Decentralized control

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Outline

- Multivariable plants
- RGA
- Decentralized control
- Pairing rules
- Examples

MIMO (multivariable case)

Distillation column

"Increasing L from 1.0 to 1.1 changes y_D from 0.95 to 0.97, and x_B from 0.02 to 0.03"

"Increasing V from 1.5 to 1.6 changes y_D from 0.95 to 0.94, and x_B from 0.02 to 0.01"

Steady-State Gain Matrix

$$\begin{pmatrix} \Delta Y_D \\ \Delta x_B \end{pmatrix} = G(0) \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix}$$

$$G(0) = \begin{bmatrix} g_{11} & g_{12}(0) \\ g_{21} & g_{22}(0) \end{bmatrix} = \begin{bmatrix} \frac{0.97 - 0.95}{1.1 - 1.0} & \frac{0.94 - 0.95}{1.6 - 1.5} \\ \frac{0.03 - 0.02}{1.1 - 1.0} & \frac{0.01 - 0.02}{1.6 - 1.5} \end{bmatrix} = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}$$

Effect of input 1 (ΔL) on output 2 (Δx_B)

Can also include dynamics :

$$G_{(s)} = \begin{bmatrix} \frac{0.2}{1+50s} & -\frac{0.1}{1+50s} \\ \frac{0.1}{1+40s} & -\frac{0.1}{1+40s} \end{bmatrix} \begin{matrix} \rightarrow \Delta y_D \\ \rightarrow \Delta x_B \end{matrix}$$

(Time constant 50 min for y_D)

(time constant 40 min for x_B)

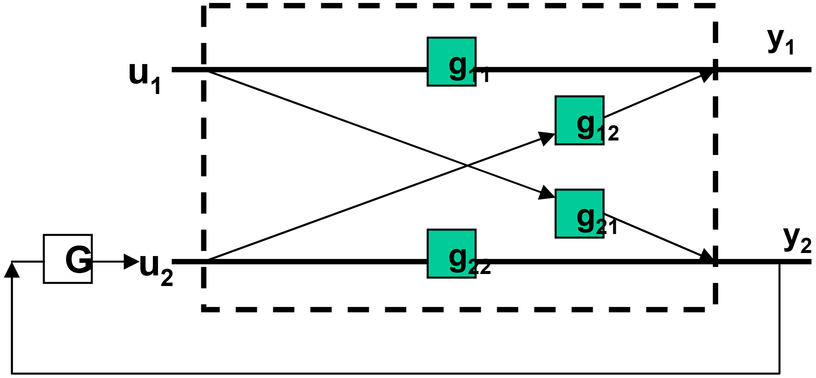
Analysis of Multivariable processes

What is different with MIMO processes to SISO:

- **The concept of "directions" (components in <u>u</u> and <u>y</u> have different magnitude"**
- **Interaction between loops when single-loop control is used**

INTERACTIONS

Process Model



"Open – loop"

$$y_1(s) = g_{11}(s)u_1(s) + g_{12}(s)u_2(s)$$

$$y_2(s) = g_{21}(s)u_1(s) + g_{22}(s)u_2(s)$$

Consider Effect of u_1 on y_1

1) **"Open-loop" ($C_2 = 0$): $y_1 = g_{11}(s) \cdot u_1$**
2) **Closed-loop" (close loop 2, $C_2 \neq 0$)**

$$y_1 = \left(g_{11}(s) - \frac{g_{12} g_{21} \cdot C_2}{1 + g_{22} \cdot C_2} \right) u_1$$

Change caused by "interactions"

Derivation:

$y_1 = g_{11} u_1 + g_{12} u_2$ where $u_2 = -c_2 y_2$

Get: $y_1 = g_{11} u_1 - g_{12} c_2 (g_{21} u_1 + g_{22} u_2)$

or: $y_1 = g_{11} (1 - c_2 g_{12} g_{21} c_2) u_1 - c_2 g_{12} g_{22} u_2$

Limiting Case $C_2 \to \infty$ (perfect control of y_2)

$$y_1 = \left(g_{11}(s) - \frac{g_{12}\, g_{21}}{g_{22}} \right) u_1$$

How much has "gain" from u_1 to y_1 changed by closing loop 2 with perfect control?

$$\text{Relative Gain} = \frac{(y_1/\mu_1)_{OL}}{(y_1/\mu_1)_{CL}} = \frac{g_{11}}{g_{11} - \frac{g_{12}\ g_{21}}{g_{22}}} = \frac{1}{1 - \frac{g_{12}\ g_{21}}{g_{11}\ g_{22}}} \stackrel{def}{=} \lambda_{11}^{RGA}$$

The relative Gain Array (RGA) is the matrix formed by considering all the relative gains

$$RGA = \Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \frac{(y_1/u_1)_{OL}}{(y_1/u_1)_{CL}} & \frac{(y_1/u_2)_{OL}}{(y_1/u_2)_{CL}} \\ \frac{(y_2/u_1)_{OL}}{(y_2/u_1)_{CL}} & \frac{(y_2/u_2)_{OL}}{(y_2/u_2)_{CL}} \end{bmatrix}$$

Example from before

$$G = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad \lambda_{11} = \frac{1}{1 - \underbrace{\frac{0.1 \quad 0.1}{0.2 \quad 0.1}}_{0.5}} = 2$$

$$RGA = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Property of RGA:

- **Columns and rows always sum to 1**
- **RGA independent of scaling (units) for u and y.**

Note: RGA as a function of frequency is the most important for control!

Use of RGA:

(1) Interactions

From derivation: Interactions are small if relative gains are close to 1

Choose pairings corresponding to RGA elements close to 1

Traditional: Consider Steady-state

Better: Consider frequency corresponding to closed-loop time constant

But: Avoid pairing on negative steady-state relative gain – otherwise you get instability if one of the loops become inactive (e.g. because of saturation)

Example:

$$G(0) = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}$$

$$y_1 = 0.2 \cdot u_1 - 0.1 \cdot u_2$$
$$y_2 = 0.1 u_1 - 0.1 u_2$$

$$\textbf{RGA} = \begin{bmatrix} \textbf{2} & \textbf{-1} \\ \textbf{-1} & \textbf{2} \end{bmatrix}$$

Only acceptable pairings :

$u_1 \leftrightarrow y_1$

$u_2 \leftrightarrow y_2$

Not recommended :

$u_1 \leftrightarrow y_2$

$u_2 \leftrightarrow y_1$

With integral action :
Negative RGA ⇒ individual loop unstable + overall system unstable when loops saturate

(2) Sensitivity measure

But RGA is not only an interaction measure:

Large RGA-elements signifies a process that is very sensitive to small changes (errors) and therefore fundamentally difficult to control

example

Large (BAD!)

$$G=\begin{bmatrix} 1 & 1 \\ 0.9 & 0.91 \end{bmatrix} \quad RGA=\begin{bmatrix} 91 & -90 \\ -90 & 91 \end{bmatrix}$$

$$\text{Relative change} = \overbrace{-\frac{1}{\lambda_{21}}}^{+\frac{1}{90}=+1.1\%} \text{ makes matrix singular!}$$

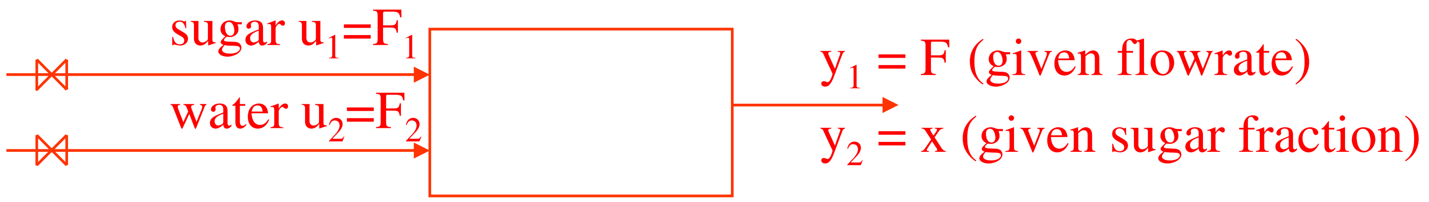
$$\text{Then } \hat{g}_{12} = \underbrace{g_{12}}_{0.9}\left(1+\frac{1}{90}\right) = 0.91$$

Singular Matrix: Cannot take inverse, that is, decoupler hopeless.

