$$

and $y(s) = g(s)u(s)$.
Consider a step change in the input:

$$u(t) = \begin{cases} 0 & \text{for} \quad t < 3 \\ 5 & \text{for} \quad t \geq 3 \end{cases} \tag{2}$$

Use the template in Fig.3 to sketch $u(t)$ and the resulting $y(t)$. Indicate the time constant, delay, and steady state value in your sketch.

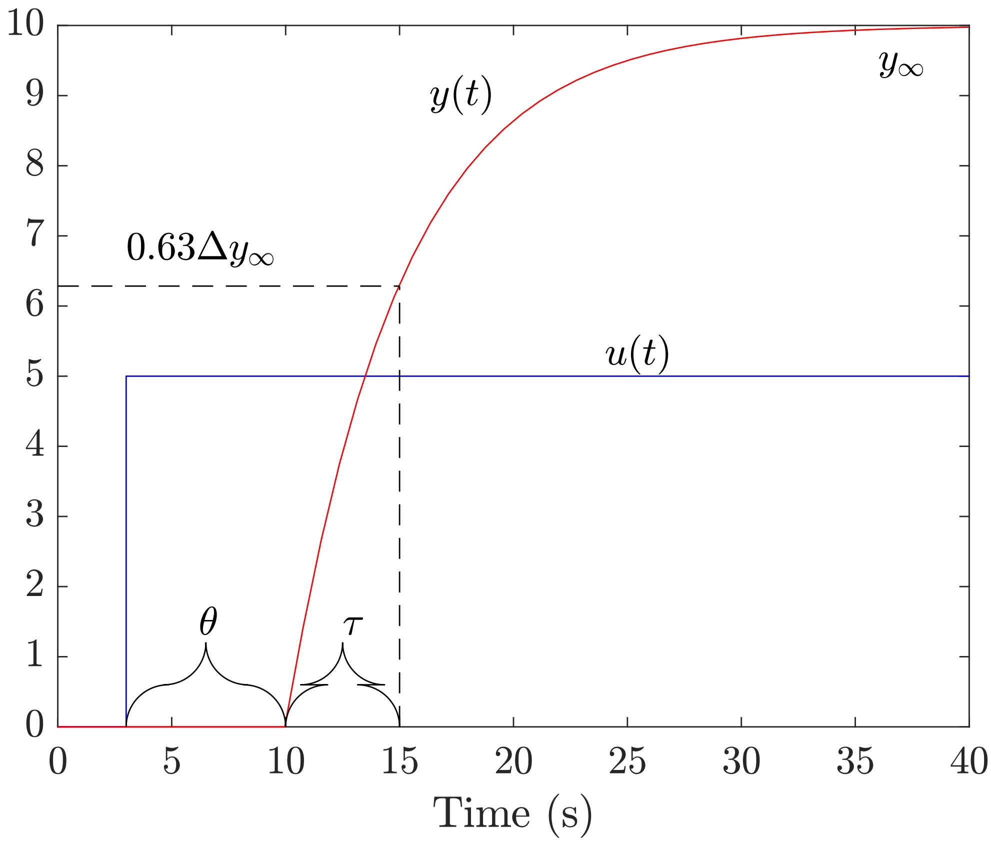


Figure 3: Step Response

3. (20 points) Consider the system in Figure 4.

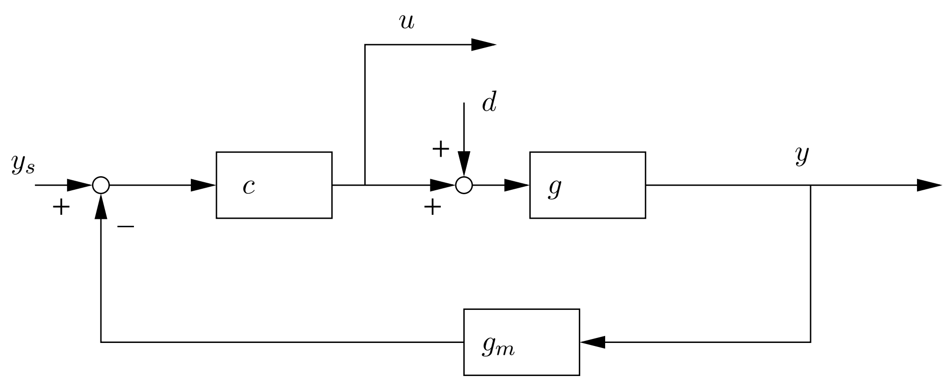


Figure 4: Block diagram

(a) (4 points) Find the closed loop transfer function $T(s)$ from $y_s(s)$ to $y(s)$. Use symbols $(c(s)$,$g(s)$,$g_m(s))$.

