SPECIALIZATION PROJECT TKP4550
Department of Chemical Engineering, NTNU, Trondheim
Project title: Modeling petroleum wells with first principles and optimizing a two-well manifold
By Rannei Solbak Simonsen
Supervisor: Sigurd Skogestad
Co-supervisor: Chriss Grimholt
Date: 20 Dec. 2013

Abstract

An important part of operation of petroleum wells is to to maximize the oil production from wells, and insure that they are optimally operated at all times. To avoid the need to re-optimize every time a disturbance occur, it is desired to construct a simple control structure for well manifolds. The goal is to keep the system at optimal operation by control rather than re-optimization.

The scope of this project is to develop a well model based on first principles and physical equations and modified the model to give the GOR-behavior we want to examine. Each model is tested and optimized in a two-well manifold. In the optimization problem the oil flow rate is maximized, subject to an active gas constraint. The optimal solutions found from simulations are compared against the following optimization criteria:

$$\left(\frac{\partial q_g}{\partial q_o}\right)_i = \left(\frac{\partial q_g}{\partial q_o}\right)_j$$

It is known that keeping this ratio equal for all the wells ensures optimal operation of the manifold. The overall goal is to find variables that can be used to control this relation when disturbances occur in the petroleum wells. This project a first step towards reaching this goal.

Preface

I would like to thank Sigurd Skogestad for the project assignment and guidance throughout the project. I would like to thank Chriss Grimholt for excellent help and guidance in this project, for always being available and patient. I would also like to thank Nina Helene and Ingrid for always answering my questions and offer comforting words when needed. And last but not least, I would like to thank my parents for motivation and support.

I declare that this is an independent work according to the exam-regulations of the Norwegian University of Science and Technology..

Contents

Chapter 1

Introduction

The operation of petroleum reservoirs is a large and advanced area and reservoir management is conducted in many timescales. The superior planning for a reservoir expands from a month to a year perspective. This step sets the general operational goals for the whole reservoir. This involves decisions like setting production rates form the wells and strategical planning like well locations [1]. On a short time horizon, typically a day or a few hours, the operation of the wells is decided. This is the time scale of interest for us.

A petroleum production system is often build up by many well-manifolds, which connects several wells from the same reservoir [2]. This causes the production from one of the wells to affect the rest of the wells in the same manifold. The challenge is to decide how to operate the wells together in order to maximize the total oil production from the manifold. A simple sketch on how the whole system can look is given in Figure 1.1.

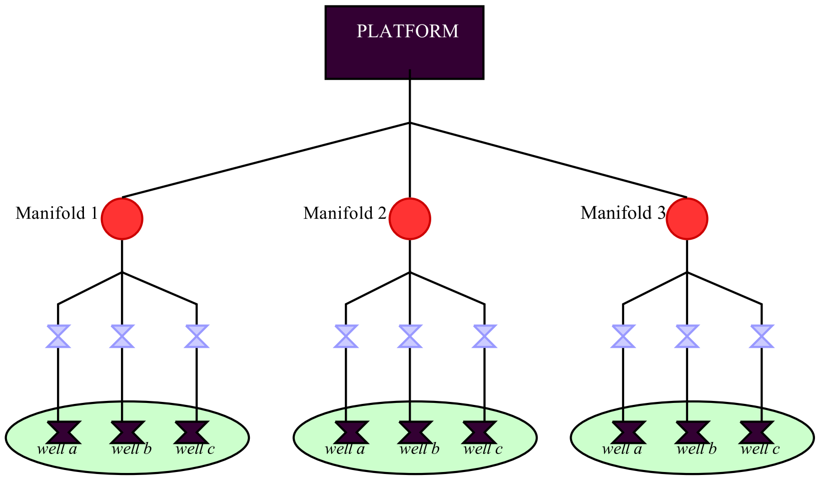


Figure 1.1: A simple sketch of a petroleum well system, where three manifolds are connecting three wells each. And all the manifold are connected to the same platform.

Both oil, gas and water is extracted form the wells, and transported through pipelines to a platform connecting all the manifolds of a sub-sea petroleum system. On board the platform the three phases are separated. Separation processes are expensive and resources on board a platform are usually limited. Thus, there is usually a limitation on how much gas such a separation facility can handle. The operation of the manifold are therefore subject to a constraint on the gas flow.

When optimizing the operation of petroleum well-manifolds we want to maximize the the oil rate and minimize the gas flow rate. The gas-oil-ratio (GOR) is of significant importance since it indicates how much gas is produced per amount of oil. In the case where we want to maximize the oil production and restrain the gas production, it will be optimal to produce at the highest rate from the well with the lowest GOR.

Optimization of the manifolds is usually performed under the assumption that the gas-oil-ratios (GOR) in the wells are constant. The optimization involves getting operational data from the operators in the north sea and run the optimization through with a complex model of the process. It is time consuming and normally conducted only once or twice per day.

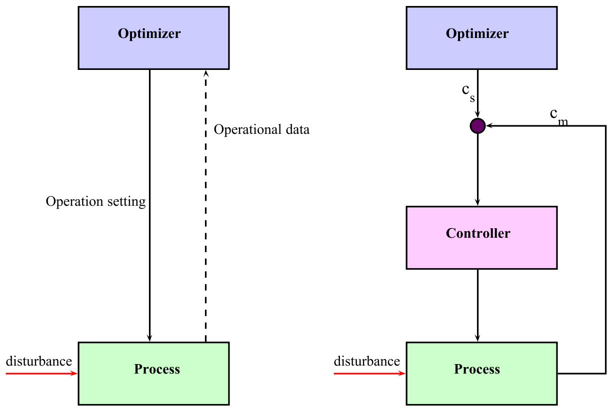


Figure 1.2: Normal operation of petroleum wells (to the left), and desired operational scheme for the petroleum wells (to the right).

The problem is that the GOR changes due to disturbances in the well reservoir. When a disturbance occur, the operation of the wells are no longer optimal, however the disturbance is not accounted for until the process is re-optimized. Since the optimization is executed so seldom, this leads to longer periods of non-optimal operation of the manifold. It is therefore desirable to construct a simple control structure for the wells in a manifold, to control the operation back to optimal after a disturbance has occurred.

In the case where the manifold is operated under constrained conditions, we already know the optimal solution. The optimization criteria is described in Urbanczyk and Wattenbargers article *Optimization of well Rates under Gas Coning Conditions* [3]. Where it is said that the optimal way to operate a manifold, is to keep the change in the gas rate with respect to an infinite small change in the oil flow equal for all the wells. However, we do not know how to control the wells in order to keep the manifold at this condition. The problem is that we do not have a way to express the derivative of the gas rate with respect to the oil rate. In order to control the system we need a way to either directly measure this change rate or express the rate by variables we can measure.

The scope of this project is two folded. Firstly we want to develop a well model based on first principles and physical equations, and connect two wells into a manifold. Second we want to study the well behavior for different types of wells and optimize the two-well manifold. We will compare the behavior for wells based on different GOR-expressions. The models are therefore modified to give the behavior we want to examine. The manifold is also optimized for disturbances in the GOR-expression. The optimal solutions are tested against the optimal criteria given in Urbanczyk and Wattenbargers article.

The development of the model is done in Chapter 2 and Chapter 3. The modeling process involves both the derivation of the equations for each part of the system and solving the system of equations.

The optimization part of the project starts with its own introduction in Chapter 4. When the models are complete they will be modified to account for different GOR expression, this is done in Chapter 5, and a two-well manifold is put together. The well models are all calibrated from the same test case-wells, in order to compare the different well behaviors. The test-case wells is presented in 6 together with an analysis of the different well types the models represent. The manifold is optimized in Chapter 7, and the analysis of the optimal solutions are also given there.

Chapter 2

Developing the Model

2.1 The well system

To predict the behavior of different well types we need a model. Most of the time models are constructed by using experimental data and/or fundamental chemical and physical equations. A model is normally a simpler presentation of the real world, describing the behavior of the system. We will base our models on simple physical equations and first principles. In total four different well types will be studied, all based on different GOR expressions. In this chapter and the next, equations for the physical model are derived and solved.

In the first place the system consists of a reservoir, one well, a pipeline with one valve, and an outlet, as illustrated in Figure 2.1. The system is divided into nodes, where each sub-system between the nodes can be described by the same equation. The four system nodes are
Node 1: The reservoir outer wall, short: R
Node 2: The well bottom hole, short: wf
Node 3: The inlet of the valve. short: wh
Node 4: The outlet after the valve: short out

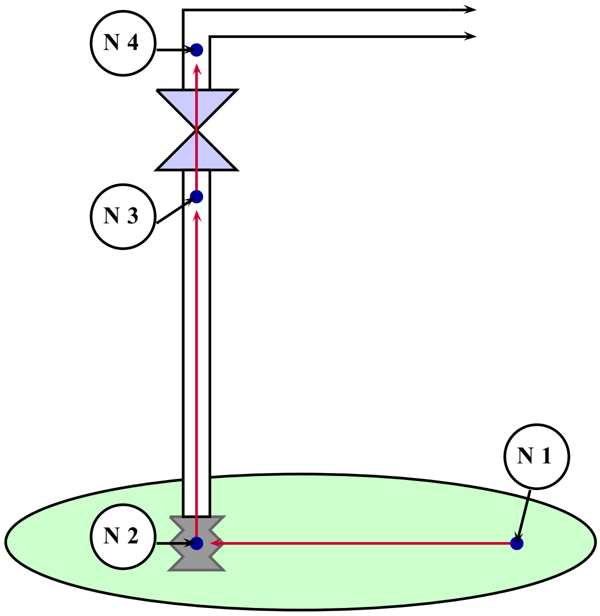


Figure 2.1: The petroleum well with one well, a pipeline with one valve, and an outlet.

2.2 From Node 1 to Node 2 - Darcy's law

Oil and gas does not lay in reservoirs as lakes or rivers, the fluids are stored in pores in the rock beneath the seafloor. This means that the oil and gas flow through porous media from the reservoir and to the well bottom hole. To describe this flow one can use Darcy's law [5]. With the right assumptions one can describe the flow through porous media with a quite simple expression, where the flow velocity is directly proportional with the pressure difference across the porous media. The system is illustrated in Figure 2.2. Darcy's law valid for fluid such as oil and gas can be expressed as:

$$v = \frac{q}{A} = -\frac{k}{\mu}\left(\frac{\partial P}{\partial L} - 0.433\gamma\cos\alpha\right) \tag{2.1}$$

Where
v = apparent fluid velocity, cm/sec
A = cross-sectional area of flow, cm^2
k = permeability of porous medium, darcies
μ = fluid viscosity, centipoise (cp)
$\frac{\partial P}{\partial L}$ = pressure gradient over the length of the flow path, atm/cm
γ = fluid spesific gravity
α = angel of dip measured counterclockwise between the vertical direction downward and the inclined plane of the fluid flow.

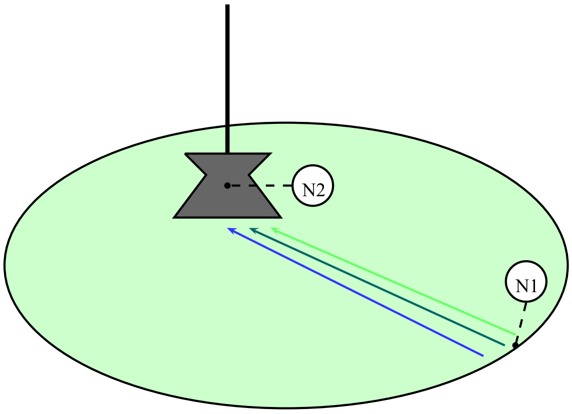


Figure 2.2: The flow through the reservoir to the well head.

This equation can be simplified by assuming linear fluid flow (i.e $\alpha = 0$), that the flow occurs in a horizontal direction, in a laminar regime, no chemical reaction and that there is only one fluid present in the system of pores. Rearranging and then integration Equation 2.1 gives under the mentioned conditions a very simple expression which gives the relationship between the pressure drop between two point in the porous media and the resulting *linear* flow rate between the point.

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{\partial p}{\partial L} \tag{2.2}$$

$$q\int_0^L dL = -\frac{kA}{\mu}\int_{p_1}^{p_2} dp$$

$$q = -\frac{kA}{\mu L}\Delta P \tag{2.3}$$

Where
L = length of linear flow path, cm
ΔP = pressure drop $(p_1 - p_2)$ of flowing fluid over the length L, atm

If one assume that the density ρ is constant for liquid, one can rewrite Equation 2.3 to mass basis:

$$m = q\rho = -\frac{kA\rho}{\mu L}\Delta P \tag{2.4}$$

Accounting for a compressible fluid

In the integration of Equation 2.2 it is assumed that the flow is not a function of the pressure. This is however a rather poor assumption for the gas phase flow. So the expression needs to be modified to account for the pressure dependence of the gas phase.

It is now necessary to take into account that q is a function of the pressure. This is done by first assuming ideal gas, and substituting the following equation for q in Equation 2.2 :

$$q = \frac{m_g}{\rho} = \frac{nRT}{PA} = \frac{m_g}{M}\frac{RT}{PA}$$

Which gives the following expression to be integrated:

$$\frac{m_g}{M}\frac{RT}{PA} = -\frac{k}{\mu}\frac{\partial p}{\partial L} \tag{2.5}$$

Integration of Equation 2.5 gives the relationship between the pressure difference in the reservoir and the gas mass flow.

$$m\int_0^L dL = -\frac{kMA}{\mu RT}\int_{p_1}^{p_2} Pdp$$

$$m = k_1\left(p_2^2 - p_1^2\right) \quad \text{where} \quad k_1 = \text{constant} = -\frac{kMA}{\mu RT\Delta L} \tag{2.6}$$

Calculating the constants

The constants in Darcy's law are system specific and can be found by using a well test case. If the mass flow rates for all the three phases for a specific well flow pressure and reservoir pressure are know, the constants can be calculated. Thus, the values of the parameters making up the constant is not needed explicit. The equations for the flow in the reservoir is then simply:

$$\boxed{m_o = k_o(P_2 - P_1)} \tag{2.7}$$

$$\boxed{m_w = k_w(P_2 - P_1)} \tag{2.8}$$

$$\boxed{m_g = k_g(P_2^2 - P_1)^2} \tag{2.9}$$

Where the k-values must be decided specifically for each system at hand. The way the pressure difference is stated here ($\Delta P = P_2 - P_1$) the k-values will be negative.