↑

Control difficult

Exercise. Blending process



- Mass balances (no dynamics)
  - Total: $F_1 + F_2 = F$
  - Sugar: $F_1 = x\,F$

(a) Linearize balances and introduce: $u_1=dF_1$, $u_2=dF_2$, $y_1=F_1$, $y_2=x$,

(b) Obtain gain matrix G ($y = G\,u$)

(c) Nominal values are x=0.2 [kg/kg] and F=2 [kg/s]. Find G

(d) Compute RGA and suggest pairings

(e) Does the pairing choice agree with "common sense"?

Decentralized control

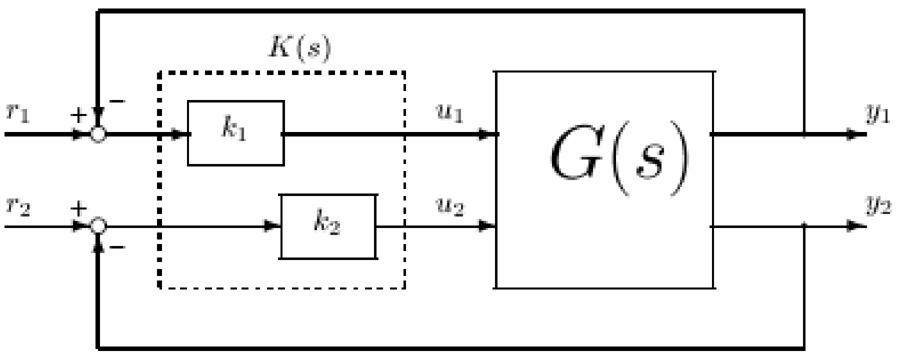


Figure 10.14: Decentralized diagonal control of a 2×2 plant

$$K(s) = \mathrm{diag}\{k_i(s)\} = \begin{bmatrix} k_1(s) & & & \\ & k_2(s) & & \\ & & \ddots & \\ & & & k_m(s) \end{bmatrix}$$

Two main steps

- Choice of pairings (control configuration selection)
- Design (tuning) of each controller

Design (tuning) of each controller $k_i(s)$

- Fully coordinated design
  - can give optimal
  - BUT: requires full model
  - not used in practice
- Independent design
  - Base design on “paired element”
  - Can get failure tolerance
  - Not possible for interactive plants (which fail to satisfy our three pairing rules – see later)
- Sequential design
  - Each design a SISO design
  - Can use “partial control theory”
  - Depends on inner loop being closed
  - Works on interactive plants where we may have time scale separation

Effective use of decentralized control requires some “natural” decomposition

- Decomposition in space
  - Interactions are small
  - G close to diagonal
  - Independent design can be used

- Decomposition in time
  - Different response times for the outputs
  - Sequential design can be used

Independent design: Pairing rules

Pairing rule 1. RGA at crossover frequencies. *Prefer pairings such that the rearranged system, with the selected pairings along the diagonal, has an RGA matrix close to identity at frequencies around the closed-loop bandwidth.*

Pairing rule 2. *For a stable plant avoid pairings ij that correspond to negative steady-state RGA elements,* $_{ij}(0) \leq 0$.

Pairing rule 3. *Prefer a pairing ij where* g_{ij} *puts minimal restrictions on the achievable bandwidth. Specifically, its effective delay* $_{ij}$ *should be small.*

Example 1: Diagonal plant

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathrm{RGA} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Simulations (and for tuning): Add delay 0.5 in each input
- Simulations setpoint changes: $r_1=1$ at $t=0$ and $r_2=1$ at $t=20$
- Performance: Want $|y_1-r_1|$ and $|y_2-r_2|$ less than 1

- G (and RGA): Clear that diagonal pairings are preferred

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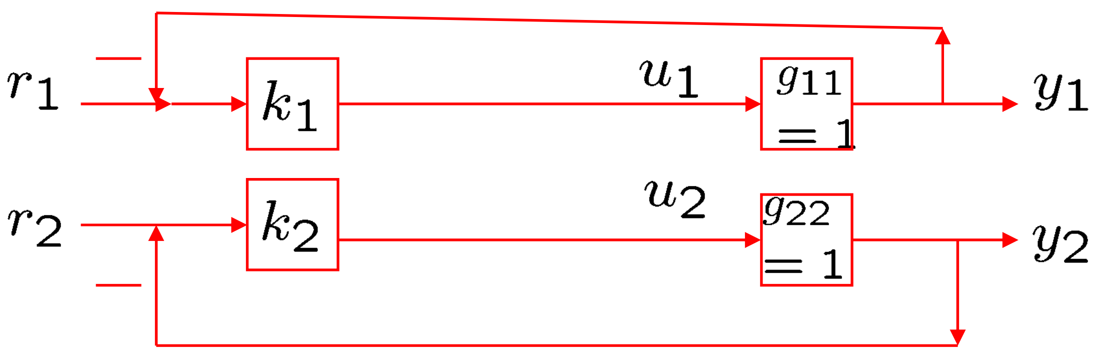
Diagonal pairings

Get two independent subsystems:



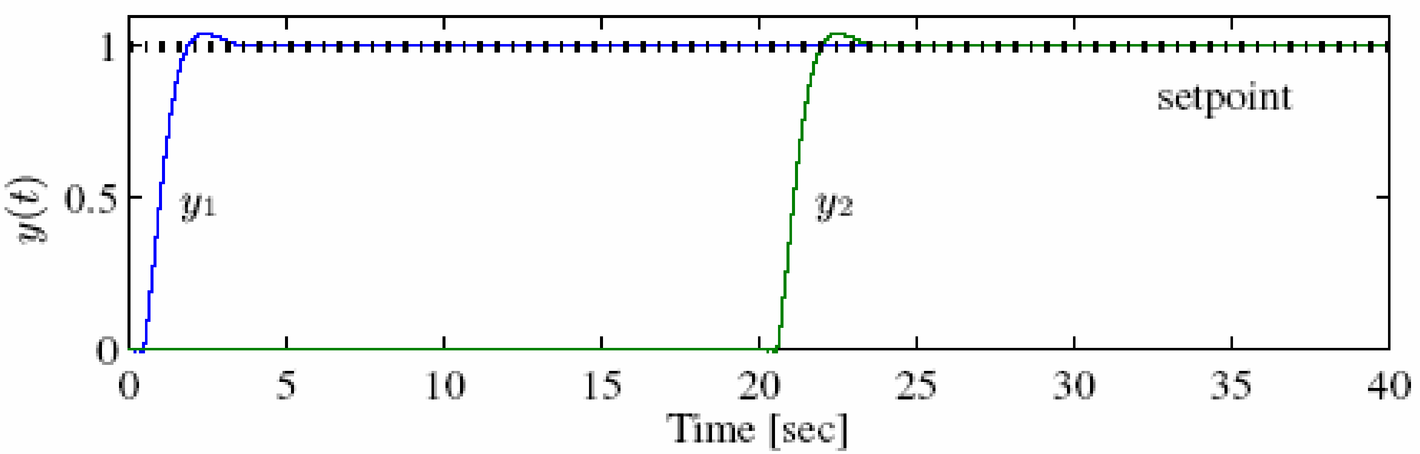
Diagonal pairings....

