Process control TKP4140 – Midterm Exam

October 2016

Student number: SOLUTION (short)

- Write your **student number** on **every** page.
- Write your answers in the designated spaces.
- Do **not** separate the sheets.
- If you need extra space, use the extra pages in the end.

Problem 1

1. (7 points) The transfer function $g(s)$ is a first order transfer function with time delay and the output is $y(s) = g(s)u(s)$. The step response $y(t)$ is plotted in Figure 1, and:

$$u(t) = \begin{cases} 0 & \text{for } t < 5 \\ 2 & \text{for } t \geq 5 \end{cases}$$

2 (a) Sketch $u(t)$ in Figure 1.

2 (b) Indicate the time delay (θ), time constant (τ), and steady state value (y_∞) in Figure 1.

3 (c) Write down θ, τ, k, and $g(s)$.

$\theta =$ 5 s $\quad \tau =$ 15 s $\quad k = \frac{\Delta y_\infty}{\Delta u} = \frac{3}{2} = 1.5$

$g(s) = \frac{1.5 e^{-5s}}{15s+1}$

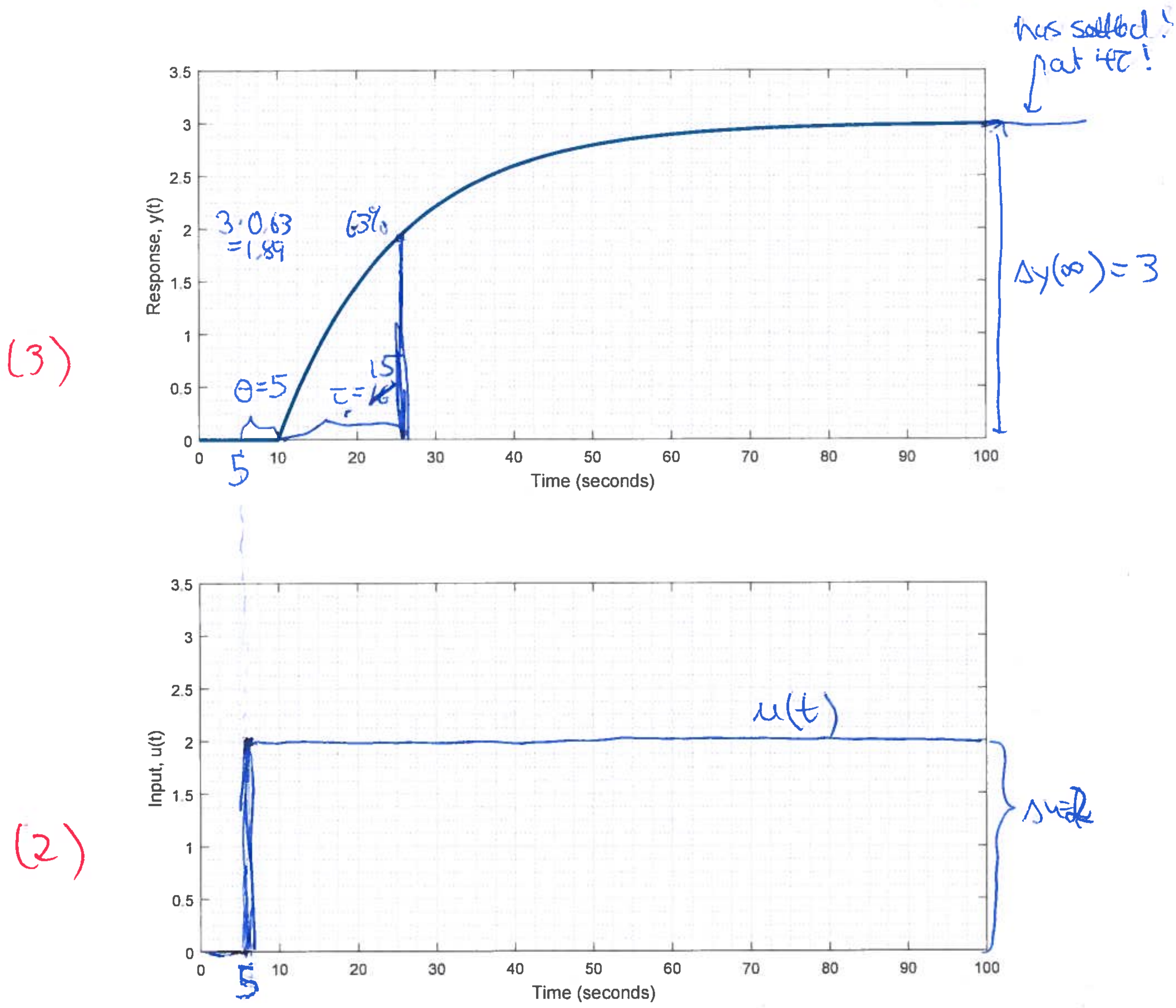


Figure 1: First order transfer function.

Problem 2

2. (18 points) Consider a heated tank with perfectly controlled level as the one in Figure 2. **Assume**: constant density (ρ), constant heat capacity (C_p), constant volume (V), perfect mixing.

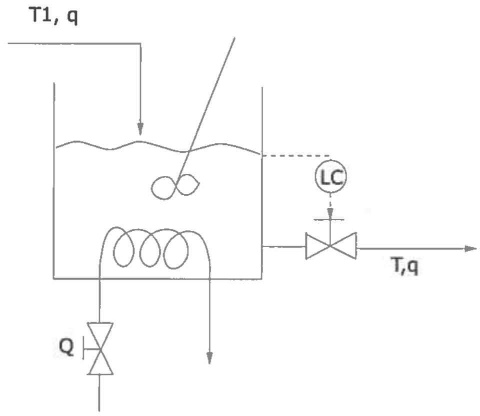


Figure 2: Heated tank

Table 1: Parameters for the heated tank.

V	=	10 000	ℓ
q	=	1.5	m^3/s
ρ	=	1000	kg/m^3
$C_v \approx C_p$	=	4200	J/kgK

(a) Formulate the dynamic energy balance of the system and write it in the form $\frac{dT}{dt} = \dots$