2.2.1 Using the gas-oil-ratio to describe the gas flow rate

Instead of using only Darcy's law, one can also apply the gas oil ratio (GOR), which describes the amount of gas produced per amount of oil. It can be expressed as:

$$GOR = \frac{m_g}{m_o} \tag{2.10}$$

Assuming that the oil flow always follows Darcy's law, we are able to relate the flow pattern of gas to the GOR-expression rather than using Darcy's law. This relation is very useful for studying wells with different GOR expressions. In cases where the GOR-expression is varied for the well model the flow rate of gas will then be expressed as:

$$\boxed{m_g = GOR \cdot m_o} \tag{2.11}$$

2.3 From Node 2 to Node 3 - The mechanical energy balance

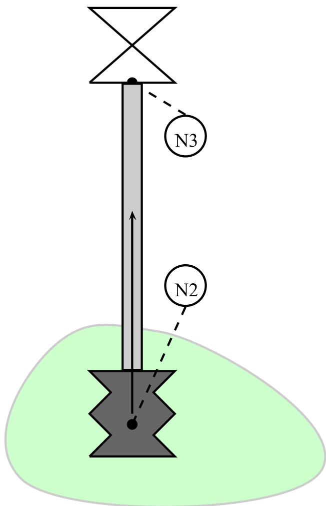


Figure 2.3: The flow through the pipeline from the well head to the valve

To describe the flow through the pipeline, the mechanical energy balance is applied, expressed for a stationary, continuous process in Equation 2.12 [6]. An illustration of the system in which there equations describer is given in Figure 2.3.

$$m\alpha_2\frac{v_3^2}{2} + mgL_2 + m\int_{p_2}^{p_3}\frac{dp}{\rho} + \Phi = m\alpha_1\frac{v_2^2}{2} + mgL_1 + W_s \tag{2.12}$$

This expression can be simplified by making certain assumptions, and even with these assumptions the equation will turn out to be quite useful. If one assume that there is no friction ($\Phi = 0$), no work is done ($W_s = 0$) and the kinetic energy is ignored ($m\alpha_2\frac{v_2^2}{2} = 0$), and write Equation 2.12 on deviation form instead of integrated form the equation becomes:

$$dp = \rho g dL \tag{2.13}$$

Where dp is the pressure difference over a small length dL, g is the gravity constant and ρ is the fluid density.

For liquids where the density is independent of pressure, this equation is solved quite easily. It is however, not that straight forward for gasses where the density is a function of pressure. If we assume ideal gas the density can be expressed as a function of pressure.

$$\rho_g^{i.g} = \frac{PM}{RT} \tag{2.14}$$

Combing Equation 2.13 and Equation 2.14 gives:

$$\frac{dp}{P} = \frac{Mg}{RT}dL$$

Which when integrated gives the pressure drop for a pure (ideal) gas flowing in a vertical pipeline.

$$[ln(P)]_{p_2}^{p_3} = \frac{PM}{RT}\Delta L$$

The problem is that the flow form the reservoir is a mix of oil, gas and water. In order to solve this problem all the phases are put together in a pseudo fluid which contains all the three phases. And we can operate with a mixed density expressed as:

$$\rho_{mix} = v_g\rho_g^{i.g} + v_o\rho_o + v_w\rho_w \tag{2.15}$$

Oil is known to be a slightly compressible fluid, but it will be assumed that both water and oil in incompressible. The densities ρ_o and ρ_w are therefore constant for the further calculations. Instead of expressing the mixed density in terms of volume factions, we want to expressed in terms of mass, and we use the correlation for the volume fraction of phase φ:

$$v_\varphi = \frac{\frac{m_\varphi}{\rho_\varphi}}{v_{tot}}$$

Where the total volume of oil, gas and water is:

$$v_{tot} = \frac{m_g}{\rho_g^{i.g}} + \frac{m_o}{\rho_o} + \frac{m_w}{\rho_w}$$

The mixed density can now be expressed on mass flow basis (Equation 2.16). The pressure dependence of the gas phase is also accounted for, by substituting the expression given in Equation 2.14 for the gas density.

$$\rho_{mix} = \frac{m_o + m_g + m_w}{v_{tot}} = \frac{m_o + m_g + m_w}{\frac{m_g RT}{MP} + \frac{m_o}{\rho_o} + \frac{m_w}{\rho_w}} \tag{2.16}$$

To find the pressure drop for the pseudo fluid in the pipeline we use the mixed density when integrating Equation 2.13.

$$\int_{p2}^{p3} \frac{1}{\rho_{mix}} dp = \int_{p2}^{p3} \left(\frac{m_g RT}{M_g P} + \frac{m_o}{\rho_o} + \frac{m_w}{\rho_w} \right) dp = m_{tot} g \int_{z2}^{z3} dL$$

To make the expression neater the constant parameters are compiled together as:

$$\boxed{D = \frac{m_g RT}{M_g} \quad \text{and} \quad B = \frac{m_o}{\rho_o} + \frac{m_w}{\rho_w}} \tag{2.17}$$

Integration gives the expression for the pressure drop over the pipe line. The integration limits are from the top of the pipeline (P_3) to the bottom (P_2), so the pressure difference $\Delta P = P_2 - P_3$ is the pressure at the well head (node 2) minus the pressure before the valve (node 3) (ref. Figure 2.3).

$$[Dln(P) + BP]_{p2}^{p3} = m_{tot} g \Delta L \tag{2.18}$$

The equation must be solved implicit due to the “ln-to-P”-part. For simplicity, we want to be able to solve the equation directly, and an approximation for the ln-expression is applied.

The expression can be approximated to a simpler expression by using a series expansion on the natural logarithm. The approximation is given in the equation below.

$$[ln(P)]_{p2}^{p3} = \ln(\frac{P_2}{P_3}) = \ln(\frac{\Delta P + P_3}{P_3}) = \ln(1 + \frac{\Delta P}{P_3}) \approx \frac{\Delta P}{P_3} \tag{2.19}$$

The pressure drop over the pipeline can then be expressed as a function of the pipe length L.

$$\boxed{\Delta P = P_2 - P_3 = \frac{m_{tot} g P_2}{D + P_2 B} L} \tag{2.20}$$

The pressure drop profile for both Equation 2.18 and Equation 2.20 are shown in Figure 2.4. We found the difference between the two to be tolerable, and the approximated expression will be used in the further calculations.

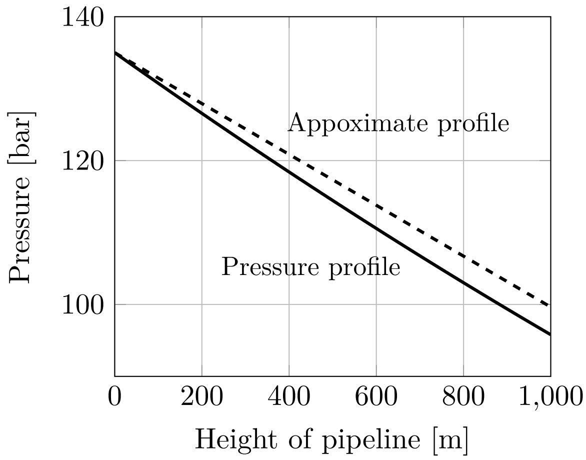


Figure 2.4: The pressure drop over the pipe line, the solid line shows the pressure drop calculated from the exact equation, while the dashed line shows the approximated pressure drop.

2.4 From Node 3 to Node 4 - The valve equation

This part of the system involves the valve. The valve will cause a pressure drop in the steam flowing through it. The system is illustrated in Figure 2.5.

2.4.1 Describing the pressure drop from the mechanical energy balance

To describe the flow over the valve both the mass balance and the mechanical energy balance is needed. First the general equation will be derived for a compressible fluid, such as a gas.

Conservation of mass balance gives:

$$w = \rho_1 v_1 S_1 = \rho_2 v_2 S_2$$

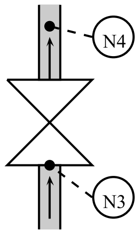


Figure 2.5: The flow over the valve. The nodes are place right after and right before the valve openings.

Where:
w is the mass flow
v is the the volumetric flow rate $[m^3/s]$
ρ is the fluid density.
and S is the cross sectional area at point 1 and 2

The mechanical energy balance is:

$$\frac{v_2^2}{2\alpha_2} - \frac{v_1^2}{2\alpha_1} + \int_{p_1}^{p_2} \frac{1}{\rho} dp + \frac{1}{2} v^2 e_v = 0$$

By combining the mass balance and the mechanical energy balance one can eliminate the flow rate v_1 and v_2. The expression for the mass flow rate as a function of the pressure drop is then found, as given in Equation 2.21.

$$w = \rho_2 S_2 \sqrt{\frac{-2\alpha_2 \int_{p_1}^{p_2} \frac{1}{\rho} dp}{1 - \frac{\alpha_2}{\alpha_1} (\frac{\rho_2 S_2}{\rho_1 S_1})^2 + \alpha_2 e_v}} \tag{2.21}$$

To make the expression simpler some assumptions can be made, and the expression will still be useful. Four assumptions that can be made are:

1. $e_v = 0$
2. The velocity profile at the inlet (index 1) is so flat that $\alpha_1 = 1$
3. The velocity profile at the outlet (index 2) is given by the approximate profile so that $\frac{1}{\alpha_2} = (\frac{S}{S_0})^2$
4. Assume that $S_2 = S_0$

With these assumptions the general expression for the mass flow rate of a compressible fluid through a valve is:

$$w = C_d \rho_2 S_0 \sqrt{\frac{-2 \int_{p_1}^{p_2} \frac{1}{\rho} dp}{1 - (\frac{\rho_2 S_0}{\rho_1 S_1})^2}} \tag{2.22}$$

To be able to solve this equation on must have the expression for the density as a function of pressure. And the expression derived earlier for the pseudo fluid mixed density ρ_{mix} given in Equation 2.16 is used. The following relation was found for the flow through the pipeline and can also be used to solve the integral in Equation 2.22.

$$\int_{p_1}^{p_2} \frac{1}{\rho_{mix}} dp = \frac{1}{m_{tot}} \int_{p_1}^{p_2} \frac{m_g RT}{MP} + \frac{m_o}{\rho_o} + \frac{m_w}{\rho_w} dp = \frac{1}{m_{tot}} [Dln(P) + BP]_{P_1}^{P_2}$$

From this we now have the mass flow rate through the valve, expressed as a function of the pressure drop (ΔP), the fluid density (ρ_{mix}(P)), the pipe dimensions (S_1) and the valve opening (S_1) (Equation 2.23).

$$w = C_d \rho_{mix,2} S_0 \sqrt{\frac{-2 \int_{p_1}^{p_2} \frac{1}{\rho_{mix}} dp}{1 - (\frac{\rho_{mix,2} S_0}{\rho_{mix,1} S_1})^2}} = C_d \rho_{mix,2} S_0 \sqrt{\frac{-2 \frac{1}{m_{tot}} [Dln(P) + BP]_{P_1}^{P_2}}{1 - (\frac{\rho_{mix,2} S_0}{\rho_{mix,1} S_1})^2}} \tag{2.23}$$

The discharge coefficient C_d is to account for errors in the assumptions made about e_v, α_1 and α_2. The coefficient must be determined experimentally, and depends on Reynolds number and the relation between S_0 and S. When one assume that $S_2 = S_0$ the α_2 is nearly unit, and it has been found that C_d is about the same for compressible and incompressible fluids. For well designed venturi meters the value is about 0.98.

Even with the assumptions made for the original equation, this expression is still quite complicated. The pressure drop will therefore be described by the valve equation instead. The derivation follows in the next section.

2.4.2 Describing the pressure drop by the valve equation

The flow through a valve can be changed by changing the valve position (here labeled z). The valve position set the valve opening and is therefore controlling the flow through the valve. The valve equation describes the relation between the flow, valve opening and the pressure drop. The valve equation for non-compressible fluids are given in Equation 2.24.

$$q = C_d f(z) A \sqrt{\frac{\Delta P}{\rho}} = C_v \sqrt{\frac{\Delta P}{\rho}} \tag{2.24}$$

On mass flow basis the relation $m = q\rho$ is used, and the equation becomes:

$$m = \rho q = C_v \sqrt{\rho \Delta P} \tag{2.25}$$

Where
C_d is the valve constant
f(z) is the valve characteristics
A is the cross section area for the valve
and C_v is the valve coefficient.

Instead of using the complicated approach explained earlier, we will make use of an average density to account for the fact that we have gas (compressible fluid) in our system. Rather than integrating the density over the pressure drop the average of the densities before and after the valve are used in the valve equation. The assumption of a pseudo one phase fluid for the multiphase flow is still applied. Thus, the mixed density are averaged as shown in Equation 2.26, where P_3 and P_3 is the density at the inlet and outlet, respectively.

$$\boxed{\bar{\rho}_{mix} = \frac{\rho_{mix,P_3} - \rho_{mix,P_4}}{2}} \tag{2.26}$$

Using this expression in Equation 2.25 results in a much simpler expression for the pressure drop over the valve. The new expression can be solved directly for the mass flow as a function of the valve opening and the pressure drop over the valve.

$$\boxed{m = C_v \sqrt{\frac{\rho_{mix,P_3} - \rho_{mix,P_4}}{2} (P_3 - P_4)}} \tag{2.27}$$

An important notion it that the valve equation gives the mass flow in kg/s. When using the valve equation in the model calculations the valve constant, C_v, must be scaled to give the mass flow in kg/d.

2.4.3 Valve parameters

The parameters needed in the valve equation are the valve constant C_v and the valve characteristic $f(z)$. The valve constant is also called the flow coefficient, it gives the valves efficiency at allowing fluid to flow through it. It depends on the valve opening z, and varies for different valves depending on each valves flow capacity. The expression for C_v is given in Equation 2.28.

$$C_v = C_d f(z) A \tag{2.28}$$

The *f(z)* gives the valve characteristic, and is a function of how the valve is opening related to the flow rate through it. The three most common valves used in control are linear valves, equal percentage vales and quick opening valves (Figure 2.6). One

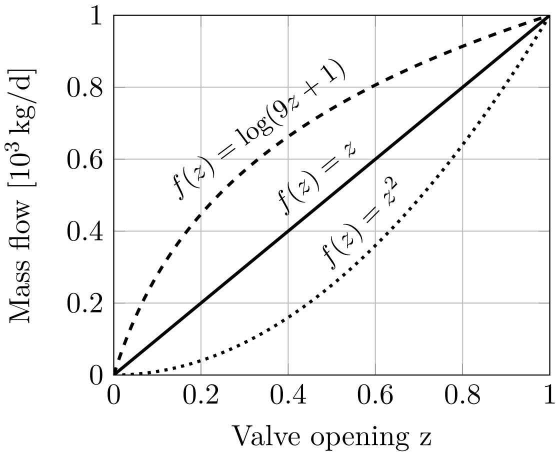


Figure 2.6: The mass flow through the valve as a function of the valve opening, dashed line for a logarithmic valve, solid line for a linear valve, and dotted line for a parabolic valve.

