# Multiple Model Predictive Control (MMPC) for Nonlinear Systems and Improved Disturbance Rejection

- Motivation & Tutorial Overview
- Multiple Model Predictive Control
  - Nonlinear Processes
  - Disturbance Rejection
- Summary

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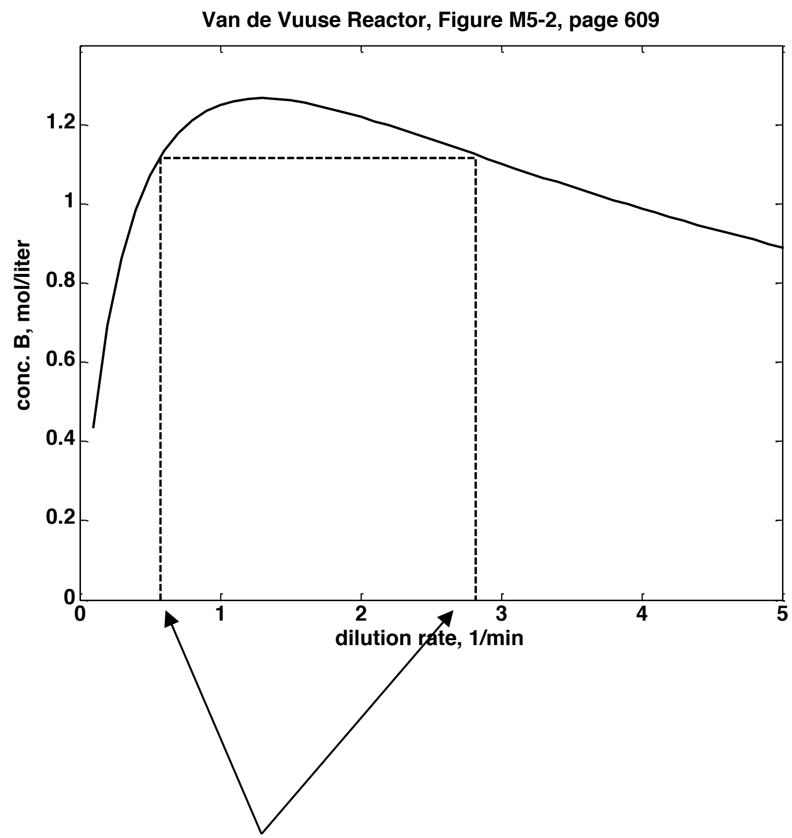
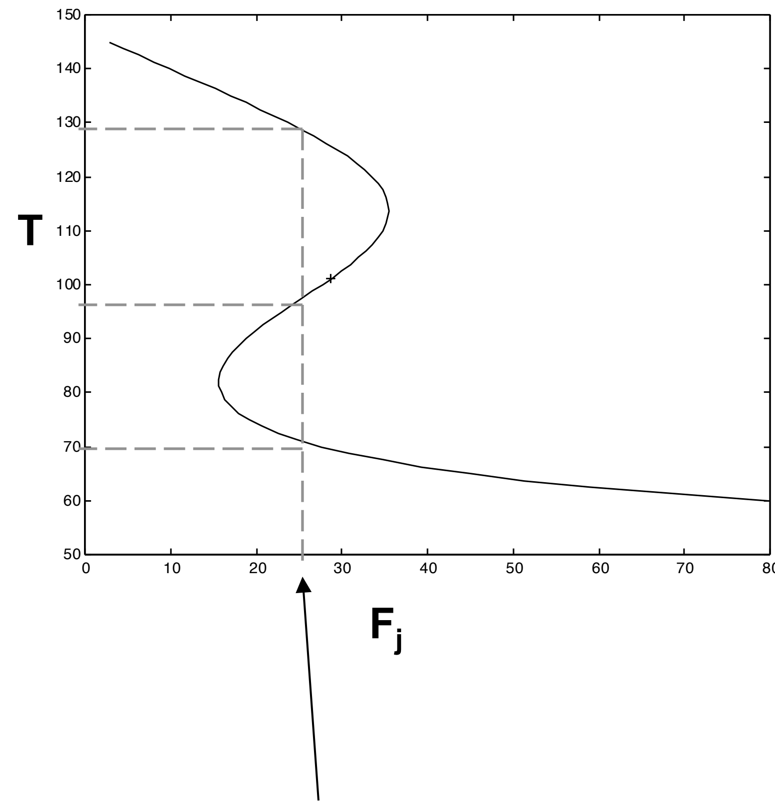
# Nonlinear Behavior: Steady-state

## Input Multiplicity

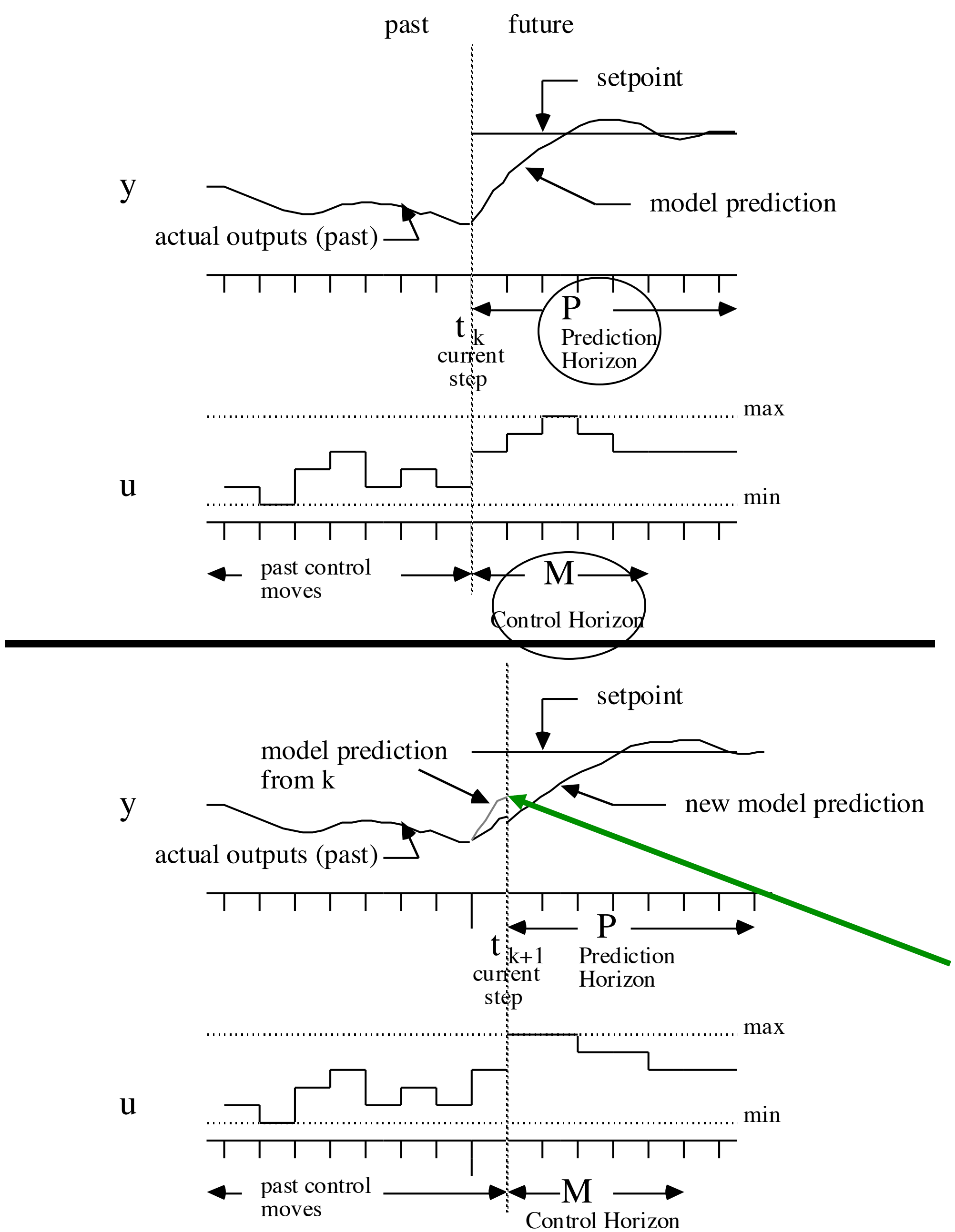



Two different input values
yield the same output

## Output Multiplicity (hysteresis)




One input can yield three
different output values




# MPC

- Constraints
- Multivariable
- Time-delays

- Objective function?
- Optimization technique?
- Model type?

- Disturbances/mismatch?
  - Current and Future
- Initial cond./state est.?

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# Our Approaches to Nonlinear MPC

- Quadratic Objective Function
- Models
  - Fundamental: numerical integration or collocation
  - Fundamental with linearization at each time step
  - **Multiple model**
  - Artificial neural network
- State Estimates/Initial Conditions
  - Additive output disturbance (e.g. DMC)
  - Estimation horizon (optimization)
  - Extended/appended state Kalman Filter
    - **Importance of stochastic states**

# Intuitive Nonlinear Model-based Strategy

Model equations

$$\dot{x} = f(x,u)$$
$$y = g(x)$$

Integrate model from time step k-1 to k

$$\hat{x}_k = F_{t_s}(\hat{x}_{k-1}, u_{k-1})$$
$$\hat{y}_{k|k-1} = g(\hat{x}_k)$$

Obtain plant measurement

$$y_k$$

Calculate model error

**(additive output disturbance)**

$$d_k = y_k - \hat{y}_{k|k-1}$$

Choose hypothetical set of current and future control moves

$$u_k, u_{k+1}, \cdots, u_{k+P-1}$$

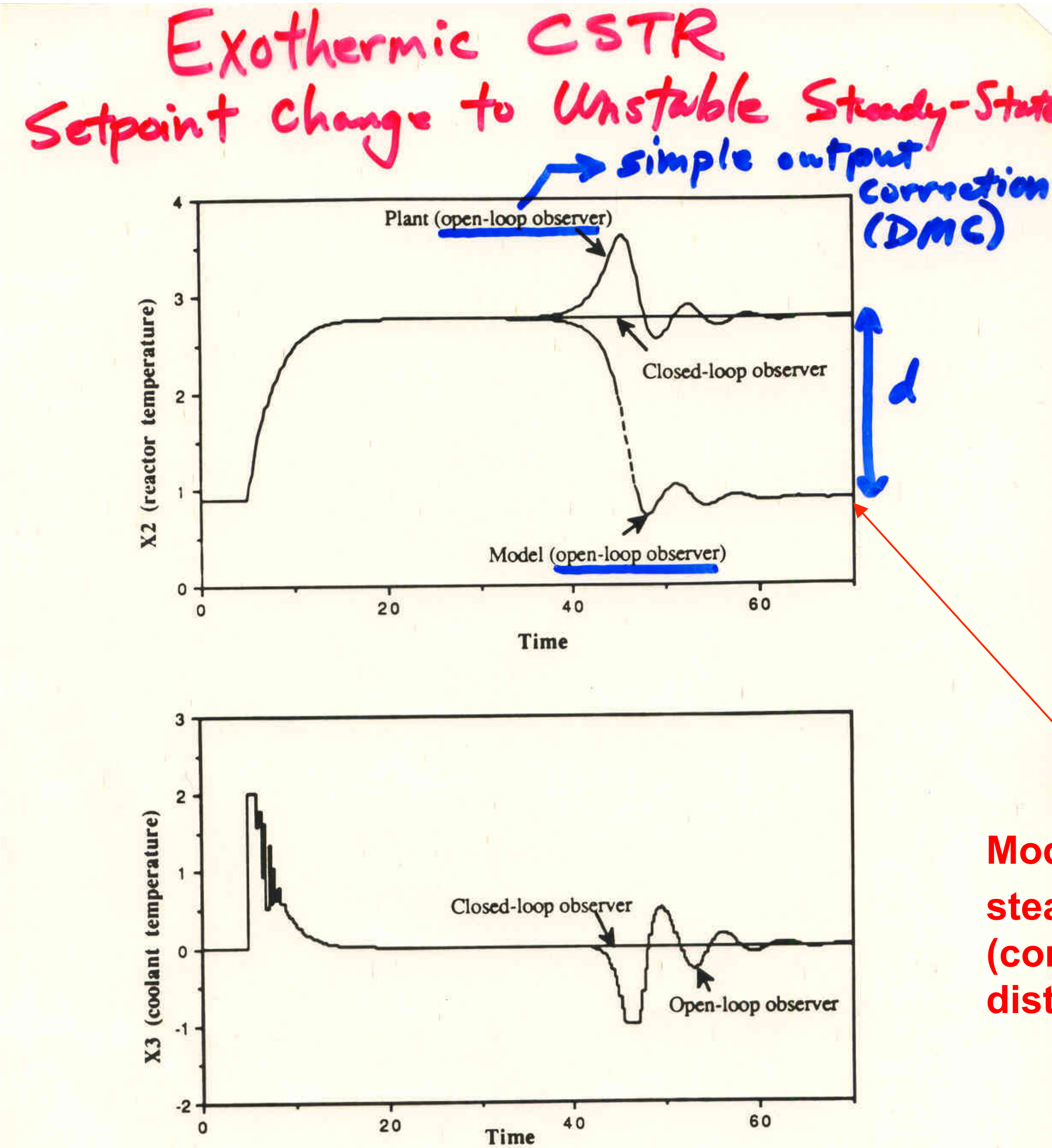
Integrate model from time step k to k +P (based on hypothetical control moves)

$$\hat{x}_{k+1} = F_{t_s}(\hat{x}_k, u_k)$$
$$\hat{y}_{k+1|k} = g(\hat{x}_{k+1}) + d_k$$

to

$$\hat{x}_{k+P} = F_{t_s}(\hat{x}_{k+P-1}, u_{k+P-1})$$
$$\hat{y}_{k+P|k} = g(\hat{x}_{k+P}) + d_k$$

Evaluate objective function and repeat until optimum is obtained




Response to a setpoint change to a open-loop unstable operating point with a perfect model. P = 10, M = 1.