$$K = \begin{bmatrix} \frac{1}{\tau_1 s} & 0 \\ 0 & \frac{1}{\tau_2 s} \end{bmatrix} \tag{10.51}$$

first-order responses

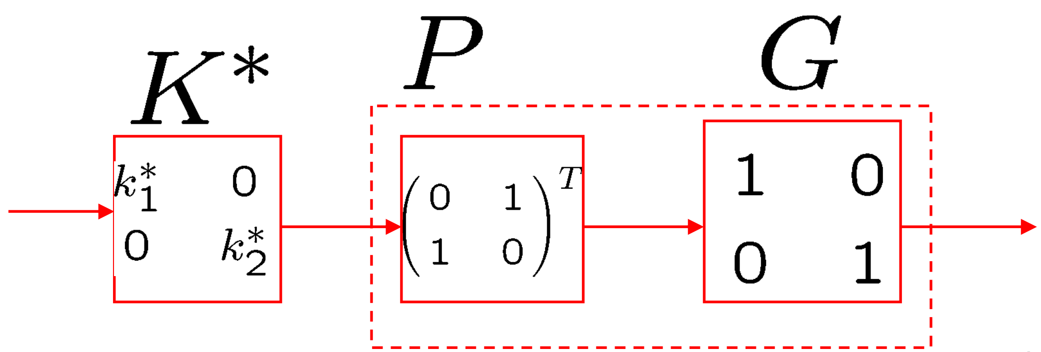
$$y_1 = \frac{1}{\tau_1 s + 1} r_1 \quad \text{and} \quad y_2 = \frac{1}{\tau_2 s + 1} r_2 \tag{10.52}$$

Simulation with delay included:



(a) Diagonal pairing; controller (10.51) with $\tau_1 = \tau_2 = 1$

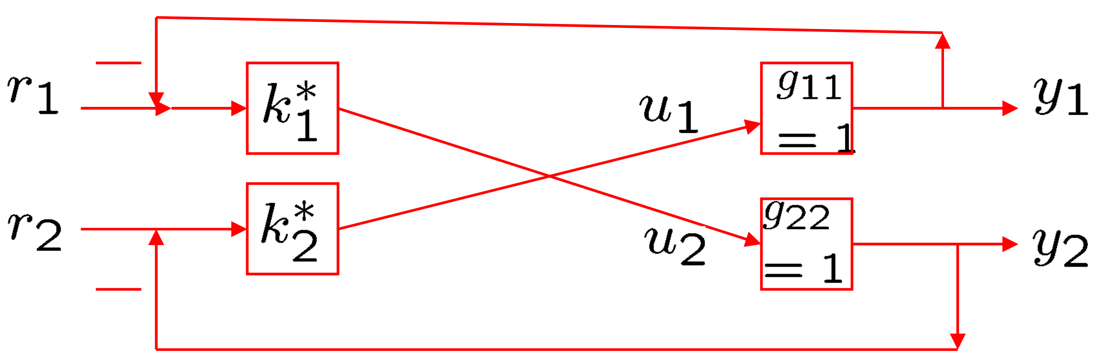
Off-diagonal pairings (!!?)



$$G^* = \text{Permuted plant} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

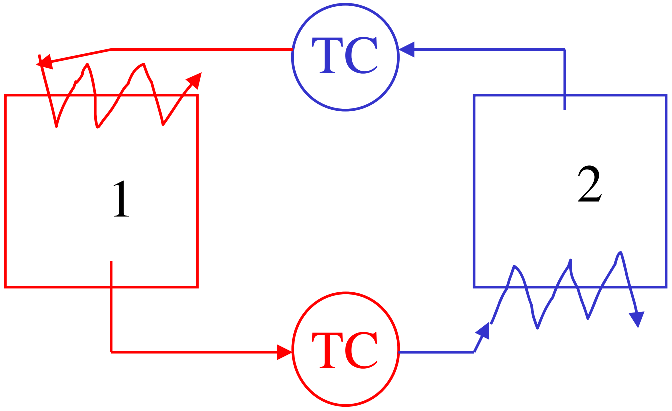
Pair on two zero elements !! Loops do not work independently!

But there is some effect when <u>both</u> loops are closed:



Off- diagonal pairings for diagonal plant

- Example: Want to control temperature in two completely different rooms (which may even be located in different countries). BUT:
  - Room 1 is controlled using heat input in room 2 (?!)
  - Room 2 is controlled using heat input in room 1 (?!)

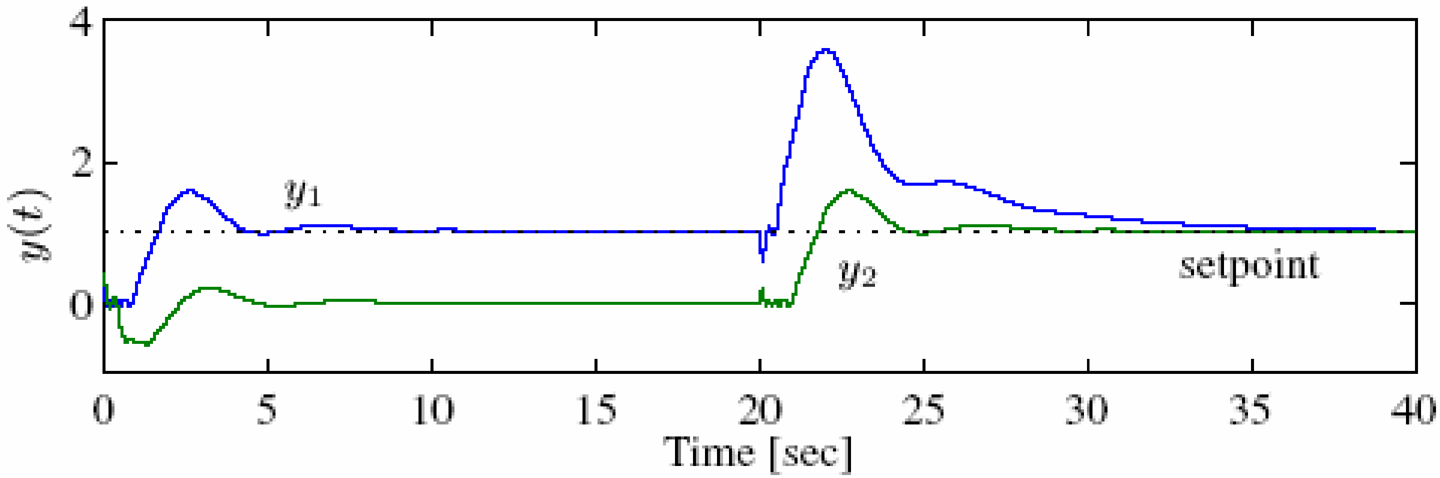


??

Off-diagonal pairings....

Controller design difficult. After some trial and error:

$$K^*(s) = \begin{bmatrix} \frac{-(0.5s+0.1)}{s} & 0 \\ 0 & \frac{(0.5s+2)}{s} \end{bmatrix} \qquad (10.54)$$



(b) Off-diagonal pairing; plant (10.53) and controller (10.54)

– Performance quite poor, but it works because of the "hidden" feedback loop $g_{12}\, g_{21}\, k_1\, k_2$!!
– No failure tolerance

Example 2: One-way interactive (triangular) plant

$$G = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix},$$

.

$$\text{RGA} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad G^{-1} = \begin{pmatrix} 1 & 0 \\ -5 & 1 \end{pmatrix}$$

- Simulations (and for tuning): Add delay 0.5 in each input

- RGA: Seems that diagonal pairings are preferred
- BUT: RGA is not able to detect the strong one-way interactions (g_{12}=5)

