PID control. Practical issues

Smith Predictor (NOT PID...) PID Controller forms Ziegler-Nichols tuning Windup Digital implementation

PID controller

"Ideal" form:
$$u(t) = u_0 + \underbrace{K_c[e(t) + \frac{1}{\tau_I} \int_0^t e(t)dt + \tau_D \frac{de(t)}{dt}]}_{\Delta u}$$

- $e(t) = y_s y_m(t)$
- P-part: MV (Δu) proportional to error
 - This is usually the main part of the controller!
 - Make sure K_c has the right sign! With negative feedback in the loop, Kc has the same sign as the process gain k.
 - Problem: Gives steady-state offset if used without I-action. Offset= 100%/(1+K_ck)
- I-part: To avoid offset, add contribution proportional to integrated error.
 - Note: Larger integral time τ_{I} gives <u>less</u> I-action (turn off by selecting tauI=9999)
 - Sometimes called "reset time" Physical interpretation: τ_{I} is essentially the time it takes to "reset" the bias (u_{0}).
 - Note: Integral term keeps changing as long as e≠0

 > Will eventually make e=0 (no steady-state offset!)
- Possible D-part: Add contribution proportional to change in (derivative of) error
 - Note: Larger derivative times more D-action (turn off by selecting taud=0).
 - Can improve control for high-order (S-shaped) response, but sensitive to measurement noise

Smith Predictor



SP looks good in theory. BUT: It's sensitive to time delay error AND we have found that well-tuned PID (with $\tau_{\rm D} = \theta/3$) is more robust and almost always better than Smith predictor controller*) **FORGET SP!**

* Chriss Grimholt and Sigurd Skogestad. <u>"Should we forget the Smith Predictor?</u>" (2018) In 3rd IFAC conference on Advances in PID control, Ghent, Belgium, 9-11 May 2018. In IFAC papers Online (2018)

Table 7.1	Common	PID	Control	lers
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Controller Type	Other Names Used	Controller Equation	Transfer Function	
Parallel	Ideal, additive, ISA form	$p(t) = \overline{p} + K_c \left(e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de(t)}{dt} \right)$	$\frac{P'(s)}{E(s)} = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$	
Parallel with derivative filter	Ideal, realizable, ISA standard	See Exercise 7.10(a)	$\frac{P'(s)}{E(s)} = K_c \left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha \tau_D s + 1} \right)$	
Series	Multiplicative, interacting	See Exercise 7.11	$\frac{P'(s)}{E(s)} = K_c \left(\frac{\tau_I s + 1}{\tau_I s}\right) (\tau_D s + 1)$	
Series with derivative filter	Physically realizable	See Exercise 7.10(b)	$\frac{P'(s)}{E(s)} = K_c \left(\frac{\tau_I s + 1}{\tau_I s}\right) \left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1}\right)$	
Expanded	Noninteracting	$p(t) = \vec{p} + K_c e(t) + K_I \int_0^t e(t^*) dt^* + K_D \frac{de(t)}{dt}$	$\frac{P'(s)}{E(s)} = K_c + \frac{K_I}{s} + K_D s$	
Parallel, with proportional and	Ideal β, γ controller	$p(t) = \tilde{p} + K_c \left(e_P(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de_D(t)}{dt} \right)$	$P'(s) = K_c \left(E_P(s) + \frac{1}{\tau_I s} E(s) + \frac{\tau_D s E_D(s)}{r} \right)$	
derivative weighting		where $e_P(t) = \beta y_{sp}(t) - y_m(t)$ $e(t) = y_{sp}(t) - y_m(t)$ $e_D(t) = \gamma y_{sp}(t) - y_m(t)$	where $E_P(s) = \beta Y_{sp}(s) - Y_m(s)$ $E(s) = Y_{sp}(s) - Y_m(s)$ $E_D(s) = \gamma Y_{sp}(s) - Y_m(s)$	

+ many more (see manual for your control system...)