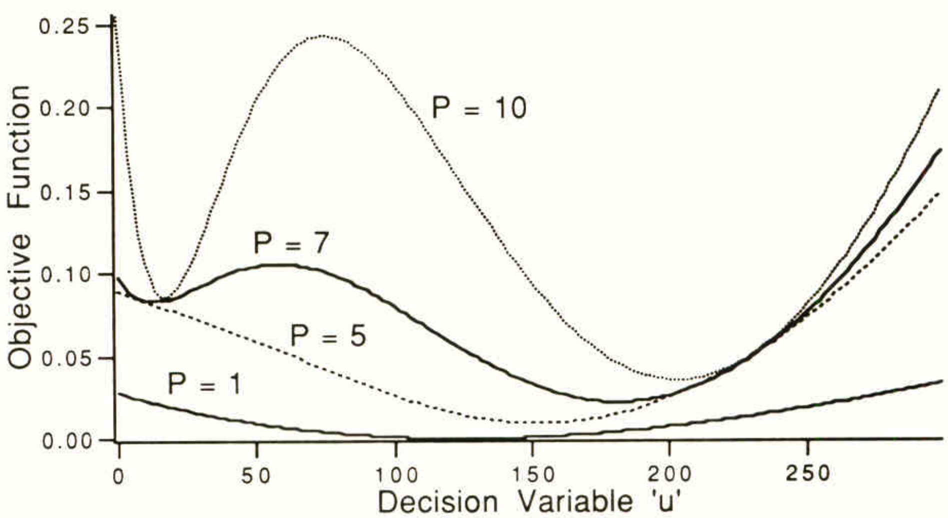
# Additive Disturbance Assumption

**Model converges to different steady-state than plant (compensated by additive disturbance term)**

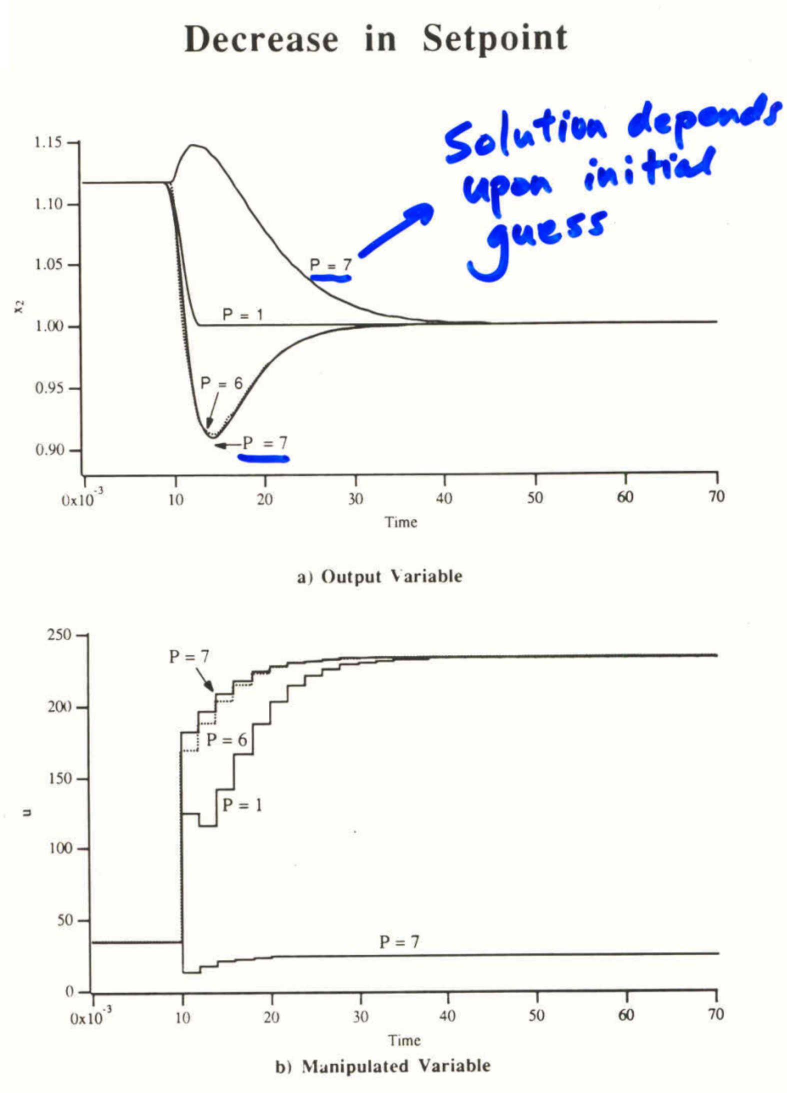
# Non-Convex Problem

**Objective Function**
**(Decrease in Setpoint)**

**M = 1, different values of P**




**Input Multiplicity ----> Multiple Minima**




Sistu and Bequette, 1992 ACC

# EKF-based NMPC (Lee & Ricker, 1994)

- Nonlinear Model, integrate from step k-1 to k
- State Estimation: Extended Kalman Filter
  - Linearized at each time step
  - Find best state estimate at time step k
- Prediction
  - One integration of NL ODEs based on set of control moves (**unforced or "free response"**) from step k to k+P
  - Perturbation (linear) model - effect of changes in control moves (**forced**)
- Optimization
  - QP, since linear model is used

**Can use linear state-space KF-MPC code!**

1530 *Ind. Eng. Chem. Res.* **1994,** *33,* 1530–1541

**Extended Kalman Filter Based Nonlinear Model Predictive Control**

**Jay H. Lee***

*Department of Chemical Engineering, Auburn University, Auburn, Alabama 36849-5127*

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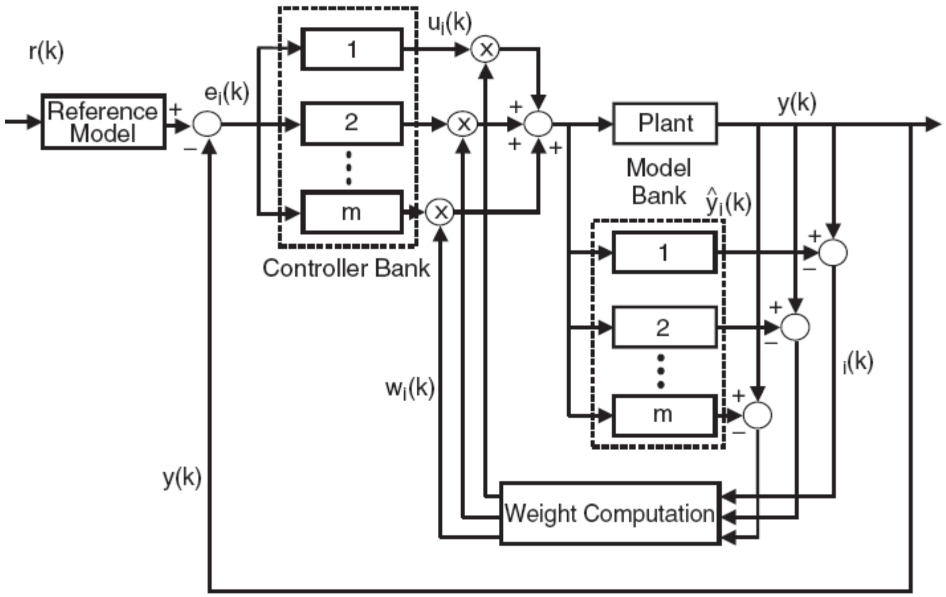
*Department of Chemical Engineering, University of Washington, Seattle, Washington 98195*

# Motivation for Multiple Linear Models

- Development time for fundamental models
  - Difficulty with physiological systems
- Much data required for artificial neural networks
  - Problems with “overfitting” and extrapolation
- At particular operating points, linear models are often a good description
  - How to switch between models?

# Multiple Model-based Control

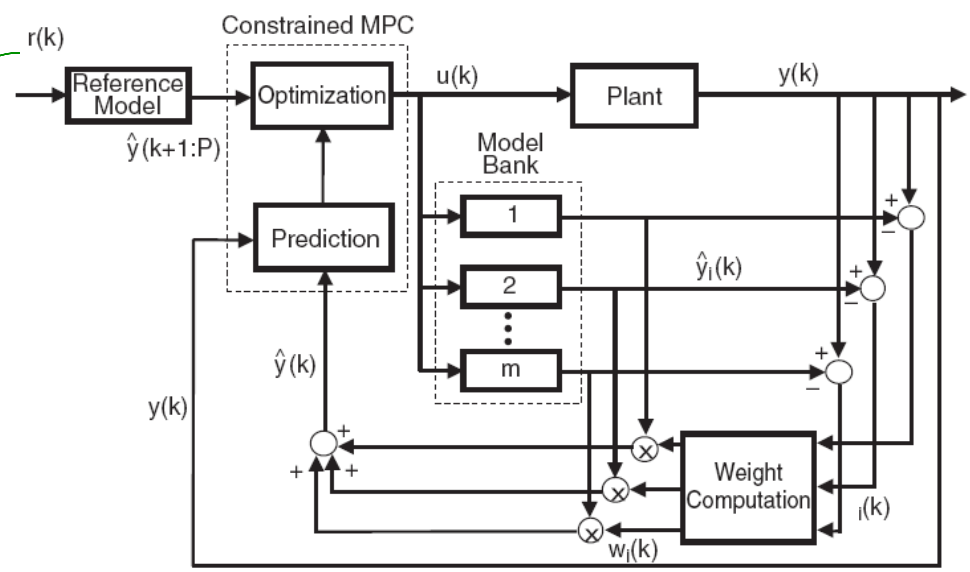



**Multiple Model Adaptive Control (MMAC)**

Athans et al. (1977) – LQG, Jet aircraft control

Roy, Kaufman - Drug infusion control

Schott, Bequette (1997) – PI, CSTR




**Multiple Model Predictive Control (MMPC)**

Rao et al. (2001, 2003)

Drug Infusion Control

Aufderheide & Bequette (2003)
Nonlinear CSTR

# First, a Concise Review of Linear MPC

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# Step Response & Additive Output

**Correction term** → $d_k = y_k - \hat{y}_{k|k-1}$

**measured output** ($y_k$) **model predicted output** ($\hat{y}_{k|k-1}$)

**The "corrected prediction" is set equal to the measured output**

$$\hat{y}_{k|k} = \hat{y}_{k|k-1} + d_k \iff \hat{y}_{k|k} = y_k$$

**"deadbeat" observer**

**The "corrected prediction" for jth future step is (step response form)**

$$\hat{y}_{k+j|k} = \underbrace{\sum_{i=1}^{j} s_i \Delta u_{k-i+j}}_{\text{effect of future control moves}} + \underbrace{\sum_{i=j+1}^{N-1} s_i \Delta u_{k-i+j} + s_N u_{k-N+j}}_{\text{effect of past control moves}} + \underbrace{\hat{d}_{k+j}}_{\text{correction term}}$$

**forced response** **free response**

$$\hat{d}_{k+j} = \hat{d}_{k+j-1} = \cdots = d_k = y_k - \hat{y}_{k|k-1}$$

**constant additive disturbance**

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# Problems with “Classical MPC” (e.g. DMC)

- Finite Step/Impulse Models Limited
  - Many parameters (~50 for each input-output relationship)
  - Limited to open-loop stable processes (there is no corrective feedback to model states)
- Additive Output Disturbance Assumption
  - Poor performance for input step disturbances
  - No explicit measurement noise trade-off

The most common criticism of MPC (Shinskey, 2002)

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# State Space Models & State Estimation

$$x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma^d d_k$$

$$y_k = C x_k$$

For perturbations to manipulated inputs

$$\underbrace{\Gamma^d = \Gamma}$$

**Assume disturbance propagation**

$$d_{k+1} = d_k$$

**Appended state formulation**

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x^a_{k+1}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k}$$

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# State Estimation Problem

$$d_{k+1} = d_k + w_k$$ Random walk

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x^a_{k+1}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + v_k$$

## Kalman Filter

$$\hat{x}^a_{k|k-1} = \Phi^a \hat{x}^a_{k-1|k-1} + \Gamma^a u_{k-1}$$ **Prediction**

$$\hat{x}^a_{k|k} = \hat{x}^a_{k|k-1} + L_k \left( y_k - C^a \hat{x}^a_{k|k-1} \right)$$ **Correction**

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# Offset-Free Performance

- Next few slides present conditions for offset-free performance
- Unmeasured disturbances estimated as either state or output disturbances
- State observer techniques can then be used

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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# Disturbance Models

## State or Input Disturbance

$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x_k^a}$$

## Additive Output Disturbance

$$\underbrace{\begin{bmatrix} x_{k+1} \\ p_{k+1} \end{bmatrix}}_{x_{k+1}^a} = \underbrace{\begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x_k^a} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \qquad y_k = \underbrace{\begin{bmatrix} C & G_p \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x_k^a}$$

DMC: $G_p = I$

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# Disturbance Models

## General Input and Output Disturbances

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \\ p_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}}_{\Phi^a} \begin{bmatrix} x_k \\ d_k \\ p_k \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 & G_p \end{bmatrix}}_{C^a} \begin{bmatrix} x_k \\ d_k \\ p_k \end{bmatrix}$$

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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# State Estimator

$$\begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}}_{\Phi^a} \begin{bmatrix} \hat{x}_{k-1:k-1} \\ \hat{d}_{k-1|k-1} \\ \hat{p}_{k-1|k-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \\ 0 \end{bmatrix}}_{\Gamma^a} u_{k-1}$$

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{d}_{k|k} \\ \hat{p}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} + \begin{bmatrix} L_x \\ L_d \\ L_p \end{bmatrix} \left( y_k - C^a \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} \right)$$

Deterministic or stochastic observer design

For offset-free performance:

# disturbances = # outputs, and augmented system must be detectable

Ref: Muske & Badgwell, J. Proc. Cont., 12: 617-632 (2002)

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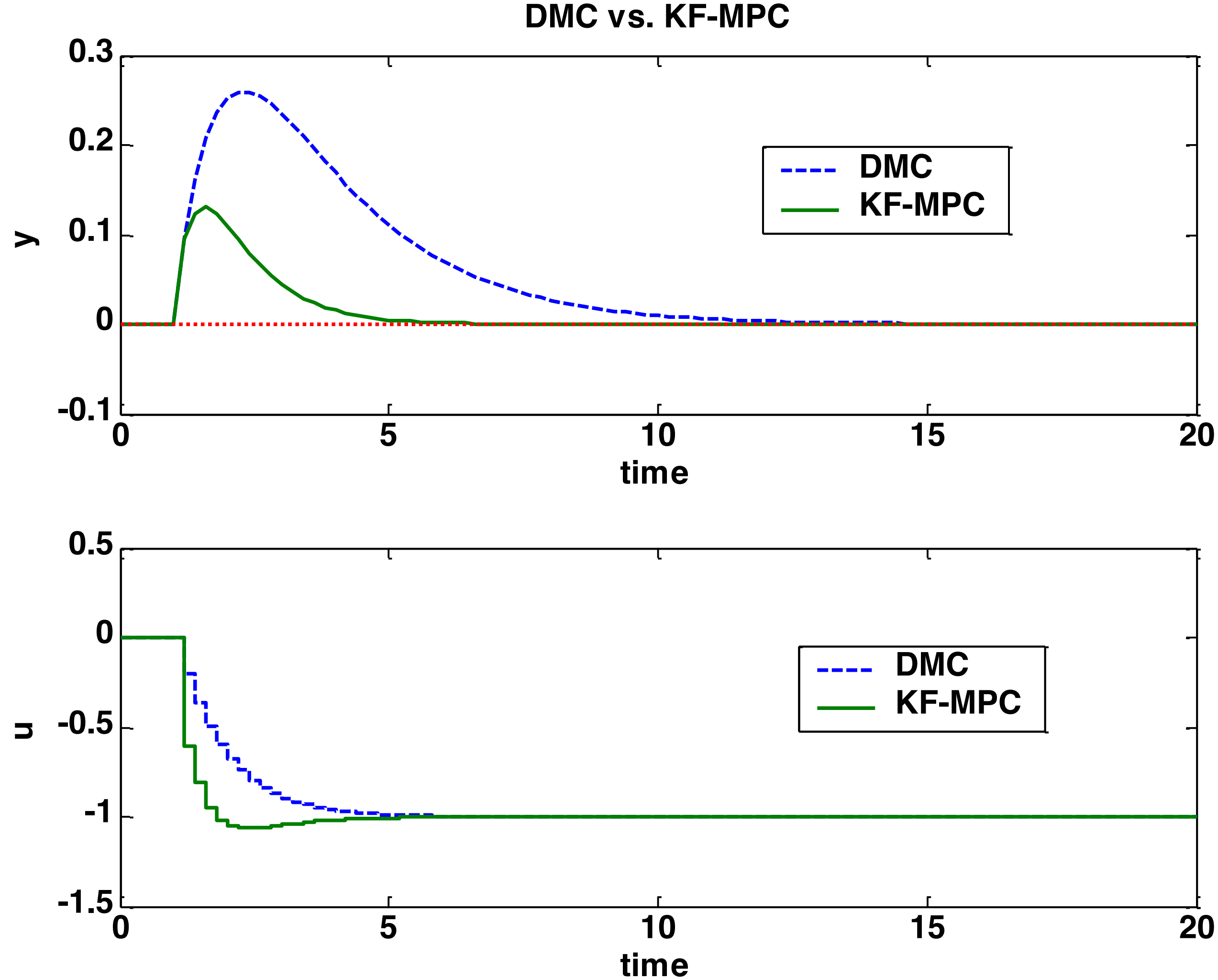
# Problem: Unmeasured Step Input Disturbance




DMC: additive output disturbance assumption (bias)

