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MODELING, CONTROL AND SIMULATION OF RECIRCULATING AQUACULTURE SYSTEMS

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Abstract: Recirculating aquaculture systems (RASs) in land based fish tanks, where the fish tank effluent is biologically treated and then recirculated back to the fish tanks, offers a possibility for an ecologically sustainable fish production. To explore the advantage of a RAS to its maximum the recirculation ratio should be as high as possible. This implies strong demands on the water treatment, i.e. the maintenance of an efficient nitrification, denitrification, organic removal and phosphorus removal. The complexity of recirculating aquaculture systems, however, implies that dynamic simulations are required for analysis and optimization of a plant with respect to configuration, effluent water quality, production, and robustness. Here we derive new dynamic models of fish growth, gastric evacuation, and nitrogen excretion compatible with the state of art modeling of waste water treatment processes. For the purpose of simulating RASs, dynamic models of moving beds used in the water treatment are also derived. *Copyright © IFAC 2004.*

Keywords: Aquaculture, biofilm, control, model, moving bed, wastewater

1. INTRODUCTION

The global harvest of wild fish has stagnated around 90 million tons a year and is not expected to rise. As a result of a steadily increasing demand for fish, aquaculture is therefore a tremendously rapid growing industry. Recirculating aquaculture systems (RAS) in land based fish tanks, where the fish tank effluent is biologically treated and the water is recycled back to the rearing tanks, eliminate most of the problems associated with traditional farming in open systems.

To fully explore the advantage of RASs to its maximum and make the systems commercially successful, the recirculation ratio should be as high as possible. To optimize by carrying out full scale experiments alone based on *ad hoc* assumptions is simply too time-consuming. For example, the growth, feed intake, respiration etc. of the fish change with age (months/years) and the nitrification bacteria in the water treatment may take months to establish stable populations after a change in conditions.

The need for dynamic modeling for deeper insight of the aquaculture performance has been identified, and during the last few years there has been a development towards the use of models for analysis and simulation of aquaculture. Most of them have their origin in ecological modeling and applies to fish ponds without wastewater treatment processes (Jamu and Piedrahita, 2002; Jimenez-Montealegre *et al.*, 2002; Li and Yakupitiyage, 2003). Because of an aquaculture standpoint the few studies on RASs, which consider wastewater treatment, use basic steady-state models of the treatment processes, where the efficiency is set to either a fixed percentage removal or a fixed removal rate (Losordo and Hobbs, 2000; Ernst *et al.*, 2000). However, since the system is dynamic with characteristic times in the same range as the transients, the dynamics of the biology in the treatment processes as well as a more diversified waste description has to

be included for realistic simulations. The complexity of RASs, due to their feedback and multivariable character, implies that nontrivial dynamic models of all important system components (the fish, feed, bacteria, rearing basins, treatment units etc.) are required. Here, we develop new dynamic models of feed requirement, fish growth, gastric evacuation and nitrogen excretion that are adapted to the state of art wastewater treatment modeling.

2. PROCESS DESCRIPTIONS

The closed aquaculture system used in the development of a simulator is illustrated in Fig. 1. The treatment comprises basically three biological steps. Most of the nitrogen waste produced by the fish is in the form of ammonium. The ammonium is oxidized into nitrite and nitrate by nitrifying autotrophic bacteria (N). In the denitrification step (D) heterotrophic bacteria degrade organic matter from faeces and feed spill using the nitrate and nitrite as electron acceptor in an anoxic environment. The remaining degradable organic matter is degraded in an aerobic environment (B), where the heterotrophic bacteria use oxygen as an electron acceptor instead.

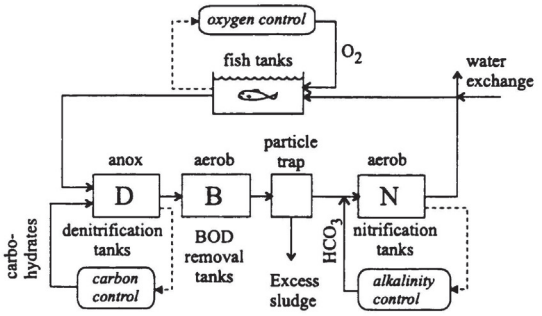

Fig. 1. A schematic picture of a RAS with control.