Table 2.1: Valve specifications

Parameter		Value
C_d	Valve constant	1×86400
A	Cross-section area	5.56 cm^2
f(z)	Valve characteristic	z

important relation between *f(z)* and z is that their limits have to match. When z is 0 the valve is completely closed, which means that $f(0) = 0$, and the same for a fully open valve when $z = 1$, $f(z) = 1$.

For simplicity the valve in the model is selected to be a linear control valve. The specifications for the valve are given in Table 2.1.

2.5 Summary of the equations, variables and parameters

The well model is now fully described by a set of equations. What still remains it to decide the different parameters and variables. Some can be found in literature, the rest on the other hand must be found from a well test case. This will be done in a later chapter (Chapter 6). The equations are summed up and labeled f_1 to f_8 in Table 2.2. The parameters are listen in Table 2.3, with an estimated value and a short explanation for each of them.

Table 2.2: The equations describing the system, and the unknown variables

Eq. nr.	Equation	Variable
f_1 :	$m_{tot} - m_g - m_o - m_g - m_w = 0$	m_{tot}, m_g, m_o, m_w
f_{2a} :	$m_g - k_g \left(P_2^2 - P_1^2\right) = 0$	m_g, P_2
f_{2b} :	$m_g - GOR \cdot m_o = 0$	m_g, m_o, P_2
f_3 :	$m_o - k_o \left(P_2 - P_1\right) = 0$	m_o, P_2
f_4 :	$m_w - k_w \left(P_2 - P_1\right) = 0$	m_w, P_2
f_5 :	$p_2 - P_3 - \frac{m_{tot} g P_2}{D + P_2 B} z = 0$	$m_{tot}, m_g, m_o, m_w, P_2, P_3$
f_6 :	$m_{tot} - C_v \sqrt{\bar{\rho}_{mix}(P_4 - P_3)} = 0$	$m_{tot}, P_3, \rho_{mix,3}, \rho_{mix,4}$
f_7 :	$\rho_{mix,3} = \frac{m_{tot}}{\frac{D}{P_3} + B}$	$m_{tot}, m_g, m_o, m_w, P_3, \rho_{mix,3}$
f_8 :	$\rho_{mix,4} = \frac{m_{tot}}{\frac{D}{P_4} + B}$	$m_{tot}, m_g, m_o, m_w, \rho_{mix,4}$
Where		
D:	$\frac{m_g R T}{M_g}$	
B:	$\frac{m_o}{\rho_o} + \frac{m_w}{\rho_w}$	
$\bar{\rho}_{mix}$	$\frac{\rho_{mix,3} + \rho_{mix,4}}{2}$	

Table 2.3: Parameters in the system equation

Parameter	Value	Explanation	Units
M_g	16.04×10^{-3}	Molar weight of natural gas (assumed pure CH_4)	kg/mol
ρ_{gas}	0.656	Density of natural gas in the reservoir	kg/m^3
ρ_{oil}	800	Density of oil at STP	kg/m^3
ρ_{water}	1000	Density of water at STP	kg/m^3
A	5.56	Cross-sectional area of valve	cm^2
k_i		Constant in Darcy's law, system specific	
R	8.314	Gas constant	$J\,mol/K = m^3\,Pa\,mol/K$
L	1000	Length of the pipeline	m
T	373	Temperature, assumed constant	K
P_1	300	Reservoir pressure	bar
P_4	100	Pressure after valve	$Pa = kg\,s^2/m$
g	9.8	Gravity constant	m/s^2
C_d	0.98	Discharge coefficient	*no units*
z	[0, 1]	Valve opening	*no units*
f(z)	z	Valve characteristic	*no units*

Chapter 3

Solving the Model Equations

3.1 Constructing the incidence matrix

The complete system of equations describing the flow from the reservoir through the top valve was given in the previous chapter. They are repeated here in Table 3.1. In this chapter the equation system will be solved.

The equation set is a sparse set of non linear equations with 8 equations and 8 unknown variables

For these type of equations it is often possible to find subsets of the equations [7]. Some of these subsets can be solved by themselves and the solution of one subset can be used to solve another until all the subsets are solved. The underlying structure of these subsets can be found by constructing the equations incidence matrix. Where the rows of the matrix is the equations, and each column represent a unknown variable. If a variable occur in one of the equations, it will get a "1" in that row, if the space is filled with a "0" it indicates that an equation is not depending on that variable. The incidence matrix of the well model equation is given in Figure 3.1

There are different approaches to locate the subset of equations. One approach is to move the rows and columns around. The goal is to get a lower triangular matrix by moving the row with the most "1's" to the bottom and the row with the least"1's" to the top. In addition the to moving the columns around as well. A more systematic approach is following an algorithm starting by dedicating one variable to each equation, and then drawing up a node for each equation (also called graph theory), and connecting all the equations depending on the same variable. The graph theory approach was done for this system of equations, however all the nodes collapsed into one node, indicating that the equations cannot be divided into subsets to be solve individually.

Since the systematic graph theory approach failed to give a simple way to solve the sytem of equations, the rows and columns are moved around in order to sort the incidence

Table 3.1: The equations describing the system, and the unknown variables

Eq. nr.	Equation	Variable
f_1 :	$m_{tot} - m_g - m_o - m_g - m_w = 0$	m_{tot}, m_g, m_o, m_w
f_{2a} :	$m_g - k_g\left(P_2^2 - P_1^2\right) = 0$	m_g, P_2
f_{2b} :	$m_g - GOR \cdot m_o = 0$	m_g, m_o, P_2
f_3 :	$m_o - k_o\left(P_2 - P_1\right) = 0$	m_o, P_2
f_4 :	$m_w - k_w\left(P_2 - P_1\right) = 0$	m_w, P_2
f_5 :	$p_2 - P_3 - \frac{m_{tot} g P_2}{D + P_2 B} z = 0$	$m_{tot}, m_g, m_o, m_w, P_2, P_3$
f_6 :	$m_{tot} - C_v\sqrt{\bar{\rho}_{mix}(P_4 - P_3)} = 0$	$m_{tot}, P_3, \rho_{mix,3}, \rho_{mix,4}$
f_7 :	$\rho_{mix,3} = \frac{m_{tot}}{\frac{D}{P_3} + B}$	$m_{tot}, m_g, m_o, m_w, P_3, \rho_{mix,3}$
f_8 :	$\rho_{mix,4} = \frac{m_{tot}}{\frac{D}{P_4} + B}$	$m_{tot}, m_g, m_o, m_w, \rho_{mix,4}$
<u>Where</u>		
D:	$\frac{m_g RT}{M_g}$	
B:	$\frac{m_o}{\rho_o} + \frac{m_w}{\rho_w}$	
$\bar{\rho}_{mix}$	$\frac{\rho_{mix,3} + \rho_{mix,4}}{2}$	

matrix into a lower triangular matrix. Form the matrix (Figure 3.1))we see that equation f_2, f_3, f_4 contains only two 1's and will be placed on the top. Equation f_7 contains six 1's and will be placed on the bottom, and the rest of the equations will be placed in order between them. The sorted incidence matrix is shown in Figure 3.2. As the graph theory already indicated, the matrix shows that it is not possible to create subset of equations to be solved for this system. All the equations must therefore be solved at the same time.

$$
\begin{array}{c}
\\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} m_{tot} & m_o & m_g & m_w & P_2 & P_3 & \rho_{mix,3} & \rho_{mix,4} \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

Figure 3.1: The incidence matrix of the well model equations

$$
\begin{array}{c}
\\
f_2\\ f_3\\ f_4\\ f_1\\ f_5\\ f_7\\ f_8\\ f_6
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} P_2 & m_g & m_o & m_w & m_{tot} & P_3 & \rho_{mix,3} & \rho_{mix,4} \end{array}\\
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0\\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 0\\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$

Figure 3.2: The incidence matrix of the well model equations

3.2 Solving the system by iteration

The trick to solve this set of equations is to realize that the mass flow is a preserved quantity. Consequently, the total mass flow found in the reservoir form equation Equation f_1 (Darcy's law), must be equal the mass flow given in for example Equation f_6 (valve equation). This can be used to iterate on. To find a good iterative variable one can examining the incidence matrix closer. The Equation f_6 is taken out of the system and this equation will function as a control equation. The iterative variable is chosen to be P_2. One can now iterate on the error between the total mass flow found in equation f_1 and the mass flow found the f_6 at a given pressure in node 2. If the error is zero the guessed P_2 is correct.

The iterative variable P_2 and the control equation are indicated as a blue column and blue row respectively (Figure 3.2). With this variable and equation parted from the equation system the "inner matrix" is a lower triangular matrix, which can easily be solved. The iteration is done by *fzero* in Matlab.

Chapter 4

Introduction Optimizing the Manifold

Now that the modeling part of the project is done, the well behavior will be studied by using two test cases. A manifold consisting of two wells are constructed in order to study how the two wells are effecting each other. This also makes it possible to optimize the manifold operation, by finding the best way to operate the two wells together.

In addition to the well model based only on first principles, three different GOR expressions are used as a basis for well models. The GOR expression will affect the expression for the gas flow rate. This is done in order to study different types of wells. The models will be named after which GOR expression is used in the model.

The well behavior can be indicated by constructing a plot with the boundaries for when the well is a pure oil or pure gas well. The wells examined are mix-phase wells and will lay somewhere in between these boundaries. When studying a manifold it can be useful to know if the well is closer to a pure liquid- or gas-phase well. These plots give information on how the production of gas from the mixed-phase wells are compared to a pure-phase well.

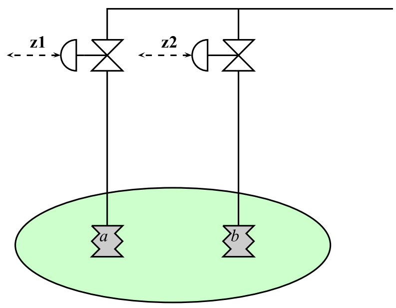


Figure 4.1: The manifold consisting of two wells

The main focus in this section is the optimization of the manifold, where the aim is to maximize the oil production. The production is however subject to limitations on the gas flow. The goal with the optimization is to find the optimal way to operate the two wells together, in order to produce as much oil as possible without violating the gas constraint.

In the case where the manifold is operated under constrained conditions, we already know the optimal solution. The optimization criteria is described in Urbanczyk and Wattenbargers article *Optimization of well Rates under Gas Coning Conditions* [3]. They state that the optimal way to operate a manifold is to keep the following ratio equal for all the wells

$$\left(\frac{\partial q_g}{\partial q_o}\right)_{well\ a} = \left(\frac{\partial q_g}{\partial q_o}\right)_{well\ b} \tag{4.1}$$

In the article the optimization condition is given on a volumetric basis. The well models developed here are expressed in terms of mass, thus the optimization condition will also be given on a mass basis.

$$\left(\frac{\partial m_g}{\partial m_o}\right)_{well\ a} = \left(\frac{\partial m_g}{\partial m_o}\right)_{well\ b} \tag{4.2}$$

The two well manifold will always be operated under constrained conditions. This means that the optimal solutions found after optimizing the manifold should always follow the optimization condition. The expressions (Equation 4.2) for the well models will therefore be found, and tested with the values obtained from optimization of the system.

Chapter 5

The Gas-Oil-Ratio

The gas-oil-ratio (GOR) is the relation between the amount of gas out of a well system over the amount of oil. This relation can be express either on volumetric basis or on mass basis, as shown in Equation 5.1 and 5.2 respectively. Since all the equations for the physical model is expressed in terms of mass, the GOR based on relation between the mass flows will be used.

$$\mathrm{GOR}_q = \frac{q_g}{q_o} \tag{5.1}$$

$$\mathrm{GOR}_m = \frac{m_g}{m_o} \tag{5.2}$$

The GOR expresses how much gas i produced per amount of oil from a manifold. Usually, it is desired to produce as little gas per amount of oil as possible (i.e a low GOR). Different GOR models will effect the behavior of the system, and therefore how to run it at optimal conditions.

In this report four different GOR models will be studied. The first is the GOR relation which is directly derived from the equations for mass flow expressed by Darcy's law. The three others are made up to give the behavior we want to investigate, namely an increase in the GOR when the production increases. Meaning that when the oil production goes up, so does the production of gas in the system.

The GOR relations to be studied at are presented under, where GOR_D is the GOR derived by using Darcy's law, and GOR_{1-3} are ratios we made up.

