Input-output Controllability Analysis

Idea: Find out how well the process can be controlled - without having to design a specific controller

Note: Some processes are impossible to control

Will not say so much about it this year
Some of the old exam questions are not relevant!

Reference: S. Skogestad, ``A procedure for SISO controllability analysis - with application to design of pH neutralization processes'', *Comp.Chem.Engng.*, **20**, 373-386, 1996.

Example: First-order with delay process

$$g(s) = k\frac{e^{-\theta s}}{1+\tau s}; \quad G_d(s) = k_d\frac{e^{-\theta_d s}}{1+\tau_d s}$$

$+$ Measurement delays: $\theta_m, \quad \theta_{md}$.

Problem: What values are desired for good controllability?

Qualitative results:

	Feedback control	Feedforward control
k		
τ		
θ		
k_d		
τ_d		
θ_d		
θ_m		
θ_{md}		

WANT TO QUANTIFY!

Example: First-order with delay process

$$g(s) = k\frac{e^{-\theta s}}{1+\tau s}; \quad G_d(s) = k_d\frac{e^{-\theta_d s}}{1+\tau_d s}$$

+ Measurement delays: θ_m, θ_{md}.

Problem: What values are desired for good controllability?

Qualitative results:

	Feedback control	Feedforward control
k	Large	Large
τ	Small	Small
θ	Small	Small
k_d	Small	Small
τ_d	Large	Large
θ_d	No effect	Large
θ_m	Small	No effect
θ_{md}	No effect	Small

Want fast and large response from input (MV) to output (CV) (rows k, τ, θ)

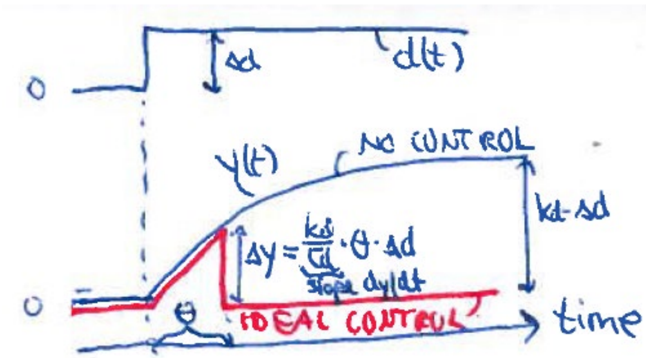
Want slow and small response from disturbance (DV) to output (CV) (rows k_d, τ_d, θ_d)

WANT TO QUANTIFY!

Quantify: Controllability requirements

- Assume process (g) has effective delay θ
- Assume maximum allowed output change (error) is Δy_{max}
- Consider response to disturbance, $g_d = k_d/(\tau_d\ s+1)$
  - For step Δd : Output reaches $\Delta y = (k_d\ \theta\ /\tau_d)\ \Delta d$ at time θ (approximately)
  - If this is larger than acceptable (Δy_{max}) then we are in trouble
  - To be controllable, we must require

    $(k_d\ \theta\ /\tau_d) < \Delta y_{max}/\ \Delta d$



- More generally:
  - Define ω_d as frequency where $|g_d(j\omega_d)|=\Delta y_{max}/\Delta d$
  - Then the controllability requirement is (Rule 1/2)
- $\omega_d\ \theta\ < 1$

- In addition we must avoid input saturation. We have: $\Delta y = g_d\ \Delta d\ + g\ \Delta u$
- So to get Δy=0 without exceeding constraint Δu_{max}, we must require (Rule 4)

At all frequencies $\omega<\omega_d$ (where we need control) :	$\lvert g(jw)\ \Delta u_{max}\ \rvert > \lvert g_d(jw)\ \Delta d \rvert$
At steady state:	$\lvert k\ \Delta u_{max}\ \rvert > \lvert k_d\ \Delta d \rvert$
Initial response (approximately):	$\lvert k\ /\tau\ \Delta u_{max}\ \rvert > \lvert\ k_d/\tau_d\ \ \Delta d \rvert$

Controllability analysis

- Use controllability analysis
  - To avoid spending time on impossible control problem
  - To help design the process (e.g., size buffer tanks)
- Also useful for tuning.
  - τ_c = SIMC tuning parameter
  - Must for acceptable controllability have:

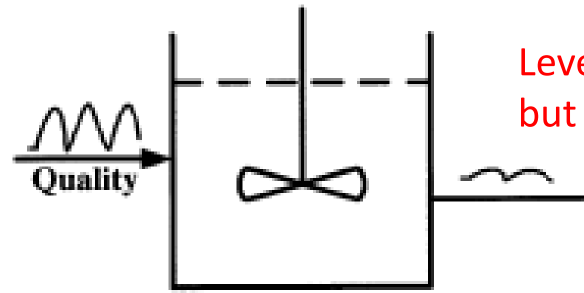
$$\theta \leq \tau_c \leq \frac{1}{\omega_d}$$

- Note
  - Tight control: $\tau_c = \theta$
  - "Smooth" control: $\tau_c = 1/\omega_d$

ω_d is defined as frequency where $|g_d(j\omega_d)| = \Delta y_{max}/\Delta d$

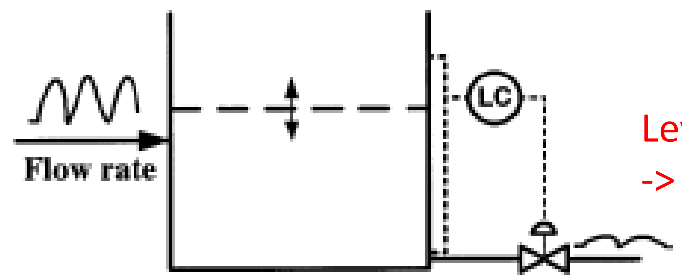
If process is not controllable: Need to change the design

- For example, dampen disturbance by adding buffer tank:



Level control unimportant, but need good mixing

(I) Averaging by mixing (mixing tank)



Level control is NOT tight -> level varies

Integral action is not recommended for averaging level control

(II) Averaging level control (surge tank)

Figure 1. Two types of buffer tanks.