2.1 Rearing Tanks

Generally, fish of different age and size have to be separated due to intra species competition. The fish is therefore graded by size with regular intervals. Most fish move one fish tank 'up-size' every grading. Hence, the number of tanks is typically equal to the number of gradings within a production cycle (average time from fingerling to slaughter). Every grading the first tank is restocked with new fingerlings.

2.2 Moving Beds

Moving beds has shown to have some of the advantages of the two traditional techniques for biological treatment, i.e. activated sludge and fixed media biofilm reactors. In the treatment tanks are suspended carriers entrapped, for example small plastic tubes with fins and a cross inside such as Kaldnaes and ANOX (Ødegaard *et al.*, 2000), on which biofilm can grow. The suspension of the biofilm carriers prevents clogging and the fact that the bacteria are attached to the carriers implies that there is no need for sludge recycle. The mixing can be assumed complete as a result of either aeration or stirring (in the anoxic tanks).

2.3 Particle Trap

At least one particle trap to remove feed residues, particulate faeces and excess sludge is needed in the system. The exact location of this process in the loop (see Fig. 1) can be discussed but it should be placed such that the amount of heterotrophic sludge in the nitrifying reactors is small, since the nitrifying efficiency will otherwise decrease. The default trap referred to here is a sand-filter with a presumed removal efficiency of 90%.

3. THE MODELS

All models in the simulator are based on dynamic mass balances. The notation and units follow the standard in wastewater treatment (Grau *et al.*, 1982), with S used for concentrations of soluble substances and X for particulate matter, and the units gN/m^3 for nitrogen compounds, gCOD/m^3 for organic matter, gP/m^3 for phosphorus, mole HCO_3^-/m^3 for alkalinity and rates expressed per day (d^{-1}). The compounds modeled are the ones used in the activated sludge model no. 1 (ASM1) (Henze *et al.*, 2000) extended with total phosphorus, CO_2 and NO_2^- (see Table 1).

3.1 Fish Tanks

The fundament of the fish tank modeling is that the contents of carbon (COD), nitrogen (N) and phosphorus (P) in the feed, fish, waste and respired air should add up in a mass balance, i.e., the total produced waste at time t of compound i in Table 1 is

$$w_i = w_{Loss,i} + w_{F,i} + w_{g,i} + w_{r,i}, \qquad (1)$$

where $w_{Loss,i}$ is the waste due to feed loss, $w_{F,i}$ is the waste caused by feed digestion, $w_{g,i}$ is the (negative) waste due to accumulation in the fish, and $w_{r,i}$ is the effect of the respiration. The contents of a salmon and a typical feed are listed in Table 2. Knowing the composition of the feed and the faeces, the fish growth rate, the feeding rate and the oxygen and carbon dioxide respiration rate we can then determine w_i with (1). The fish growth is temperature dependent

Table 1. Variables and Waste Production (kg/kg) Matrix

i	Model Variables Not.	Description	Feed in water (per kg feed)	Digested feed (per kg feed)	Fish growth (per kg)	Respiration (per kg fish)
1	S_I	Inert soluble organic material	$0.5I_{Feed}$	$0.5I_{Feed}$	$-0.5I_{Fish}$	0
2	S_S	Readily biodegradable substrate	$0.7COD_{Feed}$	$0.15COD_{Feed}$	$-0.15COD_{Fish}$	$-0.15r_O$
3	X_I	Inert particulate organic material	$0.5I_{Feed}$	$0.5I_{Feed}$	$-0.5I_{Fish}$	0
4	X_S	Slowly biodegradable substrate	$0.3COD_{Feed}$	$0.15COD_{Feed}$	$-0.15COD_{Fish}$	$-0.15r_O$
5	X_{BH}	Active heterotrophic biomass	0	$0.5COD_{Feed}$	$-0.5COD_{Fish}$	$-0.5r_O$
6	X_{BA}	Active autotrophic biomass	0	0	0	0
7	X_p	Part. products from biomass decay	0	$0.2COD_{Feed}$	$-0.2COD_{Fish}$	$-0.2r_O$
8	S_O	Dissolved oxygen	0	0	0	$-r_O$
9	S_{NO}	Nitrate and nitrite nitrogen	0	0	0	0
10	S_{NH}	Ammonium and ammonia nitrogen	0	$0.5N_{Feed}$	$-0.8N_{Fish}$	0
11	S_{ND}	Soluble biodegradable organic nitrogen	$0.5N_{Feed}$	$0.20N_{Feed}$	$-0.10N_{Fish}$	0
12	X_{ND}	Part. biodegr. organic nitrogen	$0.5N_{Feed}$	$0.30N_{Feed}$	$-0.10N_{Fish}$	0
13	S_{Alk}	Alkalinity (as HCO_3^--equivalents)	0	0	0	0
14	S_{CO2}	Dissolved carbon dioxide	0	0	0	$(44/32)r_O$
15	S_P	Phosphorus	P_{Feed}	P_{Feed}	$-P_{Fish}$	0
16	TSS	Total solid substance	$I =$	content of inert matter (kgCOD/kg)		
17	Q	Flow	$N =$	nitrogen content (kgN/kg)		
18	K_La	Oxygen mass transfer coefficient	$COD =$	carbon content (kgCOD/kg)		
19	S_{NO2}	Nitrite concentration	$P =$	P = phosphorus content (kgP/kg)		
20	L	Biofilm thickness	$r_O =$	r_O oxygen respiration rate (kg O_2/kg·d)		