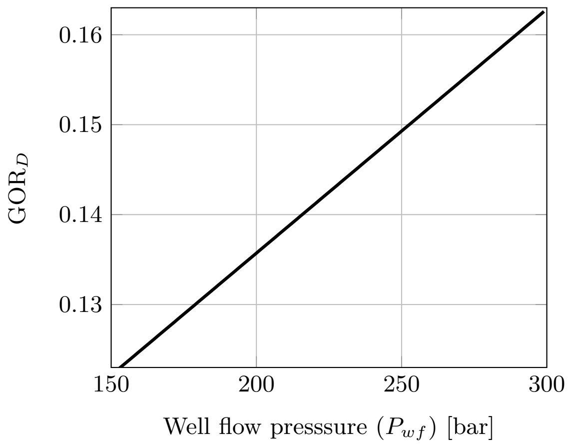
$$GOR_D = \frac{k_g}{k_o}(P_{wf} + P_R) \tag{5.3a}$$

$$GOR_1 = \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \tag{5.3b}$$

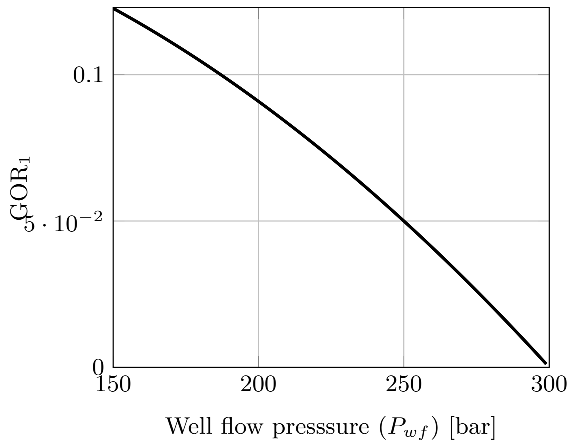
$$GOR_2 = \frac{k_g}{k_o}(P_{wf} - P_R) \tag{5.3c}$$

$$GOR_3 = \frac{k_g}{k_o}(P_{wf} - P_R)^2 \tag{5.3d}$$

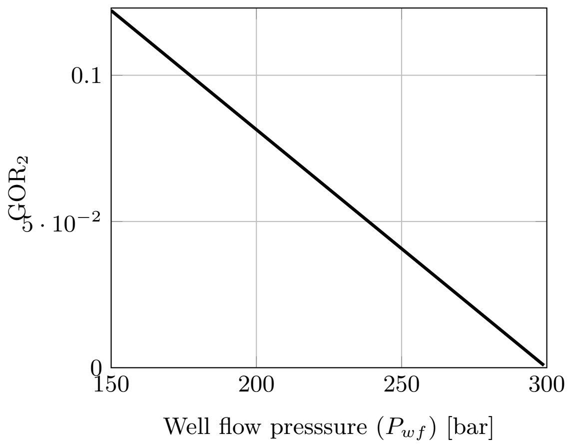
The expressions describe the change in GOR with increasing production ($P_{wf} \rightarrow 0$). The change in GOR can be predicted merely by studying the expressions. Increasing the production will cause GOR_D to decrease. For GOR_1 the increase will be the difference of the pressures squared. The expression for GOR_2 indicates that GOR_2 is linear increasing with production, while GOR_3 is quadratic increasing with production. The change in the GOR expressions with respect to increasing well production are also shown in Figure 5.1.



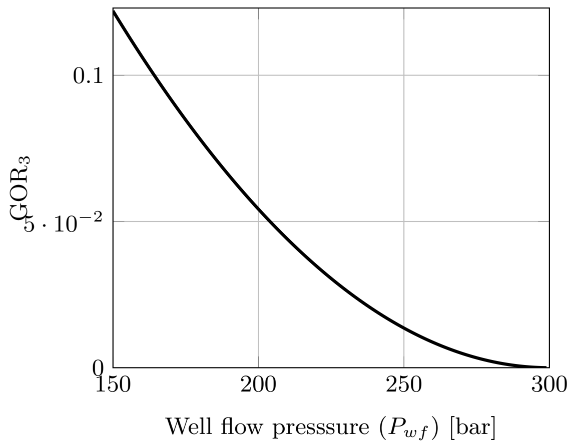
a) GOR$_D$ profile for physical model



b) Profile for model GOR$_1$



c) Profile for model GOR$_2$



d) Profile for model GOR$_3$

Figure 5.1: The gas-oil-ratio for each of the GOR models used in this report plot against the pressure at the wellhead. The plots show how the GOR changes with the production rare, where in this case maximum production is at P$_{wf}$ = 150, and no production is when $P_{wf} = P_R = 300$

Chapter 6

Case Studies

In the case study a two well manifold will be applied. Each manifold will consist of two wells noted well a and well b, the system is presented in Figure 6.1.

For all the well case studies two common base-wells will be used to calibrate the k-values used in the expressions for the mass flow rates in the reservoir. The two base-wells are presented in Table 6.1. Base-well 1 will be used to calibrate the a wells and base-well 2 will be used to calibrate the b wells for the models.

The mass flow rates for the oil and water will follow Darcy's law regardless of the GOR expression used for the well. The k-values for the oil and water expressions will therefore not change, and can be calculated only once. The expression for the gas flow rate change when the GOR changes and new k-values must be calculated for each GOR used for the well. The calculations are given next.

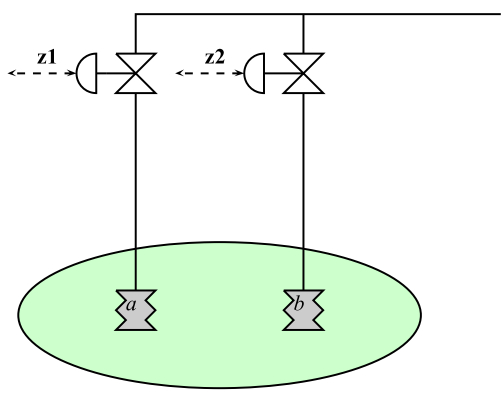


Figure 6.1: The two well model used in the test cases

Table 6.1: Base-wells used to calibrate the case studies

	Well 1	Well 2	Units
P_R	300	300	bar
P_wf	150	162	bar
m_g	56 429	98 892	kg/d
m_o	461 997	531 841	kg/d
m_w	144 599	501 450	kg/d

6.1 The mass flow rate expressions

The relation between the mass flow rates and the GOR_m is given in equation 6.1

$$GOR = \frac{m_g}{m_o} \tag{6.1}$$

The mass flow of the oil will be assumed to follow Darcy's law as before, and can therefore be expressed as:

$$m_o = k_o(P_{wf} - P_R) \tag{6.2}$$

Since the oil flow rate follow Darcy's law in all the cases, this means that when the GOR expressions changes, the expression for the mass flow rate must be adjusted accordingly. Rearranging Equation 6.1, the mass flow rate can be expressed as:

$$m_g = GOR \cdot m_o \tag{6.3}$$

The expression for the gas flow rate can then be found by applying the different GOR expressions to be studied (Equation 6.4a-d). The k_g-values can then be calculated using the two base-wells. All the k-values for all the different GORs are given in Table 6.2

$$GOR_D = \frac{k_g}{k_o}(P_{wf} + P_R) \tag{6.4a}$$

$$GOR_1 = \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \tag{6.4b}$$

$$GOR_2 = \frac{k_g}{k_o}(P_{wf} - P_R) \tag{6.4c}$$

$$GOR_3 = \frac{k_g}{k_o}(P_{wf} - P_R)^2 \tag{6.4d}$$

Table 6.2: k-values for the different wells cases

	Well a	Well b	Units
k_o	-3080	-3854	kg/bar
k_w	-964	-3634	kg/bar
$k_{g,GORD}$	-0.836	-1.551	kg/bar^2
$k_{g,GOR1}$	0.0056	0.0112	kg/bar^3
$k_{g,GOR2}$	2.508	5.1928	kg/bar^3
$k_{g,GOR3}$	-0.0167	-0.0376	kg/bar^3

6.2 Well types

When examining a well, it can be useful to know what type of well one is studying. One way to describe a well behavior is by plotting the relative pressure to the relative mass flow. For this part it is assumed that the well only produces oil and gas, meaning that the presence of water is ignored. This did not affect the resulting behavior of the wells significantly, and made the calculations easier.

The relative mass flow is the total mass flow over the maximum mass flow from the well.

$$\frac{m_{tot}}{m_{tot}^{max}} \tag{6.5}$$

The maximum flow rate will occur when the well flow pressure is at minimum (P_{wf}=0), and is the maximum oil flow plus the maximum gas flow.

$$m_{tot}^{max} = m_o^{max} + m_g^{max} \tag{6.6}$$

By setting $P_{wf} = 0$ in the oil and gas flow expressions for the different well models, the maximum well flow is:

$$\text{GOR}_D: \quad m_{tot_D}^{max} = k_o P_R + k_g P_R^2 \tag{6.7a}$$
$$\text{GOR}_1: \quad m_{tot_1}^{max} = k_o P_R + k_g P_R^3 \tag{6.7b}$$
$$\text{GOR}_2: \quad m_{tot_2}^{max} = k_o P_R + k_g P_R^2 \tag{6.7c}$$
$$\text{GOR}_3: \quad m_{tot_3}^{max} = k_o P_R + k_g P_R^3 \tag{6.7d}$$

If the well only produced oil, k_g is zero, and likewise if the well only produced gas, k_o is zero. This can be used to graph the boundary-curves for the wells in the plot, one reference curve for only liquid flow and one for only gas flow. The graph for the two-phase wells we are investigating will be somewhere in between these two reference curves.

The relative mass flow rate expression will vary for the different well models depending on the GOR expression. The expressions describing the well behavior are given below (Equation 6.8a-d). The complete derivation of the expressions can be found in Appendix A.

$$\text{GOR}_D: \quad \frac{m_{tot_D}}{m_{tot_D}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\frac{P_{wf}}{P_R}\right)^2 \tag{6.8a}$$

$$\text{GOR}_1: \quad \frac{m_{tot_1}}{m_{tot_1}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{P_{wf}}{P_R}\right)^2 - \left(\frac{P_{wf}}{P_R}\right)\right) \tag{6.8b}$$

$$\text{GOR}_2: \quad \frac{m_{tot_2}}{m_{tot_2}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{P_{wf}}{P_R}\right)^2 - \left(\frac{2P_{wf}}{P_R}\right)\right) \tag{6.8c}$$

$$\text{GOR}_3: \quad \frac{m_{tot_3}}{m_{tot_3}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{3P_{wf}}{P_R}\right)^2 + \left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{3P_{wf}}{P_R}\right)\right) \tag{6.8d}$$

Where a is defined as:

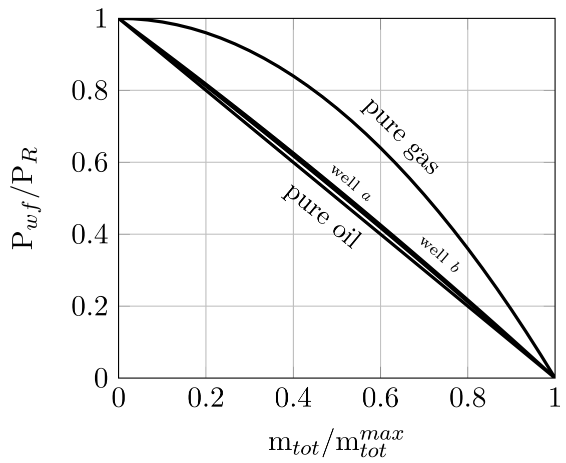
$$\text{a} = \frac{m_o^{max}}{m_g^{max} + m_o^{max}} \quad \text{and likewise} \quad \text{(1-a)} = \frac{m_g^{max}}{m_g^{max} + m_o^{max}} \quad , \quad 0 \leq a \leq 1$$

A typical well type is a “Solution-Gas Drive Well”, the properties of such wells are described in Vogel’s article *Inflow Performance Relationships for Solution-Gas Drive Wells* from 1960 [9]. In this type of wells the GOR decreases when the production increases ($P_{wf} \rightarrow P_R$). This corresponds with the GOR behavior for the well based on first principles, which was linearly decreasing with increasing production.

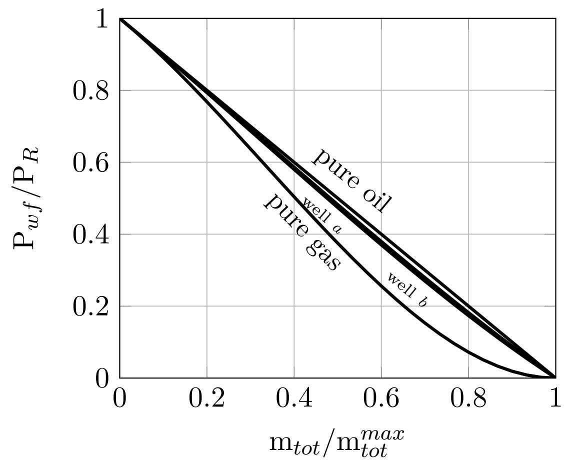
The three other models shows an opposite behavior. When the production is increased, the amount of gas per amount of oil increases. This behavior was also found when analyzing the wells based on GOR_1, GOR_2 and GOR_3 in Chapter 5.

The well based on an increase of the quadratic difference, GOR_1, resembles a pure liquid well (Figure 6.2b), whereas the linear increasing (GOR_2) (Figure 6.2c) is closer to a pure gas phase well model. The well described by the quadratic increasing relation (GOR_3) is placed somewhere in between and will resemble a mixed-phase well (Figure 6.2d).

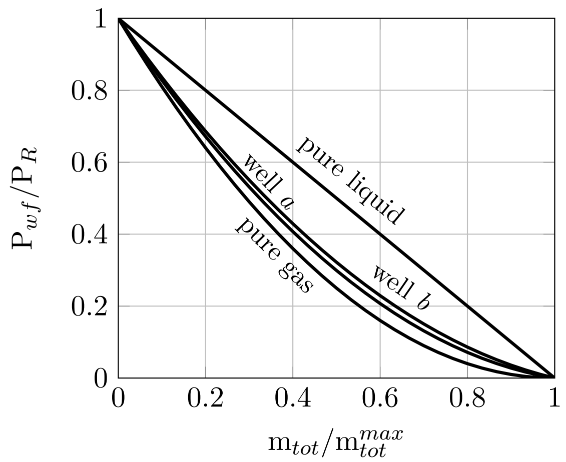
An interesting observation is that the behavior of the well depends strongly on the GOR expression used for the model. The well models are all calibrated from the the same base-well (Table 6.1), meaning that the amount of oil and gas present in the reservoir are the same, the well shows different behaviors depending on the GOR expression.



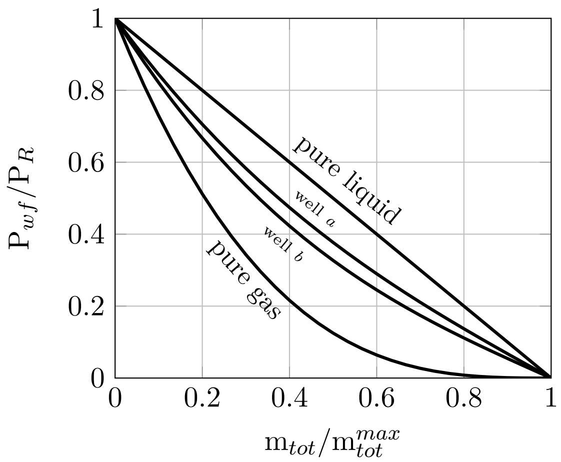
a) Well behavior for GOR_D well *a* and *b*



b) Well behavior for GOR_1 well *a* and *b*



c) Well behavior for GOR_2 well *a* and *b*



d) Well behavior for GOR_3 well *a* and *b*

Figure 6.2: In this figure the relative pressure ($\frac{p_{wf}}{P_R}$) is plot against relative mass flow rate for well the different models. The boundary conditions for a pure liquid and a pure gas phase well is also plotted. The well models studied are placed in between the boundaries of a pure-phase well.