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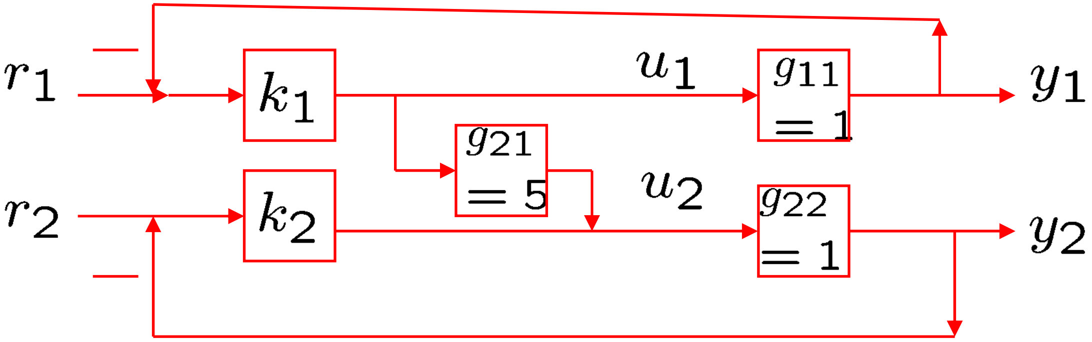
Diagonal pairings

One-way interactive:



Diagonal pairings....

$$K = \begin{bmatrix} \frac{1}{\tau_1 s} & 0 \\ 0 & \frac{1}{\tau_2 s} \end{bmatrix}$$

Closed-loop response (delay neglected):

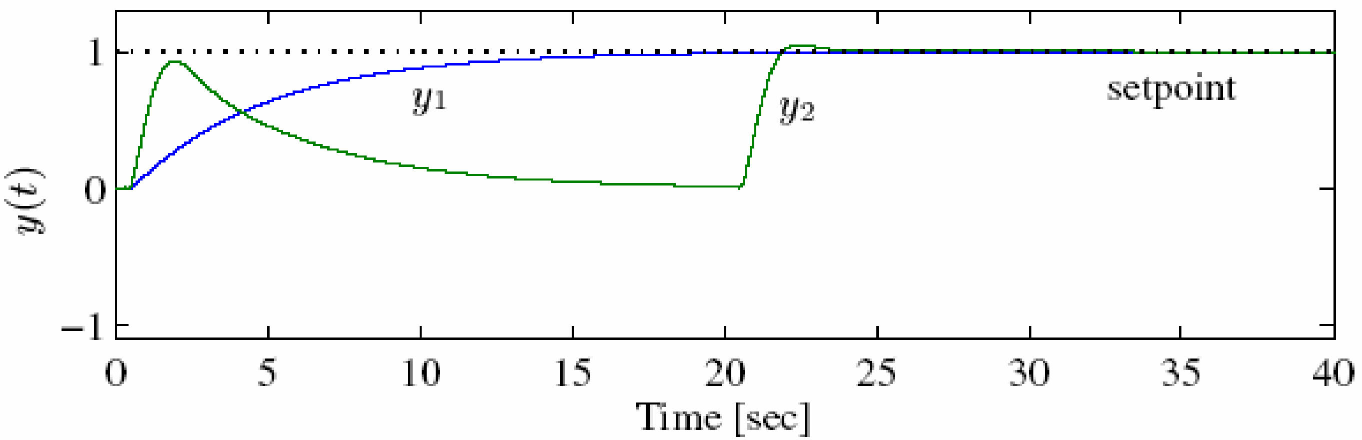
$$\begin{aligned} y_1 &= \frac{1}{\tau_1 s + 1} r_1 \\ y_2 &= \frac{5\tau_2 s}{(\tau_1 s + 1)(\tau_2 s + 1)} r_1 + \frac{1}{\tau_2 s + 1} r_2 \end{aligned}$$

With $_1 =$ $_2$ the “interaction” term (from r_1 to y_2) is about 2.5

Need loop 1 to be “slow” to reduce interactions: Need $_1 \geq 5$ $_2$

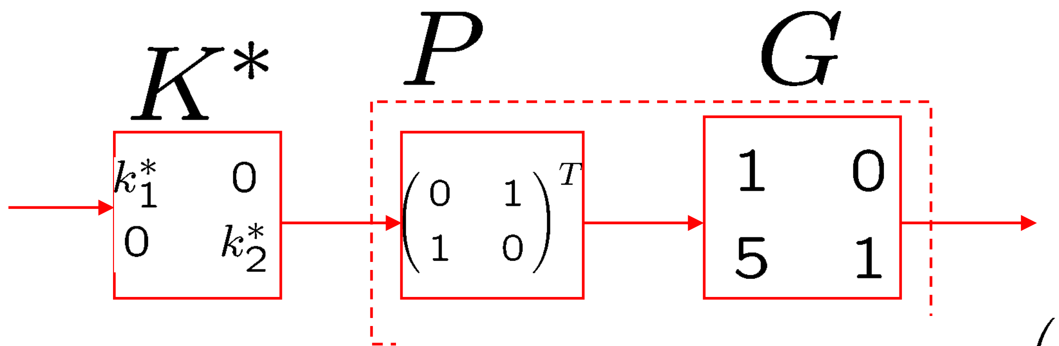
Diagonal pairings.....

$$K = \begin{bmatrix} \frac{1}{\tau_1 s} & 0 \\ 0 & \frac{1}{\tau_2 s} \end{bmatrix} \qquad (10.56)$$



(a) Diagonal pairing; controller (10.56) with $\tau_1 = 5$ and $\tau_2 = 1$

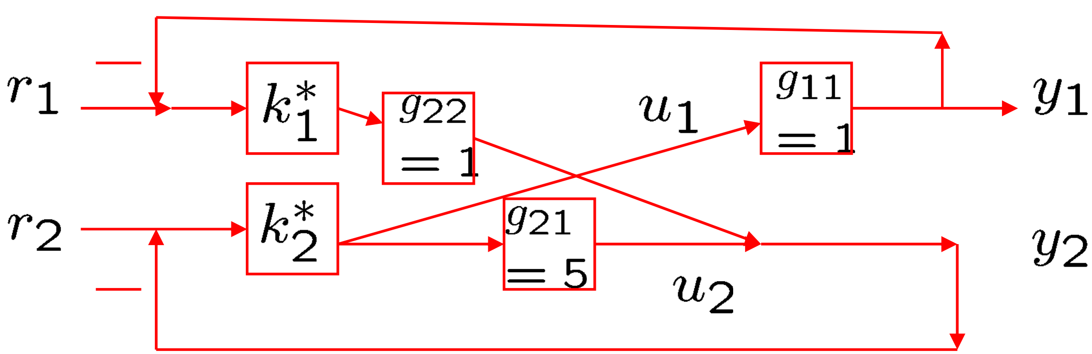
Off-diagonal pairings



$$G^* = \text{Permuted plant} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$$

Pair on one zero element ($g_{12}=g_{11}^*=0$)

BUT pair on $g_{21}=g^*_{22}=5$: may use sequential design: Start by tuning k_2^*



Off-diagonal pairings using sequential design. *The permuted plant is*

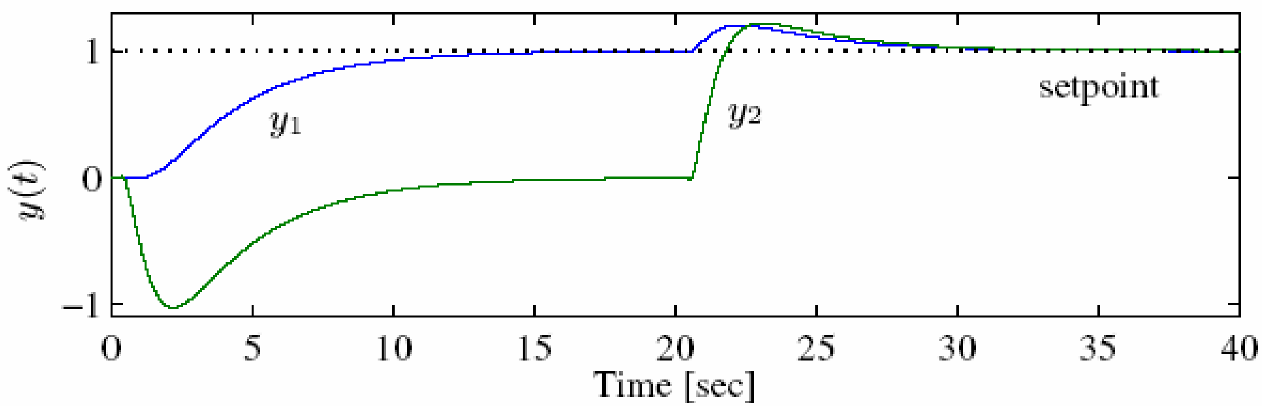
$$G^* = G\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}^T = \begin{bmatrix}0 & 1\\ 1 & 5\end{bmatrix} \qquad (10.59)$$