KF-MPC: appended state, estimated step input disturbance

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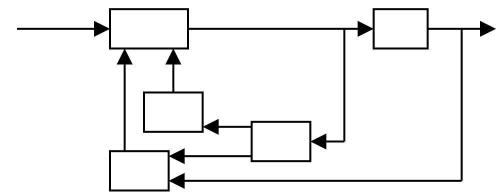
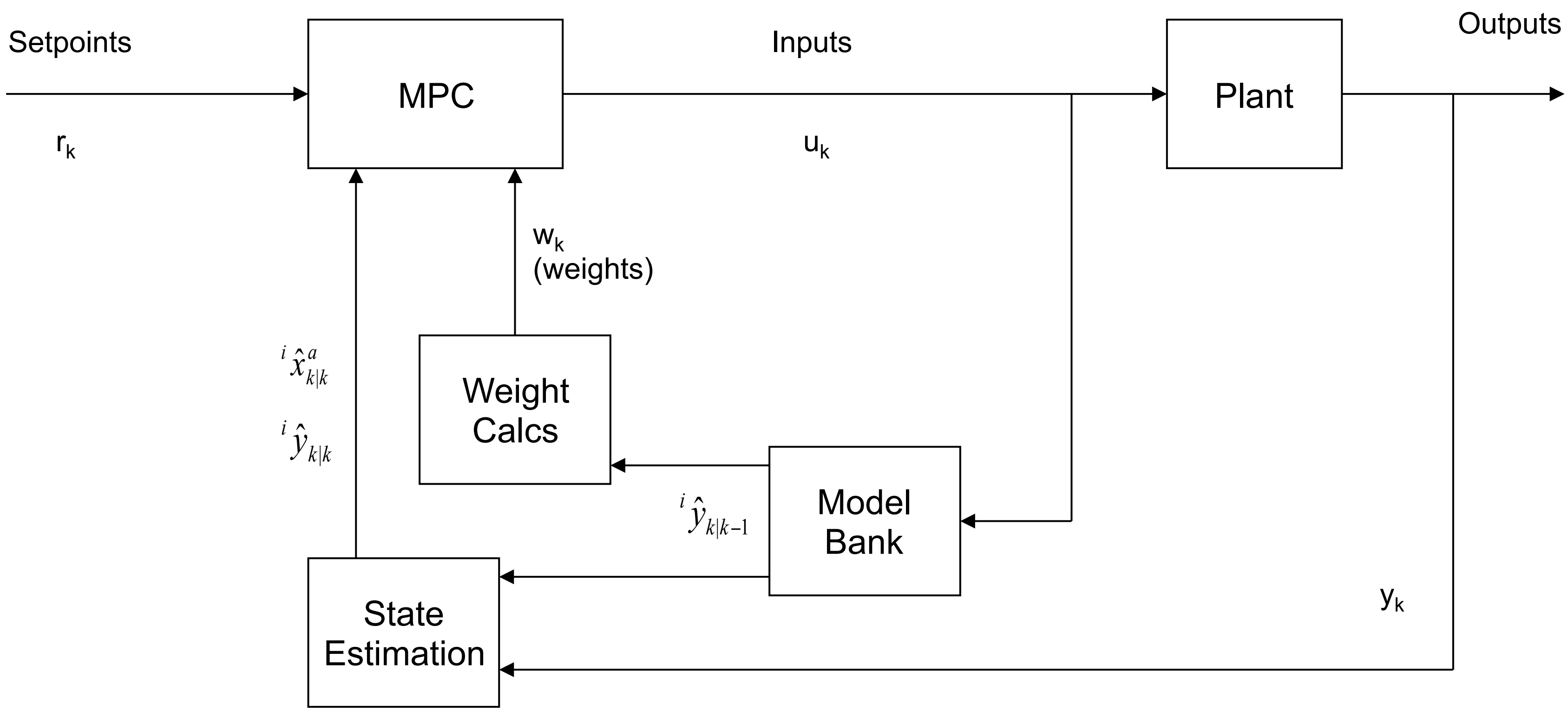
# Structure of MMPC



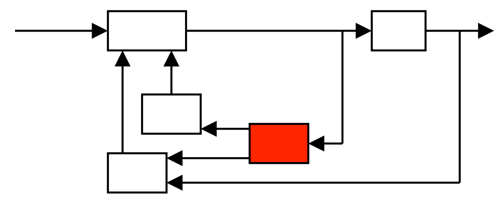


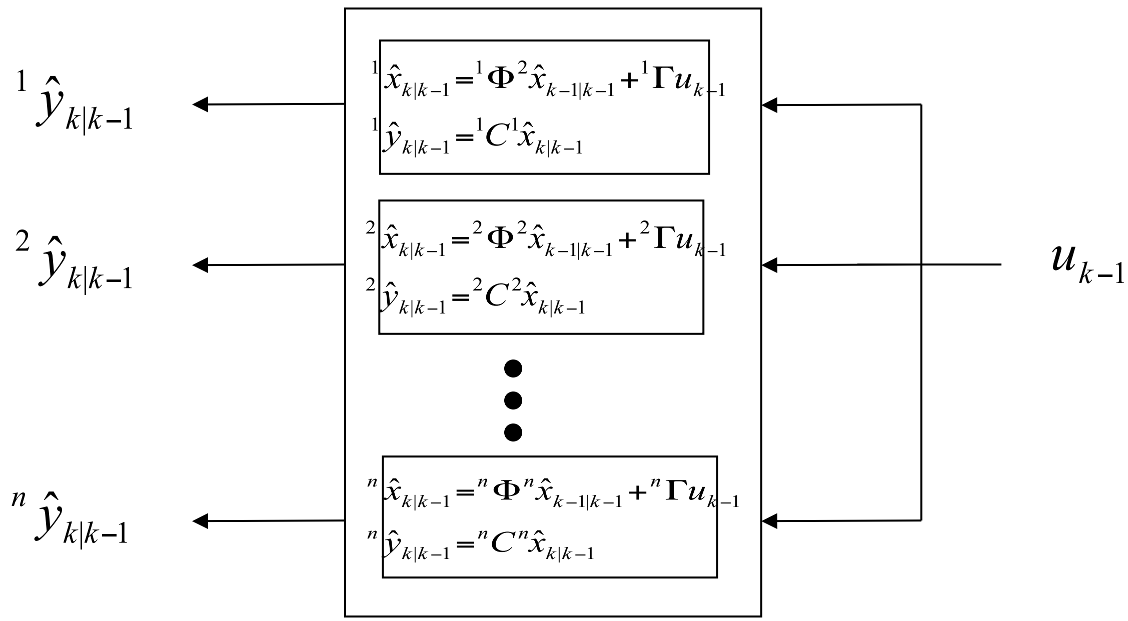

Multiple Model Predictive Control of Nonlinear Systems

Matthew Kuure-Kinsey† and B. Wayne Bequette‡

Int. Workshop on Assessment and Future Directions of NMPC
Pavia, Italy, September 5-9, 2008

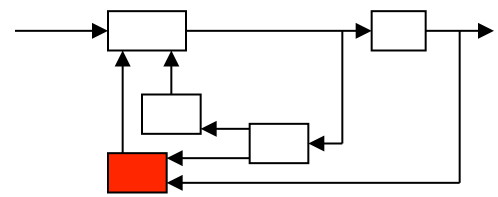
# Model Bank Details






- Each of the n linear models represents the system at a specified operating condition
- A model predicted output is calculated for each model using the current manipulated input

# State Estimation



- Need a way to account for model uncertainty and mismatch.

$$ {}^{i}x_{k+1} = {}^{i}\Phi\ {}^{i}x_k + {}^{i}\Gamma u_k $$
$$ {}^{i}y_k = {}^{i}C\ {}^{i}x_k $$

**Additive output disturbance**

$$ {}^{i}x_{k+1} = {}^{i}\Phi\ {}^{i}x_k + {}^{i}\Gamma u_k $$
$$ {}^{i}d_{k+1} = {}^{i}d_k $$
$$ {}^{i}y_k = {}^{i}C\ {}^{i}x_k + {}^{i}d_k $$

**Step input disturbance**

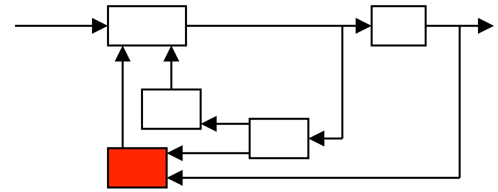
$$ {}^{i}x_{k+1} = {}^{i}\Phi\ {}^{i}x_k + {}^{i}\Gamma u_k + {}^{i}\Gamma^{d}\ {}^{i}d_k $$
$$ {}^{i}d_{k+1} = {}^{i}d_k $$
$$ {}^{i}y_k = {}^{i}C\ {}^{i}x_k $$

$$ \begin{bmatrix} {}^{i}x_{k+1} \\ {}^{i}d_{k+1} \end{bmatrix} = \begin{bmatrix} {}^{i}\Phi & G_x \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^{i}x_k \\ {}^{i}d_k \end{bmatrix} + \begin{bmatrix} {}^{i}\Gamma \\ 0 \end{bmatrix} u_k $$

$$ {}^{i}y_k = \begin{bmatrix} {}^{i}C & G_y \end{bmatrix} \begin{bmatrix} {}^{i}x_k \\ {}^{i}d_k \end{bmatrix} $$

| | |
|---|---|
| $G_x = {}^{i}\Gamma^{d}$ | **Step input** |
| $G_x = 0$ | **Additive output** |
| $G_y = 0$ | **Step input** |
| $G_y = I$ | **Additive output** |

# State Estimation



- Kalman predictor/corrector equations

$$\begin{bmatrix} {}^i x_{k+1} \\ {}^i d_{k+1} \end{bmatrix} = \begin{bmatrix} {}^i\Phi & G_x \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix} + \begin{bmatrix} {}^i\Gamma \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ {}^i\Gamma^w \end{bmatrix} {}^i w_k$$

$${}^i y_k = \begin{bmatrix} {}^i C & G_y \end{bmatrix} \begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix} + v_k$$

$${}^i x_{k+1}^a = {}^i\Phi^a \, {}^i\hat{x}_k^a + {}^i\Gamma^a u_k + {}^i\Gamma^{1,w} \, {}^i w_k$$

$${}^i y_k = {}^i C^a \, {}^i x_k^a + v_k$$

$${}^i\hat{x}_{k|k-1}^a = {}^i\Phi^a \, {}^i\hat{x}_{k-1|k-1}^a + {}^i\Gamma^a u_{k-1}$$ Prediction

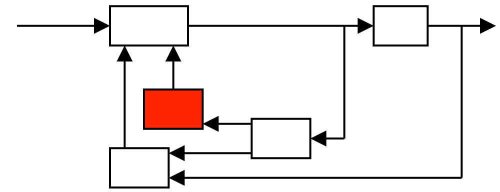
$${}^i\hat{x}_{k|k}^a = {}^i\hat{x}_{k|k-1}^a + {}^i L \left( y - {}^i C^a \, {}^i\hat{x}_{k|k-1}^a \right)$$ Correction

$${}^i\hat{y}_{k|k} = {}^i C^a \, {}^i\hat{x}_{k|k}^a$$ Updated output prediction

$${}^i L = \begin{bmatrix} 0 \\ I \end{bmatrix}$$ Additive output

$${}^i L = Kalman\ gain$$ Step input

# Weight Calculation



$${}^{i}\varepsilon_k = y_k - {}^{i}\hat{y}_{k|k}$$

Model residual

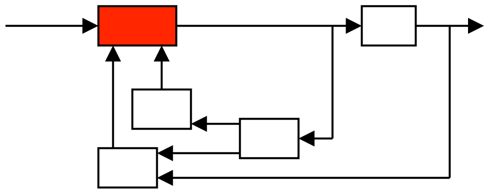
$${}^{i}p_k = \frac{\exp\left(-0.5\ {}^{i}\varepsilon_k^T\ {}^{i}\Lambda\ {}^{i}\varepsilon_k\right){}^{i}p_{k-1}}{\sum_{j=1}^{n} \exp\left(-0.5\ {}^{j}\varepsilon_k^T\ {}^{i}\Lambda\ {}^{j}\varepsilon_k\right){}^{j}p_{k-1}}$$

Bayesian probability, $i^{th}$ model

$${}^{i}w_k = \begin{cases} \dfrac{{}^{i}p_k}{\sum_{j=1}^{n} {}^{j}p_k} & {}^{i}p_k > \delta \\ 0 & {}^{i}p_k \le \delta \end{cases}$$

Weight, $i^{th}$ model

# Model Predictive Control



- “Model average” for the output prediction

$$\bar{y}_{k+j|k} = \sum_{i=1}^{n} {}^{i}w_k \; {}^{i}\hat{y}_{k+j|k}$$

- Quadratic objective function

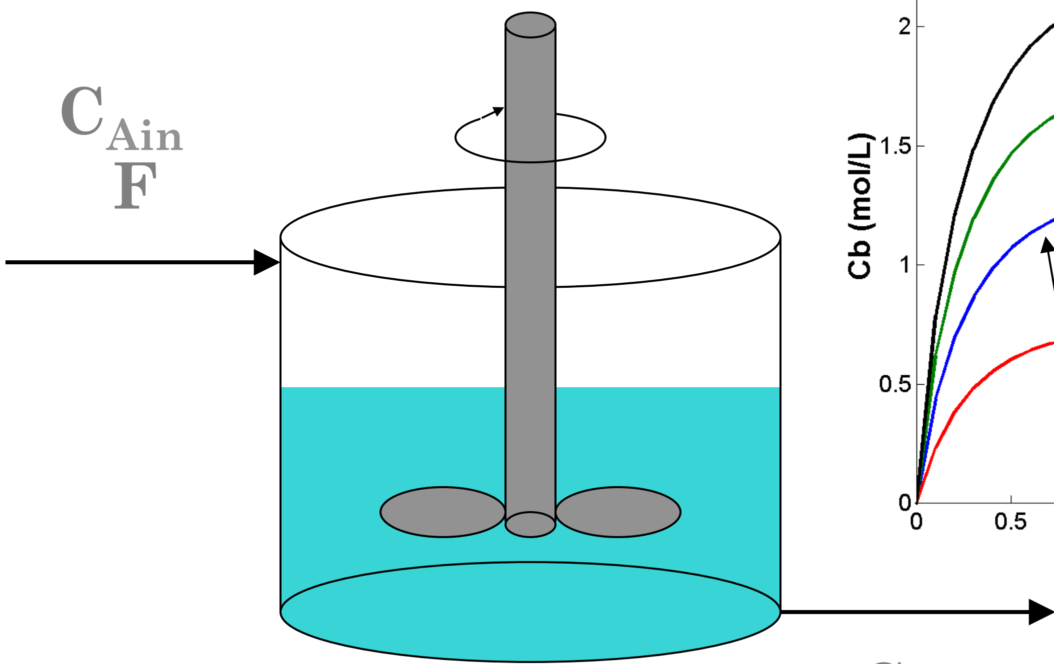
$$\min \Phi = \left(R - \bar{Y}\right)^T W_y \left(R - \bar{Y}\right) + \Delta U^T W_u \Delta U$$

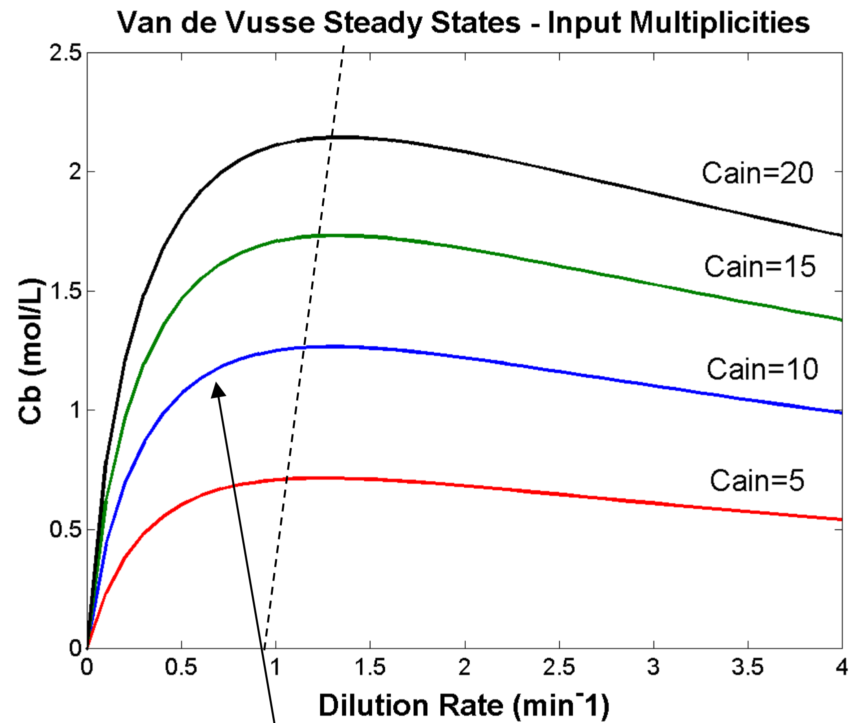
**Analytical Solution**

- Constraints on inputs and outputs

**Quadratic Program (QP)**

# Van de vuuse isothermal reactor




Constant V,T, ρ

$A \longrightarrow B \longrightarrow C$

$A + A \longrightarrow D$

Region with RHP zeros
(nonminimum phase behavior)