Series to ideal form

Series (cascade) PID:

$$c(s) = K_c \frac{(\tau_I s + 1)(\tau_D s + 1)}{\tau_I s} = \frac{K_c}{\tau_I s} (\tau_I \tau_D s^2 + (\tau_I + \tau_D)s + 1)$$

The settings given in this paper (K_c, τ_I, τ_D) are for the series (cascade, "interacting") form PID controller in (1). To derive the corresponding settings for the ideal (parallel, "non-interacting") form PID controller

Ideal PID:
$$c'(s) = K'_c \left(1 + \frac{1}{\tau'_I s} + \tau'_D s \right) = \frac{K'_c}{\tau'_I s} \left(\tau'_I \tau'_D s^2 + \tau'_I s + 1 \right)$$
 (35)

we use the following translation formulas

$$K'_{c} = K_{c} \left(1 + \frac{\tau_{D}}{\tau_{I}} \right); \quad \tau'_{I} = \tau_{I} \left(1 + \frac{\tau_{D}}{\tau_{I}} \right); \quad \tau'_{D} = \frac{\tau_{D}}{1 + \frac{\tau_{D}}{\tau_{I}}}$$
(36)

Derivation: See exercise

Note: The reverse transformation (from ideal to series) is not always possible because the ideal controller may have complex zeros.

Practical "Ideal" PID (parallel form)

The *parallel form* of the PID control algorithm (without a derivative filter) is given by

$$p(t) = \overline{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de(t)}{dt} \right] (7-13)$$

The corresponding transfer function is

$$\frac{P'(s)}{E(s)} = K_c \left[1 + \frac{1}{\tau_I s} + \tau_D s \right]$$
(7-14)



- 1. To get realizable controller and less sensitivity to measurement noise: Add filter on the measurement: $f(s) = \frac{1}{\tau_f s + 1}$, where $\tau_f = \alpha \tau_d$. Typical: $\alpha = 0.1$
- 2. To avoid "derivative kick" do not take derivative of setpoint

illustrate the elimination of derivative kick, consider the parallel form of PID control in Eq. 7-13. Replacing de/dt by $-dy_m/dt$ gives

$$p(t) = \overline{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* - \tau_D \frac{dy_m(t)}{dt} \right]$$
(7-17)

3. To avoid integral "windup" when input saturates (at max or min), use "anti" windup. Simplest: Stop integration while input saturates

Block diagram of practical "ideal" PID



Series (cascade) PID

Commercial versions of the series-form controller have a derivative filter that is applied to either the derivative term, as in Eq. 7-12, or to the PD term, as in Eq. 7-15:

$$\frac{P'(s)}{E(s)} = K_c \left(\frac{\tau_I s + 1}{\tau_I s}\right) \left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1}\right)$$
(7-15)

Typical: α =0.1



do not take derivative of setpoint

Integral windup

 Problem: Integrator "winds up" u(t) when actual input has saturated



Actual input is m. m=u if no saturation

$$u(t) = u_0 + K_c e(t) + \underbrace{\frac{K_c}{\tau_I} \int_0^t e(t) dt}_{\underline{\tau_I}}$$

Keeps changing when $e(t) \neq 0$

Anti-windup

- Approaches to avoid windup
 - 1. Stop integration (e.g. set τ_I =9999) when saturation in input occurs (requires logic)
 - 2. Make integrator track true input using feedback correction *(see Example and Exercise)*
 - 3. Use discrete controller in *velocity form*
 - 4. Use Sigurd's discrete controller with bias adjustment

Example anti-windup (Approach 2)



Disturbance (d): Pulse from 0 to 0.2 and back to 0 at t=10

File: tunepidantiwindup.mdl



t=0: Disturbance starts t=10: Disturbance ends

y = output process u = output controller m = sat(u) = input process Blue = without anti-windup
 Red = with anti-windup

Bumpless transfer

- We want a "soft" transition when the controller is switched between "manual" and "auto"
 - or back from auto to manual
 - or when controller is retuned
- Simple solution: reset bias u₀ as you switch, so that u(t) = u_{manual}(t).

