Self-Optimizing Control for Recirculated Gas lifted Subsea Oil Well Production

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Toronto, July 2024
Optimization in Oil & Gas Industry
Main Research Questions

How to optimize the operation of a

- complex, large-scale oil and/or gas production system,
- varying timescales,
- numerous potential constraints,

*Preferably* utilizing simple tools like

- PID controllers,
- selectors,
- and small-scale solvers (if necessary)?
Outline

• Conventional RTO

Put optimization into control layer:
• Self-optimizing control (SOC)
  – Marathon runner
• Case study using SOC
• New results on gradient-based control for changing active constraints
  – Primal-dual using Lagrange multipliers
  – Region-based with selectors
Optimal Operation

RTO = real-time optimization
MPC = model predictive control
ARC = advanced regulatory (PID) control
Optimal Operation

- Traditional RTO

Issue: Steady-state wait time

Issue: Non-transparent constraint control

Issue: Complex, need on-line model
Optimal Operation

- Self-optimizing control: Select good CV

*Advantage: Transparent and simple*

*Advantage: Fast*

*Issues: Nonlinearity (some loss in optimality) + not optimal if constraints change*

CV = controlled variable
Example: Optimal operation of runner

- Cost to be minimized, $J=T$
- One degree of freedom ($u=\text{power}$)
- What should we control ($CV$)?

Self-optimizing $CV$?

- **Sprinter (100m):**
  - «Run as fast as you can»
  - **Active constraint control**
  - $CV=u$ (no controller needed), $CV_s = \text{max}$
Example: Optimal operation of runner

- Marathon (40 km)

\[ J = T \]

\[ u_{\text{opt}} \quad u = \text{power} \]

\( CV_1 = \) distance to leader of race
\( CV_2 = \) speed
\( CV_3 = \text{heart rate} \)
\( CV_4 = \) level of lactate in muscles
Conclusion Marathon runner

CV = heart rate

• CV = heart rate is a good “self-optimizing” variable
• Disturbances are indirectly handled by keeping a constant heart rate
• May have infrequent adjustment of setpoint ($c_s$)
Gas-Lifted Optimization Problem
Recirculated Gas-Lifted
Steady-state optimization problem

\[
\min_{\mathbf{u}} \quad J(\mathbf{u}, \mathbf{d}) = -p_{\text{oil}}w_{\text{os}} + p_{\text{en}}\Phi_{gl}
\]

s.t. \[
\begin{align*}
g_{z_{gl,i}}(\mathbf{u}, \mathbf{d}) : & \; z_{gl,i} - 1 \leq 0 & i = 1, \ldots, 6, \\
g_{z_{s,i}}(\mathbf{u}, \mathbf{d}) : & \; -z_{s,i} + 0 \leq 0 & i = 1, \ldots, 3, \\
g_{s_i}(\mathbf{u}, \mathbf{d}) : & \; s_i - \bar{s}_i \leq 0 & i = 1, \ldots, 3, \\
g(\mathbf{u}, \mathbf{d}) : & \; w_{gs} - \bar{w}_{gs} \leq 0
\end{align*}
\]

\[
\mathbf{y} = \begin{bmatrix} p_{bh,2} & p_{wh,2} & \boxed{p_{d,3}} & p_s \end{bmatrix}^T
\]

\[d = GOR_2\]

Maximize oil revenue \hspace{1cm} Minimize gas lift cost
GLC has max. opening \hspace{1cm} SCV has min. opening
Surge constraints \hspace{1cm} Max export/produced gas constraints
Available measurements \hspace{1cm} Disturbances
Self-optimizing Control Structures

- **Structure 1**
  - Keep the **valve positions constant** ($\mathbf{u} = \mathbf{u}^*$)

- **Structure 2**
  - **Control active constraints**
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
Self-optimizing Control Structures

- **Structure 3**
  - **Region I**
    - Control active constraints
      - \( z_{gl,5} \rightarrow g(u, d) \)
      - \( z_{s,i} \rightarrow g_{z_{s,i}}(u, d) \)
    - Control **bottomhole pressure** as self-optimizing control variable
      - \( z_{gl,2} \rightarrow p_{bh,2} \)
  - **Region II**
    - Control active constraint
      - \( z_{s,i} \rightarrow g_{z_{s,i}}(u, d) \)
    - Control self-optimizing control variables
      - \( z_{gl,2} \rightarrow p_{bh,2} \)
      - \( z_{gl,5} = z_{gl,5}^* \)