This corresponds to pairing on a zero element $g^*_{11} = 0$. *This pairing is* not *acceptable if we use the independent design approach, because* u^*_1 *has no effect on* y_1 *so "loop 1" does not work by itself. However, with the sequential design approach, we may first close the loop around* y_2 *(on the element* $g^*_{22} = 5$*). With the IMC design approach, the controller becomes* $k^*_2(s) = 1/(g^*_{22}\tau_2 s) = 1/(5\tau_2 s)$ *and with this loop closed,* u^*_1 *does have an effect on* y_1. *Assuming tight control of* y_2 *gives (using the expression for "perfect" partial control in (10.28))*

$$y_1 = \left(g^*_{11} - \frac{g^*_{12}g^*_{21}}{g^*_{22}}\right)u^*_1 = -\frac{1}{5}u^*_1$$

The controller for the pairing u^*_1-y_1 *becomes* $k^*_1(s) = 1/(g^*_{11}\tau_1 s) = -5/(\tau_1 s)$ *and thus*

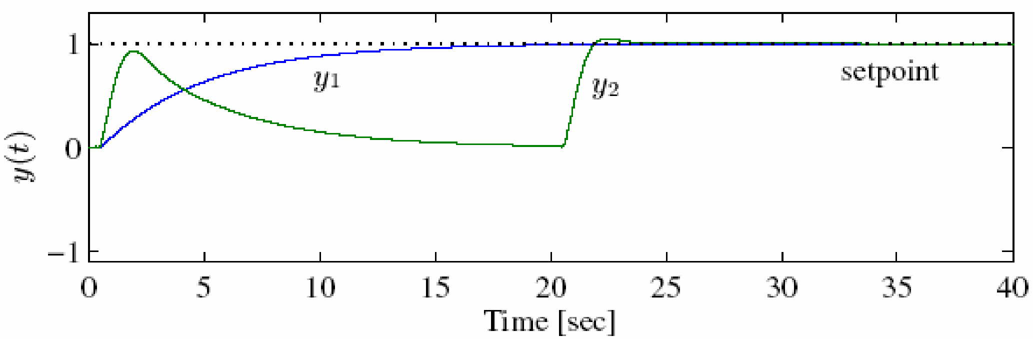
$$K^* = \begin{bmatrix}\frac{-5}{\tau_1 s} & 0\\ 0 & \frac{1}{5\tau_2 s}\end{bmatrix} \qquad (10.60)$$



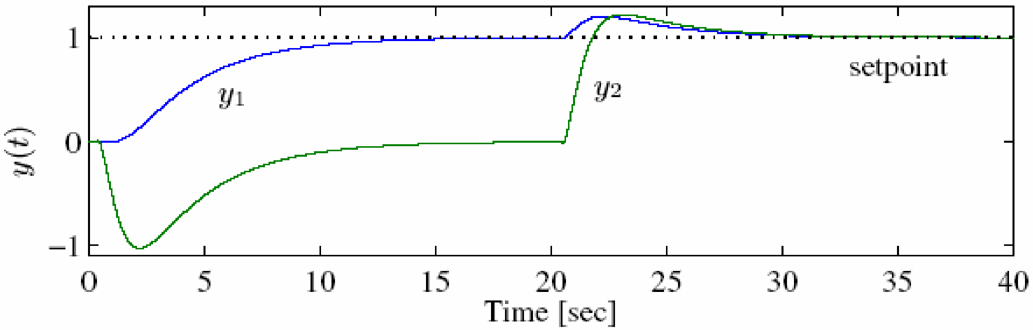
(b) Off-diagonal pairing; plant (10.59) and controller (10.60) with $\tau_1 = 5$ and $\tau_2 = 1$

Comparison of diagonal and off-diagonal pairings

$$G = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$$



(a) Diagonal pairing; controller (10.56) with $\tau_1 = 5$ and $\tau_2 = 1$



(b) Off-diagonal pairing; plant (10.59) and controller (10.60) with $\tau_1 = 5$ and $\tau_2 = 1$

Figure 10.16: Decentralized control of triangular plant (10.55)

– OK performance,
– but no failure tolerance if loop 2 fails

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Example 3: Two-way interactive plant

$$G = \begin{pmatrix} 1 & g_{12} \\ 5 & 1 \end{pmatrix}$$

- Already considered case g_{12}=0 (RGA=I)
- g_{12}=0.2: plant is singular (RGA=∞)
- will consider diagonal parings for: (a) g_{12} = 0.17, (b) g_{12} = -0.2, (c) g_{12} = -1

Controller:

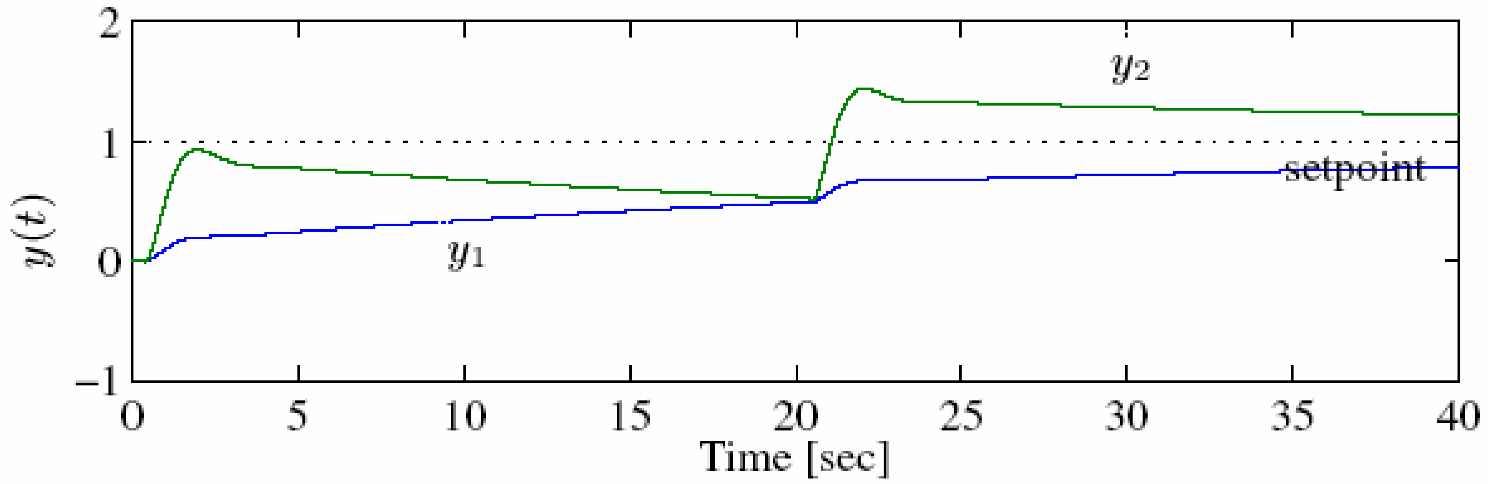
$$K = \begin{bmatrix} \frac{1}{\tau_1 s} & 0 \\ 0 & \frac{1}{\tau_2 s} \end{bmatrix} \qquad (10.62)$$

with $_1$=5 and $_2$=1

$$G = \begin{pmatrix} 1 & 0.17 \\ 5 & 1 \end{pmatrix}$$

(a) $g_{12} = 0.17$. *In this case,*

$$G^{-1} = \begin{bmatrix} 6.7 & -1.1 \\ -33.3 & 6.7 \end{bmatrix} \quad \text{and} \quad \text{RGA} = \begin{bmatrix} 6.7 & -5.7 \\ -5.7 & 6.7 \end{bmatrix}$$