$$u(t) = u_0 + \underbrace{K_c[e(t) + \frac{1}{\tau_I} \int_0^t e(t)dt + \tau_D \frac{de(t)}{dt}]}_{\Delta u}$$

Methods for <u>online</u> tuning of PID controllers

- I. Trial and error
- II. Ziegler Nichols
 - Oscillating P-control
 - Relay method to get oscillations
- III. Closed-loop response with P-control
 - Shams method

On-line tuning: Avoids an open-loop experiment, like a step input change. Advantage on-line: Process is always "under control" In practice: Both "open-loop" and "closed-loop" (online) methods are used

Optimal PID settings

- Can find optimal settings using optimization
- SIMC-rules are close to IAE-optimal for combined setpoints and disturbances*

11.3.2 Tuning Relations Based on Integral Error Criteria

Controller tuning relations have been developed that optimize the closed-loop response for a simple process model and a specified disturbance or set-point change. The optimum settings minimize an *integral error criterion*. Three popular integral error criteria are

1. Integral of the absolute value of the error (IAE)

$$IAE = \int_0^\infty |e(t)| dt \qquad (11-35)$$

where the error signal e(t) is the difference between the set point and the measurement.

2. Integral of the squared error (ISE)

$$SE = \int_0^\infty e^2(t)dt \qquad (11-36)$$

3. Integral of the time-weighted absolute error (ITAE)

$$ITAE = \int_0^\infty t |e(t)| dt \qquad (11-37)$$

*Chriss Grimholt and Sigurd Skogestad, "Optimal PI and PID control of first-order plus delay processes and evaluation of the original and improved SIMC rules", Published in: J. Process Control, vol. 70 (2018), 36-46.

Tuning of your PID controller

I. "Trial & error" approach (online)

- (a) P-part: Increase controller gain (K_c) until the process starts oscillating or the input saturates
- (b) Decrease the gain (~ factor 2)
- (c) I-part: Reduce the integral time (τ_I) until the process starts oscillating
- (d) Increase a bit (~ factor 2)
- (e) Possible D-part: Increase τ_{D} and see if there is any improvement

Very common approach, BUT: Time consuming and does not give good tunings: NOT recommended

II. Ziegler-Nichols closed-loop method (1942)

P-control only: Increase controller gain (K_c) until the process cycles with constant amplitude: ~Y(+)



- Write down the corresponding "ultimate" period (P₁) and controller gain (K₁).
- Based on this "process information" obtain PID settings:

Cycling Method			a distur-	
Ziegler-Nichols	K_c	τ ₁	τ _D	
P In the born	$0.5K_{cu}$	Intering - algore	And the line	
PI	$0.45K_{cu}$	$P_{\mu}/1.2$	and set the	
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8 <$	 PID is for idea
Tyreus-Luyben [†]	K _c	τ, *	τ_D	
PI	$0.31K_{cu}$	$2.2P_u$	- +	TL-modificatio
PID (ideal)	$0.45K_{cu}$	$2.2P_u$	$P_{\mu}/6.3$	(smaller K _c an

Table 11.4 Controller Settings based on the Continuous

Main problems ZN:

- Too aggressive (and has no tuning parameter) 1.
- Two pieces of information (Pu, Ku) is too little to capture all processes. 2. Works poorly on delay-dominant processes

[†] Luyben and Luyben (1997).

I form

on is smoothe d larger τ_I).





EXAMPLE: Process from Astrom et al. (1998)



Figure 3: Load disturbance of magnitude 2 occurs at t = 10.

$$g_0(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$$

- 1. Approximate as first-order model with k=1, $\tau_1 = 1+0.1=1.1$, $\theta=0.1+0.04+0.008 = 0.148$ Get SIMC PI-tunings ($\tau_c=\theta$): $K_c = 1 \notin 1.1/(2 \notin 0.148) = 3.71$, $\tau_l=min(1.1,8 \notin 0.148) = 1.1$
- 2. Approximate as second-order model with k=1, $\tau_1 = 1$, $\tau_2=0.2+0.02=0.22$, $\theta=0.02+0.008 = 0.028$ Get SIMC PID-tunings ($\tau_c=\theta$): $K_c = 1 \notin 1/(2 \notin 0.028) = 17.9$, $\tau_1=min(1,8 \notin 0.028) = 0.224$, $\tau_D=0.22$