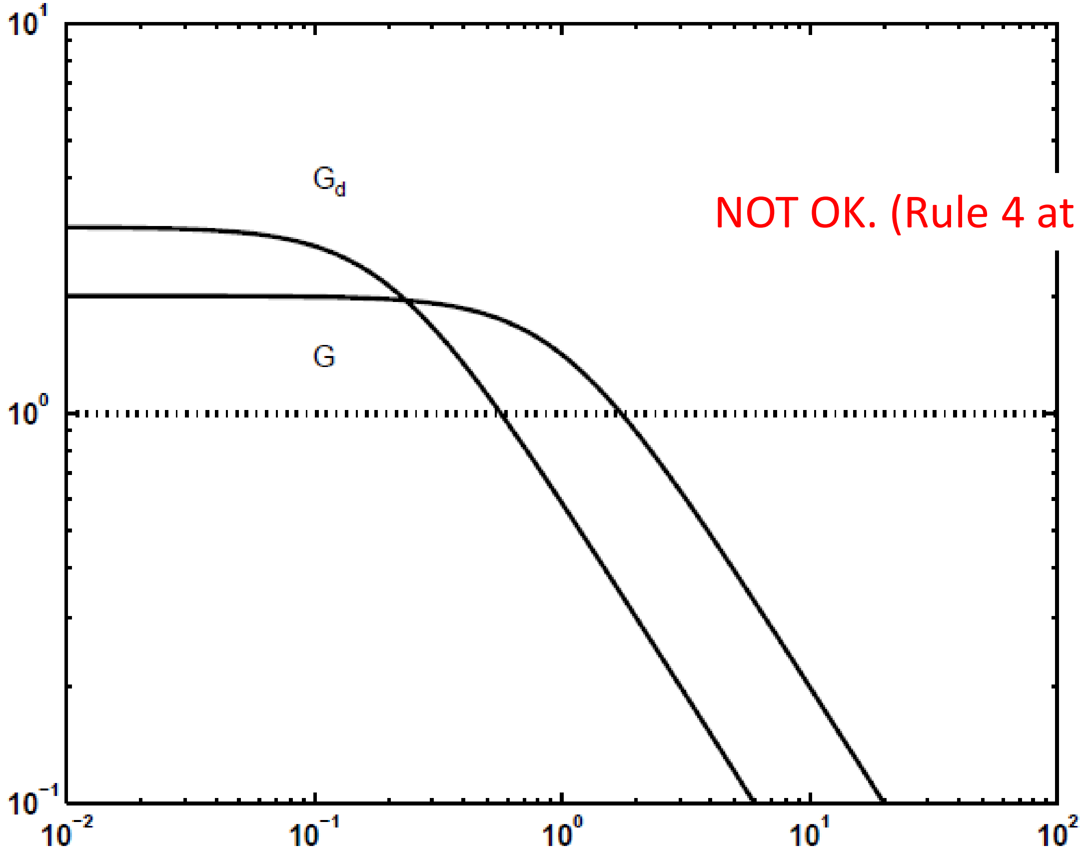
Scaled model

- In all problems, we assume that models have beed scaled such that
  - Δy_{max}=1
  - Δu_{max}=1
  - Δd =1
  - Define wd = kd/taud as frequency where |Gd(jwd)|=1

SCALED MODEL

Problem 1

$$G(s) = \frac{2}{s+1} \quad G_d(s) = \frac{3}{5s+1}$$



NOT OK. (Rule 4 at steady state)

