# TKP4140 Process control - Midterm Exam 

12. October 2017

Student number: $\qquad$

- Write your student number on every page in the indicated space.
- Write your student answers on the enclosed pages.
- Use the last two pages for details if you have too little space.
- Do not separate the enclosed pages.

1. (30 points) Consider a tank with one inflow and one outflow, as given in Figure 1. Assume constant density, $\rho=$ const.

Parameters:


$$
\begin{aligned}
A & =10 \mathrm{~m}^{2} \\
c & =10 \frac{\mathrm{~m}^{3}}{\min \sqrt{\mathrm{~m}}} \\
Q_{0}^{*} & =10 \frac{\mathrm{~m}^{3}}{\min }(\text { nominal value })
\end{aligned}
$$

Figure 1: Tank system with volume $V=A h$
(a) Formulate the dynamic mass balance and write it in the form $\frac{d h}{d t}=\ldots$
(b) What are the steady state values of $h(t)$ and $Q(t)$ ?
(c) Linearize the model and introduce deviation variables.
(d) Take the Laplace transform and derive the transfer function $g(s)$ :

$$
h(s)=g(s) Q_{0}(s)
$$

(e) What is the value of the steady state gain and the time constant in $g(s)$ ?
(f) Fill in the corresponding transfer functions in the block diagram from Figure 2.


Figure 2: Block Diagram Tank System
(g) Does the block diagram from Figure 2 correspond to an open loop or a closed loop system?
2. (10 points) Given the first-order transfer function

$$
\begin{equation*}
g(s)=\frac{2 e^{-7 s}}{5 s+1} \tag{1}
\end{equation*}
$$

and $y(s)=g(s) u(s)$.
Consider a step change in the input:

$$
u(t)=\left\{\begin{array}{l}
0 \text { for } t<3  \tag{2}\\
5 \text { for } t \geq 3
\end{array}\right.
$$

Use the template in Fig. 3 to sketch $u(t)$ and the resulting $y(t)$. Indicate the time constant, delay, and steady state value in your sketch.


Figure 3: Step Response
3. (20 points) Consider the system in Figure 4.


Figure 4: Block diagram
(a) Find the closed loop transfer function $T(s)$ from $y_{s}(s)$ to $y(s)$. Use symbols $\left(c(s), g(s), g_{m}(s)\right)$.
(b) Find the closed loop transfer function $Q(s)$ from $y_{s}(s)$ to $u(s)$. Use symbols $\left(c(s), g(s), g_{m}(s)\right)$.
(c) Find $\mathrm{T}(\mathrm{s})$ when: $g(s)=\frac{3}{5 s+1}$

$$
g_{m}(s)=\frac{-s+1}{s+1} \quad c(s)=1
$$

(d) What is the steady state gain when there is a unit step in $y_{s}$ ?
(e) Calculate the damping factor and the time constant of $\mathrm{T}(\mathrm{s})$.

Hint: the denominator in the second order transfer function is $\tau^{2} s^{2}+2 \tau \zeta s+1$.
$\zeta=\quad \tau=$
(f) Does the system oscillate?
$\qquad$
4. (20 points) Given

$$
\begin{aligned}
& g_{1}=\frac{2.5}{(6 s+1)} \\
& g_{2}=\frac{2.5(s+0.8)}{(6 s+1)^{2}} \\
& g_{3}=\frac{2.5}{\left(9 s^{2}+3 s+1\right)} \\
& g_{4}=\frac{2(-4 s+1)}{(6 s+1)^{2}} \\
& g_{5}=\frac{2 e^{-4 s}}{(6 s+1)^{2}} \\
& g_{6}=\frac{2.5}{(6 s+1)^{2}}
\end{aligned}
$$

Fill in the missing values in Table 1. In the case that the results in the table do not give a unique answer, comment on your choice.
Hints:
Initial slope of response to unit step input: $\lim _{t \rightarrow 0} y^{\prime}(t)=\lim _{s \rightarrow \infty} s g(s)$
Step response $\mathrm{y}(\mathrm{t})$ for $\mathrm{u}(\mathrm{t})=1$


Figure 5: Step responses
$\qquad$
Table 1: Problem 4; SS: steady state; TF: transfer function

| TF | Poles | Zeros | SS gain | Initial gain | Initial slope | Conclusion |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $g_{1}$ |  |  |  |  |  |  |
| $g_{2}$ |  |  |  |  |  |  |
| $g_{3}$ |  |  |  |  |  |  |
| $g_{4}$ |  |  |  |  |  |  |
| $g_{5}$ |  |  |  |  |  |  |
| $g_{6}$ |  |  |  |  |  |  |

5. (20 points) (a) SIMC tuning rules.
i. Write the SIMC PI tuning rules for a first-order plus delay process.

$$
\begin{array}{ll}
g(s)= \\
K_{c}= & \tau_{I}=
\end{array}
$$

ii. Write the SIMC tuning rules for cascade PID for a second-order plus delay process.

$$
\begin{array}{ll}
g(s)= & \\
K_{c}= & \tau_{I}=
\end{array} \quad \tau_{D}=
$$

(b) By modelling and linearization, you have derived the following process transfer function

$$
\begin{equation*}
g(s)=\frac{3(-1.5 s+1) e^{-0.5 s}}{(25 s+1)(3 s+1)(0.8 s+1)} \tag{3}
\end{equation*}
$$

i. Write the first-order plus delay approximation $g_{1}(s)$ using the half rule.
ii. Write the second-order plus delay approximation $g_{2}(s)$ using the half rule.
iii. Based on the approximations of $g_{1}(s)$ and $g_{2}(s)$ give the SIMC PI and PID settings. Use the standard choice $\tau_{c}=\theta$, where $\theta$ is the effective delay.
iv. Would you recommend a PI or a PID controller? Explain briefly.
(c) What would the SIMC PI tunings be for the system in Problem 3 (given Figure 4 with the transfer function from 3(c))?

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## Extra space if needed

Please indicate clearly which problem the solution belongs to.

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