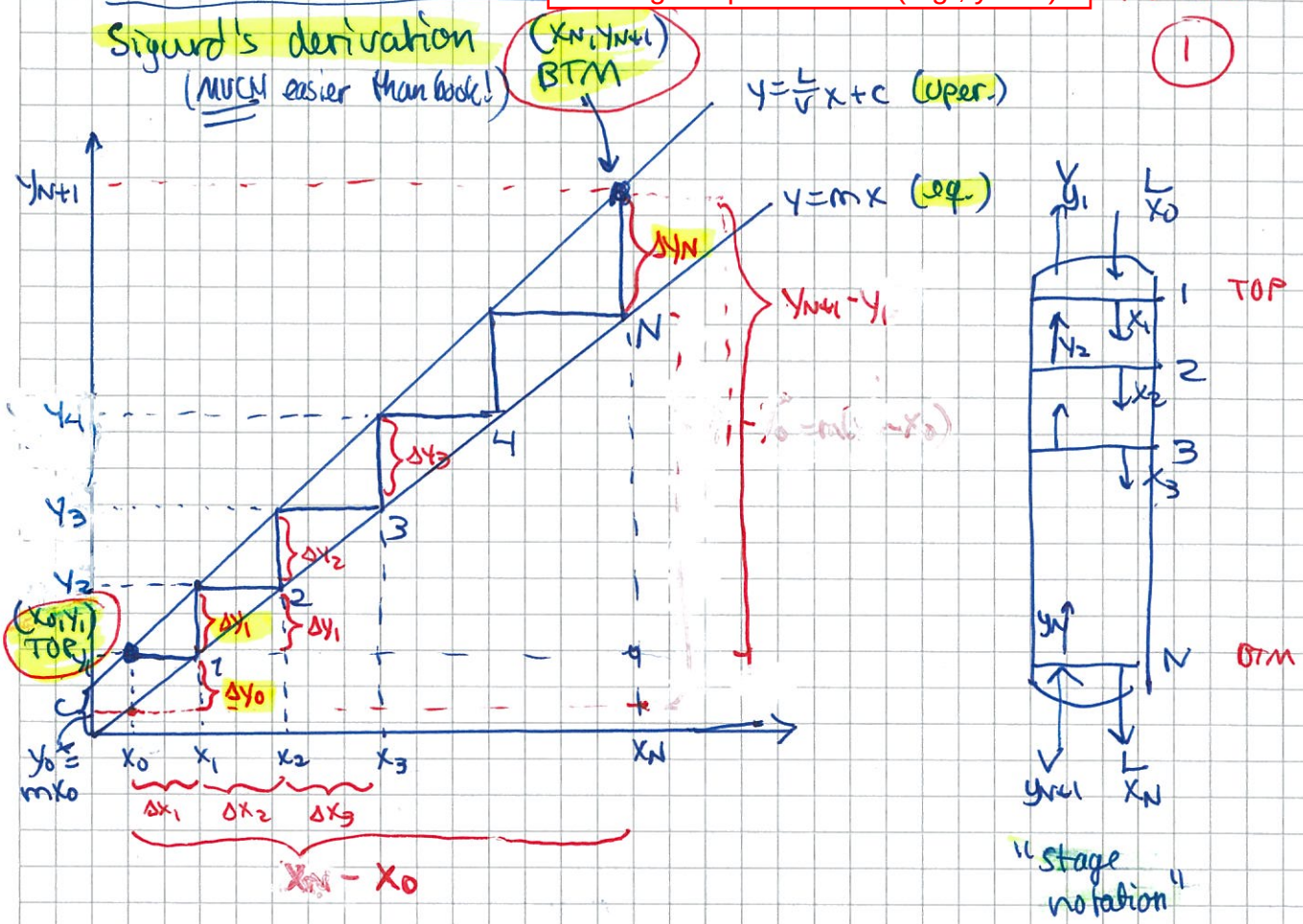
Kremser equation

Assumptions (hold for dilute mixtures):
1. Straight operating line (L/V constant)
2. Straight eqilibrium line (e.g., y=mx)

11/9-2013

(1)

Sigurd's derivation (MUCH easier than book!)