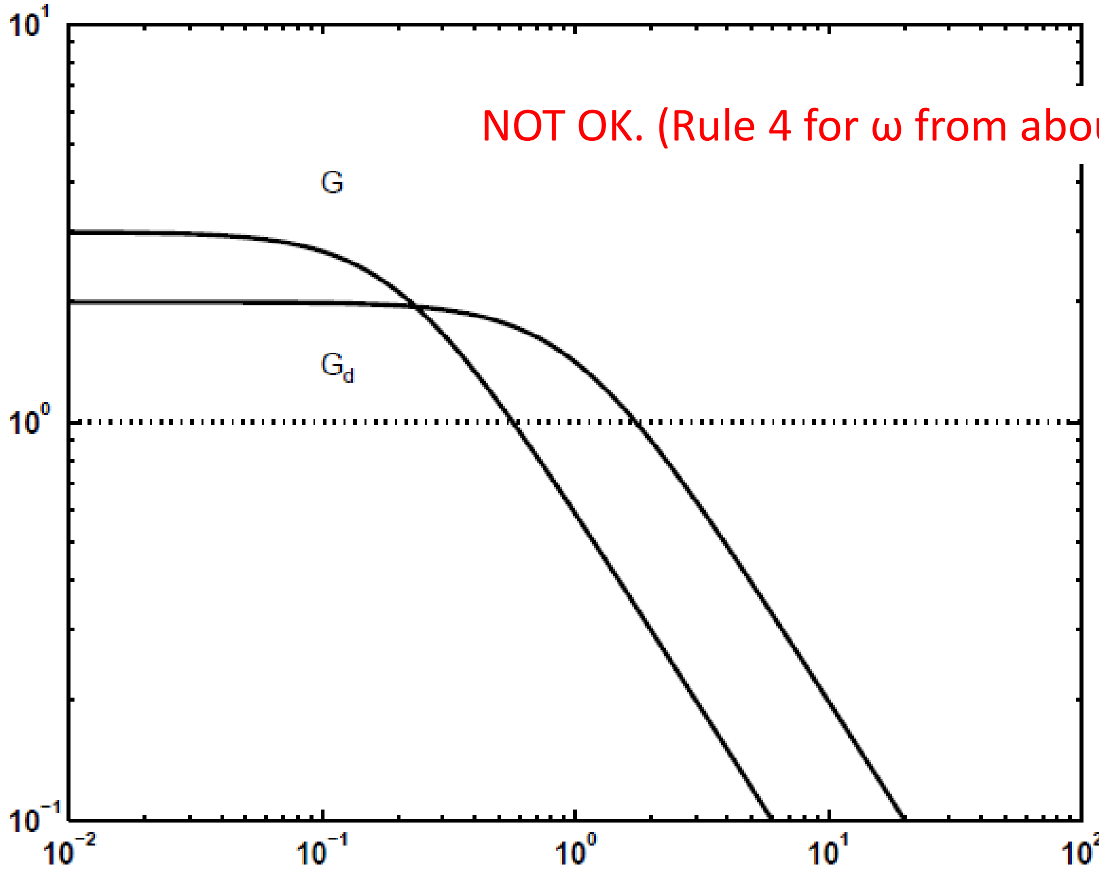
Figure 3: Magnitude of G and G_d.

SCALED MODEL

Problem 2

$$G(s) = \frac{3}{5s+1} \quad G_d(s) = \frac{2}{s+1}$$

NOT OK. (Rule 4 for ω from about 0.25 to 2)



Problem 3

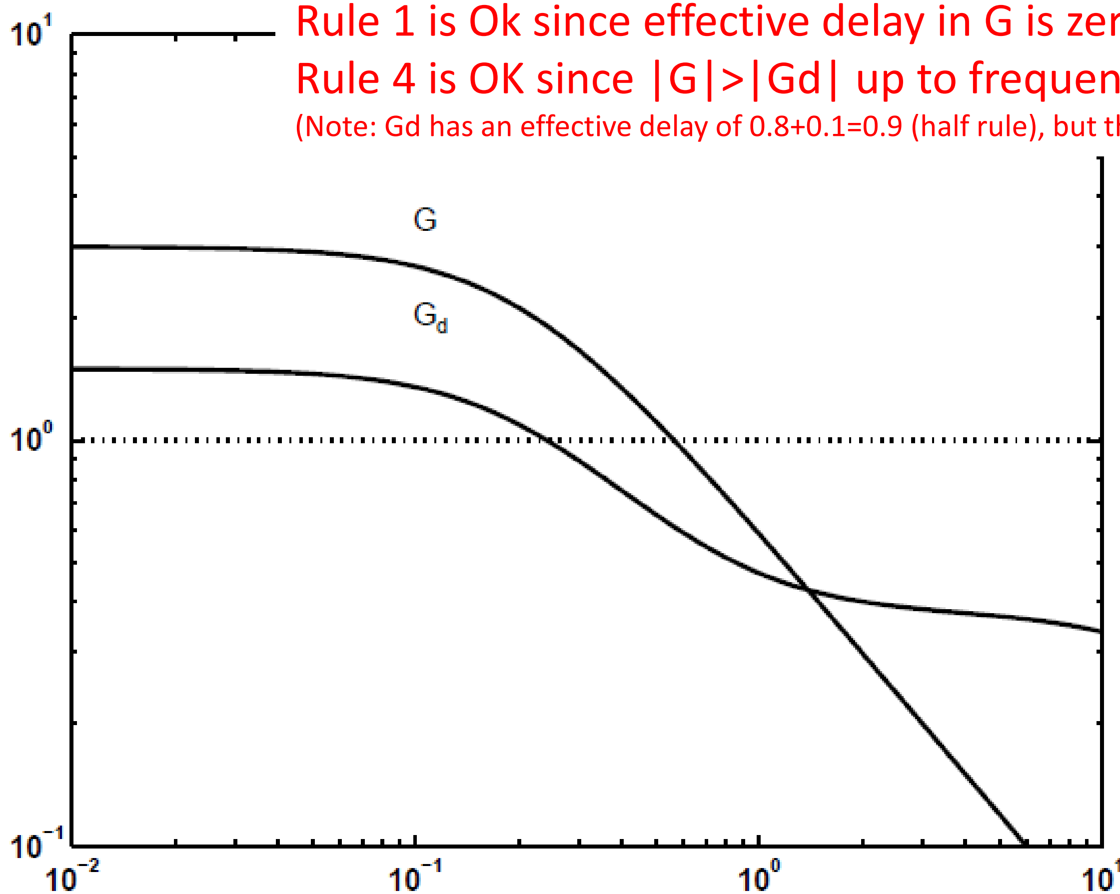
$$G(s) = \frac{3}{5s+1} \qquad G_d(s) = 7.5\frac{s - 0.8}{(s+0.2)(s+20)}$$

OK.