Energy balance: $\frac{dU}{dt} = \underbrace{H_{in} - H_{out}}_{\text{flows}} + Q + \underbrace{W_s}_{\approx 0}$

6

$$\rho V \cancel{C_v} C_p \frac{dT}{dt} = \rho q C_p (T_i - T) + Q$$

$$\Rightarrow \boxed{\frac{dT}{dt} = \frac{q}{V}(T_i - T) + \frac{Q}{\rho V C_p}}$$

(b) Linearize the model and introduce deviation variables. $\Delta T = T - T^*$ etc.

Linearize: $\frac{d\Delta T}{dt} = \frac{T_i^* - T^*}{V}\Delta q + \frac{q^*}{V}(\Delta T_i - \Delta T) + \frac{1}{\rho V C_p}\Delta Q$

3

Laplace assuming $\Delta q = 0$

3

$$s\Delta T(s) = \frac{q^*}{V}\Delta T_i(s) - \frac{q^*}{V}\Delta T(s) + \frac{1}{\rho V C_p}\Delta Q(s)$$

$$\Rightarrow \Delta T(s) = \frac{q^*/V}{s + q^*/V}\Delta T_i(s) + \frac{1/\rho V C_p}{s + q^*/V}\Delta Q(s)$$

$$= \underbrace{\frac{1}{\tau s + 1}}_{g_1(s)}\Delta T_i(s) + \underbrace{\left(\frac{1}{\rho q C_p}\frac{1}{\tau s + 1}\right)}_{g_2(s)}\Delta Q(s) \quad \text{where } \tau = \frac{q^*}{V}$$

(c) Take the Laplace transform and derive the transfer functions $g_1(s)$ and $g_2(s)$:

$$T(s) = g_1 T_1(s) + g_2(s)Q(s)$$

$g_1(s) = \dfrac{1}{\tau_1 s+1}$ $\qquad$ $g_2(s) = \dfrac{k_2}{\tau_1 s+1}$

$\tau_1 = \dfrac{V}{q}$

$k_2 = \dfrac{1}{\rho q C_p}$

(d) Considering the parameters in Table 1, what is the value of the steady state gain k_1 and the time constant τ_1 of $g_1(s)$? (write the units)