and can be expressed by the use of a Temperature Growth Coefficient (TGC) (Chen, 1990):

$$\mathrm{BW}(t) = (\mathrm{IBW}^{1/3} + \mathrm{TGC} \cdot T \cdot t)^3/1000, \quad (2)$$

where BW is the fish body weight (kg), IBW is the initial body weight (g), T is the temperature (°C) and t is the time in days (d).

Table 2. Fish and Feed Content (kg/kg)

Content	Fish	Feed	COD	N	P
Protein	0.174	0.44	1.45	0.16	-
Carbohydrate	0.002	0.14	1.10	-	-
Fat	0.020	0.24	2.14	-	-
Ash	0.024	0.08	-	-	0.20
Water	0.780	0.10	-	-	-

The number of fish decrease with age due to death, which is typically expressed in p_n percent of the population per production cycle t_p (d). To numerically simplify, we allow the number of fish to be a positive real number and assume a first order death process. For an arbitrary time between fingerling and slaughter the number of fish is then

$$n(t) = n(0)e^{-kt}, \quad k = -\frac{1}{t_p}\ln(1 - \frac{p_n}{100}), \quad (3)$$

which gives a total fish mass (kg) in fish tank j

$$m_j(t) = \mathrm{BW}_j(t)n_j(t), \quad j = 1, 2, \ldots N_{FT}. \quad (4)$$

The respiration rate of a fish, expressed as gO_2/(kg fish and day), is a fairly well known quantity and the production rate of carbon dioxide is approximately equal to the oxygen respiration rate. Hence, using the mass determined by Eq. (4), we know how much of the carbon (COD) that is lost in respiration. The amounts of COD, N and P accumulated in the fish we determine from the corresponding contents in the fish (Table 2) and the mass growth rate (kg/d) in each tank, i.e.

$$\frac{d}{dt}m_j(t) = n_j(t)\left(\frac{d}{dt}\mathrm{BW}_j(t) - k\mathrm{BW}_j(t)\right) \quad (5)$$

For every combination of fish and feed there is a Feed Conversion Ratio (FCR). Based on FCR (kg feed/kg fish) the feed rate F_j in each tank (j) is determined as the product of the mass growth (5) and FCR.

Soon after the fish has been fed the waste production increases and after some time a maximum is reached after which the waste production decreases. How much waste compound is produced after a short period of constant feeding have an appearance similar to the curve in Fig. 2. The curve has been generated by a pulse u (the rapid feed intake) passing through two first order dynamic systems with time constants T_1 and T_2 that may depend on fish size, i.e., we assume

$$T_1\frac{d}{dt}x_i(t) = -x_i(t) + k_i(1 - \epsilon_{Loss})F(t) \quad (6)$$

$$T_2\frac{d}{dt}w_{F,i}(t) = -w_{F,i}(t) + x_i(t) \quad (7)$$

where ϵ_{Loss} is the fraction of the feed lost into the water column by the chewing fish, F is the (piecewise constant) feed rate (kg/d), x_i is an internal (intestine) state variable representing a mass accumulation, and k_i (kg/kg feed) determines the proportion of the feed that is converted to compound i. Equivalently to (6) and (7) we may write

$$w_{F,i} = k_i(1 - \epsilon_{Loss})G(p)F(t), \quad (8)$$

where $G(p) = 1/((1 + pT_1)(1 + pT_2))$ and p is the derivative operator.