Rule 1 is Ok since effective delay in G is zero,

Rule 4 is OK since |G|>|Gd| up to frequency ωd where |Gd|=1

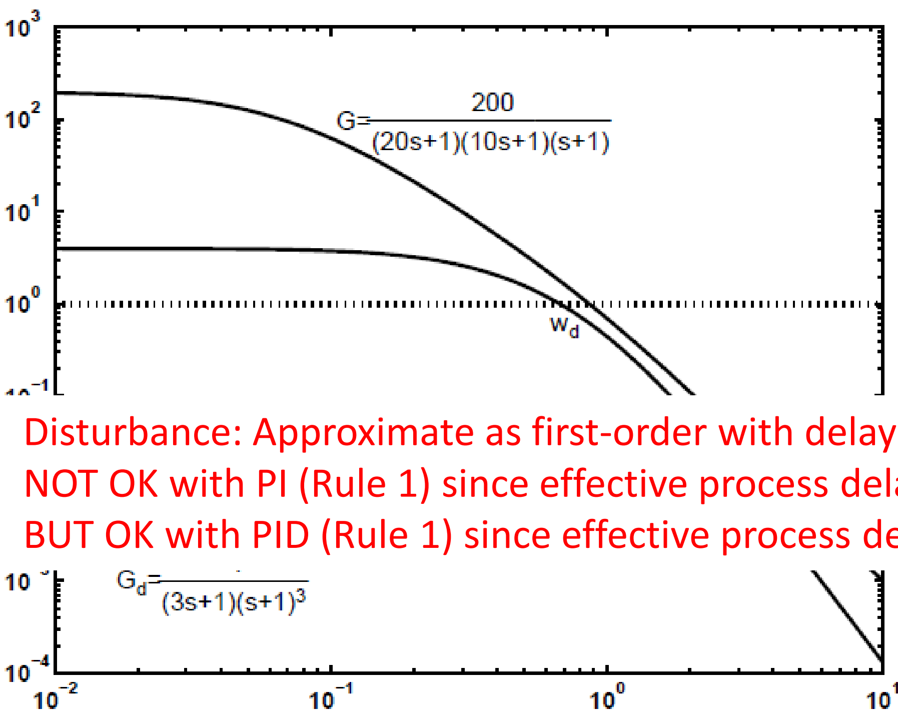
(Note: Gd has an effective delay of 0.8+0.1=0.9 (half rule), but the delay in Gd does not matter)



SCALED MODEL

Problem 4

$$G(s) = \frac{200}{(20s+1)(10s+1)(s+1)} \quad G_d(s) = \frac{4}{(3s+1)((s+1)^3}$$



(a) Magnitude of G and Gd

No problem with constraints, |G|>|Gd|

Disturbances. When does y reach 1?

What is effective delay?

Disturbance: Approximate as first-order with delay with kd=4, taud=3.5

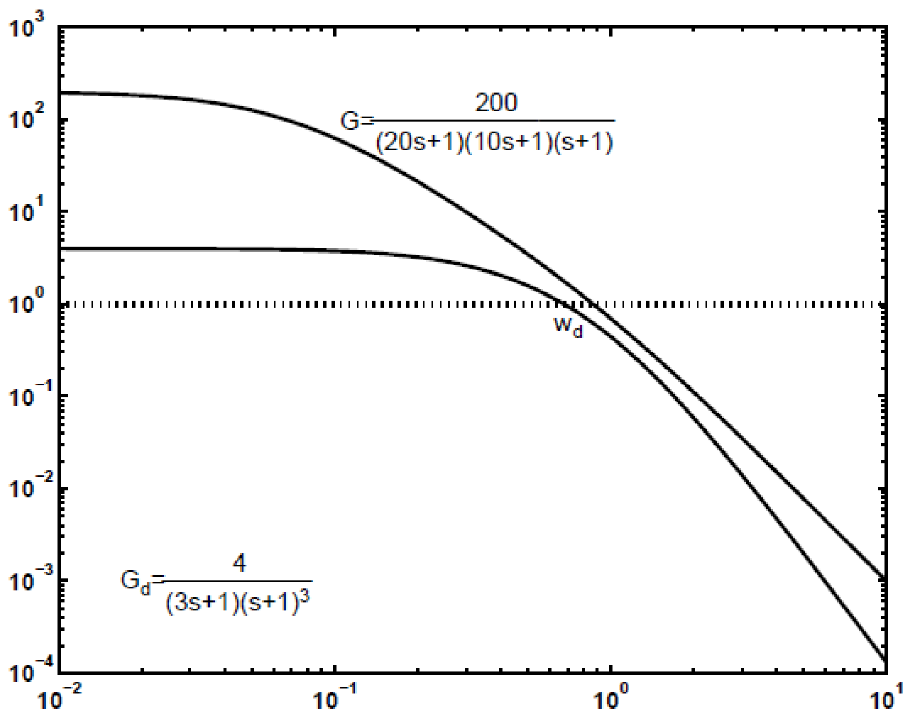
NOT OK with PI (Rule 1) since effective process delay is θ=10/2+1=6 so $k_d\,\theta\,/\tau_d = 4*6/3.5=6.9 > 1$

BUT OK with PID (Rule 1) since effective process delay is θ=0.5 so $k_d\,\theta\,/\tau_d = 4*0.5/3.5=0.6 < 1$

