Process control Q&A session

Sigurd Skogestad

14 Dec. 2021

Exam on 18 December will be digital home exam

Posted on: Tuesday, December 14, 2021 9:25:40 AM CET

The plan is that the exam on 18 December will be changed to digital home exam.

Below is given some information from the digital home exam in 2020. It will probably be the same this year.

The exam will be "open book", meaning that all books and lecture notes are permitted.

However note that the exam is is an individual, independent work.

During the exam it is not permitted to communicate with any other person about the exam questions, or distribute drafts for solutions.

Such communication is regarded as cheating. All submitted answers will be subject to plagiarism control.

I recommend that you write your answers on paper (as for a normal exam). You will get 30 min extra to scan and upload files.

Good luck with your preparations for the exam!

Best wishes from Sigurd Skogestad

About digital exam

- Whether all the answers should be written in paper and scanned and uploaded ? or we will have questions typed directly on the laptop/desktop?

- I suggest writing on paper and scanning for everything

Forum to ask questions in preparation for the exam

Posted on: Friday, November 26, 2021 8:11:06 PM CET

A Q&A forum has been created. Please, feel free to ask questions related with the exam.

Course Link /Forum/Discussion Board

Dec16, problem 6a

Hi, I am struggling with the transition: $g(s) = \frac{-0.25s+1}{10s-1} = -0.025 + \frac{0.975}{10s-1}$.

What should I mutliply with/how do I do it?

I had the same problem Dec15 1a. How do I simplify the transfer functions for sketching the response?

Sigurd Skogestad

RE: Dec16, problem 6a

Reply:

This is an example of partial fraction expansion. Here is how to do it generally:

Consider a transfer function g(s) with at least as many poles as zeros.

In general, we can then write g(s) = b0 + b1/((tau1*s+1) + b2/(tau2*s+1) + ...

Here taui are the time constants. The constants (b's) may be determined in many ways.

One simple method is to multiply by both sides by (tau1*s+1) and then set s=-1/tau1. This gives b1. Then we find b2, etc.

b0 may be found by letting s=infinity. Note that b0=0 except for systems with the same number as zeros as poles,

Example. g(s) = (-0.25s+1)/(10s-1)

This a bit different since the system is unstable, but it does not really matter.

We write (-0.25s+1)/(10s-1) = b0 + b1/(10s-1) (*)

To determine b1: Multiply on both sides of (*) by 10s-1 and set s=0.1: (-0.25s+1) = -0.025+1 = 0.975 = b1

To determine b0: Let s=infinity on both sides of (*): -0.25/10 = b0 -> b0=-0.025

Plot of disturbance response

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Continuous-time transfer function.

>> Td = 3*s/((0.4*s+1)*(0.8*s+1))

Td =

          3 s
  --------------------
  0.32 s^2 + 1.2 s + 1

Continuous-time transfer function.