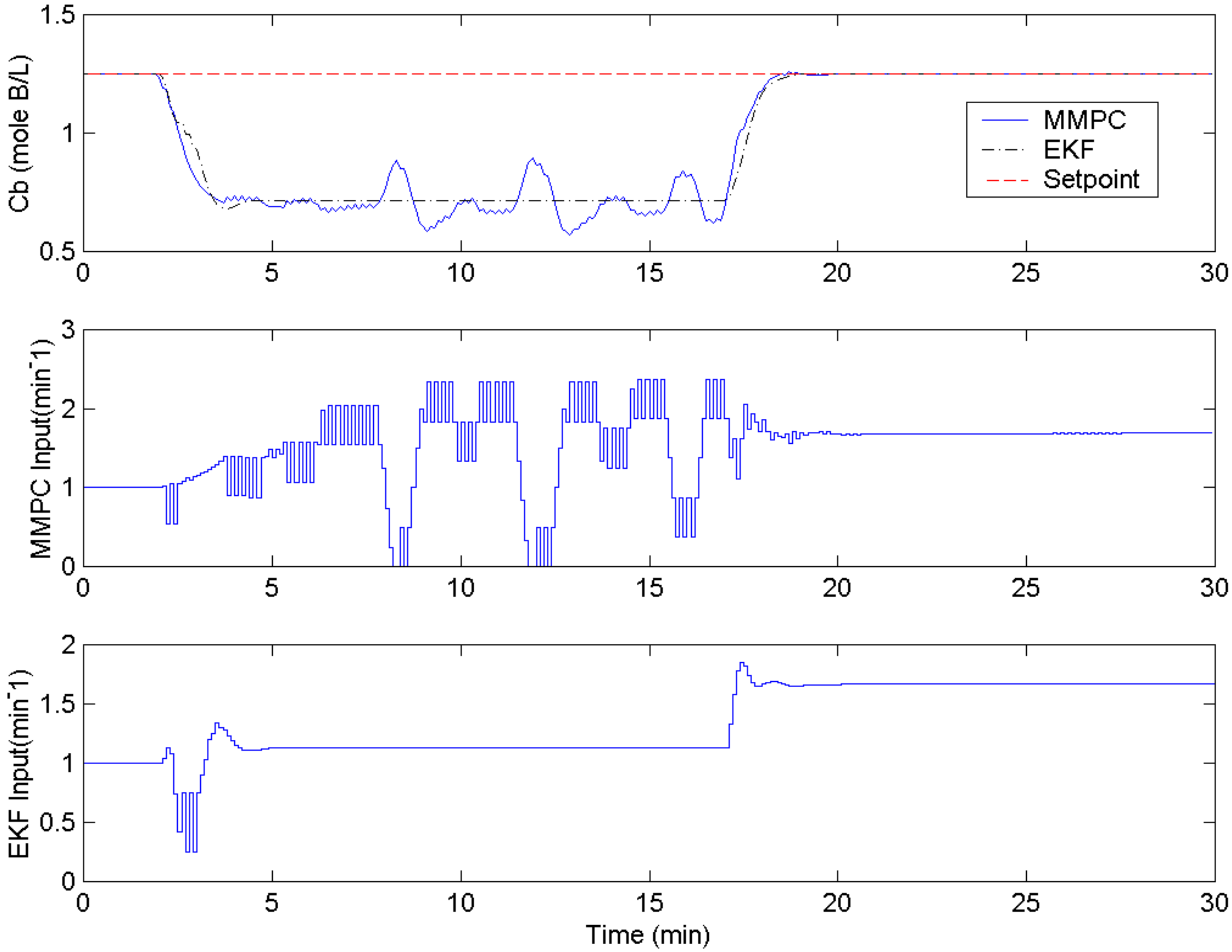
Connection between IM and RHP zeros:
Sistu & Bequette, *Chem. Eng. Sci.* (1996)

# Feed Concentration Disturbance

$C_B$ (y)

MMPC (u)

EKF-based NL-MPC (u)



# Disturbances & Propagation Into Future

- Output step
  - Generally poor  assumption for chemical processes
- Input step
  - Improved performance for many processes
- Input ramp
  - Motivated by experience with diabetes problems
- Pulse
  - Duties performed infrequently (shift change, etc.)
- Periodic
  - Poorly tuned upstream controllers, diurnal variations

# Step, Ramp, Generic

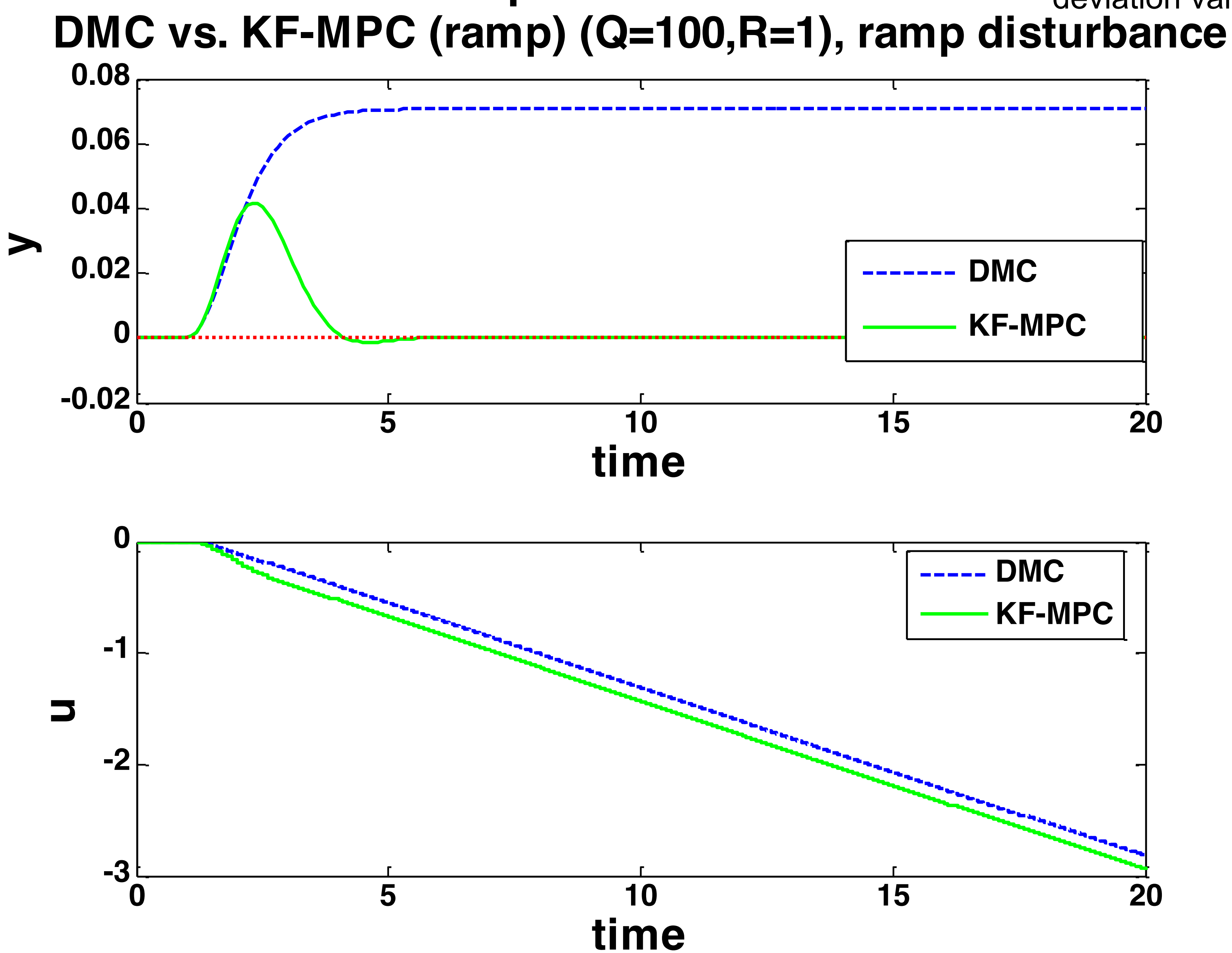
$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x^a_{k+1}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & 1 \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + v_k$$

$$\underbrace{\begin{bmatrix} x_{k+1} \\ \begin{bmatrix} d_{k+1} \\ \Delta d_{k+1} \end{bmatrix} \end{bmatrix}}_{x^a_{k+1}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ \begin{bmatrix} d_k \\ \Delta d_k \end{bmatrix} \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}}_{\Gamma^{w,a}} w_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ \begin{bmatrix} d_k \\ \Delta d_k \end{bmatrix} \end{bmatrix}}_{x^a_k} + v_k$$

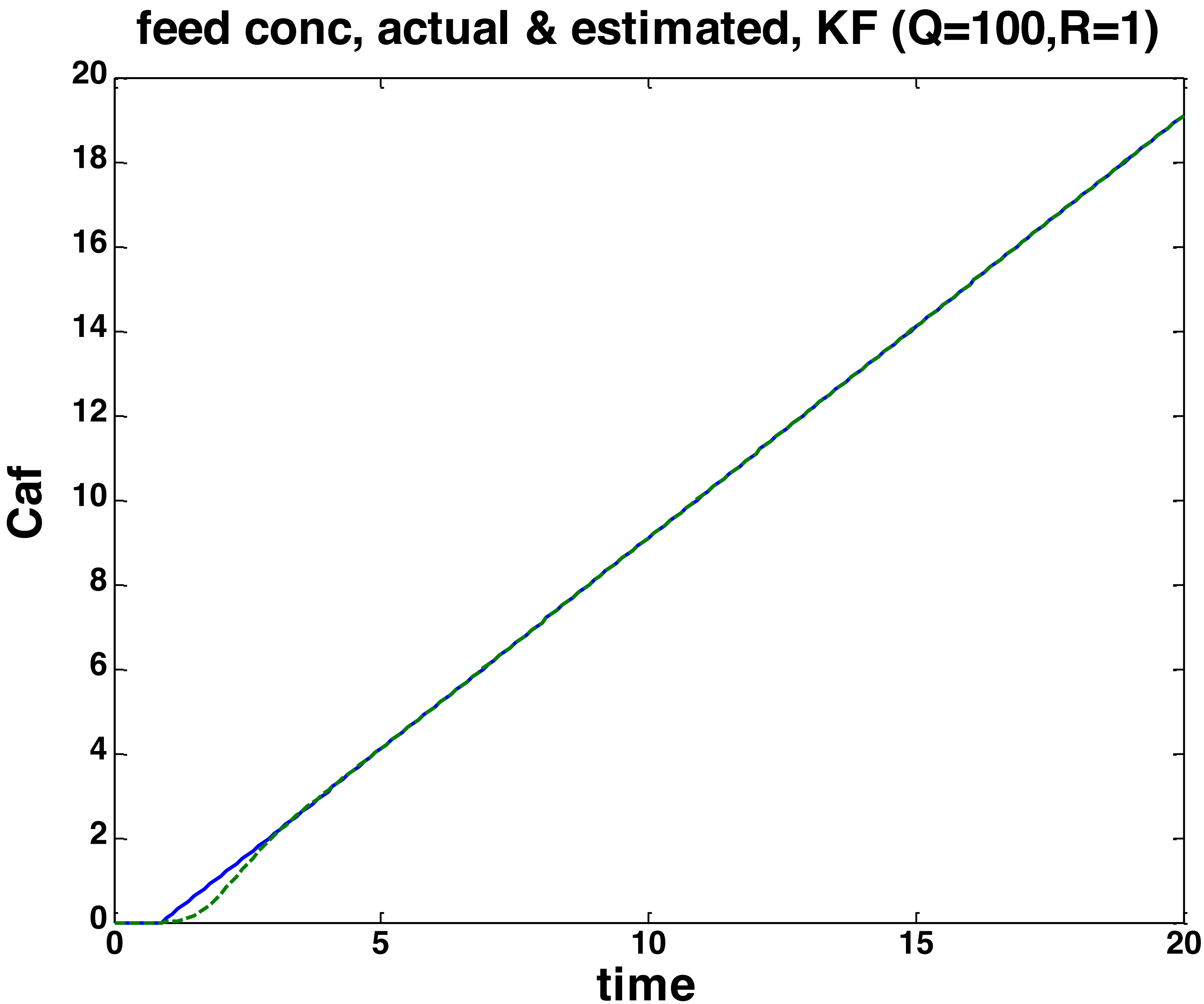
$$\underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x^a_{k+1}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & \Phi^w \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k + \underbrace{\begin{bmatrix} 0 \\ \Gamma^w \end{bmatrix}}_{\Gamma^{w,a}} w_k \qquad y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + v_k$$

# Ramp Disturbance

deviation variables



# Unmeasured Feed Concentration




deviation variables

# Periodic Disturbance

$$\begin{bmatrix}\dot{C}_A\\ \dot{C}_B\end{bmatrix}=\begin{bmatrix}-2.4048 & 0\\ 0.8333 & -2.2381\end{bmatrix}\begin{bmatrix}C_A\\ C_B\end{bmatrix}+\begin{bmatrix}7 & 0.5714\\ -1.117 & 0\end{bmatrix}\begin{bmatrix}F/V\\ C_{Af}\end{bmatrix}$$

$$\begin{bmatrix}\dot{C}_{Af}\\ \ddot{C}_{Af}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega^2 & 0\end{bmatrix}\begin{bmatrix}C_{Af}\\ \dot{C}_{Af}\end{bmatrix}+\begin{bmatrix}0\\ d\end{bmatrix}$$

Poles on imaginary axis

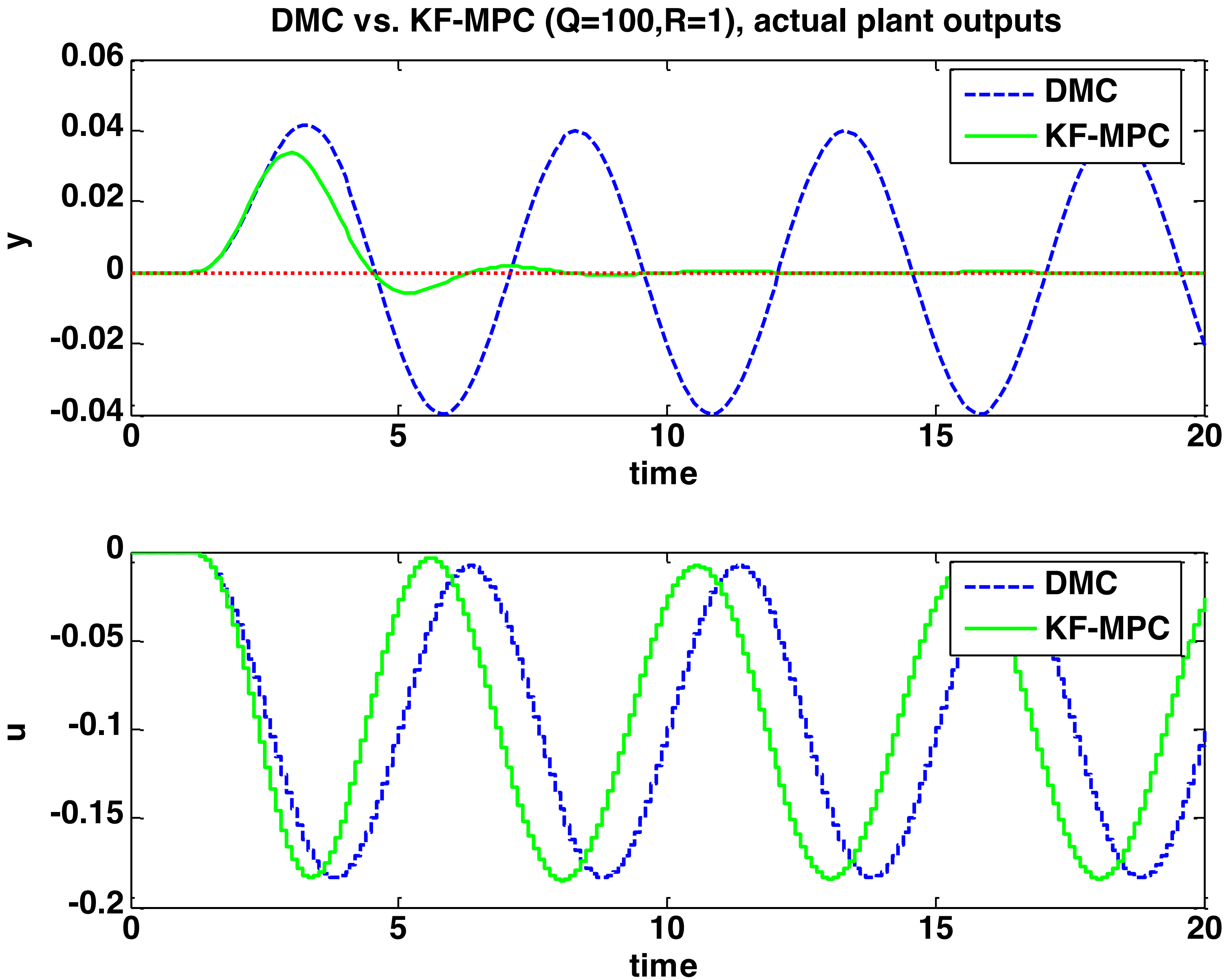
Results in sin variation in feed concentration

$$\begin{bmatrix}\dot{C}_A\\ \dot{C}_B\\ \dot{C}_{Af}\\ \ddot{C}_{Af}\end{bmatrix}=\begin{bmatrix}-2.4048 & 0 & 0.5714 & 0\\ 0.8333 & -2.2381 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\omega^2 & 0\end{bmatrix}\begin{bmatrix}C_A\\ C_B\\ C_{Af}\\ \dot{C}_{Af}\end{bmatrix}+\begin{bmatrix}7 & 0\\ -1.117 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}F/V\\ d\end{bmatrix}$$