**Allowing active constraint switching**
Self-optimizing Control Structures

- **Structure 4**
  
  **Region I**
  - Control active constraints
    - $z_{gl,5} \rightarrow g(u, d)$
    - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
    - Control **wellhead pressure** as self-optimizing control variable
      - $z_{gl,2} \rightarrow p_{wh,2}$
  
  **Region II**
  - Control active constraint
    - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \rightarrow p_{wh,2}$
    - $z_{gl,5} = z_{gl,5}^*$
Self-optimizing Control Structures

- **Structure 5**
  - **Region I**
    - Control active constraints
      - $z_{gl,5} \rightarrow g(u, d)$
      - $z_{s,i} \rightarrow g_{z_{s,i}}(u, d)$
    - Control tubing pressure as self-optimizing control variable
      - $z_{gl,2} \rightarrow \Delta p_{bw,2}$
  - **Region II**
    - Control active constraint
      - $z_{s,i} \rightarrow g_{z_{s,i}}(u, d)$
    - Control self-optimizing control variables
      - $z_{gl,2} \rightarrow \Delta p_{bw,2}$
      - $z_{gl,5} = z_{gl,5}^*$
Self-optimizing Control Structures

- **Structure 6**
  - **Region I**
    - Control active constraints
      - $z_{gl,5} \rightarrow g(u, d)$
      - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
    - Control mix of tubing and wellhead pressure as self-optimizing control variable
      - $z_{gl,2} \rightarrow c := 0.521p_{bh,2} + 0.854p_{wh,2}$
  - **Region II**
    - Control active constraint
      - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
    - Control self-optimizing control variables
      - $z_{gl,2} \rightarrow c := 0.521p_{bh,2} + 0.854p_{wh,2}$
      - $z_{gl,5} = z_{gl,5}^*$

### Null space method:
- $HF = 0$
Self-optimizing Control Structures

### Structure 7

**Region I**
- Control active constraints
  - $z_{gl,5} \rightarrow g(u, d)$
  - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
- Control two optimal self-optimizing control variables
  - $z_{gl,2} \Rightarrow c(1)$
  - $z_{gl,4} \Rightarrow c(2)$

**Region II**
- Control active constraint
  - $z_{s,i} \rightarrow g_{zs,i}(u, d)$
- Control self-optimizing control variables
  - $z_{gl,2} \rightarrow c(1)$
  - $z_{gl,4} \rightarrow c(2)$
  - $z_{gl,5} = z_{gl,5}^*$

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Null space method:

$$HF = 0$$
Simulations Results

- Steady-state monthly loss

**Table 11.2: Steady-state monthly loss**

<table>
<thead>
<tr>
<th>Control Structure</th>
<th>$-2.5% \text{GOR}_2$ (NOK)</th>
<th>$+2.5% \text{GOR}_2$ (est.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NOK 59.544</td>
<td>Inf</td>
</tr>
<tr>
<td>2</td>
<td>NOK 6116.745</td>
<td>NOK $\sim$ 3.444.831</td>
</tr>
<tr>
<td>3</td>
<td>NOK 604.897</td>
<td>NOK $\sim$ 2.810.376</td>
</tr>
<tr>
<td>4</td>
<td>NOK 686.095</td>
<td>NOK $\sim$ 3.595.481</td>
</tr>
<tr>
<td>5</td>
<td>NOK 633.027</td>
<td>NOK $\sim$ 3.065.285</td>
</tr>
<tr>
<td>6</td>
<td>NOK 124.246</td>
<td>NOK $\sim$ 1.523.036</td>
</tr>
<tr>
<td>7</td>
<td>NOK 248.667</td>
<td>NOK $\sim$ 1.817.930</td>
</tr>
</tbody>
</table>
Case study summary