>> step(Td)
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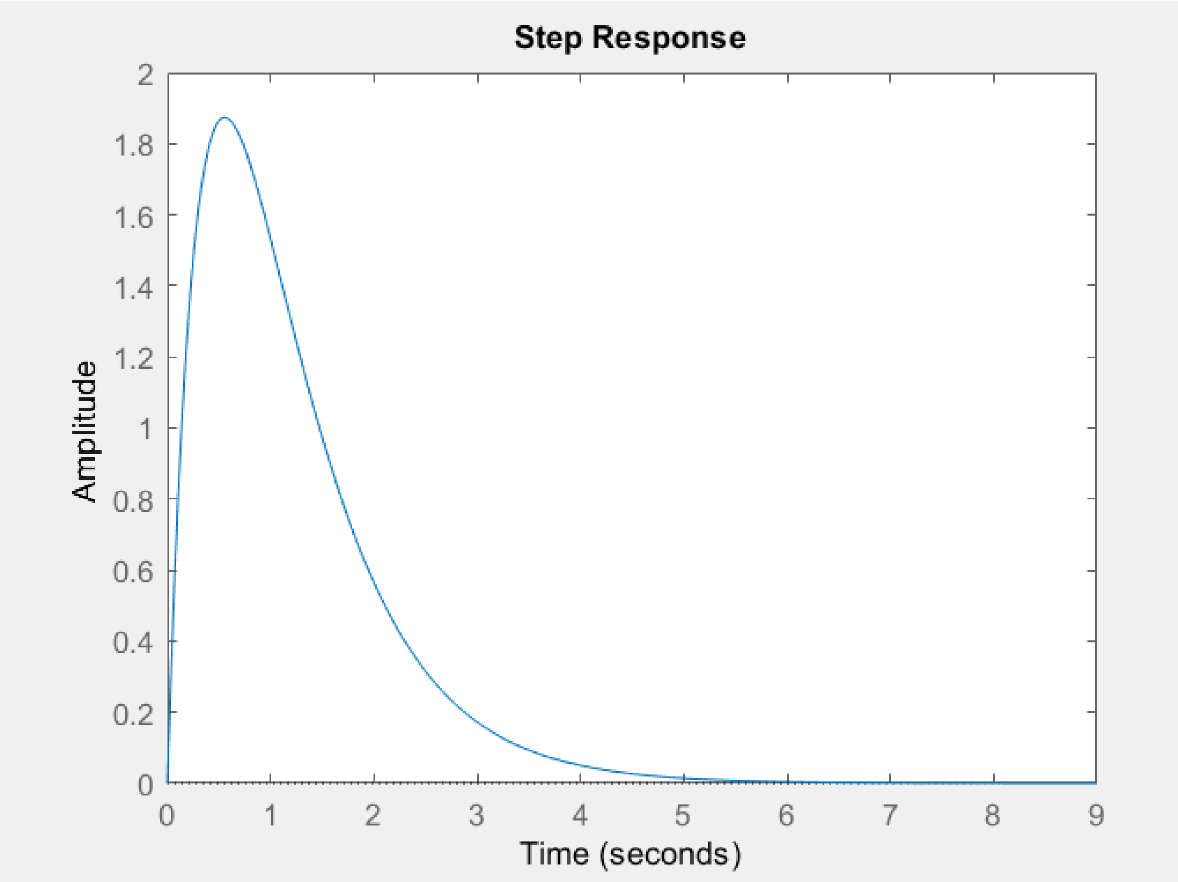
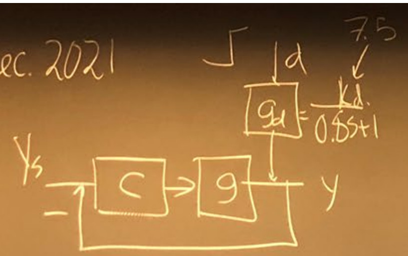


Question: What is the analytical expression for y(t)? (Simple!)

14 Dec. 2021



$$\frac{y}{d} = \frac{g_d}{1+gc} = g_d \cdot S = \frac{k_d}{(0.8s+1)}\frac{0.4s}{(0.4s+1)} = \frac{3s}{(0.8s+1)(0.4s+1)} = T_d$$

$$y = T \cdot Y_s$$

$$T = \frac{1}{0.4s+1}$$

$$S = 1 - T = 1 - \frac{1}{0.4s+1}$$

$$= \frac{0.4s}{0.4s+1}$$

$$T_d = \frac{\alpha_1}{(0.4s+1)} + \frac{\alpha_2}{(0.8s+1)}$$

$$\alpha_1 = 7.5$$
$$\alpha_2 = -7.5$$

Multiply by $(0.4s+1)$ on both sides.

$$\frac{3s}{0.8s+1} = \alpha_1 + \frac{\alpha_2}{(0.8s+1)}(0.4s+1)$$

Set $(0.4s+1) = 0 \rightarrow s = -1/0.4 = -2.5$.

$$\frac{3(-2.5)}{0.8(-2.5)+1} = \alpha$$
$$= \frac{-7.5}{-2+1} = 7.5$$

14 Dec. 2021

$$T_d = \frac{7.5}{0.4s+1} - \frac{7.5}{0.8s+1} = 7.5\left(\frac{1}{0.4s+1} - \frac{1}{0.8s+1}\right)$$