$$T(s) = \frac{\text{"direct"}}{1 + \text{"closed loop"}}$$

$$= \frac{gc}{1 + g_m gc}$$

(b) (4 points) Find the closed loop transfer function $Q(s)$ from $y_s(s)$ to $u(s)$. Use symbols $(c(s)$,$g(s)$,$g_m(s))$.

$$Q(s) = \frac{c}{1 + g_m gc}$$

(c) (5 points) Find T(s) when: $g(s) = \frac{3}{5s+1}$ $g_m(s) = \frac{-s+1}{s+1}$ $c(s) = 1$

$$T(s) = \frac{\frac{3}{5s+1} \cdot 1}{1 + \frac{-s+1}{s+1} \cdot \frac{3}{5s+1} \cdot 1} = \frac{3(s+1)}{5s^2 + 3s + 4} = \frac{3}{4} \frac{s+1}{\frac{5}{4}s^2 + \frac{3}{4}s + 1}$$

(d) (3 points) What is the steady state gain when there is a unit step in y_s?

The steady-state gain is $T(0) = \frac{3}{4} = 0.75$

(e) (2 points) Calculate the damping factor and the time constant of T(s).
Hint: the denominator in the second order transfer function is $\tau^2 s^2 + 2\tau\zeta s + 1$.

$\zeta = \frac{3}{2 \cdot 4\tau} = 0.335$ $\tau = \sqrt{\frac{5}{4}} = 1.118$

(f) (2 points) Does the system oscillate?

Yes, the system oscillates because $|\zeta| < 1$. This happens because the controller gain is a bit too large.

4. (20 points) Given

$$g_1 = \frac{2.5}{(6s+1)}$$
$$g_2 = \frac{2.5(s+0.8)}{(6s+1)^2}$$
$$g_3 = \frac{2.5}{(9s^2+3s+1)}$$
$$g_4 = \frac{2(-4s+1)}{(6s+1)^2}$$
$$g_5 = \frac{2e^{-4s}}{(6s+1)^2}$$
$$g_6 = \frac{2.5}{(6s+1)^2}$$

Fill in the missing values in Table 1. In the case that the results in the table do not give a unique answer, comment on your choice.

Hints:
Initial slope of response to unit step input: $\lim_{t\to 0} y'(t) = \lim_{s\to\infty} sg(s)$

