Process control TKP4140 – Midterm Exam

October 2016

Student number: ____________________

- Write your **student number** on **every** page.
- Write your answers in the designated spaces.
- Do **not** separate the sheets.
- If you need extra space, use the extra pages in the end.

Problem 1

1. (7 points) The transfer function $g(s)$ is a first order transfer function with time delay and the output is $y(s) = g(s)u(s)$. The step response $y(t)$ is plotted in Figure 1, and:

$$u(t) = \begin{cases} 0 & \text{for } t < 5 \\ 2 & \text{for } t \geq 5 \end{cases}$$

   (a) Sketch $u(t)$ in Figure 1.

   (b) Indicate the time delay (θ), time constant (τ), and steady state value (y_∞) in Figure 1.

   (c) Write down θ, τ, k, and $g(s)$.

   $\theta =$ $\tau =$ $k =$

   $g(s) =$

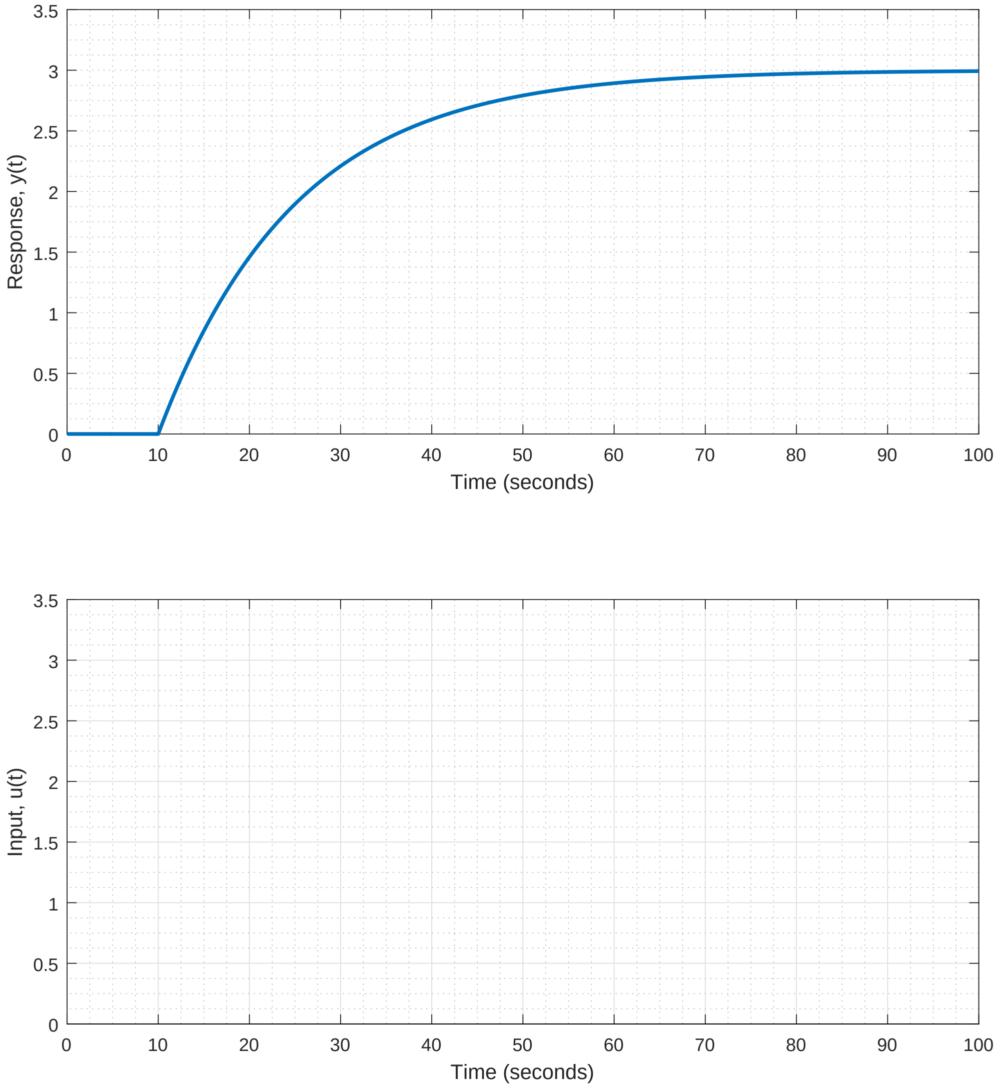

Figure 1: First order transfer function.