2

$k_1 = 1\,[K/K]$ $\qquad$ $\tau_1 = \dfrac{V}{q} = \dfrac{10\,000\,l}{1500\,l/s} = \underline{6.7\,s}$

(e) Considering the parameters in Table 1, what is the value of the steady-state gain k_2 of $g_2(s)$? (write the units)

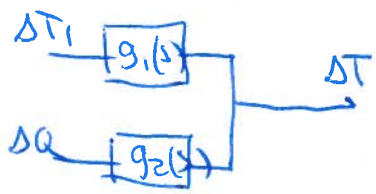
2

$k_2 = \dfrac{1}{\rho q C_p} = \dfrac{1}{1000\,\frac{kg}{m^3}\cdot 1.5\,\frac{m^3}{s}\cdot 4200\,\frac{J}{kg\,K}} = \underline{0.158\cdot 10^{-6}\,\frac{K}{J/s}}$

$= \underline{1.59\cdot 10^{-4}\,K/kW}$

(f) Draw the block diagram for the (open loop) process.

2



or

Problem 3

3. (30 points) Consider the following block diagram:

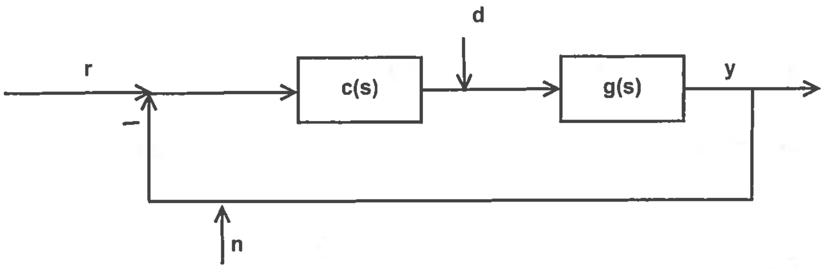


Figure 3: Closed loop block diagram

General rule

$$\mathrm{T.F} = \frac{\text{“direct”}}{\text{“1+ loop”}}$$

$$\mathbf{y = T(s)\ r + M(s)\ d + N(s)\ n}$$

(a) Write the transfer functions $T(s)$, $M(s)$ and $N(s)$. Use symbols only (c(s), g(s)).

2 $T(s) = \dfrac{gc}{1+gc}$

2 $M(s) = \dfrac{g}{1+gc}$

2 $N(s) = \dfrac{-gc}{1+gc}$

(b1) Find $T(s)$ and $M(s)$ when: $g(s) = \frac{3}{4s+1}$ and $c(s) = 2$
Write the denominator in the form: $\tau s + 1$

2 $T(s) = \dfrac{gc}{1+gc} = \dfrac{6}{4s+1+6} = \dfrac{6/7}{\frac{4}{7}s+1} = \dfrac{0.857}{0.57s+1}$

2 $M(s) = \dfrac{g}{1+gc} = \dfrac{3}{4s+1+6} = \dfrac{3/7}{\frac{4}{7}s+1} = \dfrac{0,428}{0.57s+1}$

(b2) Find the analytical expression for $y(t)$ when $r(t)$ is a unit step, $d(t) = 0$, and $n(t) = 0$

$y(t) = 0.857(1-e^{-t/0.57})$

6

Proof:

You should remember that for $g(s) = \frac{k}{\tau s+1}$ the step response is: $\Delta y(t) = k(1-e^{-t/\tau})\,\Delta r(t)$

$= 0.857(1-e^{-t/0.57})\underbrace{\Delta r(t)}_{=1}$

(c1) Find $T(s)$ when: $g(s) = \frac{3}{4s+1}$ and $c(s) = 0.5(1 + \frac{1}{s})$ $= 0.5\frac{s+1}{s}$

→ Please note that this is not a well-tuned controller.

$T(s) = \frac{gc}{1+gc} = \frac{1.5(s+1)}{(4s+1)\cdot s + 1.5(s+1)} = \frac{s+1}{\underbrace{\frac{4}{1.5}}_{2.67}s^2 + \underbrace{\frac{2.5}{1.5}}_{1.67}s + 1}$

3

Standard form $T = \frac{s+1}{\underbrace{\tau^2}_{2.67}s^2 + \underbrace{2\tau\zeta}_{1.67}s+1}$ (*)

(c2) Calculate the damping factor and the time constant of T(s).