$$y(t) = k\left(1 - e^{-t/\tau}\right) \cdot \Delta u$$

$$y(t) = 7.5\left(1 - e^{-\frac{t}{0.4}} - 1 + e^{-\frac{t}{0.8}}\right)$$

$$= 7.5\left(e^{-\frac{t}{0.8}} - e^{-\frac{t}{0.4}}\right)$$

t	0	0.4	0.8	...	∞
y(t)	0	1.79	1.74		0

Exercise 10

Problem 3

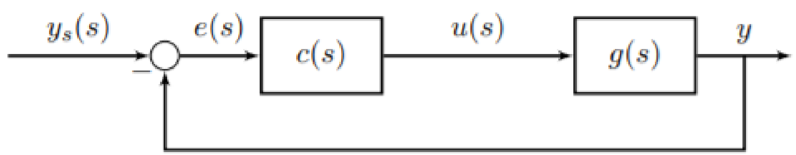


Figure 3: Closed loop control system

Given the system in from Exercise 9, Problem 1.2 c.), see Figure 3. The plant transfer function is:

$$g(s) = \frac{1}{(s+1)^3}, \tag{2}$$

and the PI controller with SIMC rules is

$$c(s) = 0.5\frac{(1+1.5s)}{1.5s}. \tag{3}$$

1. Draw the bode plot of the open loop ($L = gc$) for $\omega = 10^{-2}$ to $\omega = 10^2$ rad/s. Use Figure 4.

2. Fill in the following table.

	$\omega = 0.1$ rad/s	$\omega_c =$	$\omega_{180} =$	$\omega = 10\ rad/s$
$\|L\|$		1		
$\angle L$			$-180°$	

3. How much dead time must we add to L to have $\angle L = -180°$ at the frequency ω_c where $|L| = 1$?

Comment: In Exercise 11, we will use the results from 2 and 3 to compute gain margin, phase margin and delay margin.

Solution to Problem 3

1. The bode plot of $L(s) = g(s)c(s)$ is given in Figure 3, where $L(s)$ is given by:

$$\begin{aligned} L(s) &= g(s)c(s) \\ &= \frac{1}{(s+1)^3}\frac{0.5(1.5s+1)}{1.5(1.5s)} \\ &= 0.5\frac{1.5s+1}{s(s+1)^3} \end{aligned}$$

2. The frequency responses are given in the Table 3

	$\omega = 0.1$ rad/s	$\omega_c = 0.32$	$\omega_{180} = 1.21$	$\omega = 10\ rad/s$
$\lvert L \rvert$	3.32	1	0.15	4.94×10^{-4}
$\angle L$	$-98.6°$	$-117.7°$	$-180°$	$-256.7°$

Table 3: Frequency responses

3. At frequency $\omega_c = 0.32$ the phase shift is $-117.7°$, which means that we must add an extra phase shift of $-180 + 117.7 = -62.3°$ to have $\angle L = -180°$.

   To find out how much dead time (time delay) we can add to L, we need to compute what delay θ will give a phase shift of $-62.3°$. In order to do this, we need to find the phase shift given by a delay term $e^{-\theta s}$.

   The frequency response of the time delay can be obtained by replacing s by $j\omega$.

   Using Euler's formula:

$$G(j\omega) = e^{-\theta j\omega} = \cos(-\theta\omega) + j\sin(-\theta\omega) \qquad (3)$$

   If we consider that:

$$\angle G(j\omega) = \arctan(I/R) \qquad (4)$$

   The phase shift is then:

$$\angle e^{-\theta j\omega} = \arctan\left(\frac{\sin(-\theta\omega)}{\cos(-\theta\omega)}\right) = -\theta\omega \qquad (5)$$

   Therefore, we can find the delay that we must add by letting $-\theta\omega_c = -62.3°\frac{\pi}{180}$ [rad], which gives:

$$\theta = \frac{62.3°\frac{\pi}{180}}{0.32} = 3.4s \qquad (6)$$

   Comment: The proper unit for phase is radians. However, for convenience usually we convert it to degrees, as we have a better feel for the relative magnitudes of angles expressed in degrees rather than in radians. For example, we relate better to the angle 24° than to 0.419 radian.