Problem 2

2. (18 points) Consider a heated tank with perfectly controlled level as the one in Figure 2. **Assume**: constant density (ρ), constant heat capacity (C_p), constant volume (V), perfect mixing.

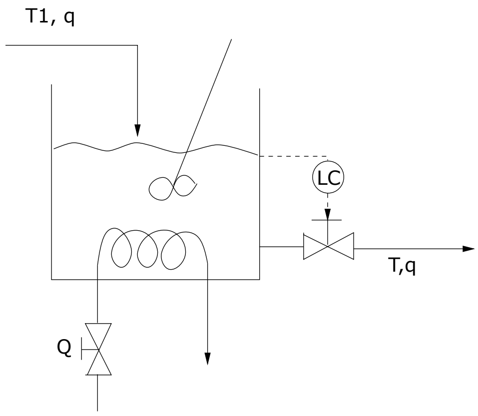

Figure 2: Heated tank

Table 1: Parameters for the heated tank.

V	=	10 000	ℓ
q	=	1.5	m^3/s
ρ	=	1000	kg/m^3
$C_v \approx C_p$	=	4200	J/kgK

(a) Formulate the dynamic energy balance of the system and write it in the form $\frac{dT}{dt} = \dots$

(b) Linearize the model and introduce deviation variables.

(c) Take the Laplace transform and derive the transfer functions $g_1(s)$ and $g_2(s)$:

$$T(s) = g_1 T_1(s) + g_2(s)Q(s)$$

$g_1(s) =$ $g_2(s) =$

(d) Considering the parameters in Table 1, what is the value of the steady state gain k_1 and the time constant τ_1 of $g_1(s)$? (write the units)

$k_1 =$ $\tau_1 =$

(e) Considering the parameters in Table 1, what is the value of the steady-state gain k_2 of $g_2(s)$? (write the units)

$k_2 =$

(f) Draw the block diagram for the (open loop) process.

Problem 3

3. (30 points) Consider the following block diagram:

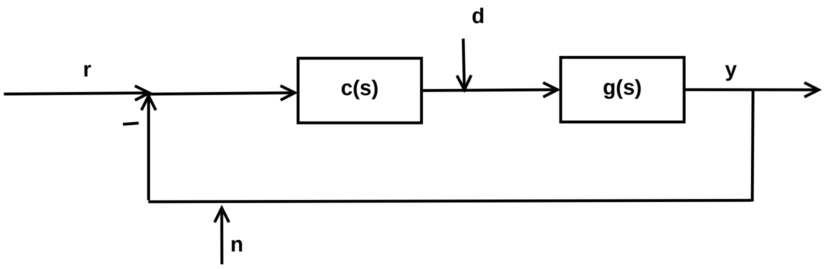

Figure 3: Closed loop block diagram

$$\mathbf{y} = \mathbf{T(s)}\ \mathbf{r} + \mathbf{M(s)}\ \mathbf{d} + \mathbf{N(s)}\ \mathbf{n}$$

(a) Write the transfer functions $T(s)$, $M(s)$ and $N(s)$. Use symbols only (c(s), g(s)).

$T(s) =$

$M(s) =$

$N(s) =$

(b1) Find $T(s)$ and $M(s)$ when: $g(s) = \frac{3}{4s+1}$ and $c(s) = 2$
Write the denominator in the form: $\tau s + 1$

$T(s) =$

$M(s) =$

(b2) Find the analytical expression for $y(t)$ when $r(t)$ is a unit step, $d(t) = 0$, and $n(t) = 0$

$y(t) =$

(c1) Find $T(s)$ when: $g(s) = \frac{3}{4s+1}$ and $c(s) = 0.5(1 + \frac{1}{s})$
→ Please note that this is not a well-tuned controller.