SCALED MODEL

Problem 4

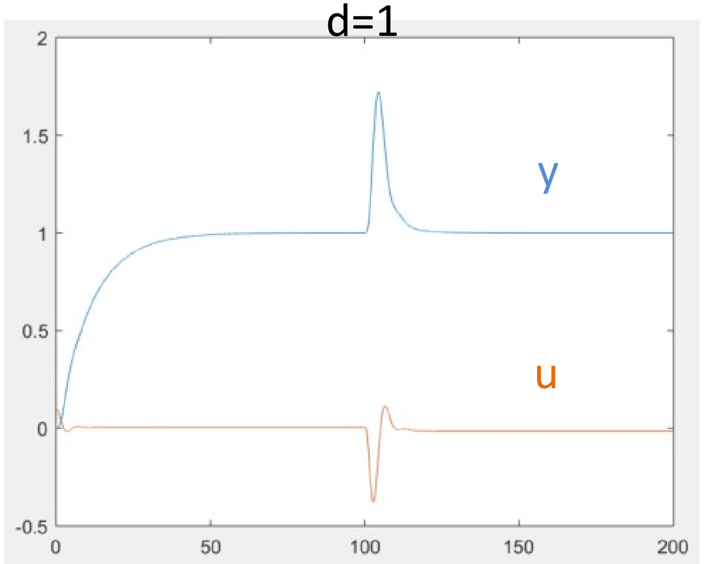
$$G(s) = \frac{200}{(20s+1)(10s+1)(s+1)} \quad G_d(s) = \frac{4}{(3s+1)((s+1)^3}$$



(a) Magnitude of G and Gd

```
%PID (w/o D-action on setpoint)
g = 200/((20*s+1)*(10*s+1)*(s+1))
gd = 4/((3*s+1)*(s+1)^3)
Kc=(1/200)*20/1,taui=20,taud=10.5
```

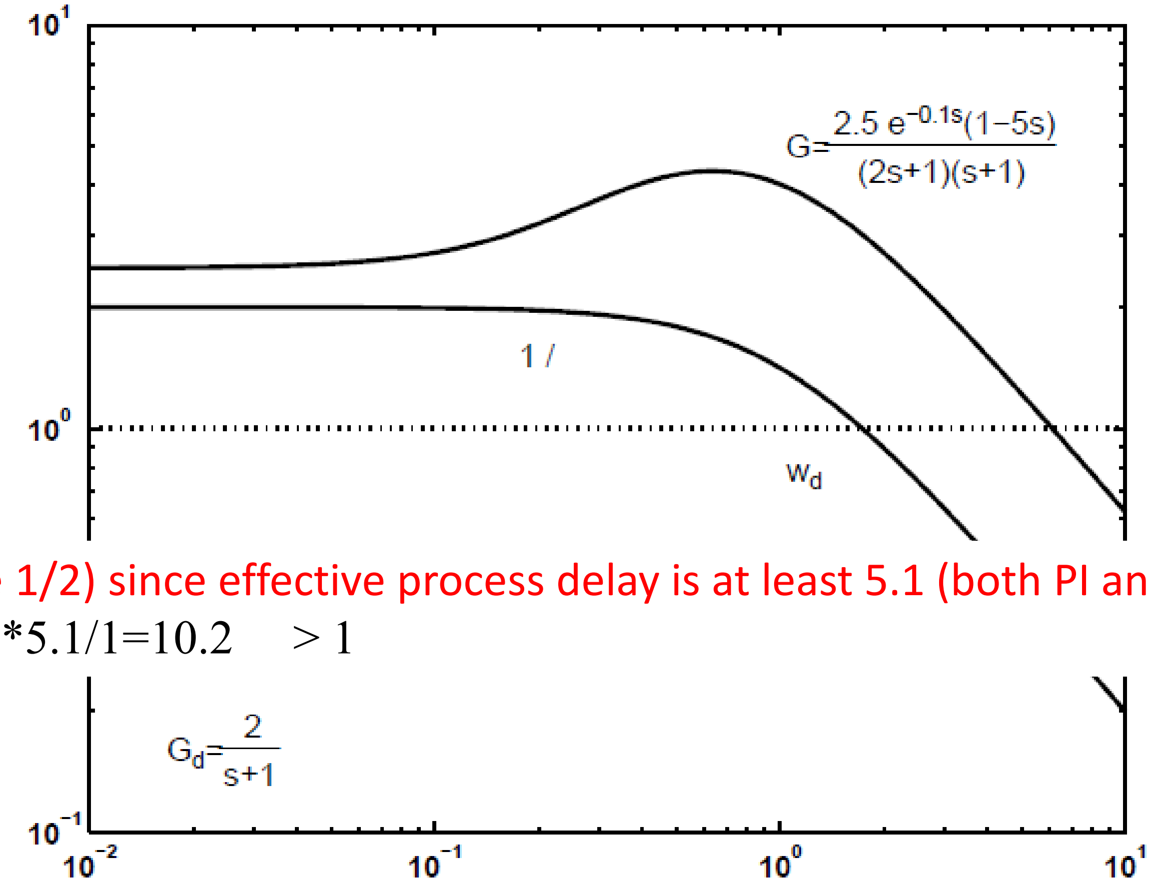
d=1



SCALED MODEL

Problem 5

$$G(s) = \frac{2.5e^{-0.1s}(1 - 5s)}{(3s+1)((s+1)^3} \qquad G_d(s) = \frac{2}{s+1}$$