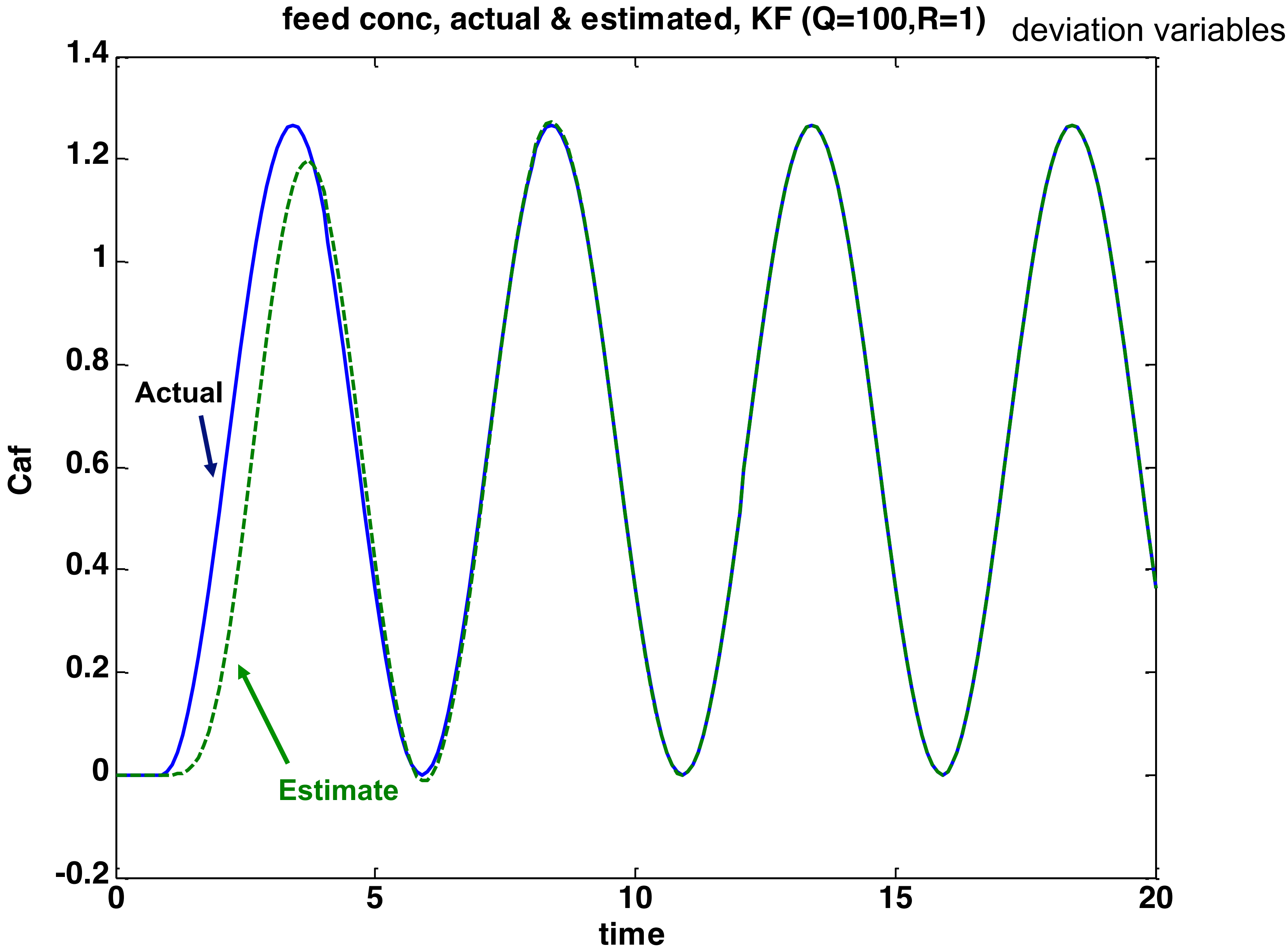
Appended state form

# DMC vs. KF-MPC

deviation variables



# Feed Concentration Estimate



# Numerous Potential Disturbances

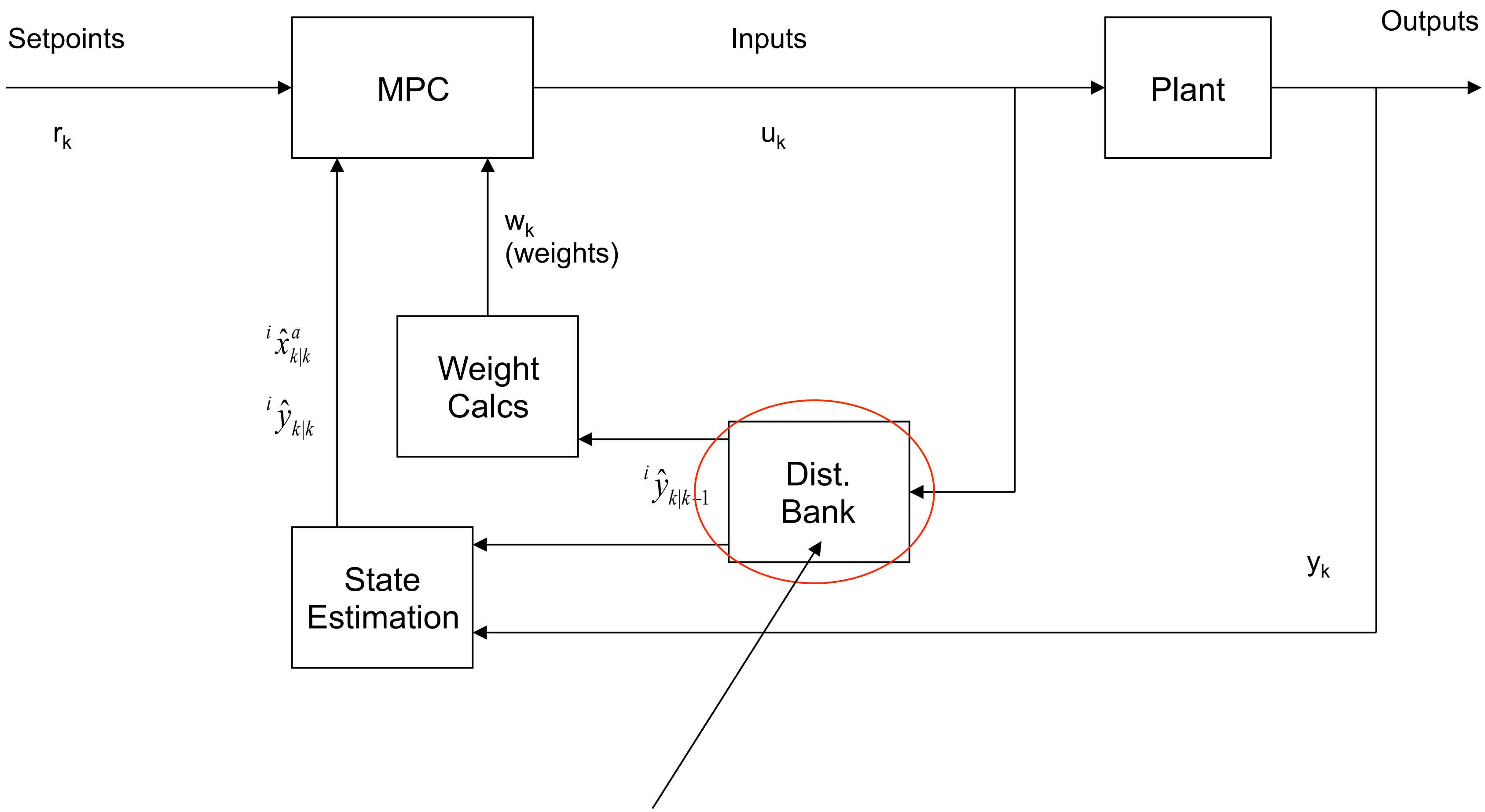
- Current Practice
  - Choose most important disturbance(s) to estimate & reject (number of disturbances = number of measurements)
- Model Bank
  - Each model associated with a different type of disturbance
  - Weighting/Blending or Switching between models

*Ind. Eng. Chem. Res.* **2010**, *49*, 7983–7989

**Multiple Model Predictive Control Strategy for Disturbance Rejection**[†,‡]

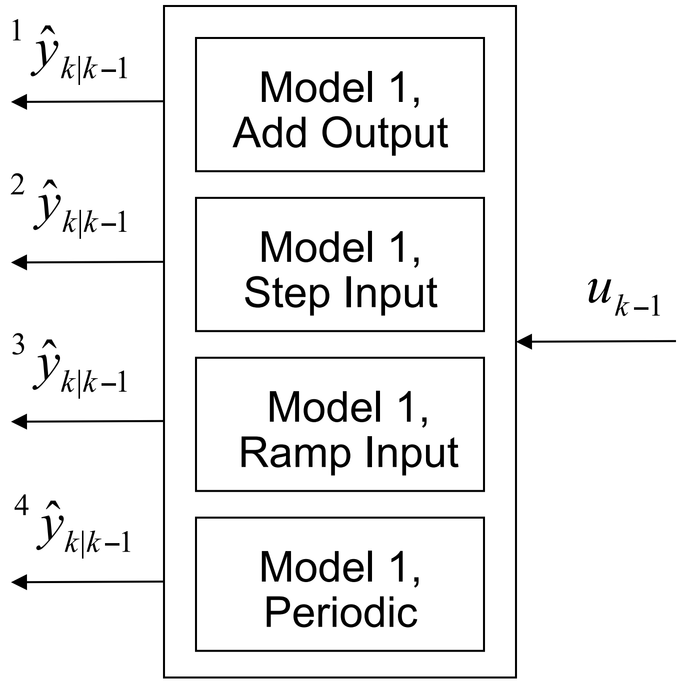
**Matthew Kuure-Kinsey**[§] **and B. Wayne Bequette***

# Disturbance Estimation - Framework




Primary difference is in the "disturbance bank"