$T(s) =$

(c2) Calculate the damping factor and the time constant of T(s).
Hint: the denominator in the second order transfer function is $\tau^2 s^2 + 2\tau\zeta s + 1$

$\zeta =$ $\tau =$

(c3) Compute $y(0)$, $y'(0)$, and $y(\infty)$ for T(s).

$y(0) =$ $y'(0) =$ $y(\infty) =$

(c4) Sketch the response $y(t)$, when $r(t)$ is a unit step, $d = 0$ and $n = 0$.
Hints:

- Consider the answers you gave in (c2) and (c3).
- Note that the period of oscillations is approximately $2\pi\tau$
- The first peak is at $t \approx 4.7s$

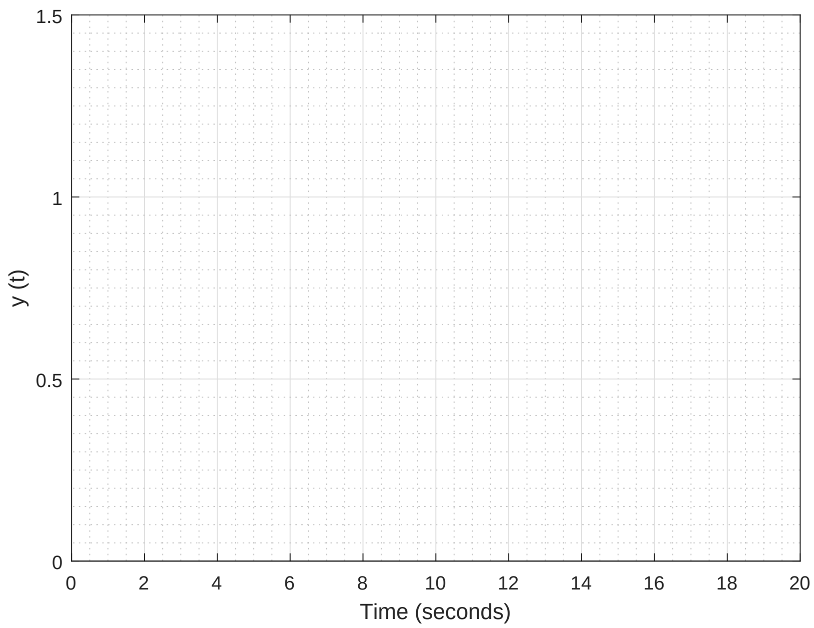

Figure 4: Step response

Problem 4

4. (15 points) Given the transfer function:

$$g(s) = \frac{(-5s+1)e^{-3s}}{(2s+1)^3(7s+1)} \tag{1}$$

(a) Write the first order plus time delay approximation $g_1(s)$ using the half-rule.

$g_1(s) =$

(b) Give the SIMC PI settings, using $\tau_c = \theta$, where θ is the effective delay.

$K_c =$ $\tau_I =$

(c) Write the second order plus time delay approximation $g_2(s)$ using the half-rule.

$g_2(s) =$

(d) Give the SIMC PID settings (cascade PID), using $\tau_c = \theta$, where θ is the effective delay.

$K_c =$ $\tau_I =$ $\tau_D =$

(e) Would you recommend a PI or a PID controller? Explain briefly.

Problem 5

5. (5 points) Given the transfer function:

$$g(s) = \frac{20e^{-3s}}{s(2s+1)(s+1)} \tag{2}$$

(a) Write the first order plus time delay approximation $g_1(s)$ using the half-rule.

$g_1(s) =$

(b) Give the SIMC PI settings. Use $\tau_c = \theta$, where θ is the effective delay.

$Kc =$ $\tau_I =$

Problem 6

6. (5 points) Indicate whether the following statements are true of false.