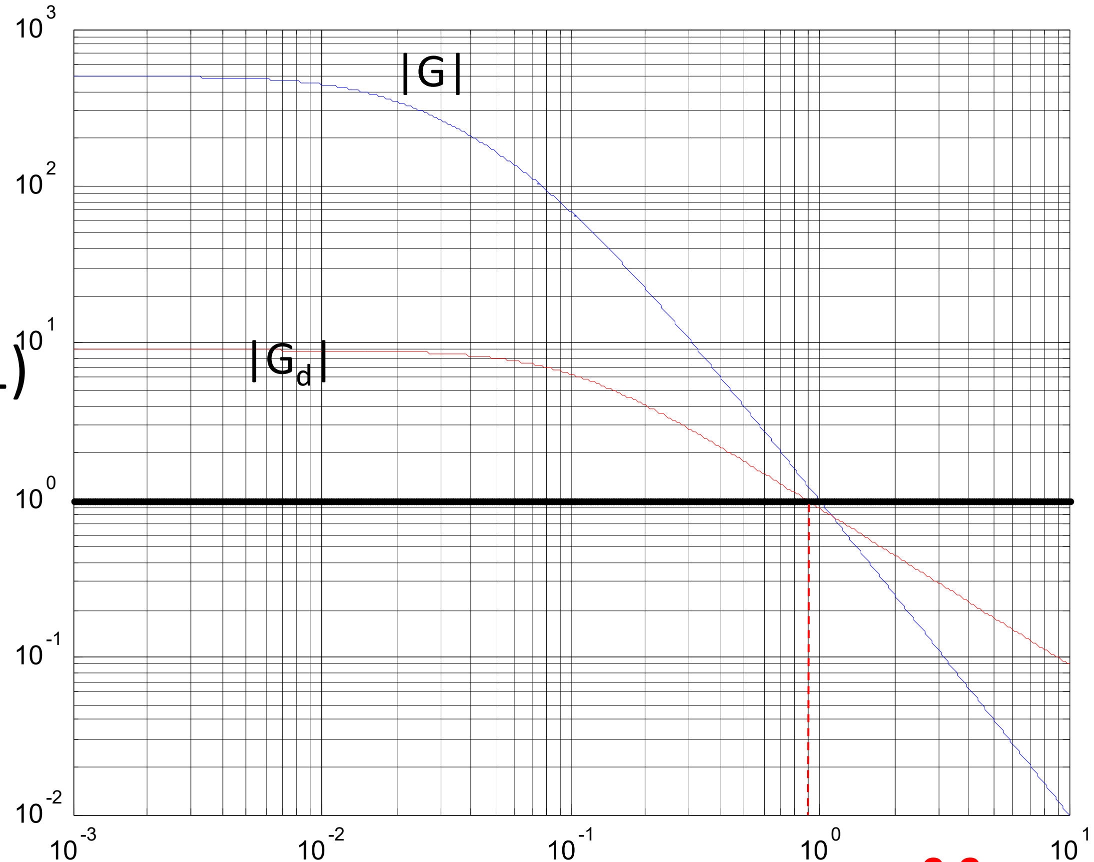


NOT OK (Rule 1/2) since effective process delay is at least 5.1 (both PI and PID),
so $k_d\,\theta\,/\tau_d = 2*5.1/1=10.2 \quad > 1$

SCALED MODEL

PROBLEM 7, g = 500/((50*s+1)*(10*s+1))

gd = 9/(10*s+1)



ω_d**=0.9**

```
s=tf('s')
g = 500/((50*s+1)*(10*s+1))
gd = 9/(10*s+1)
w = logspace(-3,1,1000);
[mag,phase]=bode(g,w);
[magd,phased]=bode(gd,w);
loglog(w,mag(:),'blue',w,magd(:),'red',w,1,'black'), grid on
```