(a) $g_{12} = 0.17$; controller (10.62) with $\tau_1 = 5$ and $\tau_2 = 1$

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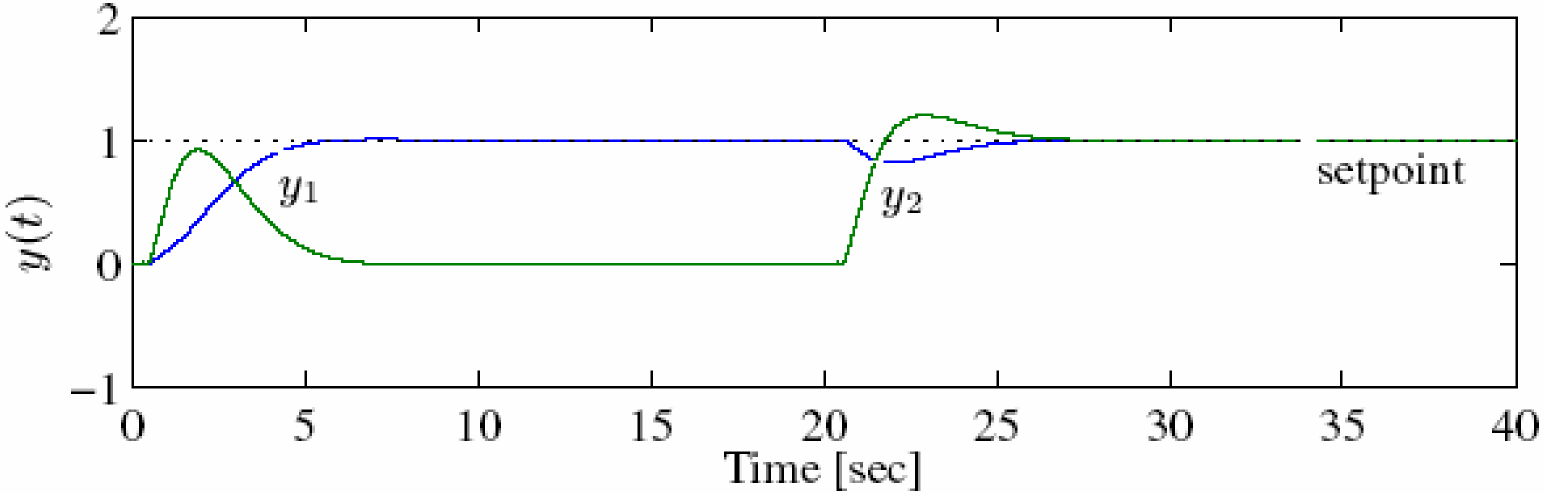
$$G = \begin{pmatrix} 1 & -0.2 \\ 5 & 1 \end{pmatrix}$$

(b) $\underline{g_{12} = -0.2}$. *In this case,*

$$G^{-1} = \begin{bmatrix} 0.5 & 0.1 \\ -2.5 & 0.5 \end{bmatrix} \quad \text{and} \quad \text{RGA} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

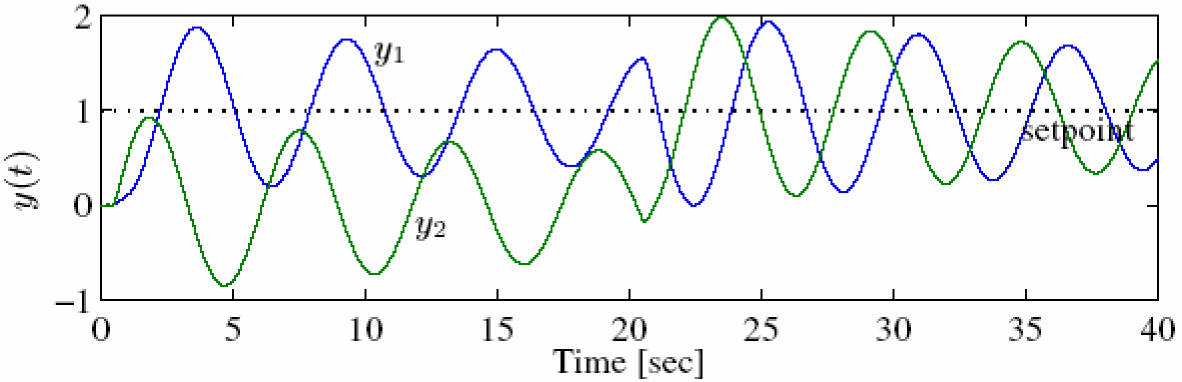


(b) $g_{12} = -0.2$; controller (10.62) with $\tau_1 = 5$ and $\tau_2 = 1$

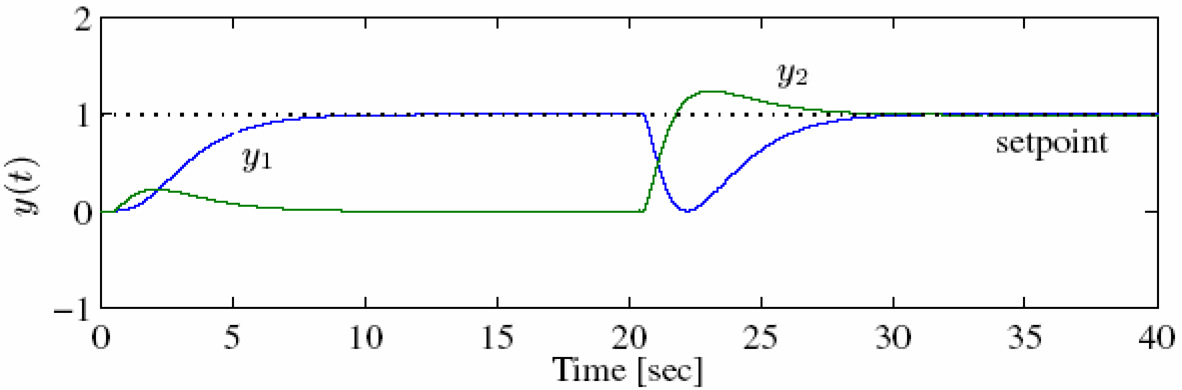
(c) $\underline{g_{12} = -1}$. *In this case,*

$$G = \begin{pmatrix} 1 & -1 \\ 5 & 1 \end{pmatrix}$$

$$G^{-1} = \begin{bmatrix} 0.17 & 0.17 \\ -0.83 & 0.17 \end{bmatrix} \quad \text{and} \quad \text{RGA} = \begin{bmatrix} 0.17 & 0.83 \\ 0.83 & 0.17 \end{bmatrix}$$



(c) $g_{12} = -1$; controller (10.62) with $\tau_1 = 5$ and $\tau_2 = 1$

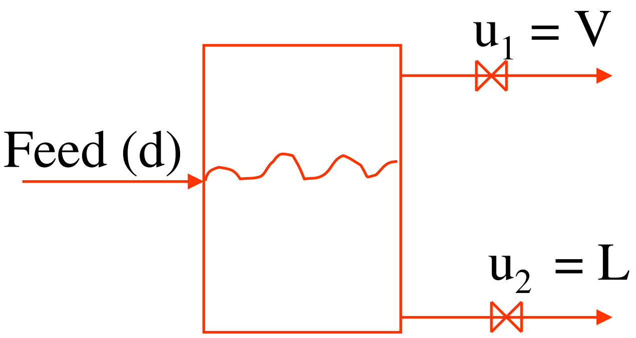


(d) $g_{12} = -1$; controller (10.62) with $\tau_1 = 21.95$ and $\tau_2 = 1$

Conclusions decentralized examples

- Performance is OK with decentralized control (even with wrong pairings!)
- However, controller design becomes difficult for interactive plants
  - and independent design may not be possible
  - and failure tolerance may not be guaranteed