Hint: the denominator in the second order transfer function is $\tau^2 s^2 + 2\tau\zeta s + 1$

3

$\zeta = 0.510$ $\tau = \sqrt{2.67} = 1.63$

Proof:

$2\tau\zeta = 1.67$

$\zeta = \frac{1.67}{2\cdot 1.63} = 0.510$

(c3) Compute $y(0)$, $y'(0)$, and $y(\infty)$ for T(s) when a unit step input $r(s) = 1/s$ is applied.

$y(0) =$ 0 $y'(0) =$ 0.37 $y(\infty) =$ 1

$$T(s) = \frac{s+1}{2.67s^2 + 1.67s + 1}$$

Step-response for unit r.

4

1 $y(\infty) = T(0) = 1$ (as expected, no steady-state offset with integral action in the controller)

2 $y'(0) = \lim_{s\to\infty} s\,T(s) = \lim_{s\to\infty} \frac{s^2}{2.67s^2} = 0.37$

1 $y(0) = \lim_{s\to\infty} T(s) = \frac{1}{\infty} = 0$

(c4) Sketch the response $y(t)$, when $r(t)$ is a unit step, $d = 0$ and $n = 0$.
Hints:

- Consider the answers you gave in (c2) and (c3).
- Note that the period of oscillations is approximately $2\pi\tau$
- The first peak is at $t \approx 4.7s$

4

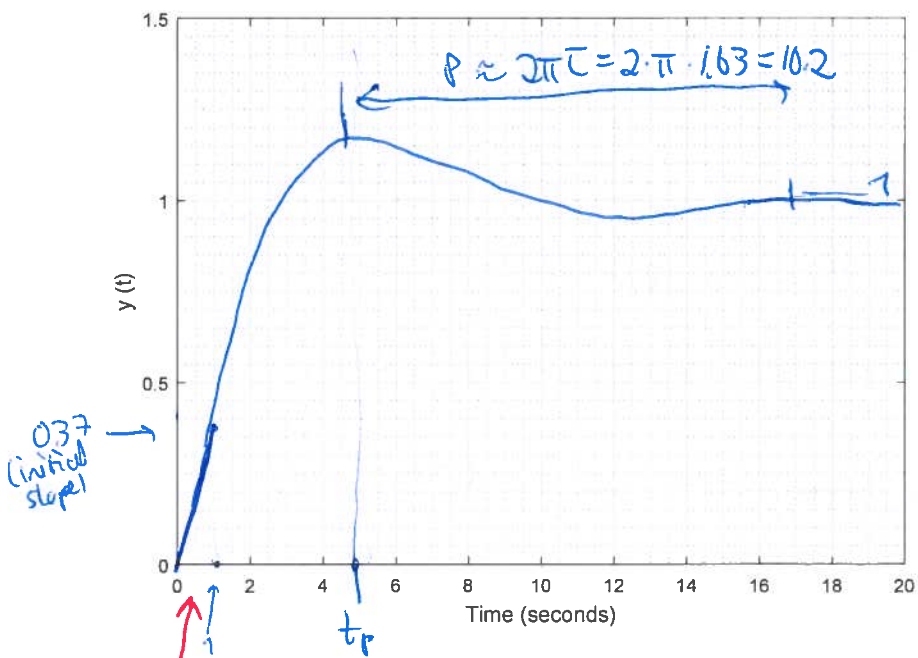


Figure 4: Step response

Start out with initial slope

0,37/1

Problem 4

4. (15 points) Given the transfer function:

$$g(s) = \frac{(-5s+1)e^{-3s}}{(2s+1)^3(7s+1)} \quad (1)$$

(a) Write the first order plus time delay approximation $g_1(s)$ using the half-rule.

$g_1(s) = \frac{e^{-13s}}{8s+1}$

$\theta = \frac{\tau_{20}}{2} + \tau_{30} + \tau_{40} + T + \theta_0$

$\tau_1 = \tau_1 + \frac{\tau_{20}}{2}$

4

$\theta = \underbrace{\frac{2}{2}}_{\tau_{20}} + \underbrace{2 \cdot 2}_{\tau_3 + \tau_4} + \underbrace{5}_{T} + \underbrace{3}_{\theta_0} = 13 \qquad \tau_1 = 7 + \frac{2}{2} = 8$

(b) Give the SIMC PI settings, using $\tau_c = \theta$, where θ is the effective delay.