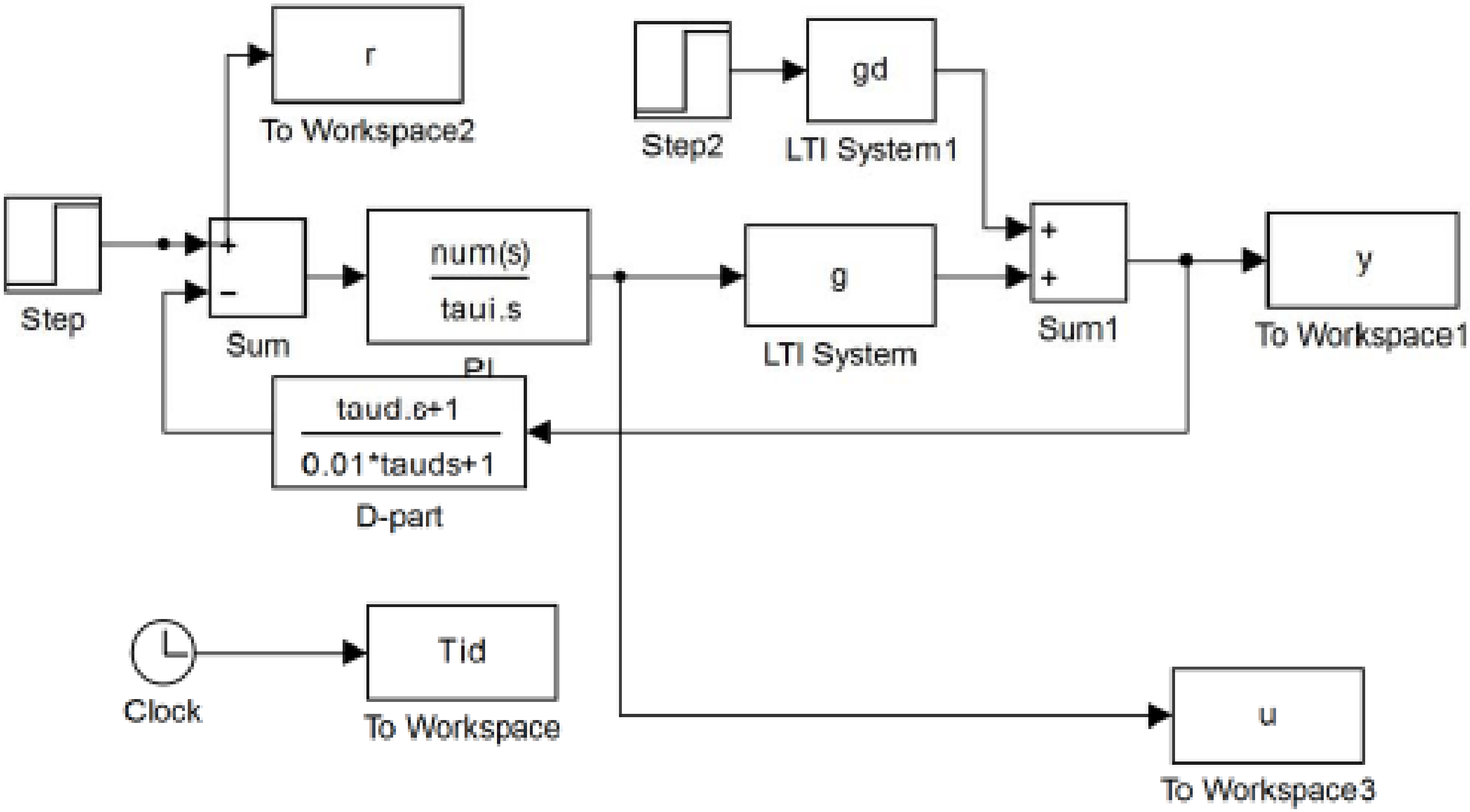
PI- control: θ_{eff} = 5 (from half rule):

$k_d\ \theta\ /\tau_d = 9*5/10 = 4.5 \quad > 1$

NOT CONTROLLABLE WITH PI!

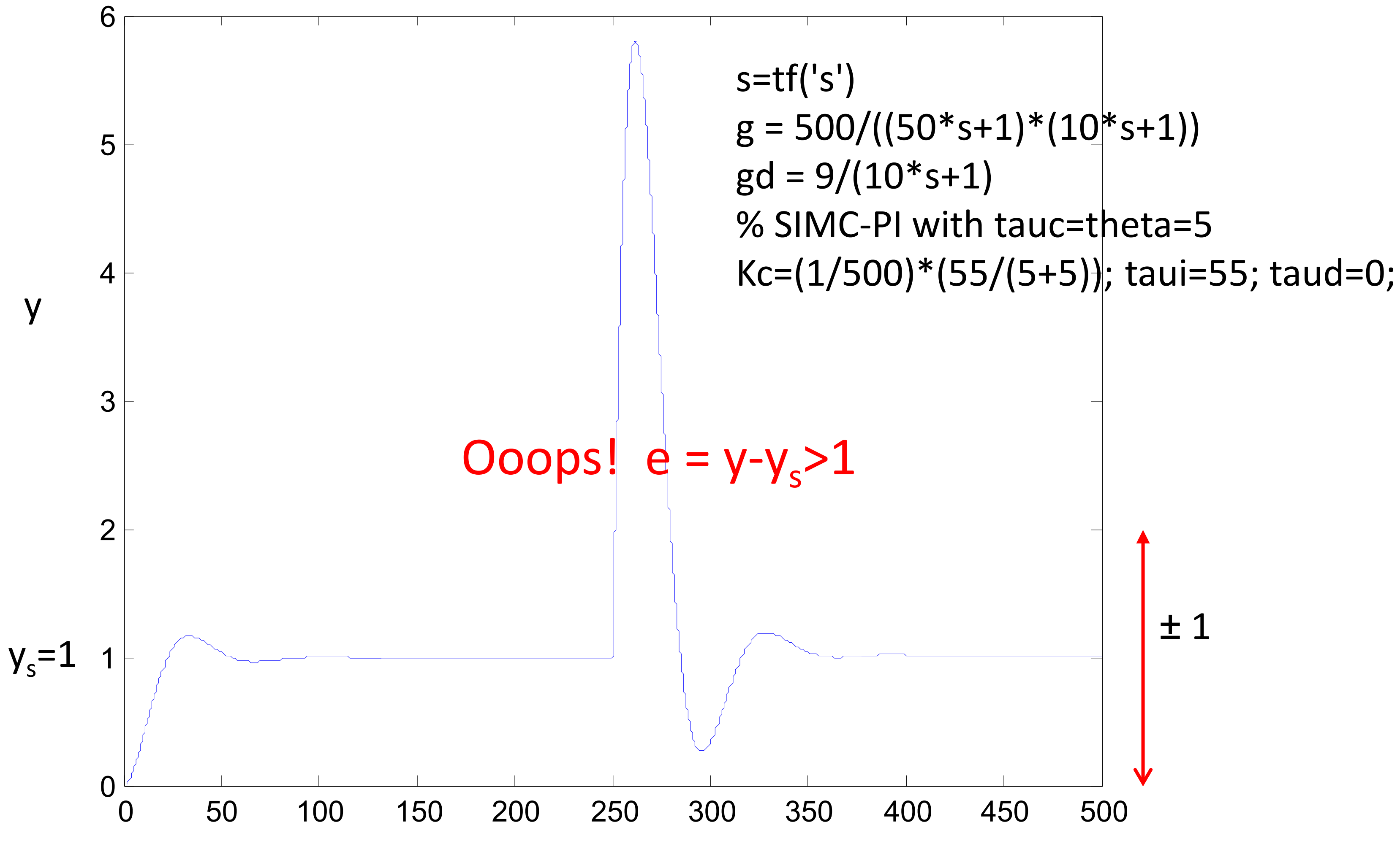
PID-control : θ_{eff} = 0. Controllable!