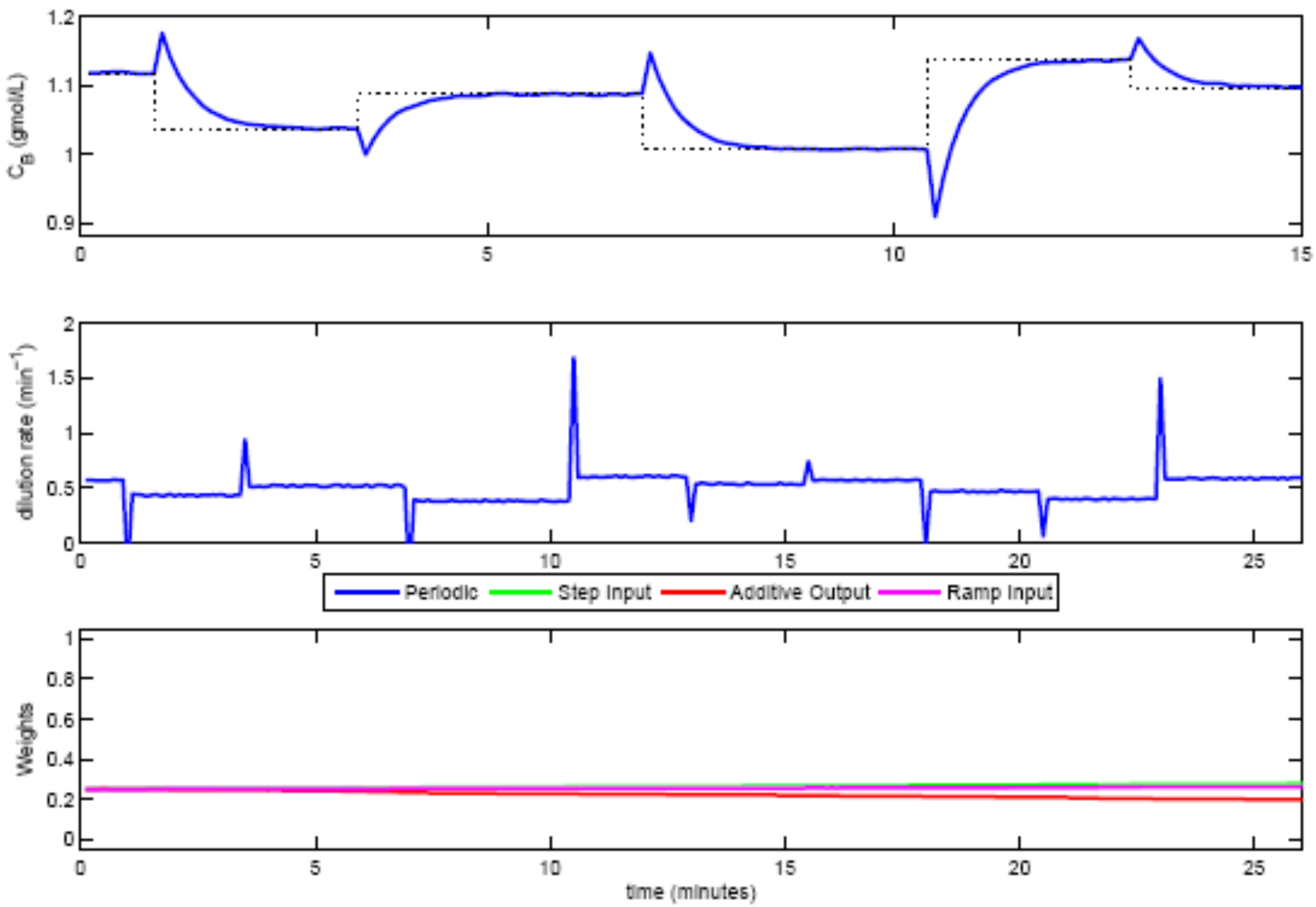
# Disturbance Bank Structure



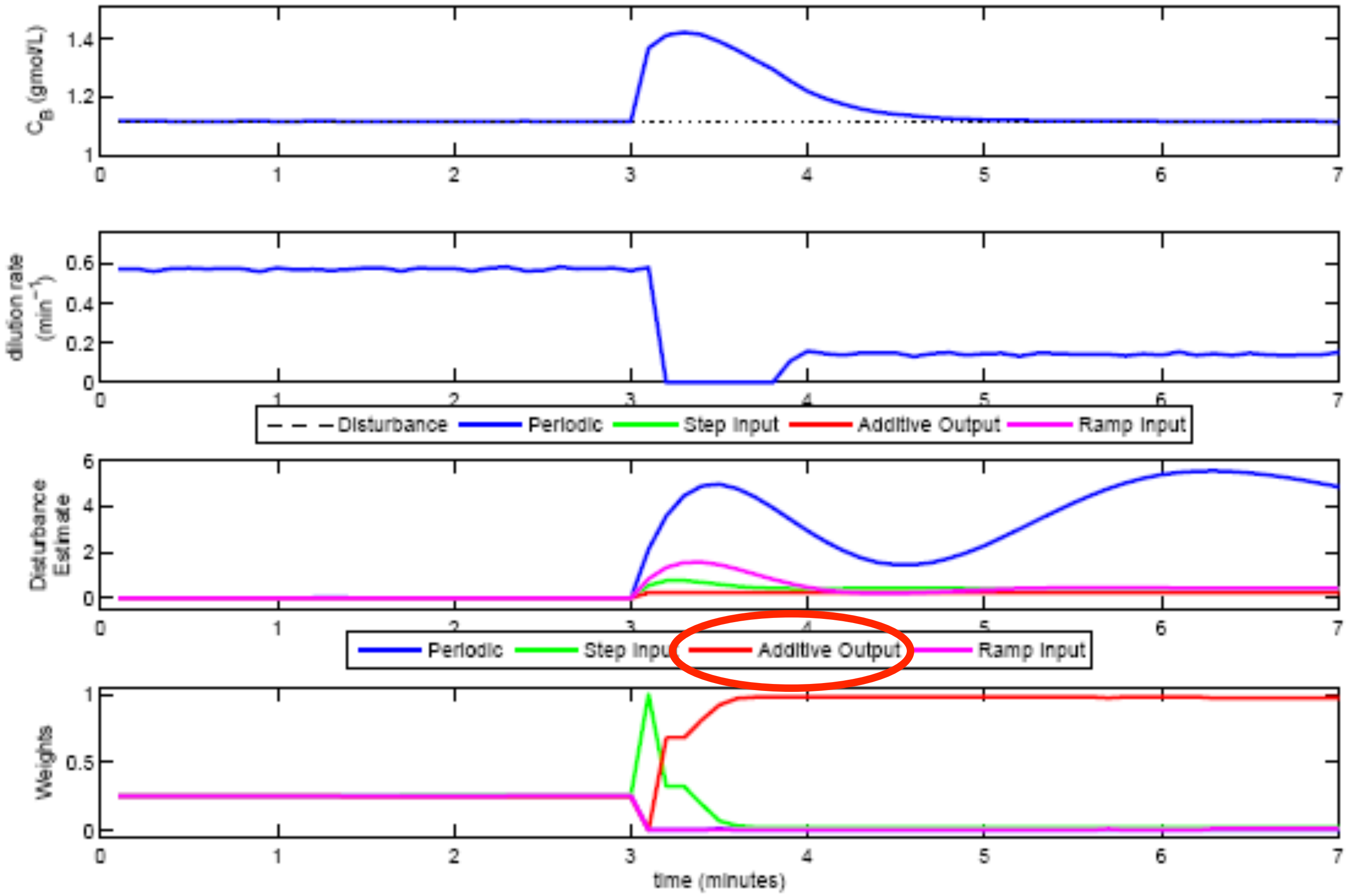

$$\underbrace{\begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix}}_{\hat{x}^a_{k+1|k}} = \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & \Phi^w \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} \hat{x}_{k-1|k-1} \\ \hat{d}_{k-1|k-1} \end{bmatrix}}_{\hat{x}^a_{k-1|k-1}} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_{k-1}$$

$$\hat{y}_{k|k-1} = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} \hat{x}_{k-1|k} \\ \hat{d}_{k-1|k} \end{bmatrix}}_{x^a_{k|k-1}}$$

# Setpoint Tracking (No Disturbances)



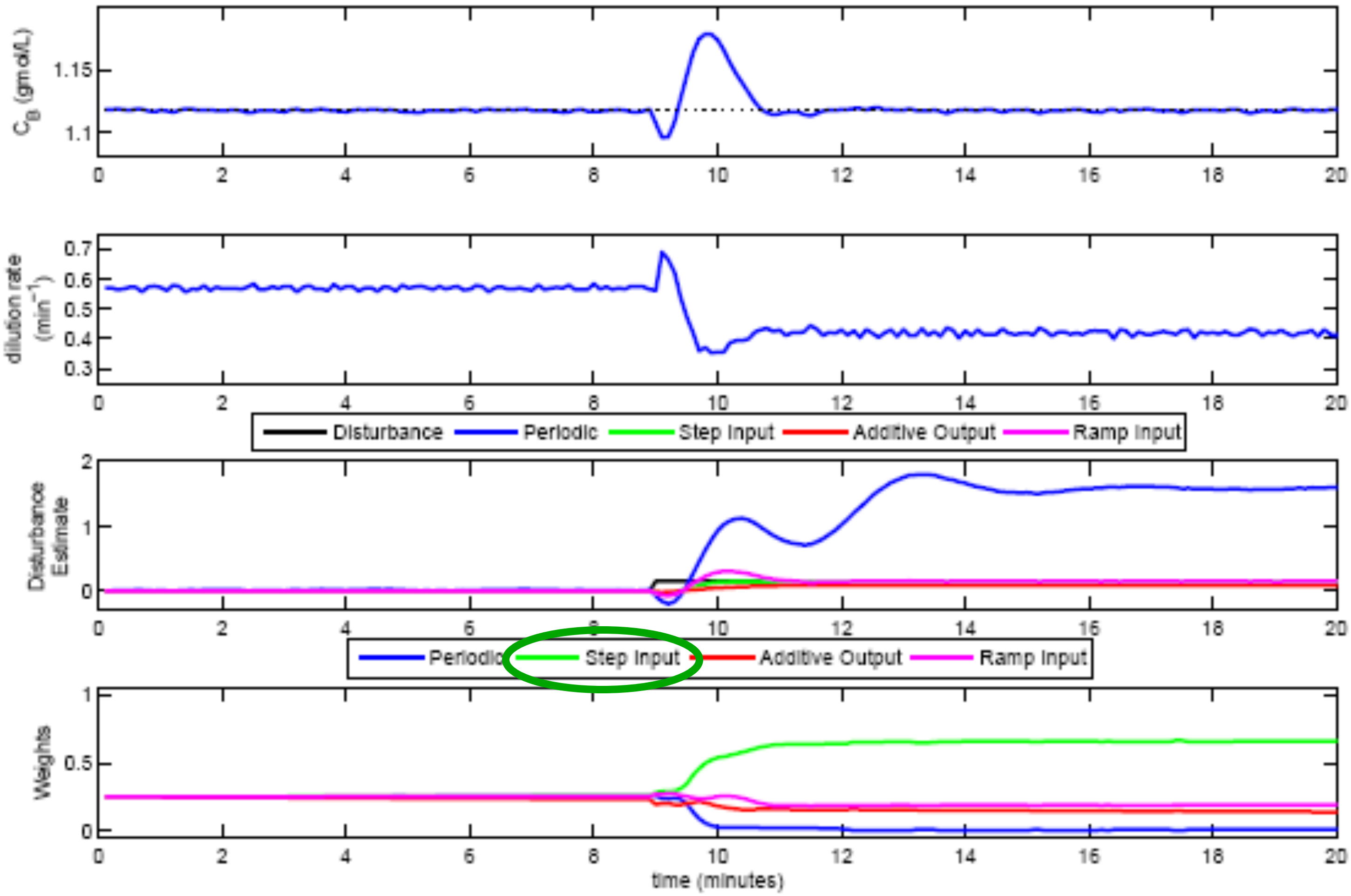
# Additive Output Disturbance




Measured output

Weights

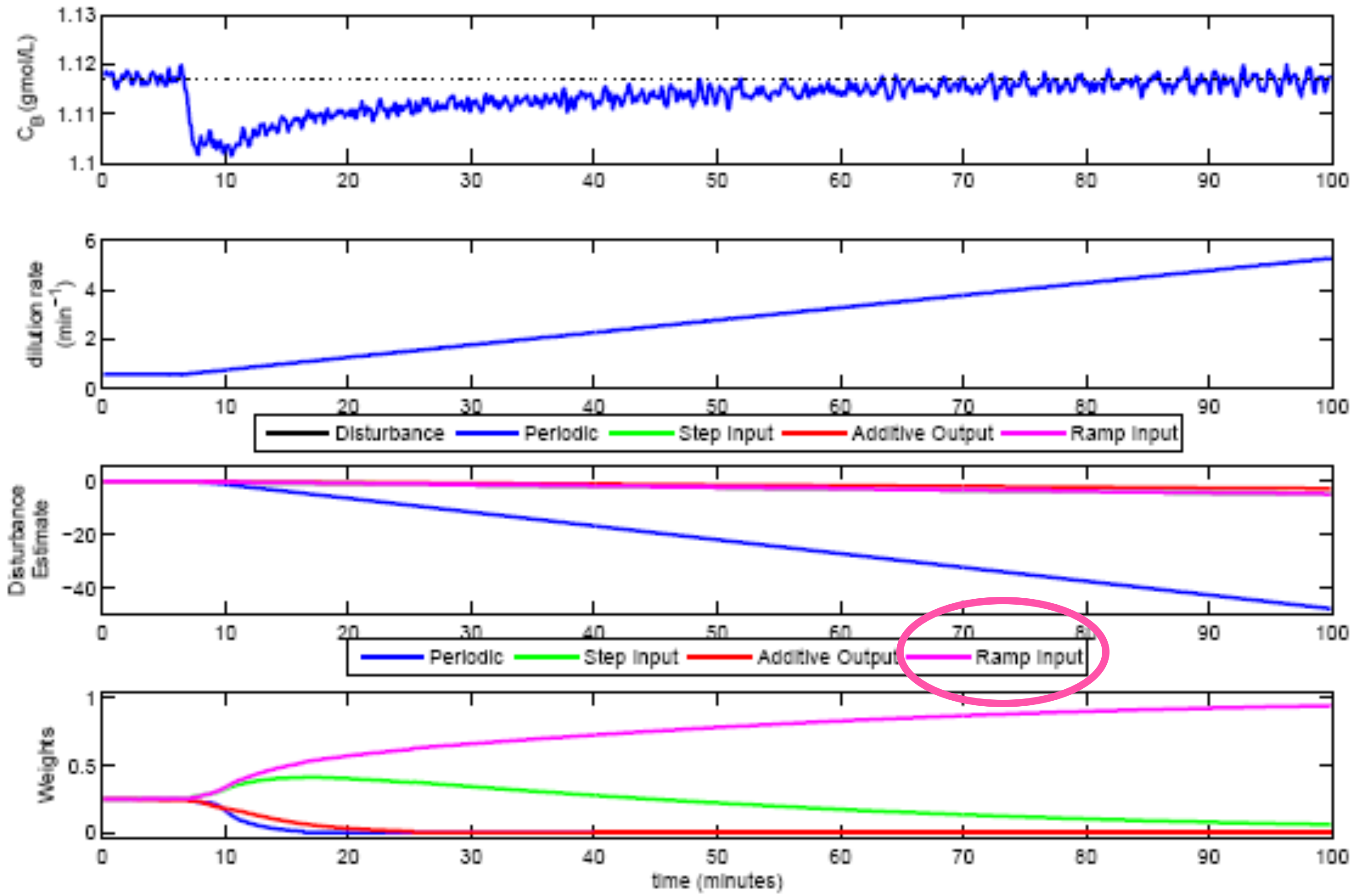
# Step Input Disturbance




Measured output

Weights

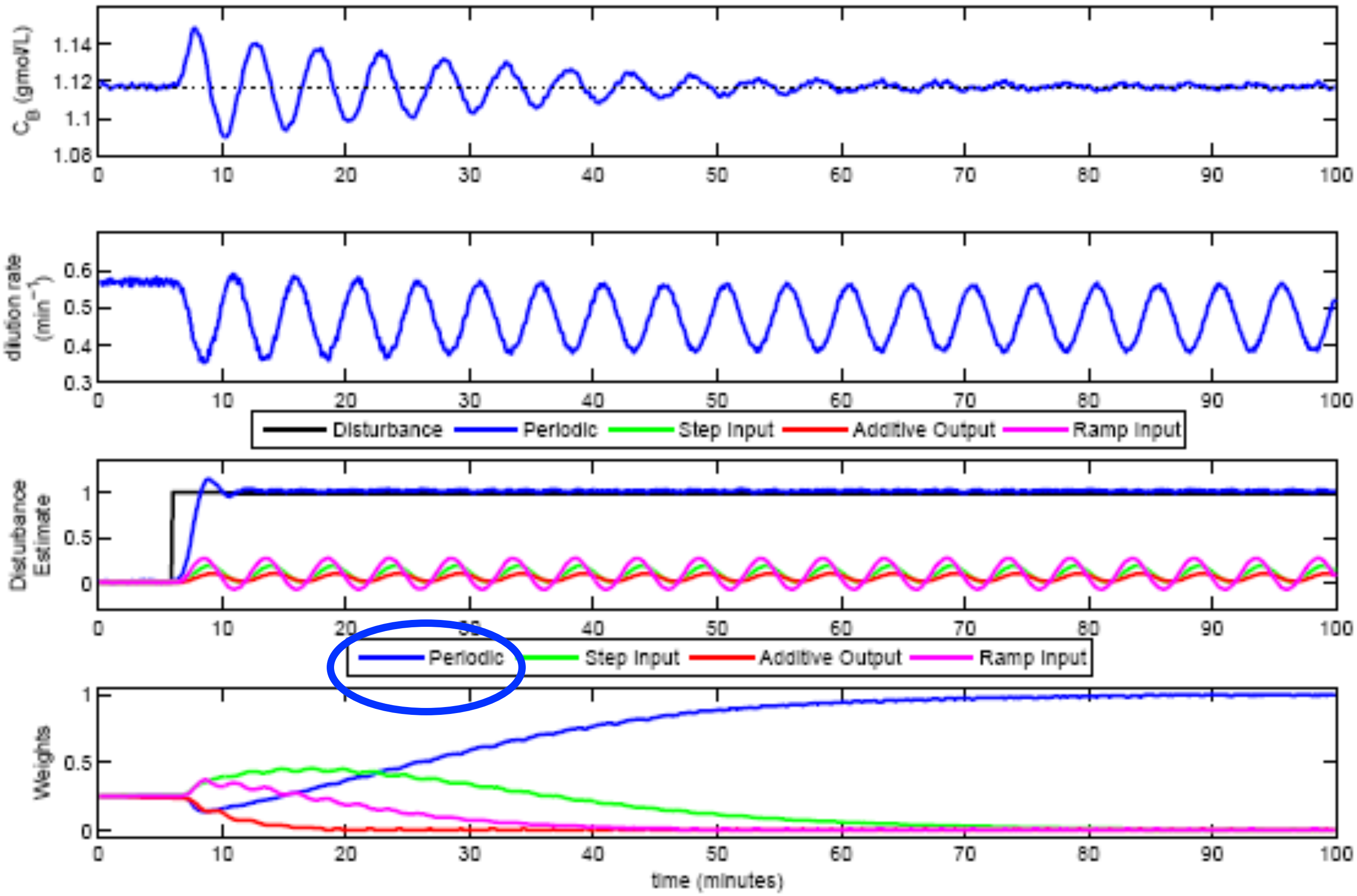
# Ramp Input Disturbance




Measured output

Weights

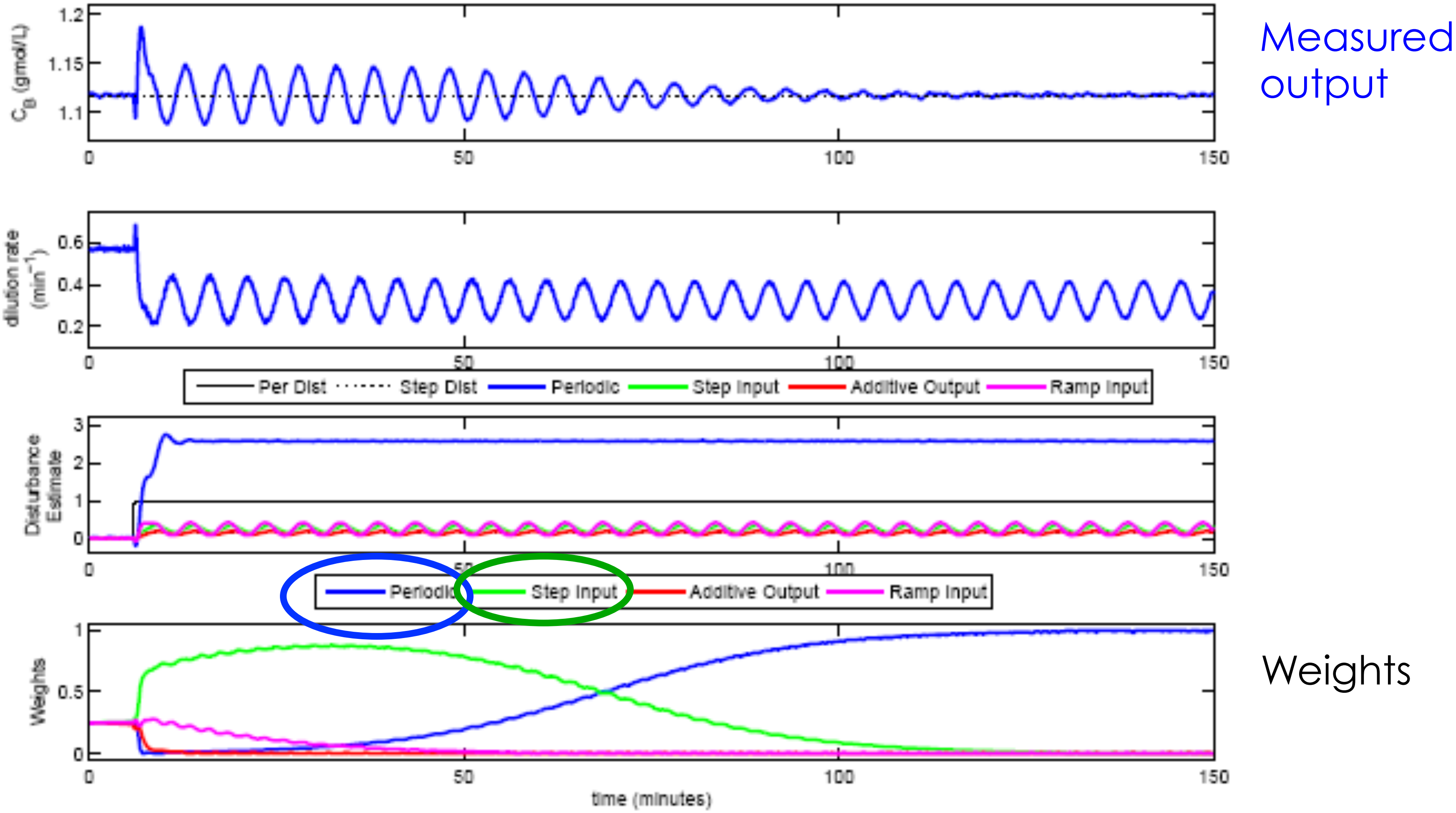
# Periodic Input Disturbance




Measured output

Weights

# Step + Periodic Input Disturbances



# Presentation Summary

- Nonlinear Model Predictive Control
  - Limitations to DMC formulation
  - State estimation-based approaches
  - Multiple linear model-based approaches
  - Formulation for different disturbances