Chapter 7

Optimizing the Manifold

7.1 Theory

In many engineering problem you are left with one (or more) degrees of freedom, i.e variables that can be manipulated to give several different solutions. In these cases there will be a numerous amount of solutions. As a consequence, the solution found depends on the value(s) of the manipulative variable(s) in the system. This extra variable can be used to optimize the process, resulting in the best performance possible of the system. In a complex problem it is not always easy to immediately say which solution is the best, and algorithms to identify the optimal solution, that is to say the optimal values of the manipulative variables, must be used.

A general optimization problem can be formulated:

$$\max_x \; f(x) \quad \text{or} \quad \min_x \; -f(x) \tag{7.1a}$$

Subject to:

$$h(x) = 0 \tag{7.1b}$$
$$g(x) \leq 0 \tag{7.1c}$$

The objective function f is the function to be minimized or maximized by varying the decision (or manipulative) variable x. The optimization is limited by the equation $h(x) = 0$ and the inequality $g(x) \leq 0$, making this a constrained optimization problem. This means that the optimal value of x must satisfy the constraints. The constraints makes up a feasible region limiting the performance for the system. Not satisfying the constraints might lead to unrealistic values of the process variables and unfeasible solutions.

7.2 The optimization problem

The optimization problem is to maximize the oil production from the system. The cost function will therefore in this case be the amount of oil, which is to be maximized. In addition, there is an upper limit on how much gas that can be extracted from the system. During the optimization this constraint is always active, meaning that the manifold is at maximum production. When the gas constraint it reached the oil production is limited since producing more oil would result in more gas as well, violating the gas constraint.

Since the goal is to maximize oil flow subject to a constraint on the gas flow, it will be optimal to produce most from the well with the lowest GOR value. However, it is not clear how to operate the wells together. As the production increase the GOR value are affected, which causes the GOR for each well to change during production. The goal with the optimization is to find the optimal ratio to produce from the two wells for a given gas constraint.

The manipulating variables of the system are the valve openings, z. The optimization solution will be how to operate the valves for the two wells, in order to produce as much oil as possible without violating the gas flow constraint. There is also a constraint considering the valves, since the valve opening can only operate between fully open or fully closed. Mathematically the problem can be stated:

Cost function

$$\max_{z_a, z_b} (m_{o,a} + m_{o,b}) = \min_{z_a, z_b} -(m_{o,a} + m_{o,b}) \tag{7.2a}$$

Subject to the following constraints:

$$m_{g,a} + m_{g,b} \leq m_g^{max} \tag{7.2b}$$

$$0 \leq z_i \leq 1 \qquad i = a, b \tag{7.2c}$$

The wells are optimized for different gas constraints, and for each constraint there is an optimal solution. The result of the optimization will therefore show how the solution varies with different constraints. For each gas constraint value the optimal solution for the valve openings, the pressure before the valve and the optimal oil flow rate out of the manifold is found.

The well optimization problem is solved using *fmincon* in Matlab. This integrated Matlab function finds the constrained minimum of a multi-variable function, and solves problems on the same form as the well optimization problem faced here. In this case we want to maximize the oil production, and must therefore try to minimize the negative value of the oil production, since *fmincon* minimizes the function value.

7.2.1 Optimal operation of the well model based on physical equations

As shown earlier, the GOR for the model based on first principals and physical equations is:

$$GOR_D = \frac{k_g}{k_o}(P_{wf} + P_R)$$

The flow rates of oil and gas are in this model described by Darcy's law.

$$\begin{aligned} m_o &= k_o(P_{wf} - P_R) \\ m_g &= k_g(P_{wf}^2 - P_R^2) \end{aligned}$$

The equations show that maximum production will be when P_{wf} is equal to 0, and the minimum production, or no production, will occur when P_{wf} is equal to P_R. This result is consistent with the anticipation that this well model is a Solution-Gas Drive Well as we found from the GOR analysis executed earlier. In the same analysis we found that for this type of well, the production is at maximum the GOR is at it's minimum.

Consequently, the well with the lowest GOR should increase the production first, as the gas constraint increases. If the first well reaches its maximum production, without violating the constraint on the gas production, the well with the second lowest GOR should start producing and so on. Because of the simplicity of this well model the optimal solution is trivial, and optimization is not really needed as long as the GOR is known for the wells.

In order to see the effect different limits on the maximum gas mass flow will have on the system (Figure 7.1a-c), the problem is still solved as an optimization problem using *fmincon* in Matlab.

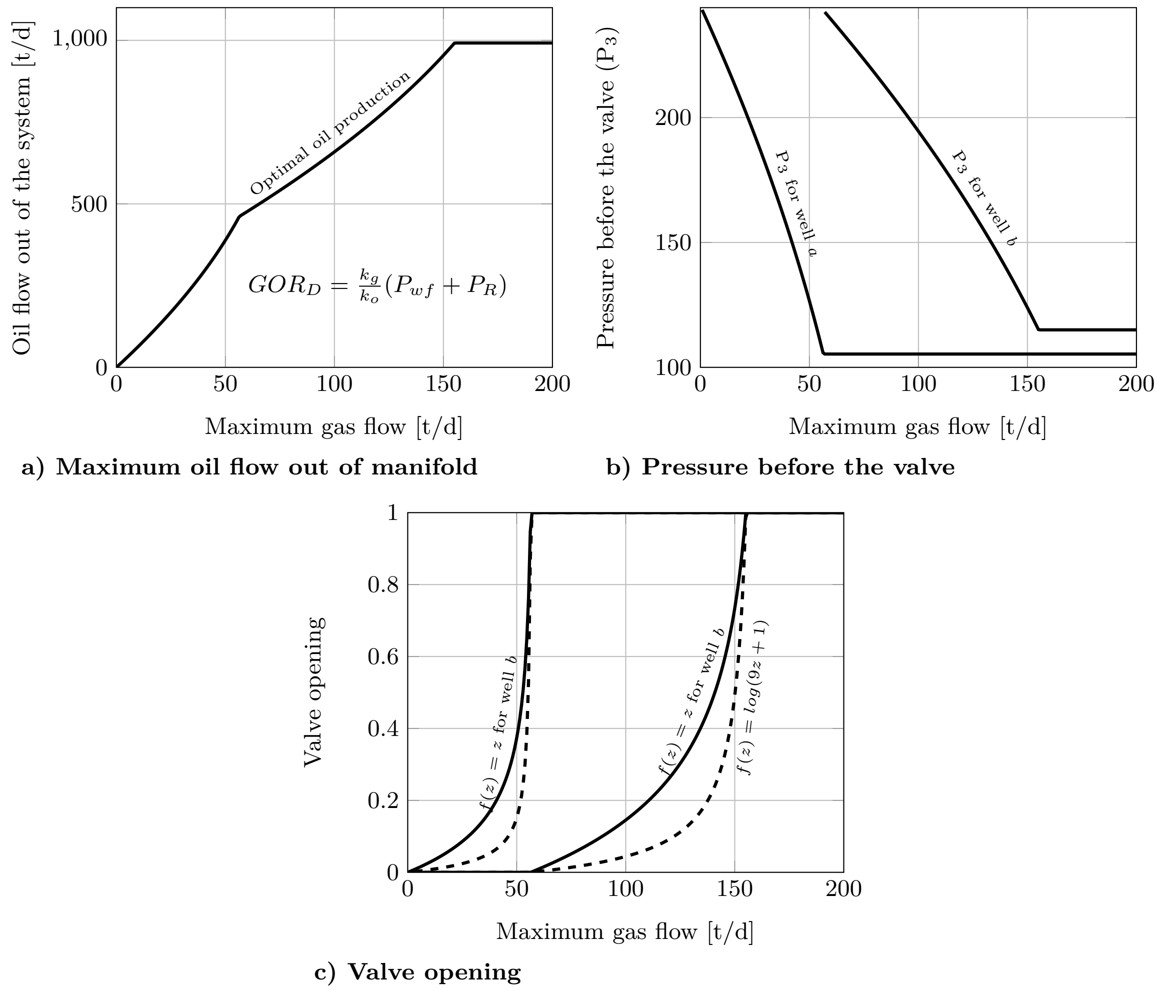


Figure 7.1: The optimal solution for the manifold where the well models are based on physical equations. Plot a) shows the maximum oil production as a function of the gas constraint value. In plot b) the pressure profile for the pressure before the control valve is plot, and the actual opening of the valve is shown in plot c).

7.2.2 Optimization of the models based on a gas-oil-ratio

The same optimization scheme was run for the three models the GOR expression:

$$
\begin{aligned}
GOR_1 &= \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \qquad (7.5a)\\
GOR_2 &= \frac{k_g}{k_o}(P_{wf} - P_R)\\
GOR_3 &= \frac{k_g}{k_o}(P_{wf} - P_R)^2
\end{aligned}
$$

The solution is in these cases more complex than to max-out the well with the lowest GOR value first and then start producing from the well with the second lowest GOR. When the total production increases, the solutions will depend on the change in the GOR for both of the wells. The goal is to find the optimal way to produce form both of the wells simultaneously to maximize the total oil flow, while staying within the constraint for the gas production.

The solution is divided into two regions and they are divided by a blue line in the plots (Figure 7.2-7.4). One region occur when the two wells are increasing the production simultaneously. In this region the optimal solution is non-trivial. The optimal solution involves adjusting both wells at the same time as the gas constraint is raised. As the gas constraint increases so does the production form both the wells, causing the GOR to change. In this region optimization is needed in order to find the optimal way to operate the wells.

The other region is after one of the wells (well a) has reached maximum production. The optimal solution is now trivial. As the gas constraint increases, the only way to further increase the total production, is to increase the production from the other well (well b).

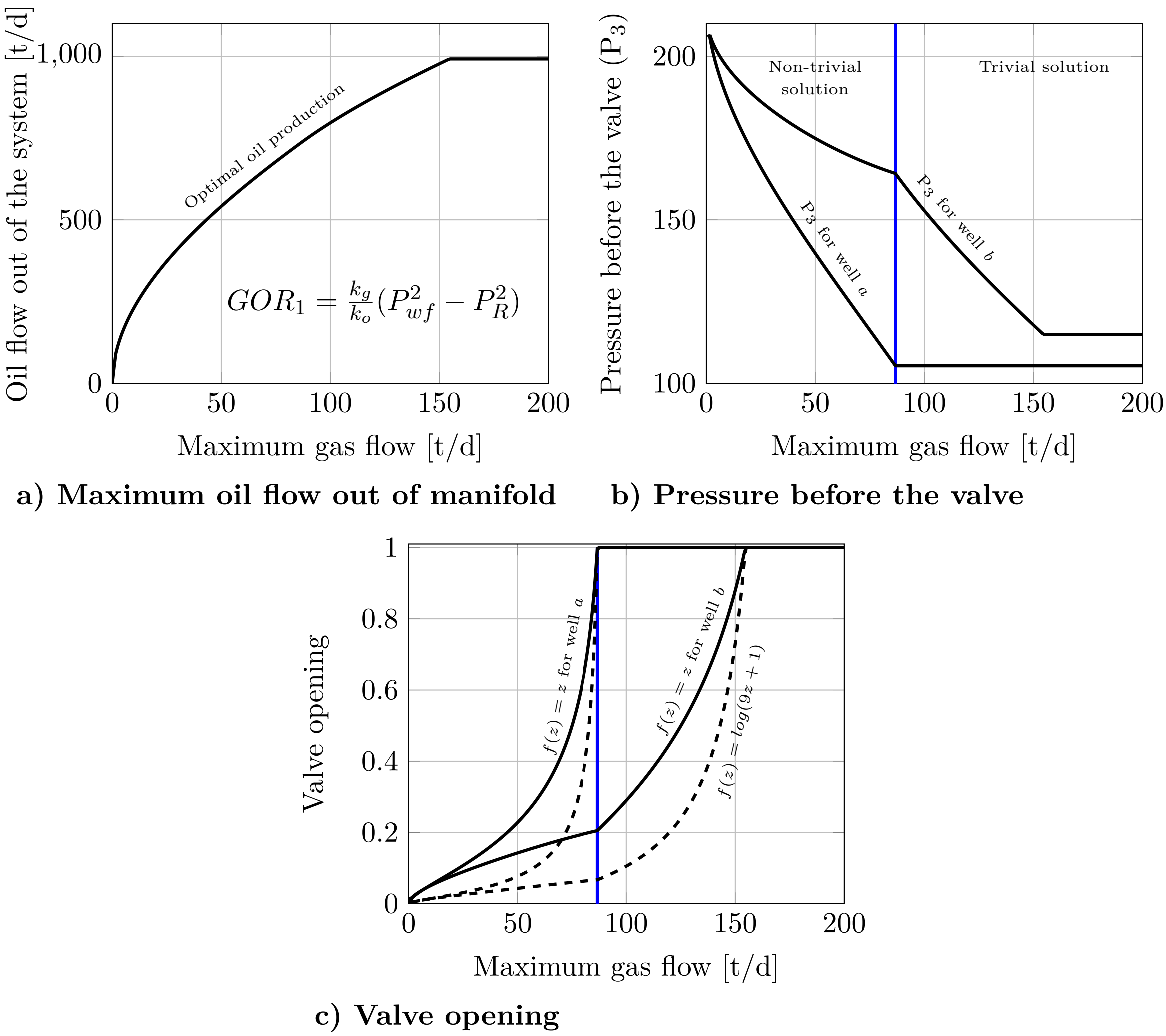
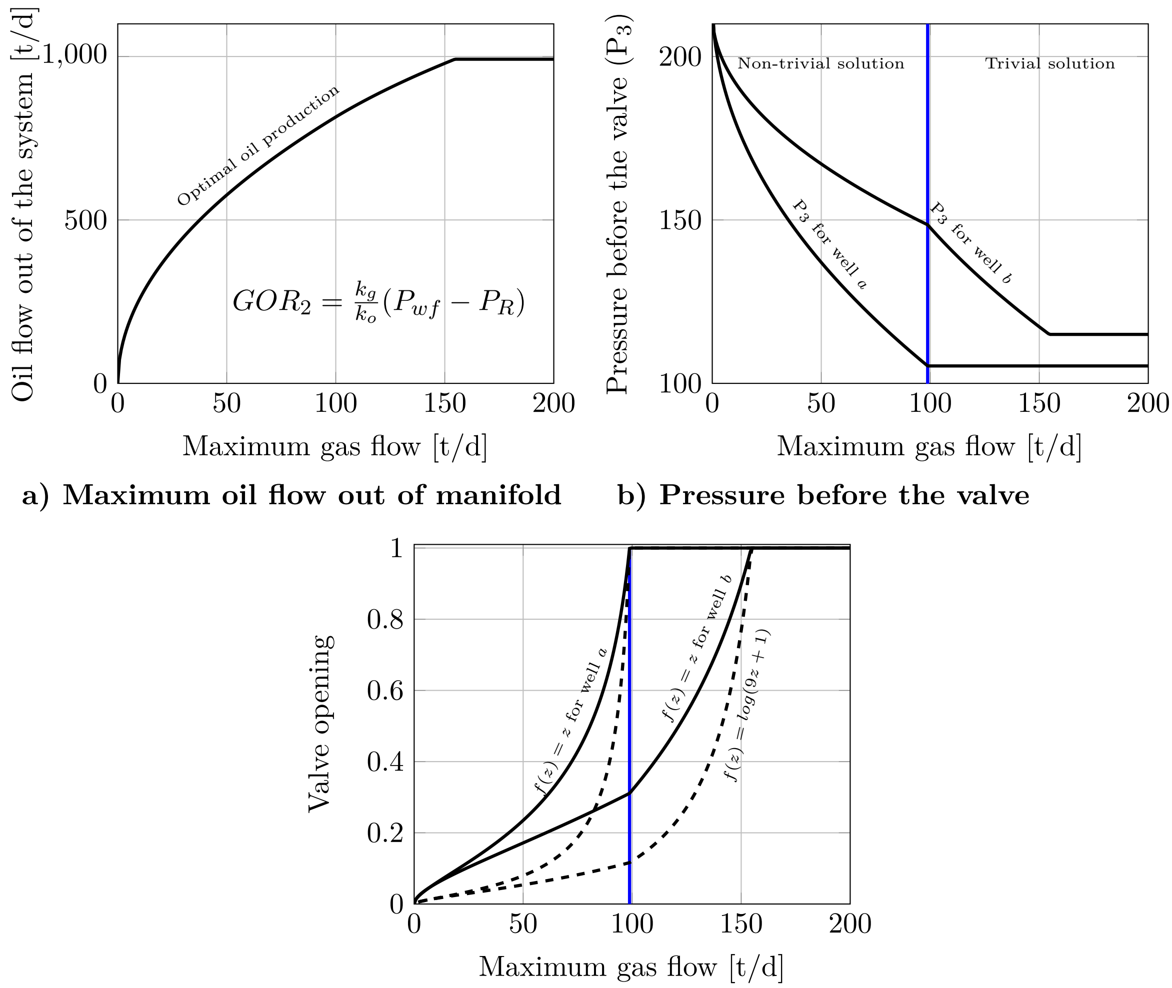