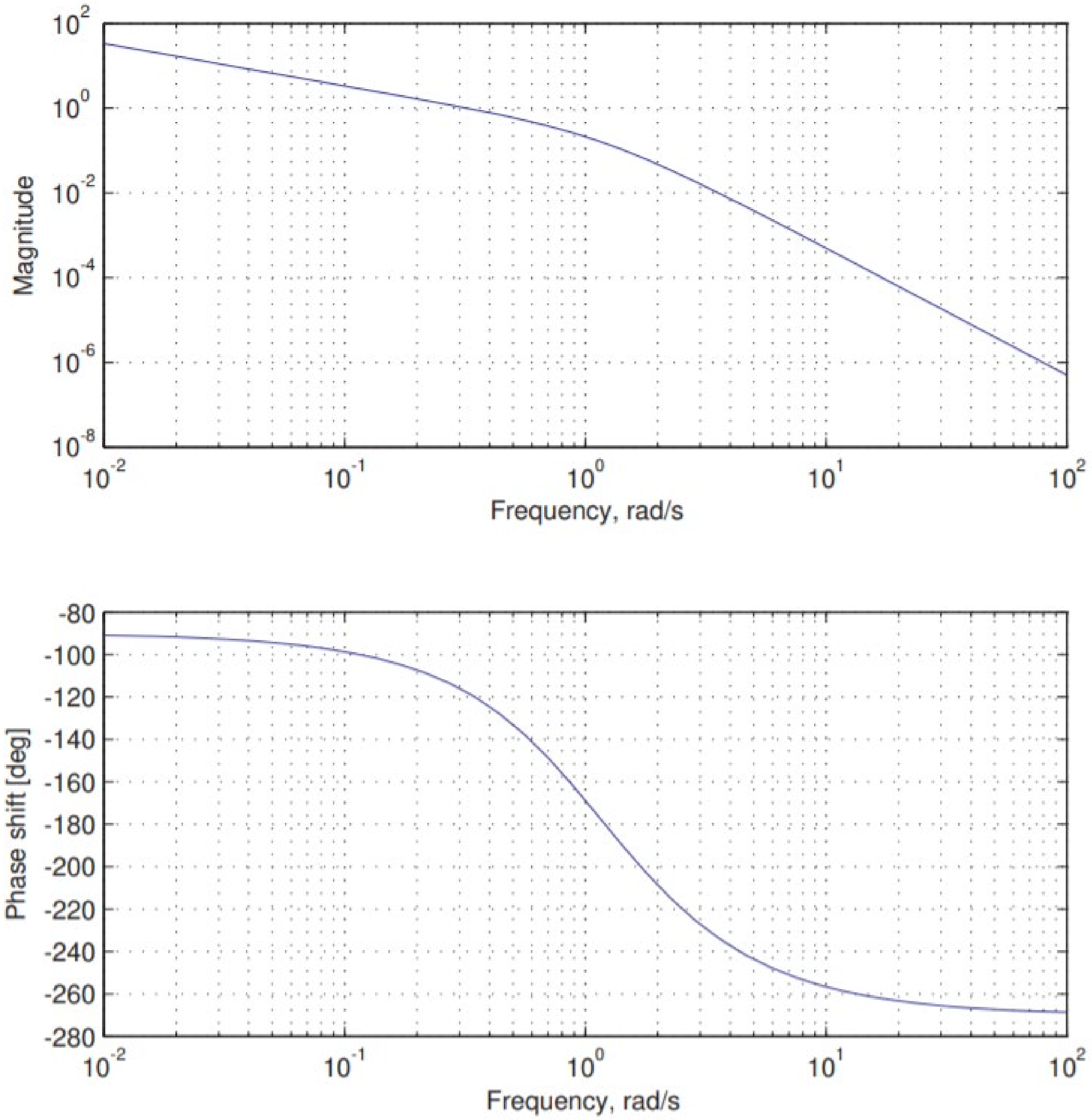


Figure 3: Bode plot of $L(s)$ for Problem 3

14 Dec. 2021

$$g(s) = \frac{1}{(s+1)^3} \left(\approx \frac{e^{-1.5s}}{1.5s+1} \right), \quad K_c = \frac{1}{k}\,\frac{\tau}{\tau_c+\theta} = 0.5$$

($\tau \leftarrow 1.5$, $k \leftarrow 1$, $\tau_c + \theta \leftarrow 3$)

$$c(s) = 0.5\,\frac{1.5s+1}{1.5s}$$

$$L(s) = g \cdot c \qquad |L(j\omega)| = \underbrace{\frac{0.5}{\sqrt{(\omega^2+1)}^{\,3}}}_{|g(j\omega)|} \cdot \frac{\sqrt{(1.5\omega)^2+1}}{1.5\omega}$$

$$\angle L(j\omega) = -3\,\text{atan}(\omega 1) + \text{atan}(1.5\omega) - \underbrace{\frac{\pi}{2}}_{\text{integrator}} \quad [\text{rad.}]$$