Example 3.10: Separator (pressure vessel)

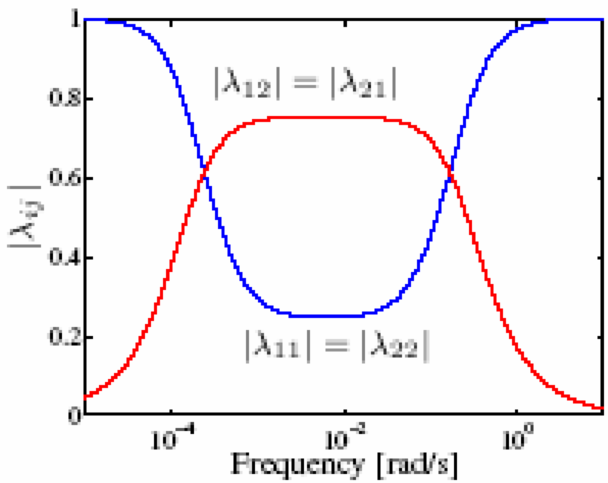


y_1 = pressure (p)

y_2 = level (h)

$$G(s) = \frac{0.01e^{-5s}}{(s+1.72\cdot 10^{-4})(4.32s+1)}\begin{bmatrix} -34.54(s+0.0572) & 1.913 \\ -30.22s & -9.188(s+6.95\cdot 10^{-4}) \end{bmatrix}$$

- Pairings? Would expect y_1/u_1 and y_2/u_2
- But process is strongly coupled at intermediate frequencies. Why?
- Frequency-dependent RGA suggests opposite pairing at intermediate frequencies



(a) Magnitude of RGA elements

Separator example....

Outline of solution: For tuning purposes the elements in $G(s)$ are approximated using the half rule to get

$$G(s) \approx \begin{bmatrix} -0.0823\frac{e^{-\theta s}}{s} & 0.01913\frac{e^{-(\theta+2.16)s}}{s} \\ -0.3022\frac{e^{-\theta s}}{4.32s+1} & -0.09188\frac{e^{-\theta s}}{4.32s+1} \end{bmatrix}$$

For the diagonal pairings this gives the PI settings

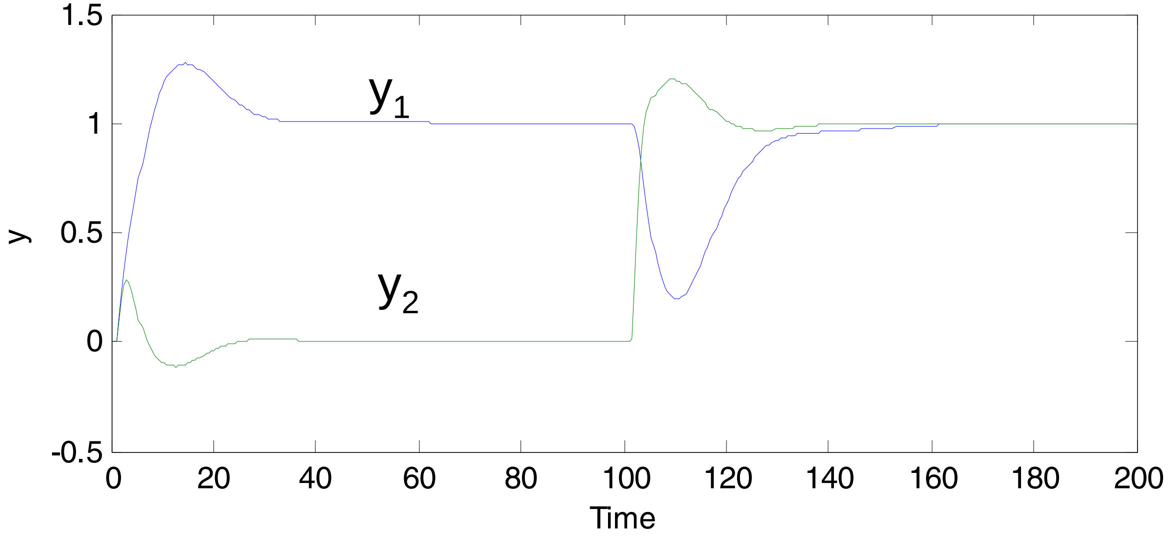
$$K_{c1} = -12.1/(\tau_{c1} + \theta), \tau_{I1} = 4(\tau_{c1} + \theta); K_{c2} = -47.0/(\tau_{c2} + \theta), \tau_{I2} = 4.32$$

and for the off-diagonal pairings (the index refers to the output)

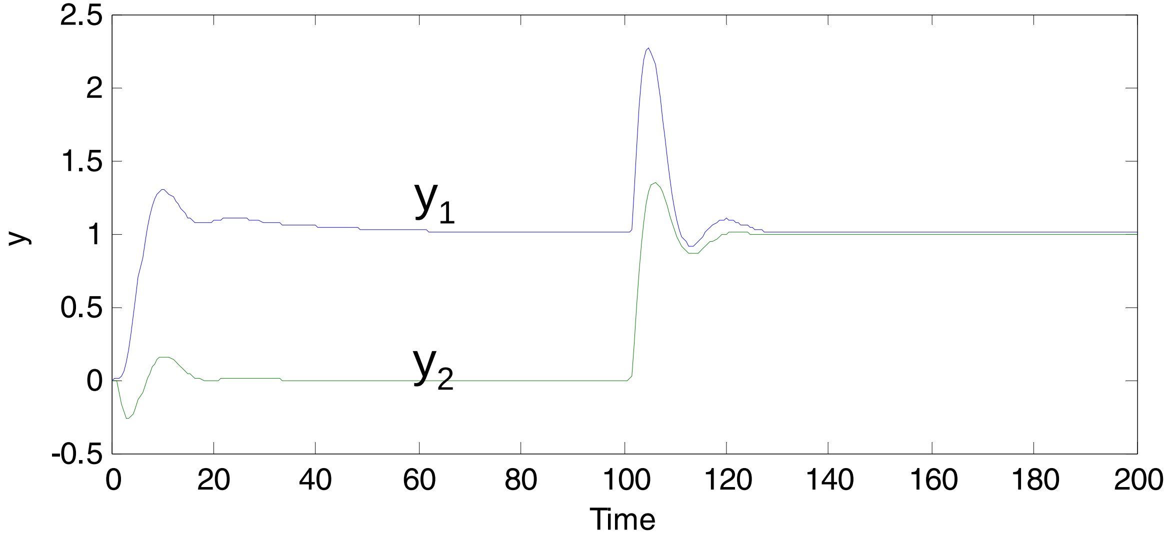
$$K_{c1} = 52.3/(\tau_{c1} + \theta + 2.16), \tau_{I1} = 4(\tau_{c1} + \theta + 2.16); K_{c2} = -14.3/(\tau_{c2} + \theta), \tau_{I2} = 4.32$$

For improved robustness, the level controller (y_1) is tuned about 3 times slower than the pressure controller (y_2), i.e. use $\tau_{c1} = 3\theta$ and $\tau_{c2} = \theta$. This gives a crossover frequency of about $0.5/\theta$ in the fastest loop. With a delay of about 5 s or larger you should find, as expected from the RGA at crossover frequencies (pairing rule 1), that the off-diagonal pairing is best. However, if the delay is decreased from 5 s to 1 s, then the diagonal pairing is best, as expected since the RGA for the diagonal pairing approaches 1 at frequencies above 1 rad/s.