Figure 7.2: The optimal solution for the model based on the GOR_1 expression. Plot a) shows the maximum oil production as a function of the gas constraint value. In plot b) the pressure profile for the pressure before the control valve is plotted, and the actual opening of the valve is shown in plot c).

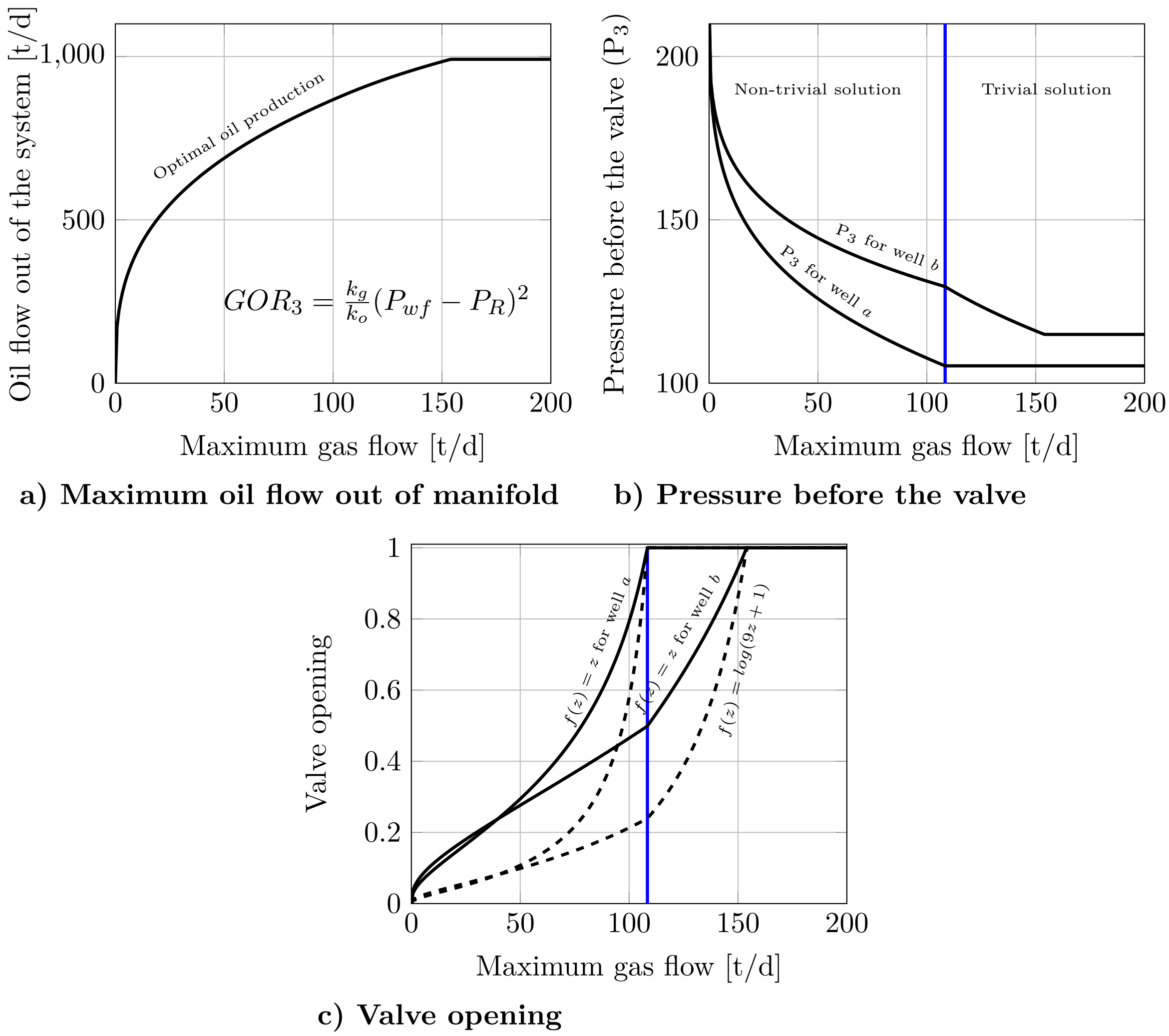


a) Maximum oil flow out of manifold

b) Pressure before the valve

c) Valve opening

Figure 7.3: The optimal solution for the model based on the GOR_2 expression. Plot a) shows the maximum oil production as a function of the gas constraint value. In plot b) the pressure profile for the pressure before the control valve is plotted, and the actual opening of the valve is shown in plot c).



a) Maximum oil flow out of manifold **b) Pressure before the valve**

c) Valve opening

Figure 7.4: The optimal solution for the model based on the GOR_3 expression. Plot a) shows the maximum oil production as a function of the gas constraint value. In plot b) the pressure profile for the pressure before the control valve is plotted, and the actual opening of the valve is shown in plot c).

7.2.3 Discussion

An important finding from the optimization is that the pressure profile for the pressure before the valve (P_3) is independent of the valve characteristic. This is valid for all the models studied. The optimal solutions for the valve opening are different depending on if a linear of logarithmic valve is used. Whereas the pressure profile before the valve is the same for both the valve types. To control the flow through the valve one can therefore control the pressure before it without taking into consideration the valve characteristic. For this reason, the further analyzes of the system will only take into account the change in the pressure before the valve.

Because the wells are calibrated and based on the same base-wells, they have the same maximum production of oil, which is around 1000 t/d. The optimization problem becomes unconstrained after the limit on gas production reaches just above 150 t/d. After this point the valves saturate, and even if we raise the allowance on maximal gas flow rate the oil production is constant. With this high allowance on the gas production the solution is simply to produce max from both the wells.

The model based on a decreasing GOR with increasing production, GOR_D

As expected, the optimal solution for the well based on physical principles is trivial. Until the pressure before well_a is constant, and maximum production for this well is reach (Figure 7.1b), the pressure before the valve in well_b is unchanged. This means that the valve in well_b does not start to open until the valve in well_a is fully open. Well_a is the well with the lowest GOR value, and as the gas constraint becomes higher the production from well_a increases until it reaches maximum production. Once well_a reaches maximum production, well_b starts producing. The production keep increasing until well_b also reaches maximum production.

The models based on an increasing GOR with increasing production

More interesting are the solutions for the three other well models, where the GOR are increasing when the production increases. The shift between the non-trivial and trivial solution is clearly visible in both the pressure profile and the valve opening plot. In all three cases both of the valves starts to open form the beginning (Figure 7.2c - 7.4c). As the limit on the maximum gas flow increases the production increases from both of the wells simultaneously.

The profile for the optimal oil production is even for the three GOR models, and does not show the same shift in the production rate as the physical model does. The shift between the two different regions,marked with a blue line, appears in the pressure profile plot (Figure 7.2b-7.4b). In the region with the non-trivial solution the pressure profiles are decreasing together in a certain ratio. Once well_a reach maximum production the

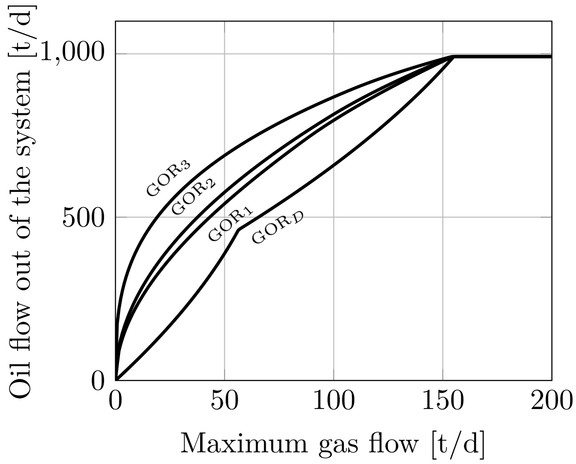


Figure 7.5: The optimal oil flow for all the four well models. The plot shows how the different oil flow patters varies depending on the model used to describe the system.

pressure profile flattens. After this point the pressure profile for well$_b$ is near linear, similar to the pressure profiles for the well based on first principles (Figure 7.1b).

Comparing the optimal oil flow profile for the three manifolds, the oil production rate from the wells are very different. The total oil production is highest from the manifold based on the GOR increasing with the quadratic difference in pressure (GOE$_3$). The wells in this manifold are increasing the production simultaneously much longer compared to the other manifold models. The effect is that the maximum oil production profile increases more rapidly for GOR$_3$ and is consistently higher than for the three other oil profiles. This result becomes very clear when the optimal oil flows are plot together (Figure 7.5). The effect diminishes as the limit on the gas production increases to the point where the problem becomes unconstrained. In this case the total oil production from the four different manifold models are the same.

This is however, not the same as saying that the GOR$_3$ model is better than the three others. A direct comparison between the well models is not possible, since they are based on different GOR expressions. We assume that the oil flow follow Darcy's law regardless of the GOR expression used as a basis, thus only the gas flow rate pattern is affected. This might explain why the oil flow out of the system are higher for GOR$_3$. The curves does however give a nice picture on how the different well models behave in terms of oil production as a function of different gas constraints (Figure 7.5).

7.3 Disturbing the system

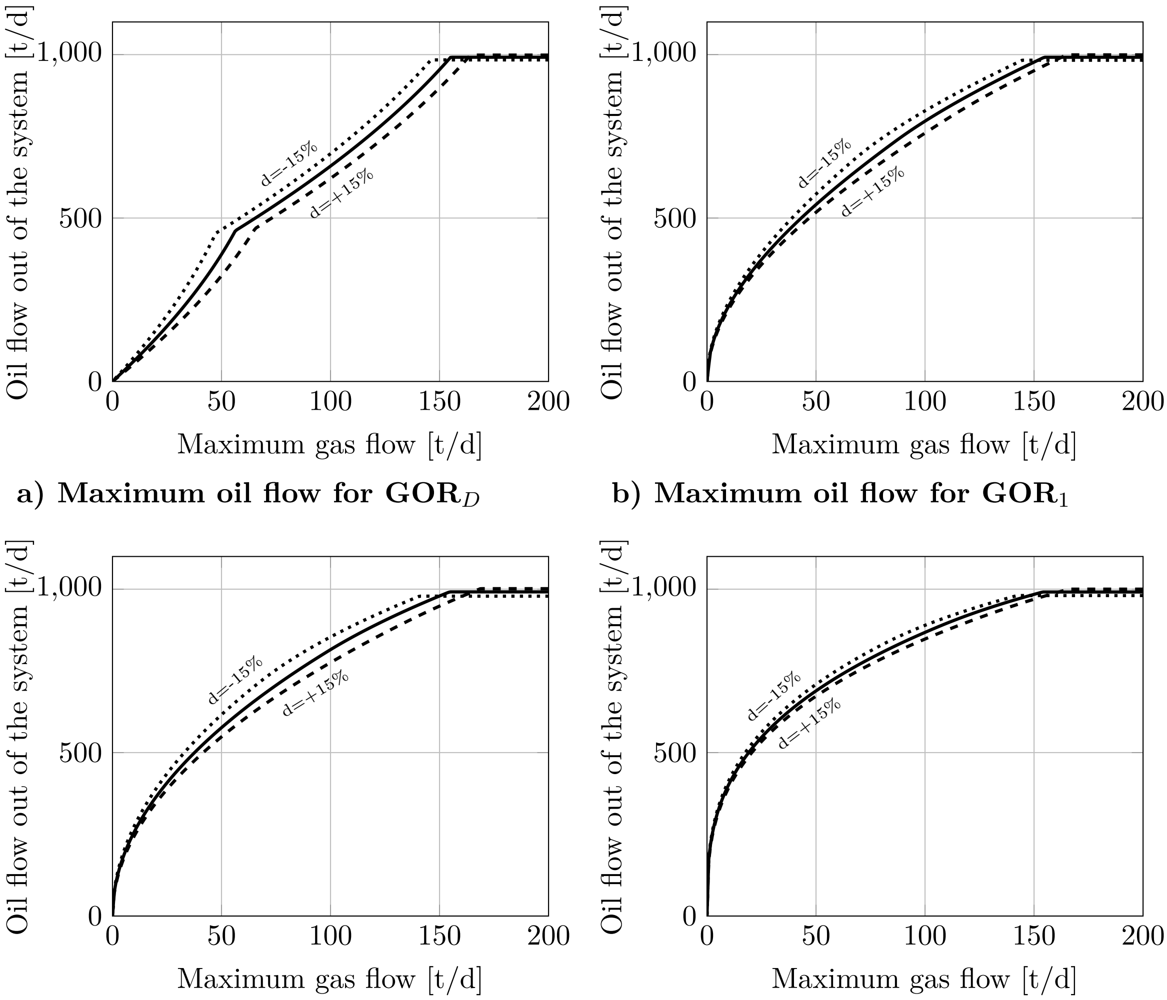
The optimization of the four different models were run with disturbances in the GOR for $well_a$. The GOR was increased with 15%, and then decreased with 15%. A decrease in the GOR leads to less gas produced per oil, which is a desired effect. The total oil production out of the manifold increases when the GOR decrease, compared to the situation with no disturbance. An increase in the GOR corresponds to more gas per amount of oil produced, in other words an undesired disturbance, and the total oil production decrease.