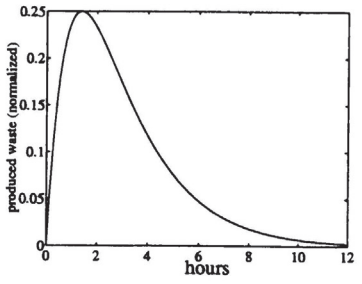

Fig. 2. Normalized production of waste from a fish modeled with $T_1 = 1$ h and $T_2 = 2$ h.

Since w_F approach zero if the feeding times are sparse and the accumulation and respiration terms in (1) are negative we may get a negative waste production w_i of nitrogen, COD and phosphorus if we let w_g and w_r be independent of the feeding rate. To avoid this we assume w_g and w_r to follow the same dynamic response as w_F.

The main steps in the calculation of the production of the waste compounds in Table 1 in each fish tank during a period between two gradings are

(1) Given temperature T, the time t_g between two gradings, the number of fish tanks ($N_{FT} = t_p/t_g$), IBW, the fish body weight (BW) immediately after a grading is determined from (2) evaluated for $t = t_g, 2t_g, \ldots, (N_{FT} - 1)t_g$.
(2) Determine the number of fish $n_j(0)$ in each tank immediately after grading using (3).
(3) Using (2) to (5), FCR and the specified feeding times (e.g. 06:00-06:15), the mass $m_j(t)$, $dm_j(t)/dt$ and $F_j(t)$ in each tank is calculated.
(4) A 'digested' feed $\tilde{F}_j(t) = G_j(p)F_j(t)$ in each tank j is determined using $G(p)$ in (8).
(5) Calculate $s_{F,j}(t) = G_j(p)u(t)$, where

$$u(t) = \begin{cases} 0 & \text{if } F_j = 0 \\ c & \text{if } F_j \neq 0 \end{cases} \quad \text{and} \quad \int_0^1 u(t)dt = 1$$

(6) Using a specified waste production matrix with the information in Table 1 the produced waste in each tank as function of time is the sum of

w_{Loss} : column 1 $\times$ $F_j(t)\epsilon_{Loss}$
w_F : column 2 $\times$ $\tilde{F}_j(t)(1 - \epsilon_{Loss})$
w_g : column 3 $\times$ $s_{F,j}(t)dm_j(t)/dt$
w_r : column 4 $\times$ $s_{F,j}(t)m_j(t)$

However, for oxygen and carbon dioxide column 3 and 4 should not be multiplied by the feed signal s_F because it is assumed that under normal circumstances the respiration rate is not coupled to the intestine activity.

Table 1 needs some further comments. The first column of the waste matrix describes how feed lost into the water is fractionated on the modeled compounds and the second column how it is fractionated after passing through the fish, i.e. the entries in the second column are the k_i in Eq. (6). For the mass balances to become correct the coefficients for every component (N, COD and P) should add up to unity in columns 1, 2 and 3. The correction coefficients for produced COD components due to respiration (column 4) should also add up to unity. Finally, the coefficients in columns 2, 3 and 4 should preferably be the same to ensure non-negative waste production.

The mass balance for component i in a fish tank is

$$V\frac{d}{dt}Z_i = Q(Z_{i,in} - Z_i) + w_i + u_i,$$

where Z_i is either soluble concentration S_i or particulate concentration X_i, $Z_{i,in}$ is the concentration in the tank influent, w_i is the produced waste and u_i is the amount of externally added or removed matter.

In the simulator, oxygen may either be introduced as a (liquid) addition to the tank influent, i.e. $u_8 = \dot{m}_{O_2}$ g/d, or by aeration in the tank. In the case of aeration the standard gas transfer model is used:

$$u_8 = VK_La_{O_2}(S_{O_2,sat} - S_8) \quad (9)$$

$$u_{14} = VK_La_{CO_2}(S_{CO_2,sat} - S_{14}) \quad (10)$$

where the mass transfer coefficient $K_La_{O_2}$ depends on the aeration method, the air flow rate and bulk characteristics. By default a ratio $K_La_{CO_2}/K_La_{O_2} = 0.9$ is used (Royce and Thornhill, 1991).