	True	False
The PID SIMC rule gives tunings for PID in ideal form.		
Increasing the integral term τ_I in a PID controller increases the effect of integral action.		
After tuning a PI controller using the SIMC rules with $\tau_c = \theta$, you realized that the closed loop response is faster than what you would like it to be. In order to slow down the response, you should decrease τ_c.		
A closed loop with P-control always has a steady-state offset of $\frac{1}{1+K_cK}$, where K_c is the controller gain and K is the steady state process gain.		
Windup is caused by the integral part of the PI controller.		

Problem 7

7. (20 points) Figure 5 depicts the responses of the following transfer functions to a step input:

$$g_1 = \frac{1}{7s+1} - \frac{1.1}{3s+1} \tag{3}$$

$$g_2 = \frac{1}{3s+1} - \frac{1.1}{7s+1} \tag{4}$$

$$g_3 = \frac{(10s+1)}{(7s+1)(1.5s+1)^2} \tag{5}$$

$$g_4 = \frac{1}{s^2+0.4s+1} \tag{6}$$

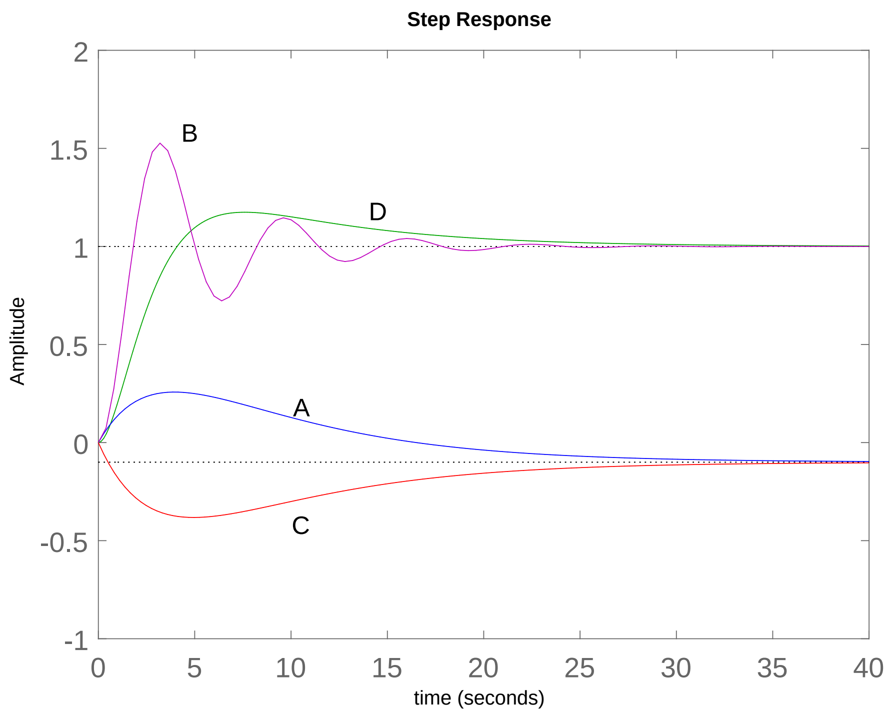

Figure 5: Step response of $g_i(s)$

Fill in the missing values in Table 2:

- Poles and zeros of $g_i(s)$
- Steady state gain (SS gain), initial gain and initial slope **when a unit step input** $u(s) = 1/s$ **is applied at** $t = 0$**.**
- As conclusion, identify the step responses in Figure 5 (A,B,C, or D).

Hints:
Initial slope of response to unit step input: $\lim_{t \to 0} y'(t) = \lim_{s \to \infty} sg(s)$
$j = \sqrt{-1}$

Table 2: Problem 7; SS: steady state; TF: transfer function

TF	Poles	Zeros	SS gain	Initial gain	Initial slope	Conclusion
g_1						
g_2						
g_3						
g_4						

Extra space if needed

Please clearly indicate which problem the solution belongs to.

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