The effect of the disturbances on the different GOR models are shown in Figure 7.6 and 7.7. Where the solid line is the optimal solution without disturbances, the dashed line is for an increase in the GOR, while the dotted line is for the decrease.

The difference in the optimal oil production for the different disturbances are largest when the constraint is around 60-80 t/d. When the problem becomes unconstrained, the effect of the disturbances are relatively small on the total oil production. The effect is also relatively small for very strict constraints on the gas flow. When comparing the oil flow profiles for the different well-models, we see that the well model based on the GOR_3-expression is least effected by the disturbances.

Examining the pressure profile for the GOR_3 model (Figure 7.7d), we see that the effect of a disturbance in $well_a$ is hardly visible on $well_b$ before the gas constraint is raised over 100 t/d. For the three other well models the disturbance in $well_a$ is noticeable in $well_b$ almost straight away.

The pressure profiles indicates how much the pressure before the valve should change (up or down) to still be at optimal operation despite a disturbance. While the optimal oil flow indicated how much more or less oil is produced when a disturbance occur and the system is optimized. This simulation plots gives a nice illustration on how sensitive the optimal solution is to disturbances. Both in terms of profit and control action needed in order to deal with the disturbances.



a) Maximum oil flow for GOR$_D$

b) Maximum oil flow for GOR$_1$

c) Maximum oil flow for GOR$_2$

d) Maximum oil flow for GOR$_3$

Figure 7.6: The optimal oil flow for all the four models with disturbances in the GOR. The dashed line is for an increase of 15% in the GOR, while the dotted line is for a 15% decrease in the GOR.

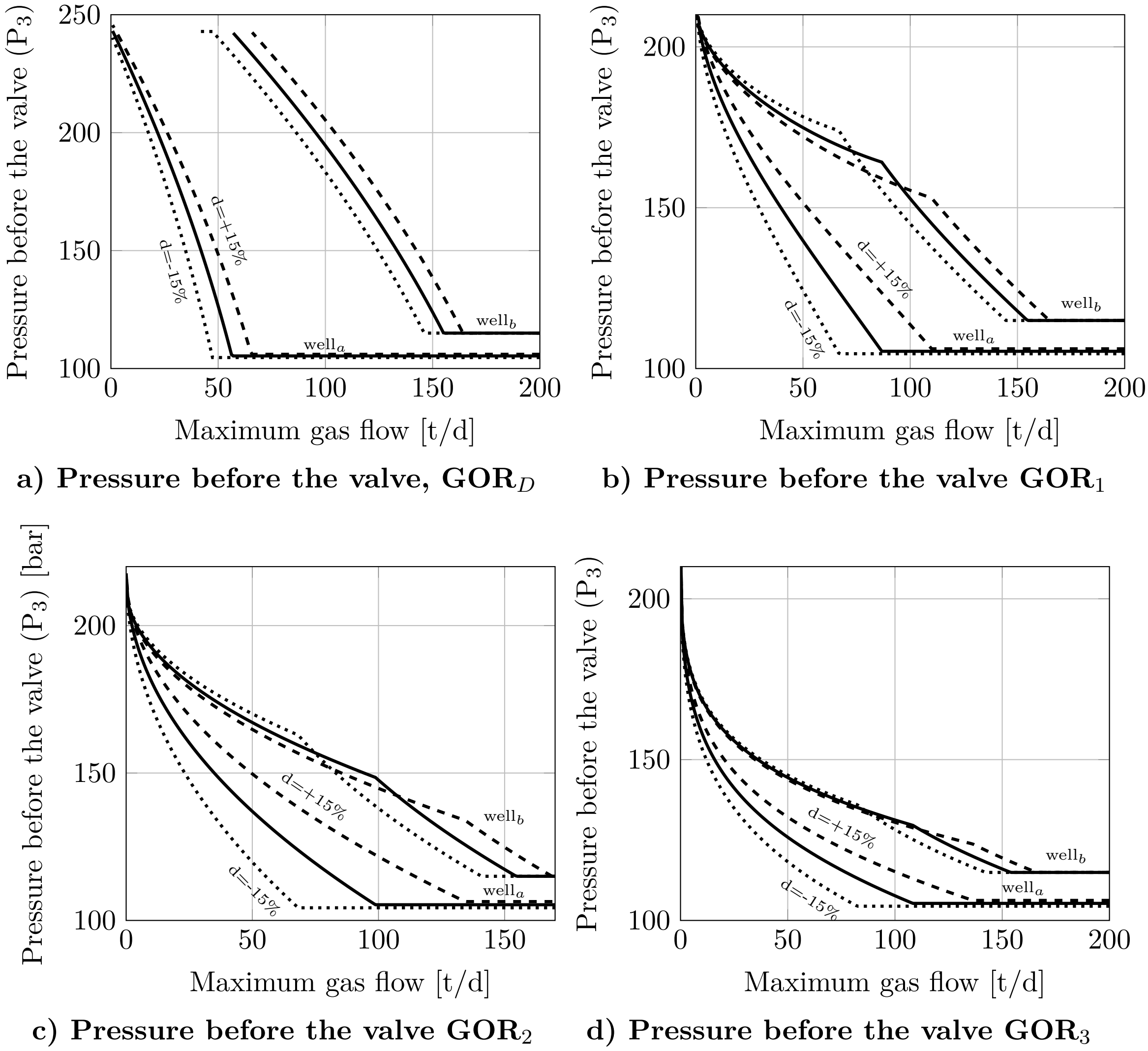


Figure 7.7: The optimal pressure before the valves in well$_a$ and well$_b$, with disturbances in the GOR. The dashed line is for an increase of 15% in the GOR, while the dotted line is for a 15% decrease in the GOR.

7.4 Optimization Criterion

7.4.1 Optimization of well rates when the gas constraint is active

When the manifold is operated under active constrain conditions the optimal solution is already known. It is derived and explained in Urbanczyk and Wattenbargers article *Optimization of well Rates under Gas Coning Conditions* [3].

An active gas constraint means that there is a certain amount of gas that can be produced from the manifold. This also limits the oil production, which is at maximum when the gas constraint is active. If the amount of gas (or oil) produced form one well increases, the equivalent amount of gas must decrease from the other wells in the manifold. If not, the total production from the manifold will violate the gas constraint. Similar, if the amount gas produced from one well decreases, the equivalent amount of gas must increase from the other wells. Otherwise the manifold is producing under the constraint limit, and therefore less oil than optimal.

To ensure optimal operation of the wells when the gas constraint is active, the change in the gas rate with respect to an infinite small change in the oil rate should be equal for all the wells in the manifold. The condition can be expressed as:

$$\left(\frac{\partial q_g}{\partial q_o}\right)_i = \left(\frac{\partial q_g}{\partial q_o}\right)_j \tag{7.6}$$

This condition holds for any number of wells in a manifold.

7.4.2 Deriving the optimization criteria for the models

In our case the optimization problem is defined such that the gas constraint is always active. Thus, the change in gas flow with respect to the oil rate in well$_a$ should be equal the change in well$_b$ for all gas constraints, as long as the two wells are increasing production simultaneously. Which is when the optimal solution is non-trivial. Since the well models used here are all expressed on mass basis, the optimization condition will be expressed in terms of mass as well.

$$\left(\frac{dm_g}{dm_o}\right)_a = \left(\frac{dm_g}{dm_o}\right)_b \tag{7.7}$$

When well$_a$ saturates at maximum production, the solution is trivial and only well$_b$ can increase the production further. Therefore, in this region only the GOR for well$_b$ changes. When this region is reached, the optimization criteria (Equation 7.7) is not valid anymore.

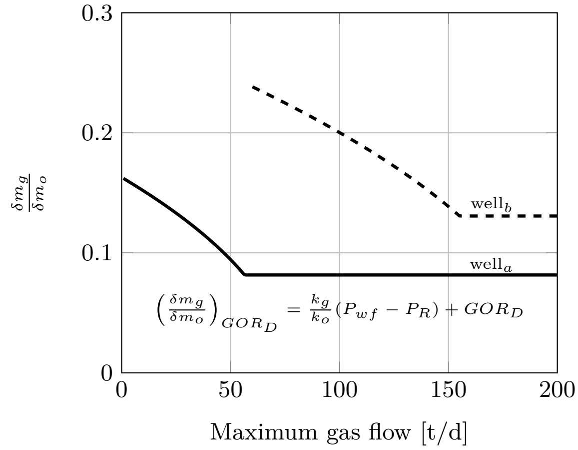
In order to test if the optimal solutions found by *fmincon* meet the optimization criteria, the derivatives (Equation 7.7) for each of the well models were derived.

$$
\begin{aligned}
GOR_D &= \frac{k_g}{k_o}(P_{wf} + P_R) \quad &: \quad \left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_D} &= \frac{k_g}{k_o}(P_{wf} - P_R) + GOR_D \\
GOR_1 &= \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \quad &: \quad \left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_1} &= 2\frac{k_g}{k_o}P_{wf}(P_{wf} - P_R) + GOR_1 \\
GOR_2 &= \frac{k_g}{k_o}(P_{wf} - P_R) \quad &: \quad \left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_2} &= \frac{k_g}{k_o}(P_{wf} - P_R) + GOR_2 = 2GOR_2 \\
GOR_3 &= \frac{k_g}{k_o}(P_{wf} - P_R)^2 \quad &: \quad \left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_3} &= 2\frac{k_g}{k_o}(P_{wf} - P_R)^2 + GOR_3 = 3GOR_3
\end{aligned}
$$

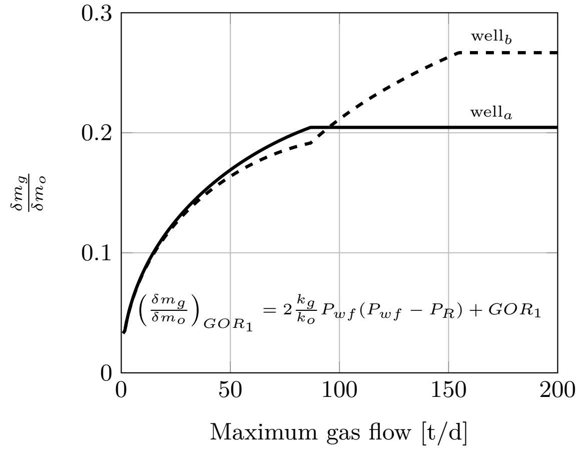
The derivation of the derivative expressions for each of the well models are presented in Appendix B.

The expressions for the derivatives shows a quite interesting result. For the well models studied here, the optimal operation of the wells are to keep the GOR for the two wells equal or close to equal.

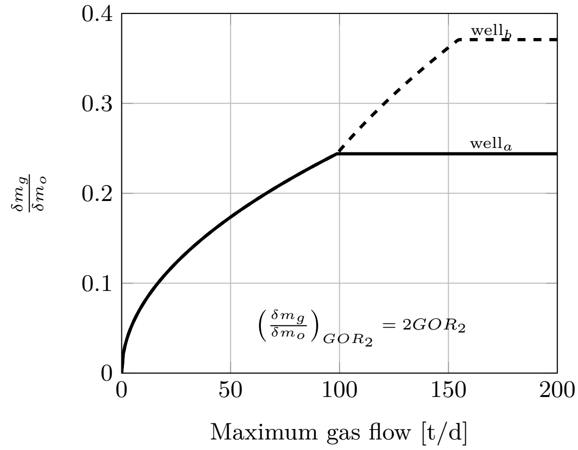
Now that the optimization criteria is derived, the optimal solutions are tested to see if they fit. All the optimal values are tested, both the case with no disturbance (Figure 7.8), and the cases where a disturbance was afflicted on the system (Figure 7.9 and 7.10).