3.2 Moving Beds

All the moving bed reactors are modeled in the same manner, except for some parameter values that have been chosen different if the film is mainly autotrophic or heterotrophic. The moving beds are modeled as biofilm reactors with fixed biofilms on the carriers and suspended sludge in the water. Due to lack of knowledge, and the fact that the movement of the carriers should enhance mass transfer, the biofilm is assumed to be homogeneous in the sense that, on the average, the biofilm and the concentrations within the film are the same at all depths of the film. The processes and the corresponding rates have been slightly changed from ASM1: (i) CO_2 and P has been added, (ii) the nitrification rate has been changed to depend on the alkalinity as in the ASM2, (iii) a Monod factor w.r.t. ammonium has been included in the growth of heterotrophs to avoid negative ammonium concentrations as in the ASM3 (Henze *et al.*, 2000), (iv) the nitrite concentration is modeled either by worst case or by balanced growth.

Let $X_{i,b}$ and $S_{i,b}$ denote the concentrations of particulates and solubles in the bulk water phase, and $X_{i,c}$ and $S_{i,c}$ denote the corresponding concentrations in the biofilm. The transfer of particulates (g/m^{2}d) from the bulk to the biofilm is assumed to be given by

$$J_i = K_aX_{i,b} - K_dL^2X_{i,c}, \quad i = 3, 4, 5, 6, 7, 12$$

where K_a and K_d are attachment and detachment rate coefficients and L is the biofilm thickness. In

the model by Maurer *et al.* (1999) the detachment is modeled as only being proportional the concentration. However, this easily results in unstable solutions. Introducing a dependence on L, which means that the thicker the biofilm the easier bacteria and other particulates detach, gives a stability in the sense that the biofilm thickness does not vary as much. From extensive testing a linear dependence was not found to be enough to give realistic variations but a squared biofilm thickness is necessary. The resulting detachment rate is then equal to what is common in models of fixed biofilms (Wik, 1999).

The flux of solubles (g/m^{2}d) from the bulk to the biofilm is assumed to depend only on the difference between the concentrations in the film and in the bulk:

$$J_i = K_x(S_{i,b} - S_{i,c}), \quad i = 1, 2, 8 \ldots 11, 13 \ldots 15.$$

The exchange transfer coefficient is assumed to be the same for all solubles and also independent of temperature. Diffusion coefficients depend on temperature and also vary some between the different solubles. The exchange coefficient, however, includes convection which is likely to dominate the diffusion in the transfer from bulk to biofilm surface because of the carrier movements.

With V_w denoting the empty bed volume minus the volume of the carriers without biofilm, a mass balance for component i in the bulk phase gives

$$\frac{d}{dt}(V_w - LA)Z_{i,b} = Q(Z_{i,b,in} - Z_{i,b}) - AJ_i + J_{i,g} + (V_w - LA)r_i$$

where A is the total area of biofilm in the reactor, $Z_{i,b,in}$ is the influent concentration, $J_{i,g}$ is the flux (g/d) from the gas phase or the surrounding air to the bulk, and r_i is the observed conversion rate (ASM1-ASM3) evaluated for the concentrations in the bulk. Only for oxygen and carbon dioxide may the flux $J_{i,g}$ meaningfully contribute, and then only in the aerated reactors. In the aerated moving bed reactors the transfer of oxygen and carbon dioxide is modeled in the same way as for the fish tanks (Eqs. (9) and (10)). Since the mass transfer coefficient depends on the air flow rate and bulk characteristics K_La is generally not constant but rather a manipulative variable used for feedback control.

Mass balances in the biofilm give

$$\frac{d}{dt}A\epsilon LS_{i,c} = AJ_i + ALr_i$$
$$\frac{d}{dt}ALX_{i,c} = AJ_i + ALr_i$$

where we note that the concentrations of solubles are defined only for the void volume in the biofilm. The biofilm thickness is given by

$$\frac{d}{dt}A(1-\epsilon)\rho_X L = \sum_{i=3}^{7} AJ_i + ALr_i,$$

where ϵ is the biofilm porosity and ρ_X is the biofilm density (gCOD/m^3). Applying the chain rule to the mass balances gives the state equations for one moving bed reactor tank:

$$\frac{d}{dt}Z_{i,b} = \frac{1}{V_w - LA}(QZ_{i,in} + (A\frac{d}{dt}L - Q)Z_{i,b} - AJ_i + J_{i,g}) + r_i(X_b, S_b)$$
$$\frac{d}{dt}S_{i,c} = \frac{1}{L}(\frac{J_i}{\epsilon} - S_{i,c}\frac{d}{dt}L) + \frac{1}{\epsilon}r_i(X_c, S_c)$$
$$\frac{d}{dt}X_{i,c} = \frac{1}{L}(J_i - X_{i,c}\frac{d}{dt}L) + r_i(X_c, S_c)$$
$$\frac{d}{dt}L = \frac{1}{\rho_X(1-\epsilon)}\left(\sum_{i=3}^{7} J_i + Lr_i(X_c, S_c)\right)$$

4. SIMULATION

4.1 Control Loops

In the simulator a couple of PI-control loops (see Fig. 1) has been implemented either for the actual regulation of the plant or to guarantee equal conditions for fair comparisons between different plant size and configurations. Except for aeration control in the aerated treatment tanks (not shown) we have oxygen control either by liquid oxygen or by aeration, alkalinity control and, if required, addition of an external carbon source to the denitrification by feedback of either the nitrate or the oxygen concentration.