$K_c = \frac{1}{k} \frac{\tau_1}{\tau_c + \theta} = \frac{8}{26} = 0.31$ ($\tau_1 = 8$, $k = 1$, $\tau_c = \theta = 13$) $\qquad \tau_I = \min(\tau_1, \underbrace{4(\tau_c + \theta)}_{104}) = 8$ ($\tau_1 = 8$)

2

(c) Write the second order plus time delay approximation $g_2(s)$ using the half-rule.

$g_2(s) = \frac{e^{-11s}}{(7s+1)(3s+1)}$

4

$\theta = \underbrace{\frac{2}{2}}_{\tau_3} + \underbrace{2}_{\tau_4} + \underbrace{5}_{T} + \underbrace{3}_{\theta_0} = 11 \qquad \tau_1 = 7 \qquad \tau_2 = 2 + \frac{2}{2} = 3$

(d) Give the SIMC PID settings (cascade PID), using $\tau_c = \theta$, where θ is the effective delay.

$K_c = \frac{1}{k} \frac{\tau_1}{\tau_c + \theta} = 0.32$ ($\tau_1 = 7$, $\tau_c = 11$, $\theta = 11$) $\qquad \tau_I = \tau_1 = 7 \qquad \tau_D = 3$

2

(e) Would you recommend a PI or a PID controller? Explain briefly.

3

PI ~~PID~~, since $\tau_2 = 3$ is much less than $\theta = 11$ ~~there~~ so there is little benefit with PID. Also note that K_c is almost the same, so little benefit.

Problem 5

5. (5 points) Given the transfer function:

$$g(s) = \frac{20e^{-3s}}{s(2s+1)(s+1)} \tag{2}$$

(a) Write the first order plus time delay approximation $g_1(s)$ using the half-rule.

$g_1(s) = \frac{20e^{-5s}}{s}$

Comment 2nd order model would be

$g_2 = \frac{20e^{-3.5s}}{s(2.5s+1)}$

This will actually be an integrating + time delay process

3

$\theta = \underbrace{\theta_0}_{3} + \underbrace{\frac{\tau_2}{2}}_{2/2} + \underbrace{\tau_3}_{1} = 5$

(b) Give the SIMC PI settings. Use $\tau_c = \theta$, where θ is the effective delay.

2

$Kc = \frac{1}{\underset{20}{K'}} \frac{1}{\underset{5+5}{\tau_c+\theta}} = \frac{1}{200}$

$\tau_I = 4(\tau_c+\theta) = 4(\overset{5}{\cancel{10}}+\overset{5}{\cancel{10}}) = \cancel{8}40$

Problem 6

6. (5 points) Indicate whether the following statements are true of false.

	True	False
The PID SIMC rule gives tunings for PID in ideal form.		X
Increasing the integral term τ_I in a PID controller increases the effect of integral action.		X
After tuning a PI controller using the SIMC rules with $\tau_c = \theta$, you realized that the closed loop response is faster than what you would like it to be. In order to slow down the response, you should decrease τ_c.		X
A closed loop with P-control always has a steady-state offset of $\frac{1}{1+K_cK}$, where K_c is the controller gain and K is the steady state process gain.	X	
Windup is caused by the integral part of the PI controller.	X	

Problem 7

7. (20 points) Figure 5 depicts the responses of the following transfer functions to a step input:

$$\underbrace{(-4.7s - 0.1)}_{} = -0.1(47s+1)$$

$$g_1 = \frac{1}{7s+1} - \frac{1.1}{3s+1} = \frac{3s+1-1.1\cdot 7s-1.1}{(7s+1)(3s+1)} \tag{3}$$

$$-0.1(-37s+1)$$

$$g_2 = \frac{1}{3s+1} - \frac{1.1}{7s+1} = \frac{7s+1-1.1\cdot 3s-1.1}{(7s+1)(3s+1)} = \frac{3.7s-0.1}{(7s+1)(3s+1)} \tag{4}$$