From figure

$$\frac{\Delta y_1}{\Delta x_1} = \frac{L}{V} \qquad \frac{\Delta y_0}{\Delta x_1} = m$$

$$\frac{\Delta y_1}{\Delta y_0} = \frac{L/V}{m}$$

Similar

$$\frac{\Delta y_2}{\Delta y_1} = \frac{L/V}{m} \qquad \frac{\Delta y_3}{\Delta y_2} = \frac{L/V}{m}$$

$$\frac{\Delta y_3}{\Delta y_0} = \left(\frac{L/V}{m}\right)^3$$

General

$$\boxed{\frac{\Delta y_N}{\Delta y_0} = \left(\frac{L/V}{m}\right)^N}$$

VERY EASY TO DERIVE!

$$\Delta y_N = y_{N+1} - y_N = y_{N+1} - m x_N$$

$$\Delta y_0 = y_1 - y_0^* = y_1 - m x_0$$

$$\frac{y_{N+1} - m x_N}{y_1 - m x_0} = \left(\frac{L/V}{m}\right)^N$$

S_{obs}

$$\Rightarrow \boxed{N = \frac{\ln \frac{y_{N+1} - m x_N}{y_1 - m x_0}}{\ln \left(\frac{L/V}{m}\right) A}} \quad (1)$$

Also note that

$$A = \frac{L/V}{m} = \frac{y_{N+1} - y_1}{m(x_N - x_0)}$$ (from figure) (2)

Can then write

$$N = \frac{\ln \dfrac{y_{N+1} - y_N^*}{y_1 - y_0^*}}{\ln \dfrac{y_{N+1} - y_1}{y_N^* - y_0^*}} \quad (*)$$

where (2)

$$y_N^* = m x_N$$
$$y_0^* = m x_0$$

Some comments.
- Same equations apply for absorption and stripping
- Can also use for exact calculations of "corners" in distillation, where equilibrium and operating lines are straight, and then McCabe-Thiele is used in middle section.

Example 10.3-3

$y_1 = 0.00101$

$y_{N+1} = 0.01$

$x_N = 0.003 \Rightarrow y_N^* = 2.53 \cdot 0.003 = 0.00759$

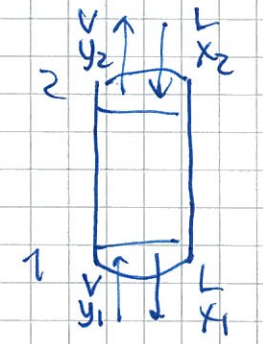
$x_0 = 0 \Rightarrow y_0^* = 0$

$$N = \frac{\ln \dfrac{0.01 - 0.00759}{0.00101 - 0}}{\ln \dfrac{0.01 - 0.00101}{0.00759 - 0}} = \frac{\ln \dfrac{2.386}{1.514}}{\ln 1.184} = 5.15$$

Cook gives 5.04 but this is because they use another value of L/V - note that L/V varies slightly.

Example 10.6-3 (end 1 - end 2 notation)

$$N = \frac{\ln \dfrac{y_1 - y_1^*}{y_2 - y_2^*}}{\ln \dfrac{y_1 - y_2}{y_1^* - y_2^*}} \quad (*)'$$



"end 1 - end 2 notation"

$y_1 = 0.022$

$x_2 = 0 \Rightarrow y_2^* = 0$

$y_2 = 0.002244$

$x_1 = 0.0218 \Rightarrow y_1^* = 0.01482$

$(m = 0.68)$

$$N = \frac{\ln \dfrac{0.022 - 0.01482}{0.002244 - 0}}{\ln \dfrac{0.022 - 0.002244}{0.01482 - 0.002244}} = \frac{\ln 3.196}{\ln 1.333} = 4.05$$

(same as book)