Based on mass balances analytical expressions how to scale the gain and integration time appropriately with flow, volumes, bacterial yield and oxygen saturation concentration have been derived. The controllers are therefore robust to almost all changes to the system. To avoid the tedious (since the simulations are fairly time consuming) tuning of the controllers every time the system or a parameter value is changed (e.g. in an optimization), the automatically tuned regulators are almost indispensable.

4.2 Simulations

The case studied here, by simulation in Simulink, is a RAS for a 5 tonnes annual production of rainbow trout with 14 parallel rearing tanks and a 30 days production cycle. To save simulation time (considerably) it was investigated if the 14 parallel tanks could be approximated by one tank having the same total volume, flow and fish mass. The approximation is illustrated in Fig. 3, where we note that the organic nitrogen is immediately available in the water after feeding due to the feed loss, while ammonium is a result of the gastric evacuation with a corresponding dynamics. We conclude that for the accuracy needed a one tank approximation is sufficient.

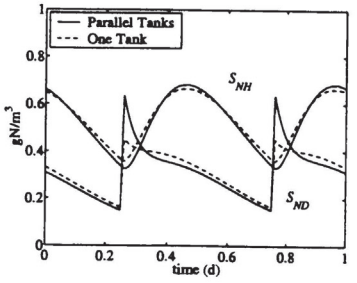

Fig. 3. Ammonium and soluble organic nitrogen in the fish tank effluent.

Table 3. Average distribution in kg/d

	Added	Waste	Fish	Respiration
COD	19.4	5.2	4.0	10.1
N	1.08	0.68	0.39	0
P	0.24	0.18	0.07	0

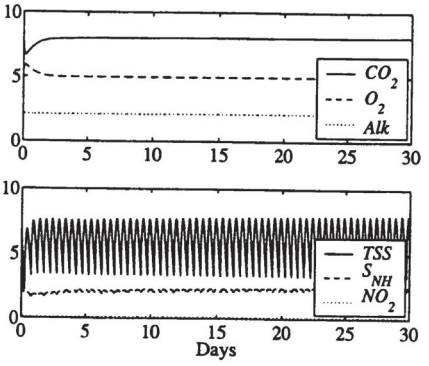

Fig. 4. Simulated concentrations in the fish tanks (oscillations caused by twice daily feeding).

The resulting average distribution over one production cycle of the carbon, nitrogen and phosphorus is listed in Table 3. Evidently a significant amount of carbon is lost in respiration. For rainbow trout the concentration limits are 12 gCO_2/m^3, 4 $gN\text{-}NH_4$ (20°,pH6.5), 10 $gTSS/m^3$, 0.05 $gN\text{-}NO_2$ and a set-point of 5 gO_2/m^3. According to the simulations the concentrations could be held below the limits with as low as a 1.5% daily exchange of water and 5 moving beds with a total volume of 50 m^3 (see Fig. 4).

5. CONCLUSIONS

A simulator for simulation of recirculating aquaculture systems has been developed. The simulator is based on mass balances, and can be applied to any combination of fish, feed and treatment provided the required data for the plant is given. Basically, the necessary fish, feed and treatment data are: (i) The content of the feed and the fish (protein, fat, carbohydrate, ash, water), (ii) the initial body weight of the fish, (iii) the time between grading of the fish and the length of the production cycle, (iv) the oxygen consumption rate, (v) the feed conversion ratio and the times of the feeding, (vi) fish tank volumes and water temperature, (vii) rough estimations of the proportions of different organic waste compounds (solids, solubles, bacteria) in the faeces, (viii) the number of treatment tanks, their volume and filling.

Simulations show that for rainbow trout it appears to be possible to operate a RAS with moving bed reactors having a total volume of $50m^3$ for an annual production of 5 tonnes rainbow trout.

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