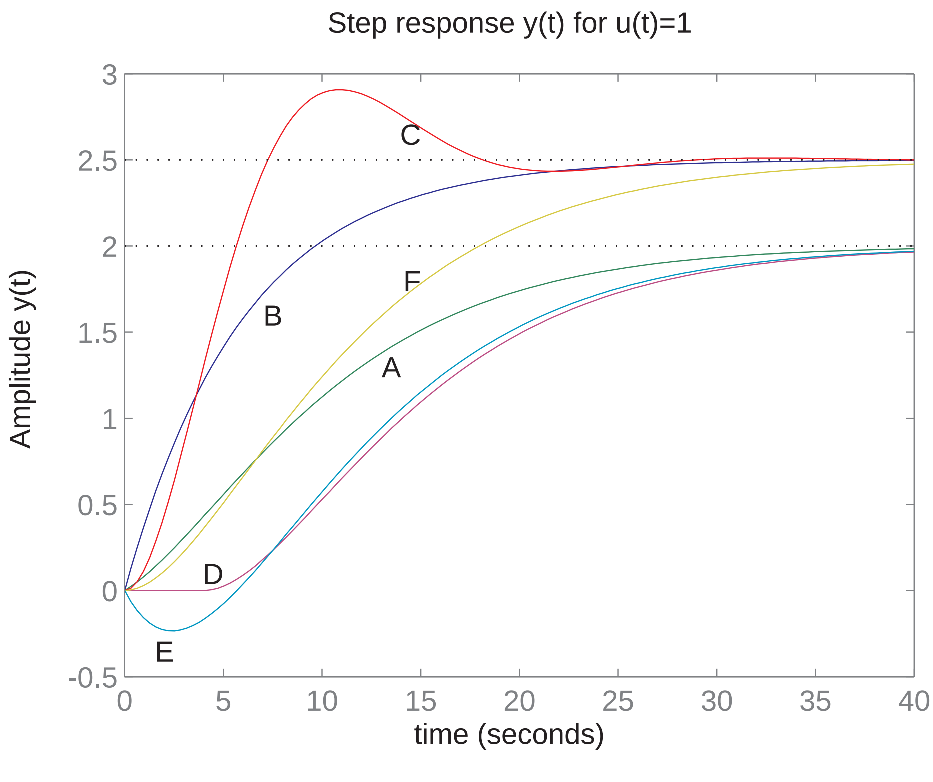


Figure 5: Step responses

Table 1: Problem 4; SS: steady state; TF: transfer function

TF	Poles	Zeros	SS gain	Initial gain	Initial slope	Conclusion
g_1	$s = -0.17$	-	2.5	0	0.41	B
g_2	$s_1 = -0.17$ $s_2 = -0.17$	-0.8	2	0	0.07	A
g_3	$-0.17 \pm 0.29i$	-	2.5	0	0	C
g_4	$s_1 = -0.17$ $s_2 = -0.17$	0.25	2	0	-0.22	E
g_5	$s_1 = -0.17$ $s_2 = -0.17$	-	2	0	0	D
g_6	$s_1 = -0.17$ $s_2 = -0.17$	-	2.5	0	0	F

5. (20 points) (a) SIMC tuning rules.

i. (3 points) Write the SIMC PI tuning rules for a first-order plus delay process.

$$g(s) = \frac{ke^{-\theta s}}{\tau s+1}$$

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c+\theta} \qquad \tau_I = \min(\tau_1, 4(\tau_c+\theta)$$

ii. (3 points) Write the SIMC tuning rules for cascade PID for a second-order plus delay process.

$$g(s) = \frac{ke^{-\theta s}}{(\tau s+1)(\tau_2+1)}$$

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c+\theta} \qquad \tau_I = \min(\tau_1, 4(\tau_c+\theta) \qquad \tau_D = \tau_2$$

(b) By modelling and linearization, you have derived the following process transfer function

$$g(s) = \frac{3(-1.5s+1)e^{-0.5s}}{(25s+1)(3s+1)(0.8s+1)} \tag{3}$$

i. (3 points) Write the first-order plus delay approximation $g_1(s)$ using the half rule.

$$\theta = 1.5 + 0.5 + \frac{3}{2} + 0.8 = 4.3$$
$$\tau = 25 + \frac{3}{2} = 26.5$$
$$k = 3$$
$$g_1(s) = \frac{ke^{-\theta s}}{\tau s+1} = \frac{3e^{-4.3s}}{26.5s+1}$$

ii. (3 points) Write the second-order plus delay approximation $g_2(s)$ using the half rule.

$$\theta = 1.5 + 0.5 + \frac{0.8}{2} = 2.4$$
$$\tau_1 = 25$$
$$\tau_2 = 3 + \frac{0.8}{2} = 3.4$$
$$k = 3$$
$$g_2(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{3e^{-2.4s}}{(25s+1)(3.4s+1)}$$

iii. (3 points) Based on the approximations of $g_1(s)$ and $g_2(s)$ give the SIMC PI and PID settings. Use the standard choice $\tau_c = \theta$, where θ is the effective delay.

PI settings:

$$K_c = \frac{1}{3} \cdot \frac{26.5}{2 \cdot 4.3} = 1.03$$
$$\tau_i = \min(26.5, 4 \cdot 2 \cdot 4.3) = 26.5$$

PID settings:

$$K_c = \frac{1}{3} \cdot \frac{26.5}{2 \cdot 4.3} = 1.74$$
$$\tau_i = \min(25, 4 \cdot 2 \cdot 2.4) = 19.2$$
$$\tau_2 = 3.4$$

iv. (3 points) Would you recommend a PI or a PID controller? Explain briefly.