- Extend gas lift model to recirculated gas-lift oil production.
- Reconfirms the SOC can be an alternative for optimization.
- Selector allows active constraint region switching.
- Structure 6 is recommended. From nullspace method:
  \[ CV = 0.52p_{bh,2} + 0.85p_{wh,2} \]
SOC: changing constraints are not handled optimally

- We have some more recent results based on KKT optimality conditions
- $\lambda = \text{Lagrange multiplier}$
- Cost gradient: $\nabla_u J \equiv J_u$

Theorem 2.3: Karush-Khun-Tucker (KKT) Optimality Conditions

Suppose that the objective function $J(u, d)$ and constraint $g(u, d)$ have subderivatives at point $u^*$. If $u^*$ is a local optimum and the optimization problem satisfies some regularity or KKT conditions (see below), then there exist constants $\lambda$, called KKT multipliers or Lagrange multipliers or dual variables, such that the following conditions hold:

\begin{align}
\nabla_u \mathcal{L}(u, d, \lambda) &= 0 \\
g_i(u, d) &\leq 0, \quad \forall i = 1, \ldots, n_g \tag{2.9a} \\
\lambda_i &\geq 0, \quad \forall i = 1, \ldots, n_g \tag{2.9b} \\
\lambda_i g_i(u, d) &= 0, \quad \forall i = 1, \ldots, n_g \tag{2.9c}
\end{align}

where

$$\nabla_u \mathcal{L}(u, d, \lambda) = \nabla_u J(u, d) + \nabla_u^T g(u, d) \lambda,$$

$$g(u, d) = [g_1(u, d) \ldots g_{n_g}(u, d)]^T,$$

$$\lambda = [\lambda_1 \ldots \lambda_{n_g}]^T.$$

Eq. (2.9a) is called stationary condition, Eq. (2.9b) is called primal feasibility condition, Eq. (2.9c) is called dual feasibility condition, and Eq. (2.9d) is called complementary slackness condition [36].
I. Primal-dual control based on KKT conditions:
Tracks active constraints by adjusting Lagrange multipliers (= shadow prices = dual variables) $\lambda$

$\mathbf{L}_u = \mathbf{J}_u + \lambda^T g_u = 0$

Inequality constraints: $\lambda \geq 0$

- Problem: Constraint control using dual variables is on slow time scale

- D. Krishnamoorthy, A distributed feedback-based online process optimization framework for optimal resource sharing, J. Process Control 97 (2021) 72–83,
II. Region-based feedback solution with «direct» constraint control (for case with more inputs than constraints)

**Process**

**Constraint controllers** (fast PID-controllers)

\[ g \text{ (constraints paired with } u_1) \]

\[ N^T g_u = 0 \]

\[ \text{Introduce } N: N^T g_u = 0 \]

**Selector on primal variables (inputs)**

**Gradient estimation**

\[ J_u \]

\[ J_{u_1} \]

\[ J_{u_2} \]

\[ \text{Changes!} \]

**Control**

1. Reduced gradient \( N^T J_u = 0 \)
   - «Self-optimizing variables»

2. Active constraints \( g_A = 0 \).

- Bernardino and Skogestad, Decentralized control using selectors for optimal steady-state operation with changing active constraints, J. Process Control, Vol. 137, 2024
New static gradient estimation based on SOC: Very simple and works well!

From «exact local method» of self-optimizing control:

\[
H^J = J_{uu} \left[ G^y T \left( \tilde{F} \tilde{F}^T \right)^{-1} G^y \right]^{-1} G^y T \left( \tilde{F} \tilde{F}^T \right)^{-1}
\]

where \( \tilde{F} = [FW_d \quad W_{ny}] \) and \( F = \frac{d\psi_{opt}}{dd} = G^y_d - G^y J_{uu}^{-1} J_{ud} \).

- Bernardino and Skogestad, Optimal measurement-based cost gradient estimate for real-time optimization, Comp. Chem. Engng., 2024
Conclusion

Move optimization into control layer by selecting good CVs

- CV = Active constraints

Unconstrained degrees of freedom:

- CV = Self-optimizing variables
- CV = Gradients

Reminder: DYCOPS conference in Bratislava (Slovakia) 16-19 June 2025. I hope to see you there!

CV = controlled variable