Separator example: Simulations (delay = 1)

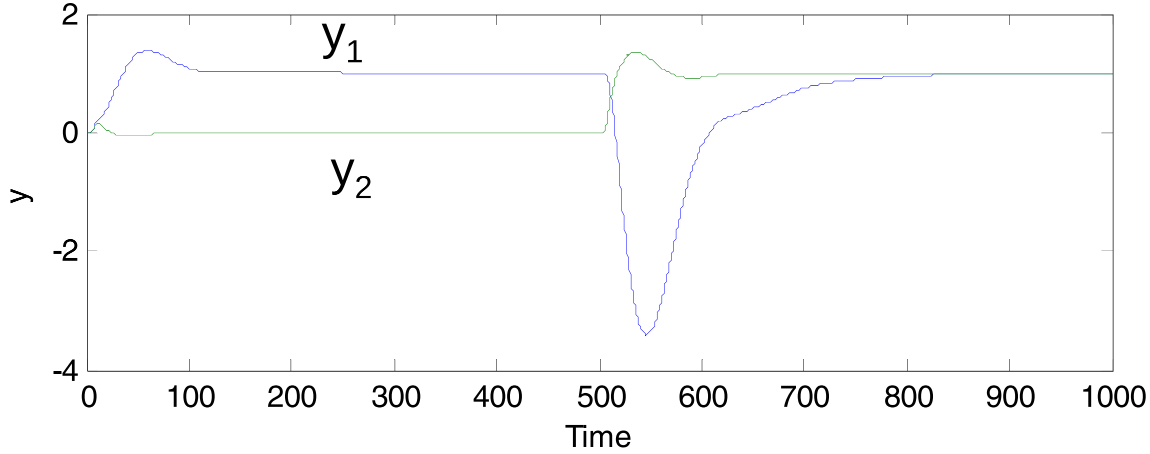


Diagonal

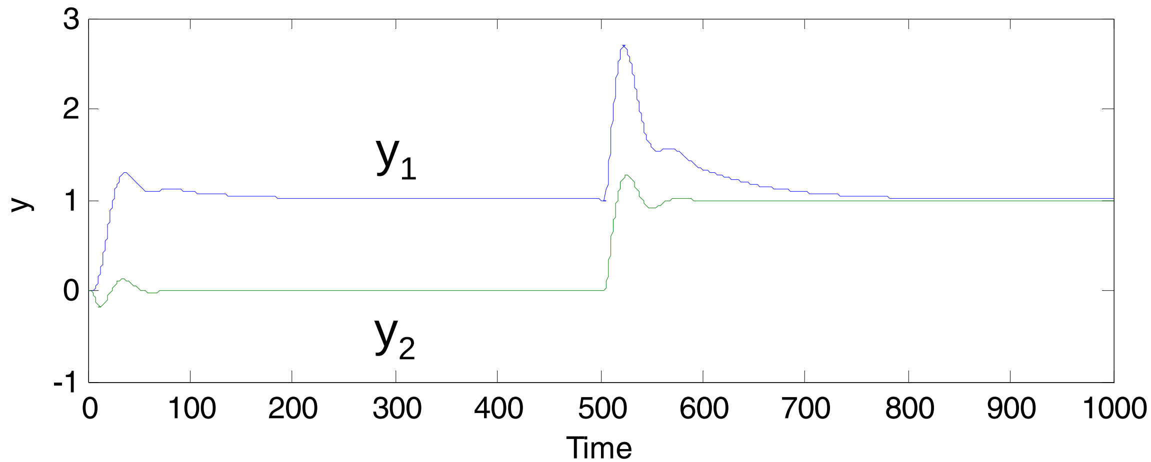


Off-Diagonal

Separator example: Simulations (delay = 5)



Diagonal



Off-Diagonal

Separator example

- BUT NOTE: May easily eliminate interactions to y2 (level) by simply closing a flow controller on u2 (liquid flow)

Iterative RGA

- For large processes, lots of pairing alternatives
- RGA evaluated iteratively is helpful for quick screening

$$\mathrm{RGA}(G) = \Lambda(G) = G \times (G^{-1})^T$$

$$\Lambda^2(G) = \Lambda(\Lambda(G))$$

$$\boxed{\Lambda^\infty = \lim_{k\to\infty} \Lambda^k(G)}$$

- Converges to "Permuted Identity" matrix (correct pairings) for generalized diagonally dominant processes.
- Can converge to incorrect pairings, when no alternatives are dominant.
- Usually converges in 5-6 iterations

Example of Iterative RGA

$$G = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 0.33 & 0.67 \\ 0.67 & 0.33 \end{bmatrix} \quad \Lambda^2 = \begin{bmatrix} -0.33 & 1.33 \\ 1.33 & -0.33 \end{bmatrix}$$

$$\Lambda^3 = \begin{bmatrix} -0.07 & 1.07 \\ 1.07 & -0.07 \end{bmatrix} \quad \Lambda^4 = \begin{bmatrix} 0.00 & \boxed{1.00} \\ \boxed{1.00} & 0.00 \end{bmatrix}$$

Correct pairing

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Stability of Decentralized control systems

- Question: If we stabilize individual loops, will the overall closed-loop system be stable?

$$\bar{G} \triangleq \text{diag}\{g_{ii}\} = \begin{bmatrix} g_{11} & & & \\ & g_{22} & & \\ & & \ddots & \\ & & & g_{mm} \end{bmatrix}$$

$$\bar{S} \triangleq (I + \bar{G}K)^{-1} = \text{diag}\left\{\frac{1}{1 + g_{ii}k_i}\right\} \quad \text{and} \quad \bar{T} = I - \bar{S}$$

$$E \triangleq (G - \bar{G})\bar{G}^{-1}$$

- E is relative uncertainty
- $\bar{T}$ is complementary sensitivity for diagonal plant

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Stability of Decentralized control systems

- Question: If we stabilize individual loops, will overall closed-loop system be stable?

Let G an $\bar{G}$ have same unstable poles, then closed-loop system stable if

$$\bar{\sigma}(\bar{T}) = \max_i |\bar{t}_i| < 1/\mu(E) \quad \forall\omega \qquad (10.72)$$

The structured singular value $\mu(E)$ is computed with respect to a diagonal structure (of $\bar{T}$).

Let G and $\bar{G}$ have same unstable zeros, then closed-loop system stable if

$$\bar{\sigma}(\bar{S}) = \max_i |\bar{s}_i| < 1/\mu(E_S) \quad \forall\omega \qquad (10.74)$$

The structured singular value $\mu(E_S)$ is computed with respect to a diagonal structure (of $\bar{S}$).

$$E_S = (G - \bar{G})G^{-1},$$

Mu-Interaction measure

Closed-loop stability if

$$\bar{\sigma}(\tilde{T}) = \max_i |\tilde{t}_i| < 1/\mu(E) \quad \forall\omega$$

At low frequencies, for integral control $\bar{\sigma}(\tilde{T}) \approx 1$,

$\Rightarrow$ $$\mu(E) < 1$$

- *Prefer pairings with* $\mu(E) < 1$ *("diagonal dominance") at frequencies within the closed-loop bandwidth.*

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Decentralized Integral Controllability

- Question: If we detune individual loops arbitrarily or take them out of service, will the overall closed-loop system be stable with integral controller?

- Addresses "Ease of tuning"
- When DIC - Can start with low gains in individual loops and increase gains for performance improvements

- <u>Not</u> DIC if

$$\lambda_{ii}(0) \geq 0 \text{ for all } i.$$

- DIC if

$$\mu(E(0)) < 1$$

Performance RGA

- Motivation: RGA measures two-way interactions only

- Example 2 (Triangular plant)

$$G = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \qquad \text{RGA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Performance Relative Gain Array

$$\Gamma(s) \triangleq \tilde{G}(s)G^{-1}(s)$$

- Also measures one-way interactions, but need to be calculated for every pairing alternative.

Example PRGA: Distillation

- see book