CHECK CONTROLLABILITY ANALYSIS WITH SIMULATIONS



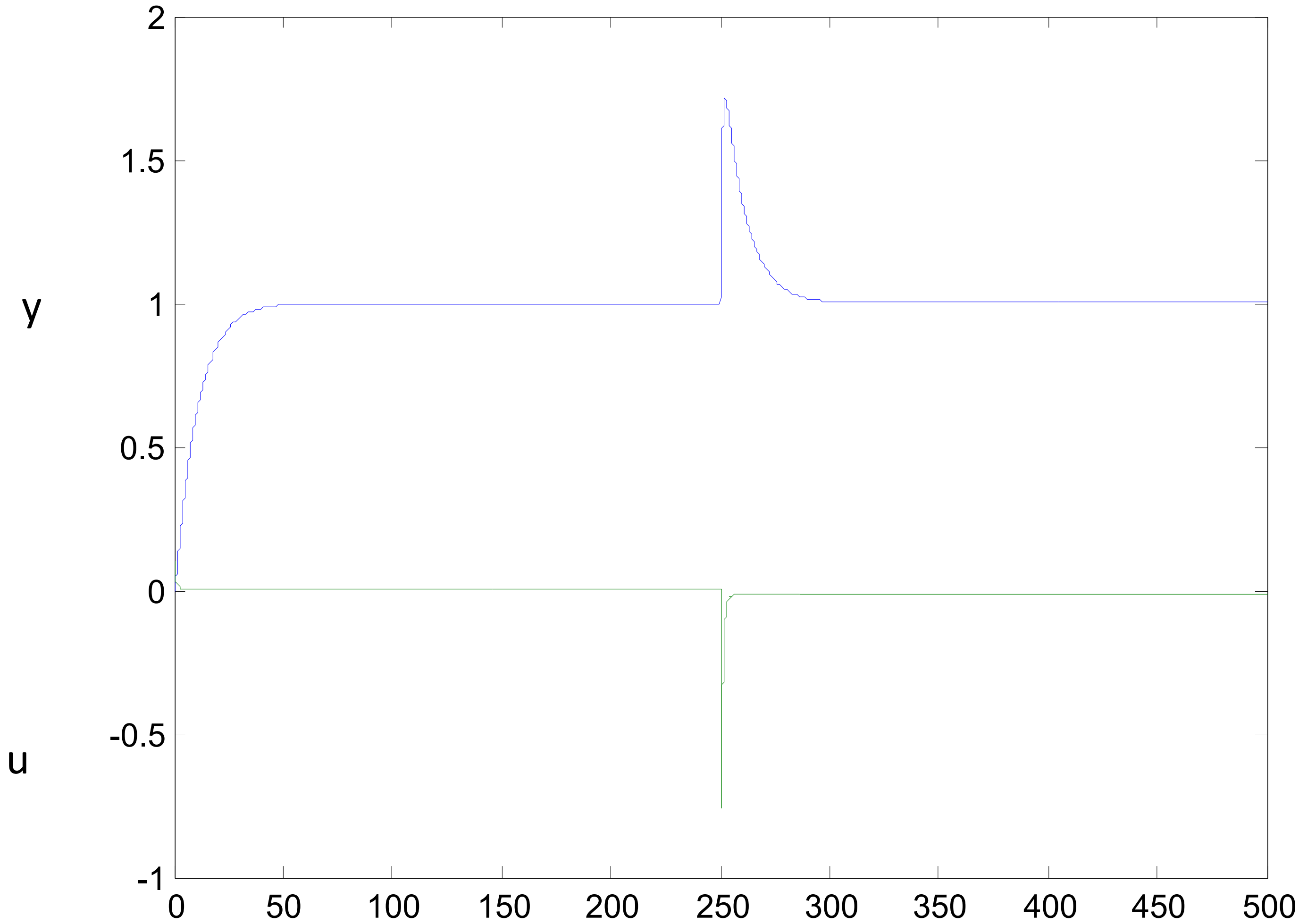
SCALED MODEL

PI control not acceptable*



*As expected since need $\omega_c > \omega_d$= 0.9, but can only achieve ω_c<1/θ = 1/5 = 0.2

PID control acceptable: y and u are within ±1



g = 500/((50*s+1)*(10*s+1))

gd = 9/(10*s+1)

%SIMC-PID (cascade form) with tauc=1/wd=1:

Kc=(1/500)*(50/(1+0)); taui=50; taud=10;

Exam.

- Saturday 18 Dec. 2021. 9-13.
- Note: Remember to state clearly all assumptions you make.
- General: Look through the whole exam before you start, read the questions carefully!

Q&A session (proposed): Tuesday 14 Dec. 10-12, (R5 ~~& Zoom~~)
(please send questions before by email: sigurd.skogestad@ntnu.no)