Åstrøm relay method (1984): Alternative approach to obtain cycling (and K_u)

- Avoids operating at limit to instability
- Use ON/OFF controller (=relay) were input u(t) varies +-d (around nominal)
- Switch when output y(t) reaches +- a₀ (deadband) (around setpoint; can use a₀=0)
- Example: Thermostat in your home



• From this obtain P_u and

 $K_u = \frac{4d}{\pi a} \underbrace{\longleftarrow}_{a: \text{ amplitude u(t) (set by user)}}_{a: \text{ amplitude y(t) (from experiment)}}$

Alternative to Ziegler-Nichols closed-loop experiment: Obtains more information and avoids cycling.

III. Shams' method: Closed-loop setpoint response with P-controller with about 20-40% overshoot



Example 2: Get k=0.99, theta =1.67, tau=3.0

Ref: Shamssuzzoha and Skogestad (JPC, 2010) + modification by C. Grimholt (PID-book 2012) See Exercise!

Example E2 (Further continued) We want to derive PI- and PID-settings for the process

$$g_0(s) = \frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

using the SIMC tuning rules with the "default" recommendation $\tau_c = \theta$. From the closed-loop setpoint response, we obtained in a previous example a first-order model with parameters k = 0.994, $\theta = 1.67$, $\tau_1 = 3.00$ (5.10). The resulting SIMC PI-settings with $\tau_c = \theta = 1.67$ are

$$PI_{cl}: K_c = 0.904, \tau_I = 3.$$

From the full-order model $g_0(s)$ and the half rule, we obtained in a previous example a first-order model with parameters $k = 1, \theta = 1.47, \tau_1 = 2.5$. The resulting SIMC PI-settings with $\tau_c = \theta = 1.47$ are

$$PI_{half-rule}$$
: $K_c = 0.850$, $\tau_I = 2.5$.

From the full-order model $g_0(s)$ and the half rule, we obtained a second-order model with parameters k = 1, $\theta = 0.77$, $\tau_1 = 2$, $\tau_2 = 1.2$. The resulting SIMC PID-settings with $\tau_c = \theta = 0.77$ are

Series PID :
$$K_c = 1.299$$
, $\tau_I = 2$, $\tau_D = 1.2$.

The corresponding settings with the more common ideal (parallel form) PID controller are obtained by computing $f = 1 + \tau_D/\tau_I = 1.60$, and we have

Ideal PID:
$$K'_c = K_c f = 1.69$$
, $\tau'_I = \tau_I f = 3.2$, $\tau'_D = \tau_D / f = 0.75$.
(5.30)



Fig. 5.6 Closed-loop responses for process E2 using SIMC PI- and PID-tunings with $\tau_c = \theta$. Setpoint change at t = 0 and input (load) disturbance at t = 10. For the PID controller, D-action is Apr only on the feedback signal, i.e., not on the setpoint y_s

Effect of sampling



- All real controllers are digital, based on sampling
- Δt = sampling time (typical 1 sec. in process control, but could be MUCH faster)
- Max sampling time (Shannon): $\Delta t < \tau_c/2$, but preferably much smaller (τ_c = closed-loop response time)
- With continuous methods: Approximate sampling time as effective delay $\theta = \Delta t/2$
- Strange things can happen if Δt is too large:



Digital implementation of first-order filter of measurement*



*Equivalent to "exponentially moving average" of time series data. See Exercise

Discrete (digital) implementation (practical in computer) of PID controller

where $\bar{u}(t) =$ "bias" term with integral action included



Digital PID implementation:

$$\bar{u}_k = \bar{u}(t) \approx \bar{u}_{k-1} + \frac{K_c}{\tau_I} e_k \Delta t$$
$$u_k = \bar{u}_k + K_c e_k - K_c \tau_D \frac{y_k - y_{k-1}}{\Delta t}$$

(Backward Euler)

This is Sigurd's recommendation = "Alt. 3" (see next page)

To avoid windup and to get «bumpless» transfer between manual and auto: Adjust bias \bar{u}_k so that we always have u_k = actual input.