PID-controller is recommended for g_2 because $\tau_2 > \theta$. Figure 6 shows the closed loop responses for both PI and PID controllers, when a unit step is given in the input u at $t = 10\,s$ (u acts as a disturbance in this case). It can be observed that the input usage is higher fro the PID controller. However, the change in the output is smaller for the PI controller.

(c) (3 points) What would the SIMC PI tunings be for the system in Problem 3 (given Figure 4 with the transfer function from 3(c))?

In practice, every measurement has a time delay. Therefore. to design the controller, one has to consider the transfer function from the input (u) to the measured output (y_m), which is:

$$gg_m = \frac{3}{5s+1} \cdot \frac{-s+1}{s+1}$$

Applying the half rule gives:

$$gg_m \approx \frac{3e^{-1.5s}}{5.5s+1}$$

Applying the SIMC PI rules, and assuming $\tau_c = \theta$:

$$K_c = \frac{1}{3} \cdot \frac{5.5.}{2 \cdot 1.5} = 0.61$$
$$\tau_i = \min(5.5, 4 \cdot 2 \cdot 1.5) = 5.5$$

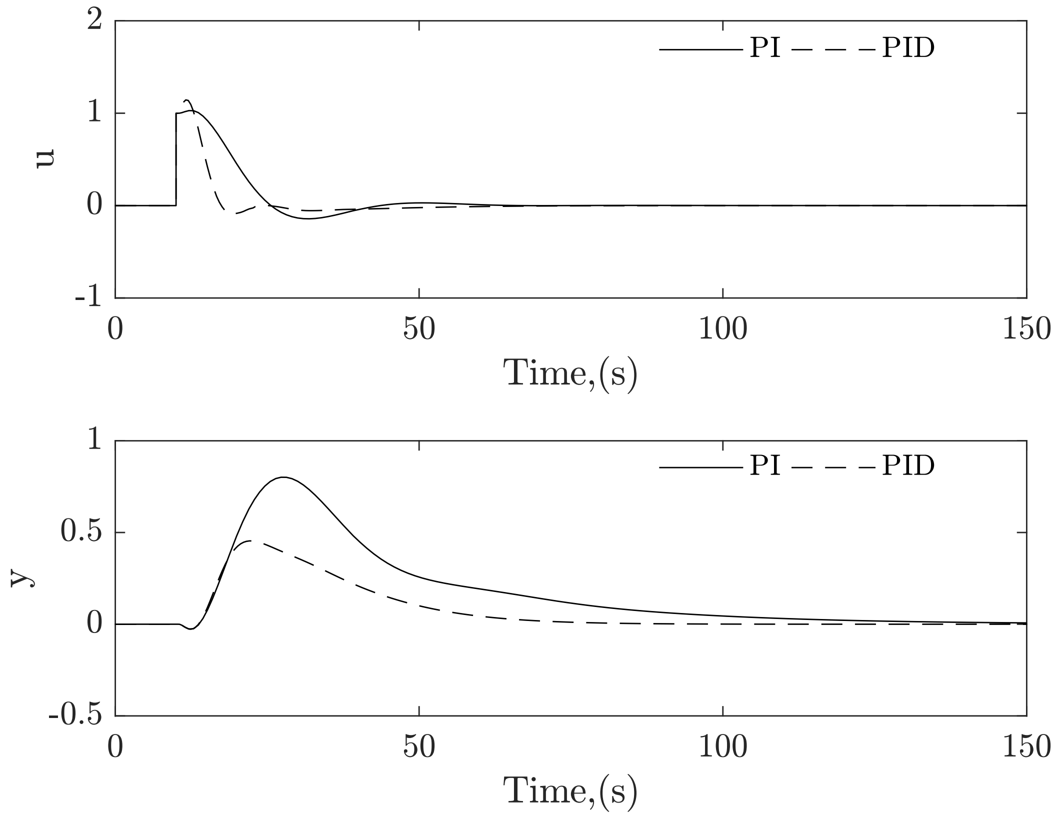


Figure 6: Closed loop response for PI and PID controllers for a unit step in u at $t = 10\,s$