# Acknowledgments

- Jing Sun (2011)
  - China

- Matthew Kuure-Kinsey (2008)
  - Cargill

- Brian Aufderheide (2002)
  - University of Trinidad and Tobago

- Ramesh Rao (2000)
  - ASPENTECH

- Clem Yu (1992)
  - Baxter

# Recent Applications of MMPC

- Integrated Gasification Combined Cycle (IGCC) Power Plants
  - Carbon Capture case
- ICU Blood Glucose Control

## 2012 AIChE Annual Meeting

Session 713: Optimization and Predictive Control II

12 AIChE Annual Meeting, Pittsburgh, PA

# Application of Linear Multiple MPC Framework towards Dynamic Maximization of Oxygen Yield in an Elevated-Pressure ASU

**Priyadarshi Mahapatra[1], Stephen E. Zitney[1] and B. Wayne Bequette[2]**

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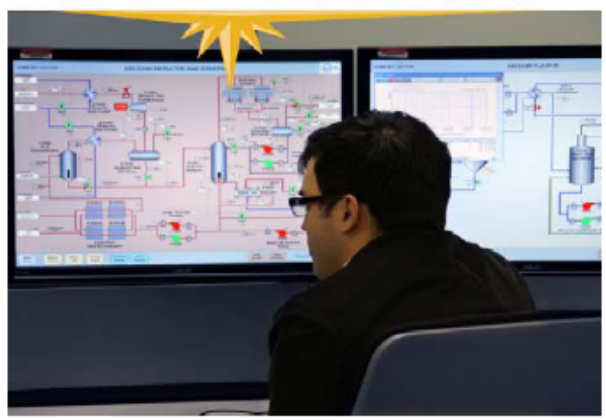
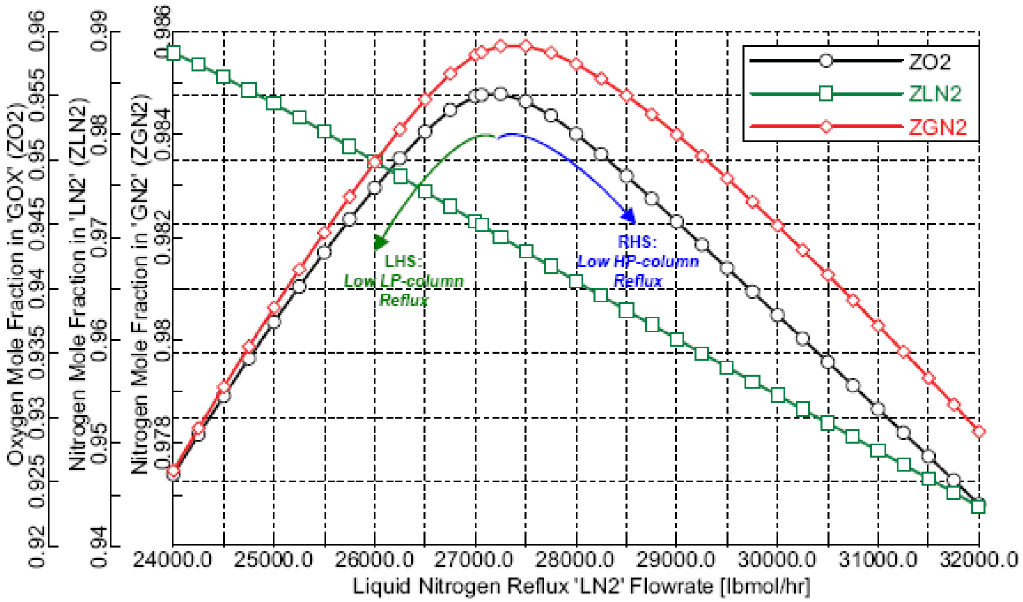


**Pittsburgh, PA**
**November 1, 2012**

# Air Separation Unit

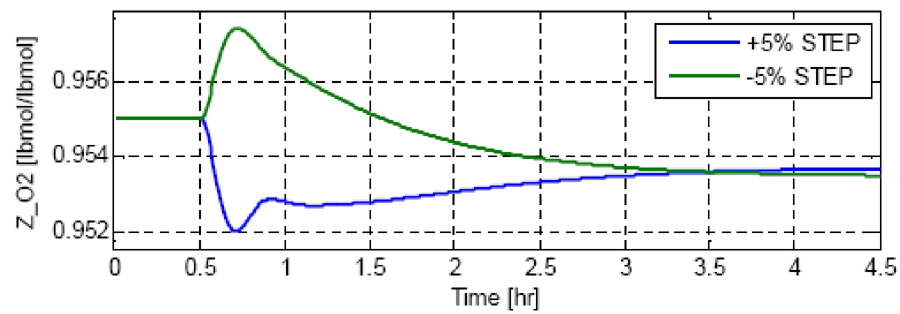
## *Problem Definition*

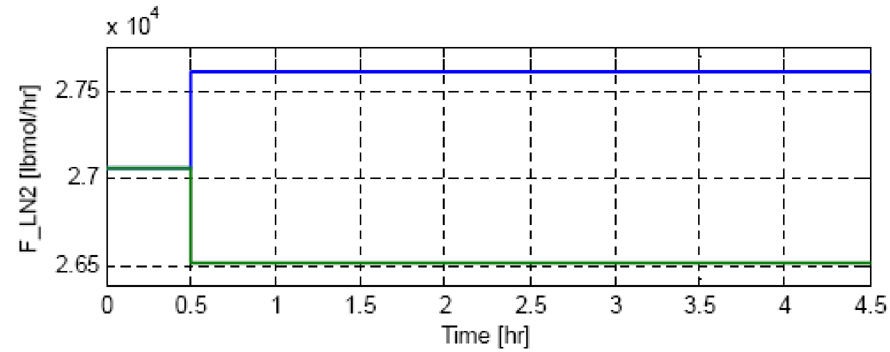



**F_LN2 Sensitivity (Steady State)**

***@ fixed F_O2 (14511.1 lbmol/hr) and F_airASU (69000 lbmol/hr)***

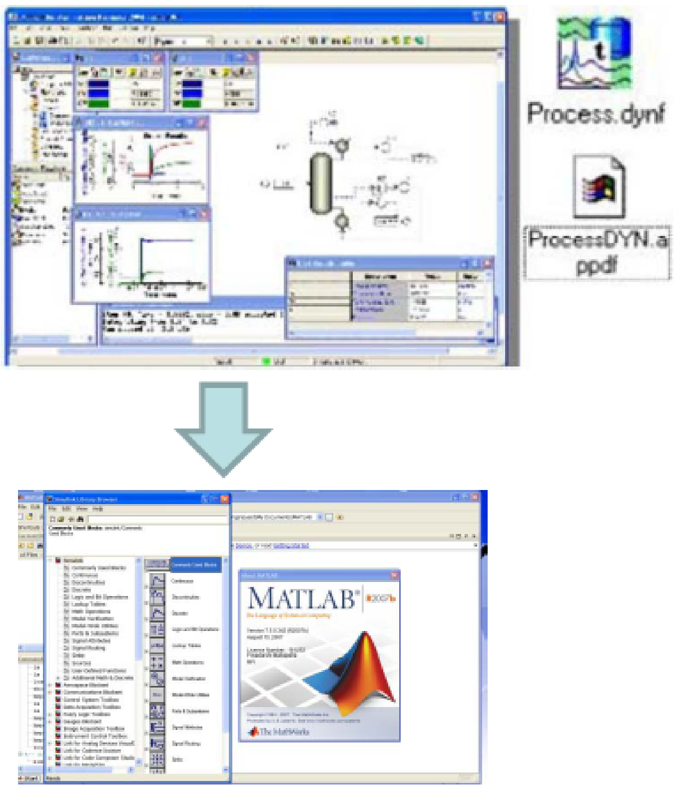
**Input Multiplicity Problem (Dynamic)**



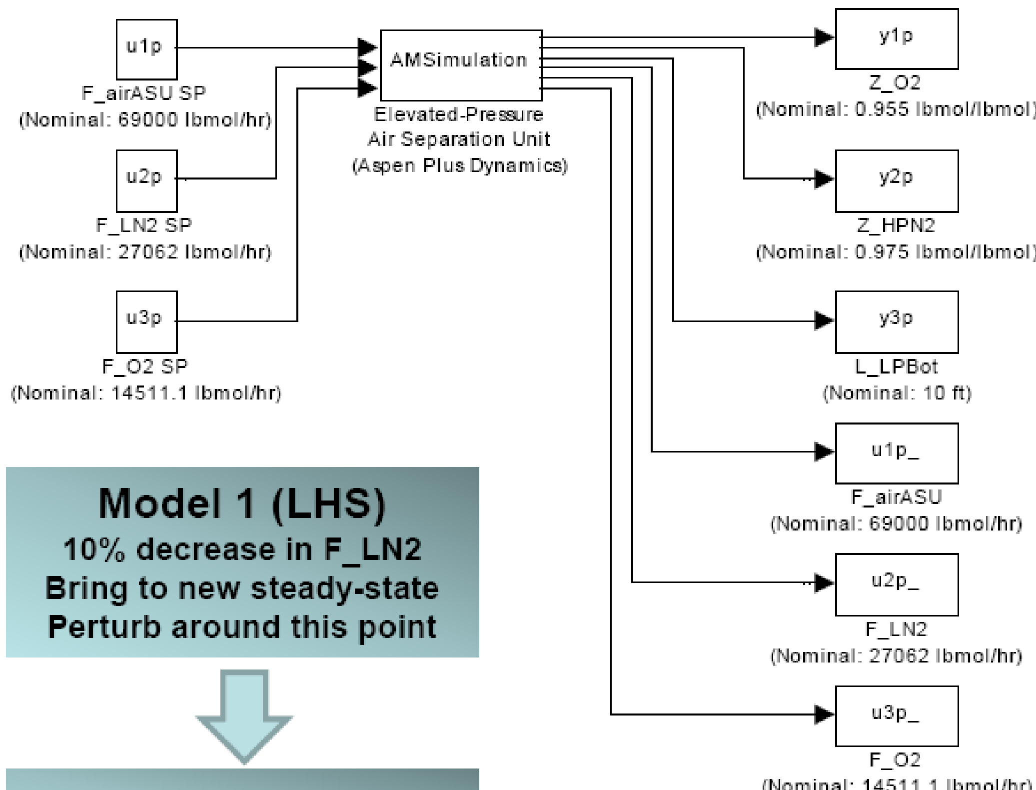



# MMPC for ASU $O_2$ maximization

## *Develop Model Bank / System Identification*




- ✓ **MIMO state-space models from 'plant' PRBS-like step-responses (parametric blackbox models)**
- × **State-space models from differential equation in Aspen using Control Design Interface (~500 states per model)**