Comment: We can «clamp» \bar{u}_k as we enter saturation (=stop integration), but it may then take a little time to get out of saturation, that is, we get some «windup».

Comparison with book: Digital implementation of PID controllers

Alt. 1 (position form) $p(t) = \vec{p} + K_c \left| e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de(t)}{dt} \right| (7-13)$

Note: p = output from controller

Finite difference approximation:

$$\int_0^t e(t^*) dt^* \approx \sum_{j=1}^k e_j \Delta t \qquad (7-24)$$
$$\frac{de}{dt} \approx \frac{e_k - e_{k-1}}{\Delta t} \qquad (7-25)$$

where

- Δt = the sampling period (the time between successive measurements of the controlled variable)
- e_k = error at the kth sampling instant for k = 1, 2, ...

Substituting Eqs. 7-24 and 7-25 into (7-13) gives the position form,

$$p_k = \overline{p} + K_c \left[e_k + \frac{\Delta t}{\tau_I} \sum_{j=1}^k e_j + \frac{\tau_D}{\Delta t} (e_k - e_{k-1}) \right]$$
(7-26) Alt. 1

 e_k = present sampled value = e(t) e_{k-1} = previous sample = $e(t-\Delta t)$ $e_{k-2} = e(t-2\Delta t)$

Alt. 3 (Sigurd's with bias as extra state, better than Alt. 1 and Alt. 2) $p_k = \bar{p}_k + K_c [e_k + \frac{\tau_D}{\Delta t} (e_k - e_{k-1})]$ where we update ("reset") the bias: $\bar{p}_k = \bar{p}_{k-1} + K_c \frac{\Delta t}{\tau_l} e_k$ To avoid windup and to get bumpless transfer: Adjust bias \bar{p}_k so that $p_k = \text{actual input}$

Alt. 2 (velocity form) In the velocity form, the change in controller output is calculated. The velocity form can be derived by writing Eq. 7-26 for the (k-1) sampling instant:

$$p_{k-1} = \overline{p} + K_c \left[e_{k-1} + \frac{\Delta t}{\tau_I} \sum_{j=1}^{k-1} e_j + \frac{\tau_D}{\Delta t} (e_{k-1} - e_{k-2}) \right]$$
(7-27)

Note that the summation still begins at j = 1, because it is assumed that the process is at the desired steady state for $j \le 0$, and thus $e_j = 0$ for $j \le 0$. Subtracting Eq. 7-27 from (7-26) gives the velocity form of the digital PID algorithm:

$$\Delta p_{k} = p_{k} - p_{k-1} = K_{c} \Big[(e_{k} - e_{k-1}) + \frac{\Delta t}{\tau_{I}} e_{k} \Big]$$

$$+ \frac{\tau_{D}}{\Delta t} (e_{k} - 2e_{k-1} + e_{k-2}) \Big]$$
(7-28) (7-28)

The velocity form has three advantages over the position form:

- 1. It inherently contains antireset windup, because the summation of errors is not explicitly calculated.
- 2. This output is expressed in a form, Δp_k , that can be utilized directly by some final control elements, such as a control valve driven by a pulsed stepping motor.
- 3. For the velocity algorithm, transferring the controller from manual to automatic model does not require any initialization of the output (\overline{p} in

A minor disadvantage of the velocity form is that the integral mode must be included. When the set point is constant, it cancels out in both the proportional and derivative error terms. Consequently, if the integral mode were omitted, the process response to a disturbance would tend to drift away from the set point.

=Bumpless transfer

? This is a major disadvantage of Alt. 2

Block diagram symbols



Figure 9-1: Types of characteristic curves