$$g_3 = \frac{(10s+1)}{(7s+1)(1.5s+1)^2} \tag{5}$$

$$g_4 = \frac{1}{s^2+0.4s+1} \tag{6}$$

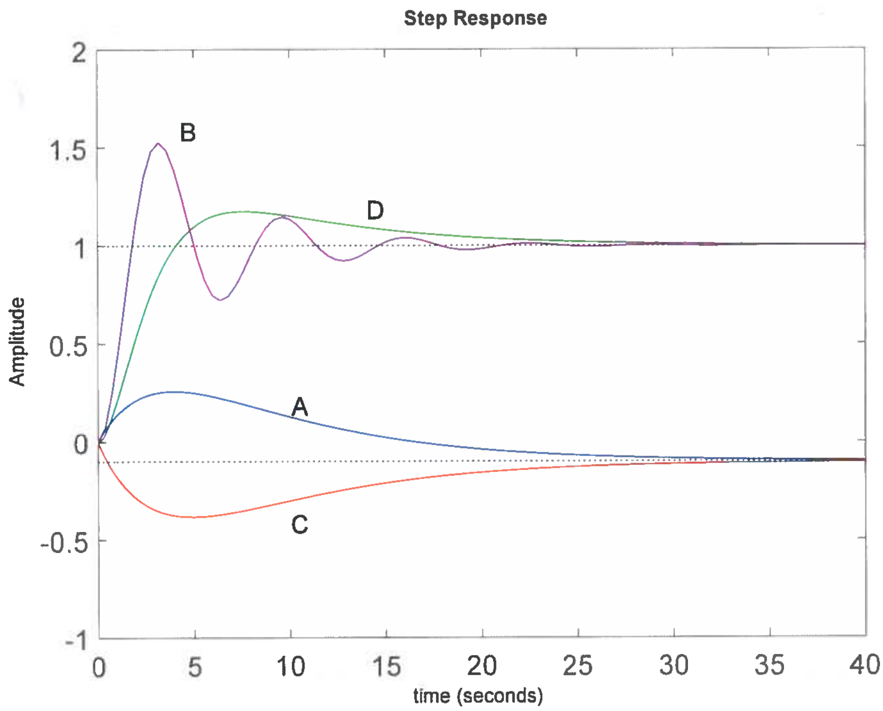


Figure 5: Step response of $g_i(s)$

Fill in the missing values in Table 2:

- Poles and zeros of $g_i(s)$
- Steady state gain (SS gain), initial gain and initial slope **when a unit step input** $u(s) = 1/s$ **is applied at** $t = 0$.
- As conclusion, identify the step responses in Figure 5 (A,B,C, or D).

Hints:
Initial slope of response to unit step input: $\lim_{t\to 0} y'(t) = \lim_{s\to\infty} sg(s)$
$j = \sqrt{-1}$

Table 2: Problem 7; SS: steady state; TF: transfer function

	TF	Poles	Zeros	SS gain	Initial gain	Initial slope	Conclusion	
5	g_1	$p_1 = -1/7$, $p_2 = -1/3$	$z_1 = -1/47$	-0.1	0	$-\frac{47}{21} \cdot 0.1 = -0.224$	C	
5	g_2	$p_1 = s_1 = -1/7$, $p_2 = s_2 = -1/3$	$z_1 = 1/37$ (RHP) (inverse response)	-0.1	0	$\frac{-0.1 \cdot (-37)}{21} = 0.176$	A	← inverse response
5	g_3	$p_1 = -1/7$, $p_2 = -1/1.5$, $p_3 = -1/5.5$	$z_1 = -1/10$	1	0	0	D	
5	g_4	complex poles $p_{1,2} = -0.2 \pm j0.98$	None	1	0	0	B ↑ oscillations	

$$s = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-0.4 \pm \sqrt{0.4^2 - 4 \cdot 1 \cdot 1}}{2}$$

$\sqrt{-3.84}$

$$= -0.2 \pm j\frac{1.96}{2} = \underline{\underline{-0.2 \pm 0.98j}}$$

Initial gain $= g(s = \infty)$

Initial slope: $y'(0^+) = \lim_{s\to\infty} s\,g(s)$

Extra space if needed

Please clearly indicate which problem the solution belongs to.