**Model 1 (LHS)**
**10% decrease in F_LN2**
**Bring to new steady-state**
**Perturb around this point**

↓

**Model Bank**

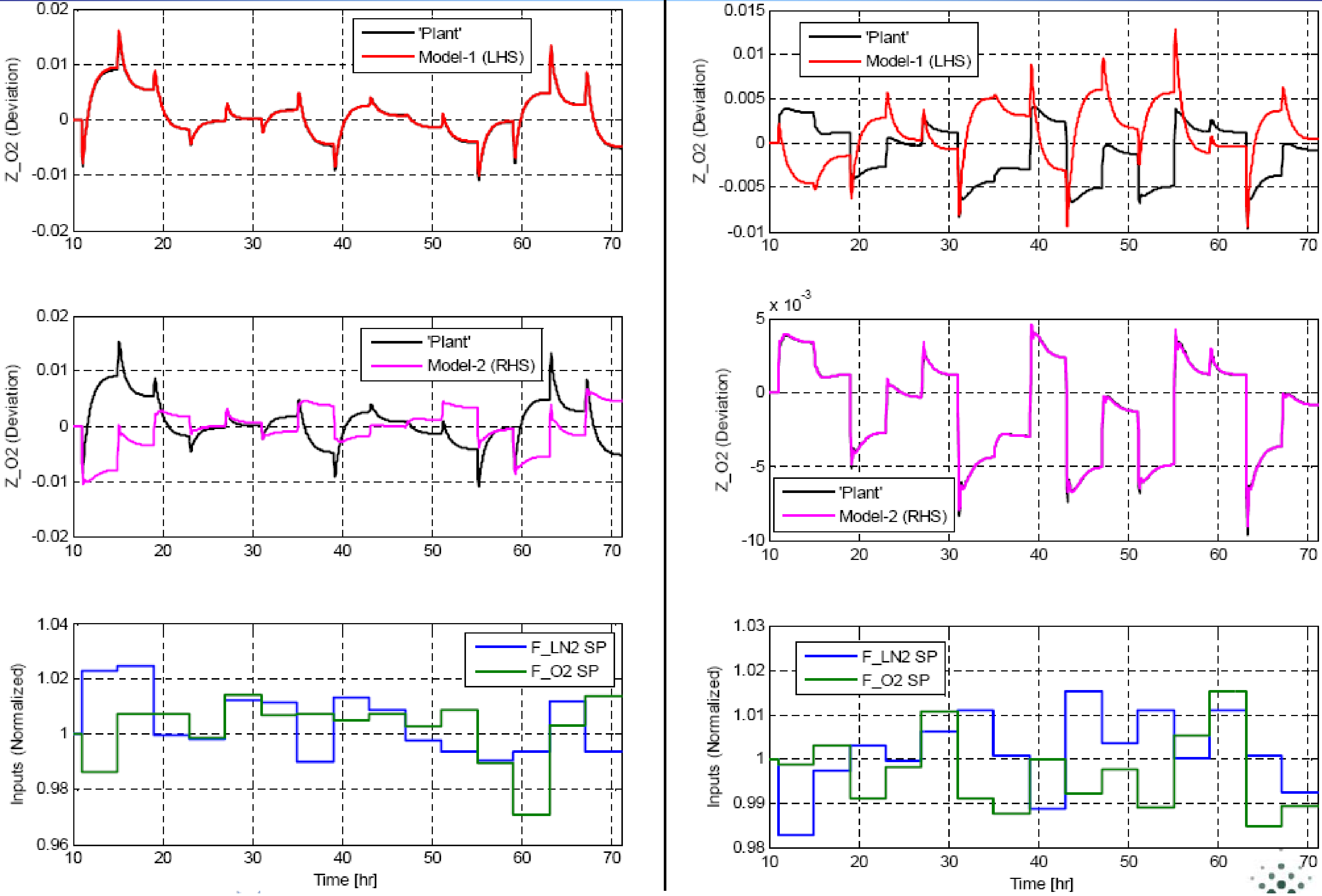
↑

**Model 2 (RHS)**
**10% increase in F_LN2**
**Bring to new steady-state**
**Perturb around this point**

$$\frac{\text{F\_airASU SP}}{\text{F\_O2 SP}} = 4.755$$

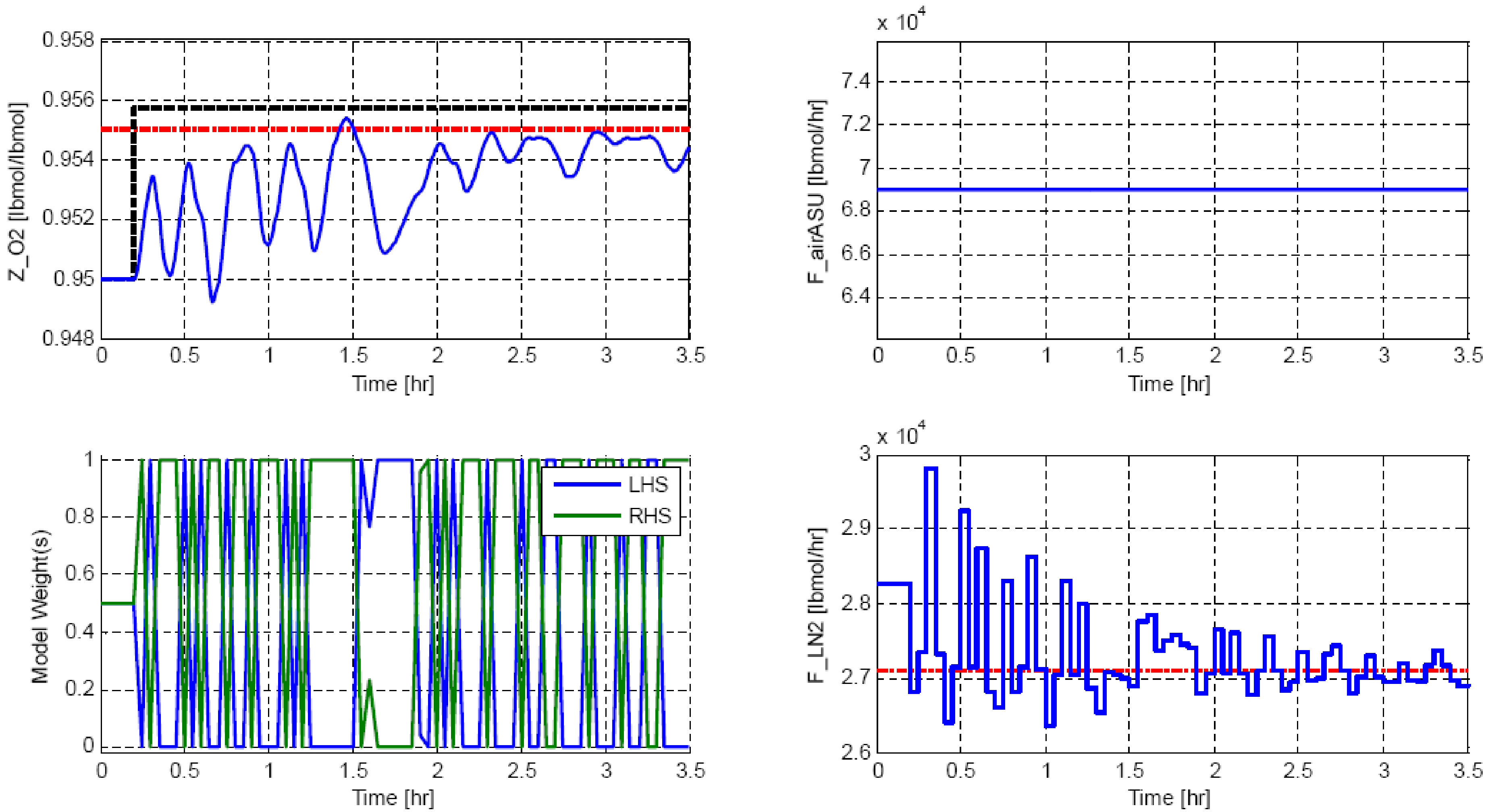
# MMPC for ASU $O_2$ maximization

## *System Identification for each model*



# MMPC for ASU $O_2$ maximization

## *Simulation Results*

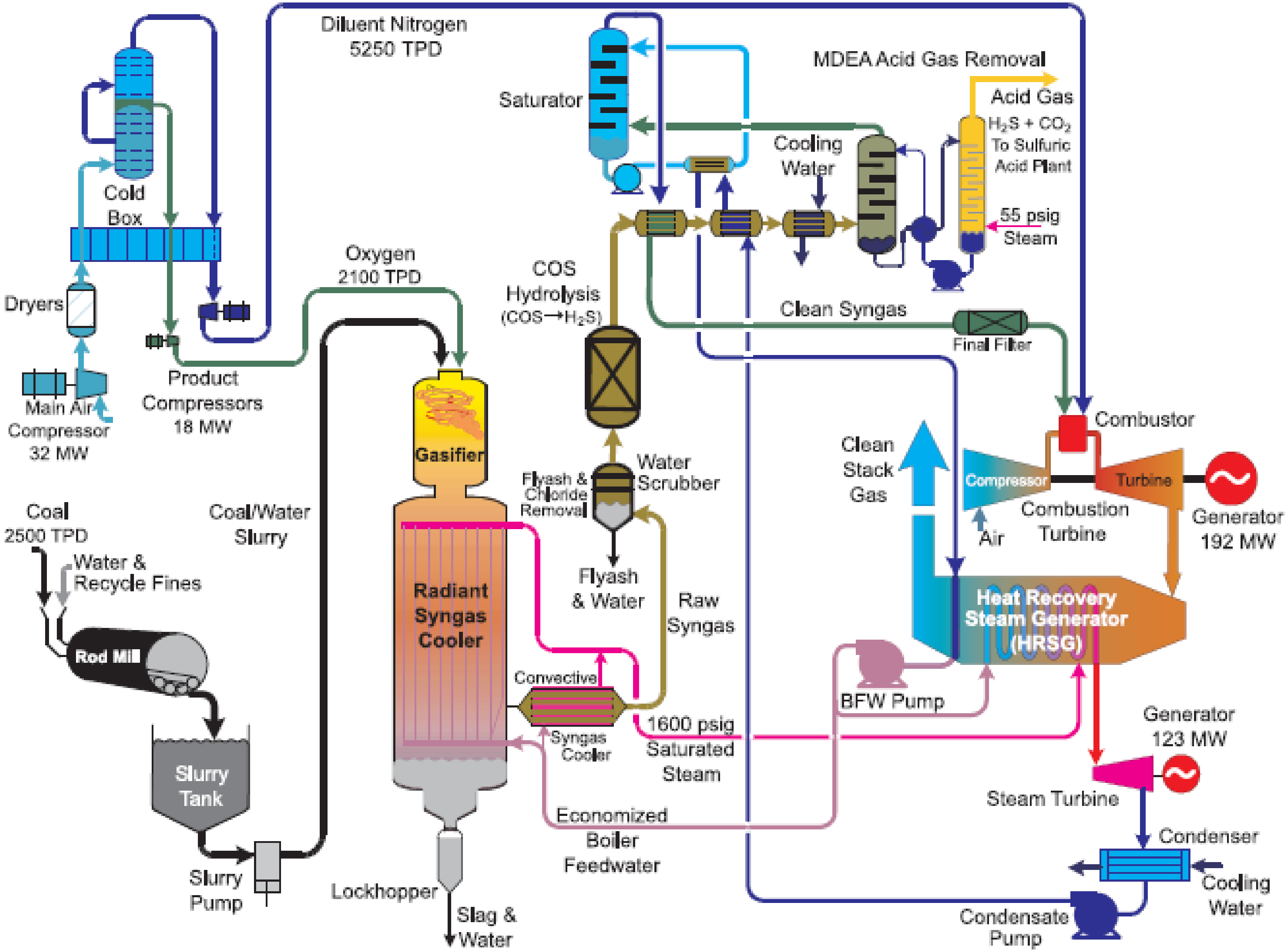



***MMPC parameters:*** $P = 75 \quad M = 20 \quad Q/R = 0.02 \quad \Delta t = 18\,\text{sec} \quad \delta = 0.01 \quad {}^{i}\Lambda = \left[10^{8}\right]$

# Summary

- MMPC algorithm could address the dynamic maximization problem
- Handles input multiplicity problem using linear controllers
- Significantly less computation time compared to first-principle based NMPC
- Prevents expensive air compression costs over a period of time

# IGCC


Diluent Nitrogen
5250 TPD
Saturator
MDEA Acid Gas Removal
Acid Gas
$H_2S + CO_2$
To Sulfuric
Acid Plant
Cooling
Water
55 psig
Steam
Cold
Box
Dryers
Main Air
Compressor
32 MW
Product
Compressors
18 MW
Oxygen
2100 TPD
COS
Hydrolysis
($COS \rightarrow H_2S$)
Clean Syngas
Final Filter
Combustor
Clean
Stack
Gas
Compressor
Turbine
Combustion
Turbine
Air
Generator
192 MW
Gasifier
Flyash &
Chloride
Removal
Water
Scrubber
Flyash
& Water
Raw
Syngas
Coal
2500 TPD
Water &
Recycle Fines
Coal/Water
Slurry
Rod Mill
Radiant
Syngas
Cooler
Heat Recovery
Steam Generator
(HRSG)
BFW Pump
Convective
Syngas
Cooler
1600 psig
Saturated
Steam
Generator
123 MW
Slurry
Tank
Steam Turbine
Economized
Boiler
Feedwater
Condenser
Cooling
Water
Slurry
Pump
Lockhopper
Slag &
Water
Condensate
Pump

# IGCC Summary

- IGCC Power Plant is a complex system to simulate
  - Significant reworking of steady-state flowsheet required adding plant details and dynamic behavior
  - 300 Units & 450 Streams
  - 80,000 equations solved – dynamic simulations
- Linear MPC is a powerful tool to control complex plants
  - Hierarchical Control Structure Design
  - Load Following / Disturbance Rejection
  - Centralized Approach is better suited for following load-demand
  - Negative effect on controllability and robustness w/ increasing level of interactions
- Limitations of this research
  - Simplifying assumptions on some units
  - Proprietary equipment details/ lack of validation data
  - Workaround for software bugs/limitations

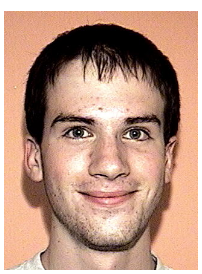
**Priyadarshi Mahapatra**
**Ph.D., 2010; NETL**

# Fuel Cell Systems

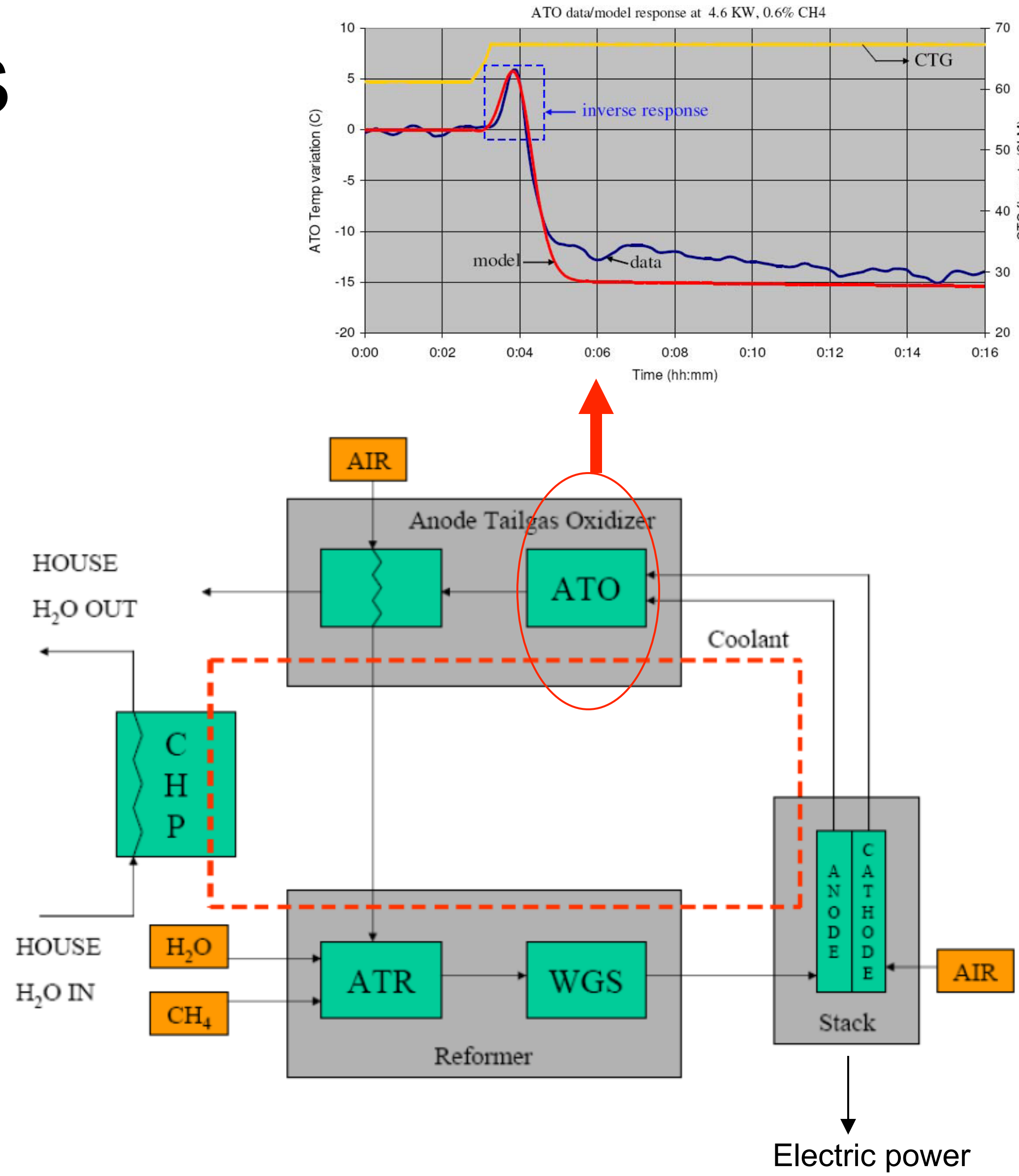
- Combined heat & power
- Nonlinear dynamics & control
- Stack condition monitoring
- Real-time optimization
- Nature-inspired membrane and catalyst design (Marc-Olivier Coppens)


Matthew Kuure-Kinsey (Dec ‘08)
Judy O’Rourke (Dec 2008)

Jeff Marquis (w/Coppens)



Matt Titus



# Closed-loop Artificial Pancreas

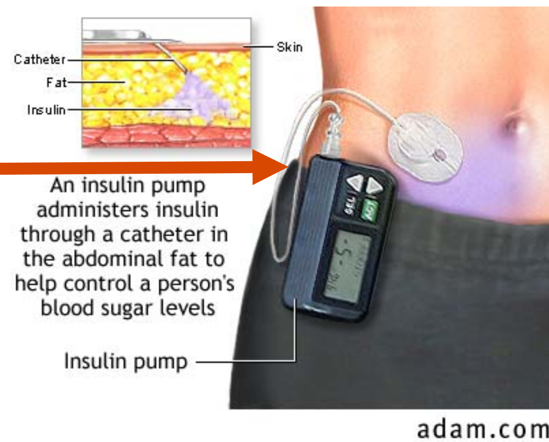
Meal knowledge: **Feedforward**

**u** Insulin infusion rate

Glucose setpoint

**r**


Controller

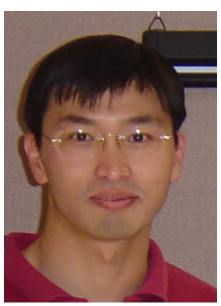



pump subject

Sensor

**y**

**Feedback** Glucose sensor signal



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Now at Shell
Development

Jing
Sun

Ruben Rojas
Fulbright Scholar
Venezuela

Fraser
Cameron