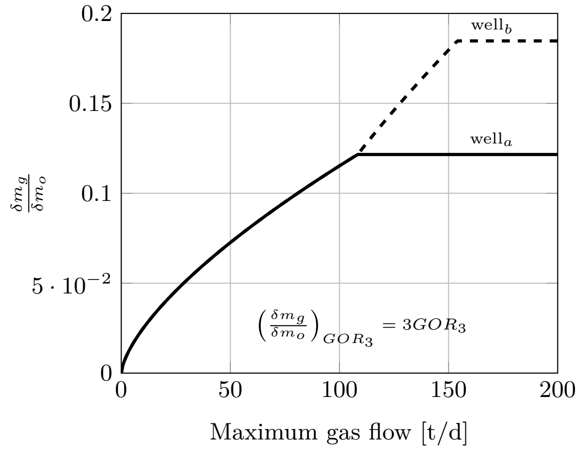
a) $\frac{dm_g}{dm_o}$ **for well model GOR**$_D$



b) $\frac{dm_g}{dm_o}$ **for well model GOR**$_1$

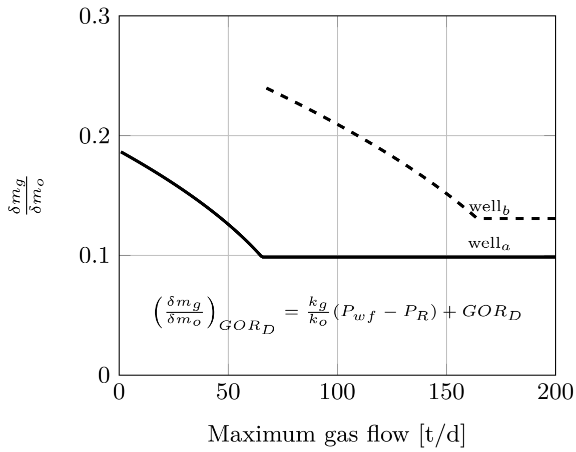


c) $\frac{dm_g}{dm_o}$ **for well model GOR**$_2$

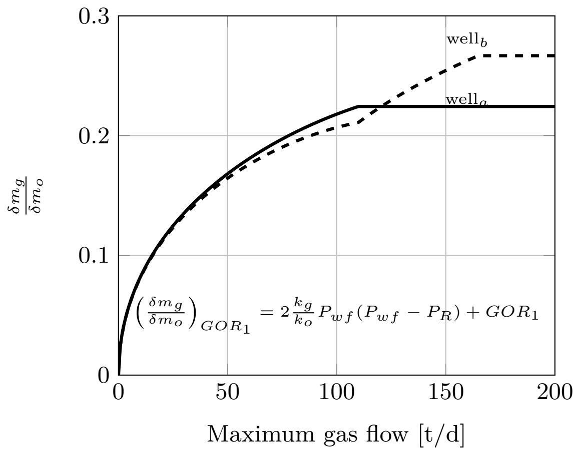


d) $\frac{dm_g}{dm_o}$ **for well model GOR**$_3$

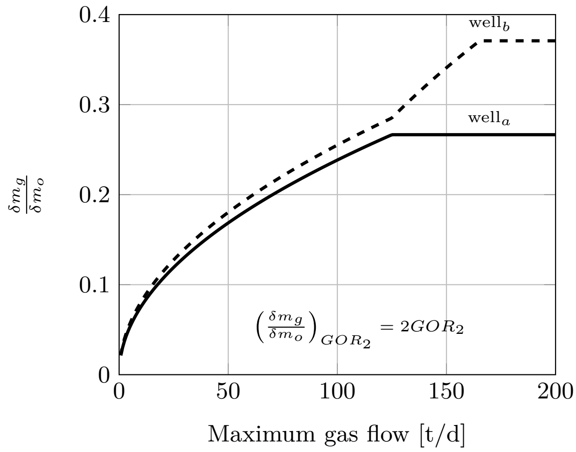
Figure 7.8: The optimization criteria for well$_a$ and well$_b$ for the four well models. Where only one slope is visible the expressions are equal and the solution is optimal. The point where they split and one on the profiles flattens is when well$_a$ has reached optimal solution.



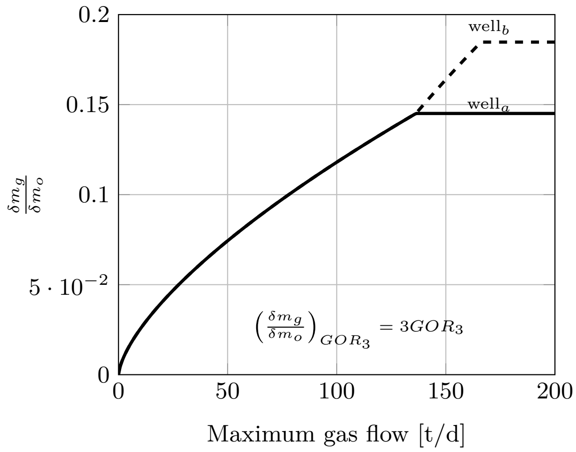
a) $\frac{dm_g}{dm_o}$ **for well model GOR**$_D$



b) $\frac{dm_g}{dm_o}$ **for well model GOR**$_1$

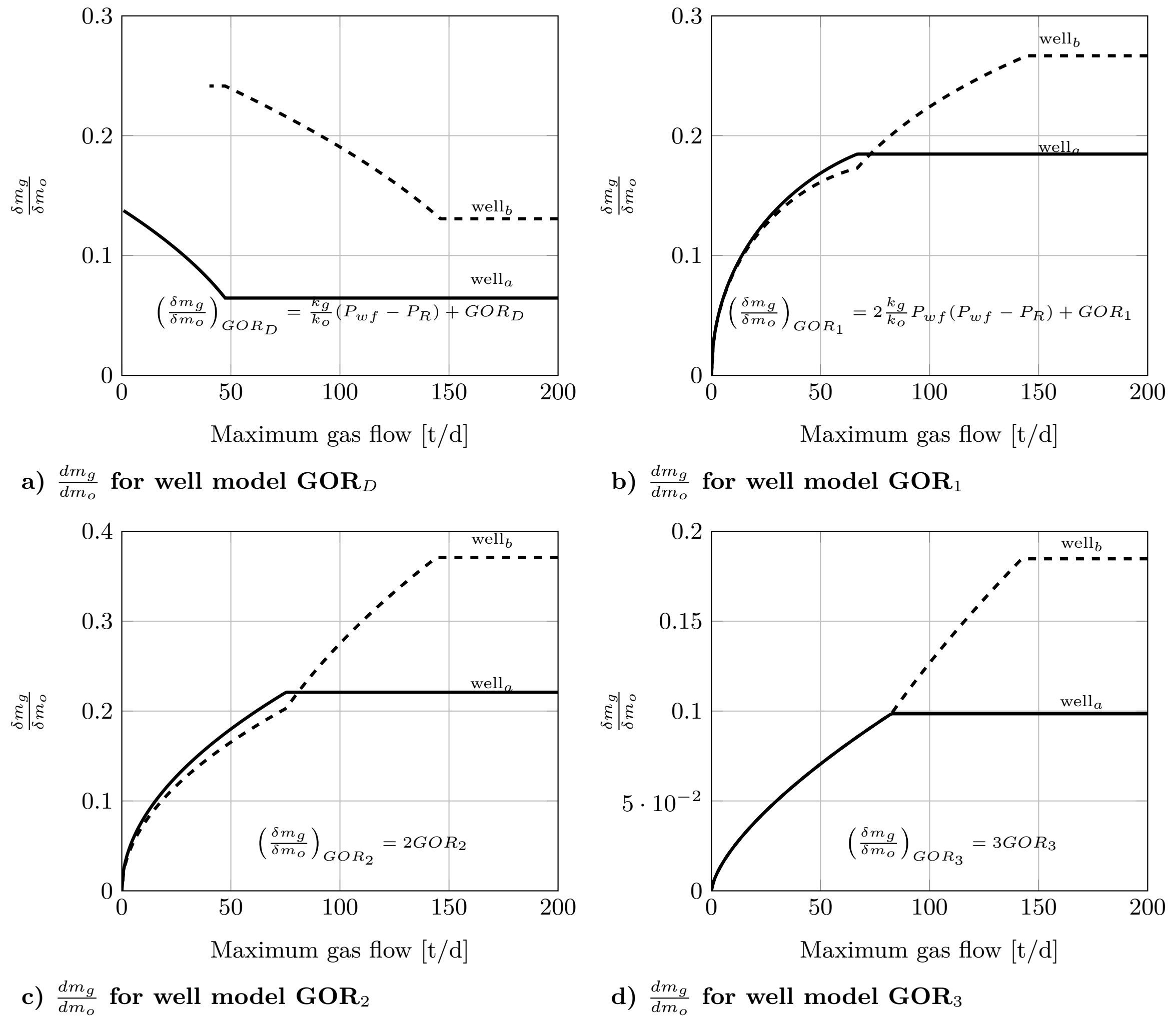


c) $\frac{dm_g}{dm_o}$ **for well model GOR**$_2$



d) $\frac{dm_g}{dm_o}$ **for well model GOR**$_3$

Figure 7.9: The optimization criteria for well$_a$ and well$_b$ for the four well models, for a +15% disturbance in the GOR in well$_a$. Where only one slope is visible the expressions are equal and the solution is optimal. The point where they split and one on the profiles flattens is when well$_a$ has reached optimal solution.



a) $\frac{dm_g}{dm_o}$ **for well model GOR$_D$**

b) $\frac{dm_g}{dm_o}$ **for well model GOR$_1$**

c) $\frac{dm_g}{dm_o}$ **for well model GOR$_2$**

d) $\frac{dm_g}{dm_o}$ **for well model GOR$_3$**

Figure 7.10: The optimization criteria for well$_a$ and well$_b$ for the four well models, for a -15% disturbance in the GOR in well$_a$. Where only one slope is visible the expressions are equal and the solution is optimal. The point where they split and one on the profiles flattens is when well$_a$ has reached optimal solution.

7.4.3 Discussion

The optimal solution found for GOR_2 and GOR_3 meet the optimization criteria precisely. For these two models the derivatives for $well_a$ and $well_b$ are equal as long as the optimal solution is non-trivial (Figure 7.8c and d). This gives reason to believe that the solution found for these well models are the correct optimal solution. If one were able to control the GOR, controlling them equal for the wells would ensure optimal operation of the manifold.

As explained earlier, the well based on first principles (GOR_D) has only a trivial solution; first let $well_a$ reach maximum production and then produce from $well_b$. Since the production is never increased simultaneously, the derivatives for $well_a$ and $well_b$ are never equal (Figure 7.8a).

For the model based on the GOR_1 expression, the derivatives are equal for strict gas constraints, however the values differ for higher gas constraints. The difference is minor and can be caused by numerical errors in the optimization. However due to the difference, it might be that the solution found is not truly the optimal solution.

The optimal solutions found with a disturbance in the GOR was tested against the criteria as well. The result should have been the same as for the non-disturbed process. However, for the well model based on GOR_2, we see that the GORs for $well_a$ and $well_b$ are not equal. The difference is minor, and we believe it is caused by a numerical mistake. Nevertheless, this difference can indicate that the solution found is not truly the optimal.

For all cases, the differences between the derivatives is made even clearer by plotting the derivative of $well_a$ minus the derivative for $well_b$. The magnitude of the difference is shown more clearly in these plots. These plots are shown in appendix B.

Chapter 8

Summary of Results

The gas-oil-ratio (GOR) for the model based of first principles was found to decrease with increasing production. This was not the behavior we were interested in, therefore three other models based on a GOR that increases with increasing production were also studied. The GOR for the models are presented below

$$
\begin{aligned}
GOR_D &= \frac{k_g}{k_o}(P_{wf} + P_R) \\
GOR_1 &= \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \\
GOR_2 &= \frac{k_g}{k_o}(P_{wf} - P_R) \\
GOR_3 &= \frac{k_g}{k_o}(P_{wf} - P_R)^2
\end{aligned}
$$

The models were used to construct a two-well manifold which was optimized. The system was optimized for both a linear valve and a logarithmic valve. The pressure drop profiles proved to be equal for both a linear and a logarithmic valve. The pressure before the valve is therefore independent of the valve characteristic for these two valve types.

We also found that the optimal solution for the two-well manifold is divided into two regions. In one of regions the optimal solution is non-trivial, this is where the well production is increased in both of the wells simultaneously. This is the region of interest. In the other region one of the wells reaches maximum production, and the solution is then trivial. As the limit on the gas flow increases, the optimal operation in this region is simply to increase the production from the second well.

The optimal solutions found for the manifold was compared to the known optimal solution. Which is to keep the following ratios equal between the wells in the manifold,

$$\left(\frac{\delta m_g}{\delta m_o}\right)_a = \left(\frac{\delta m_g}{\delta m_o}\right)_b$$

The optimization criteria was determined for all four well models by differentiation, and is presented below.

$$\begin{aligned}
\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_D} &= GOR_D + \frac{k_g}{k_o}(P_{wf} - P_R) \\
\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_1} &= GOR_1 + 2\frac{k_g}{k_o}P_{wf}(P_{wf} - P_R) \\
\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_2} &= 2GOR_2 \\
\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_3} &= 3GOR_3
\end{aligned}$$

In the region with the non-trivial solution the optimal operation for well model GOR_2 and GOR_3 is to keep the GOR for the two wells in the manifold equal. For the model based on GOR_1 it is close to optimal to keep the GOR pluss a small correction term equal between the two wells. It is hard to say why the optimal solution for the manifold with wells based on GOR_1 is deviating from the optimization criteria.

The model based on physical equations has only a trivial solution; maximize the production from the well with the lowest GOR, and then start producing from the one with the higher GOR.

Chapter 9

Conclusion

The results found in this project can be used as a first step in finding a control structure to maximize the oil production form a petroleum manifold. The next step will be to identify variables which can be used to control the manifold, in order to keep it at optimal operation despite of disturbances in the production. Even though the models here most likely do not correspond to a real petroleum well, they give a nice picture on the behavior for types of wells studied here. By optimization it was found that the optimal way to operate the manifold was to keep the GOR between the wells in a manifold equal or close to equal. Further work with the models developed here can be to find ways to measure the GOR for each of the wells in a manifold to be able to control them.

Bibliography

[1] Vidar Gunnerud and Espen Langvik *Production planning optimization for the Troll C-field.* Master Thesis at NTNU, January 2007

[2] Vidar Gunnerud and Bjarne Foss, *Oil Production optimization - a Piecewise linear model, solved woth teo decomposition strategies* NTNU, Department of Engineering Cybernetics, Trondheim, Norway, November 2009

[3] Urbanczyk and Wattenbargers *Optimization of well Rates under Gas Coning Conditions*

[4] Håvard Devold, *Oil and gas production handbook* ABB ATPA Oil and Gas, 2006

[5] Satter, Abdus; Iqbal, Ghulam M.; Buchwalter, James L. *Practical Enhanced Reservoir Engineering - Assisted with Simulation Software.* PennWell. Online version available at: http://app.knovel.com/hotlink/toc/id:kpPEREASS1/practical-enhanced-reservoir (2008)

[6] Sigurd Skogestad *Prosessteknikk - masse og energibalanse.* Tapir akademisk forlag, 3. utgave 2009

[7] A.W Westerberg, H.P Hutchinson, R.L Motard, P. Winter *Process flowsheeting.* Chapter 3, Cambridge University Press, London England, New York USA, Melbourne Australia, 1979

[8] F. Rothlauf *Design of Modern Heuristics, Natural Computing Series.* Chapter 2, Srpinger-Verlag Berling Heidelberg, 2011

[9] J.V Vogel *Inflow Performance Relationships for Solution-Gas Drive Wells.* Shell Oil Co. January 1960

Appendix A

Derivation of the relative mass flow rates

For this part it is assume that the well only produces oil and gas, so the presence of water is ignored. The total mass flow is therefore the oil flow plus the gas flow. Depending on the well model, these expression will change. We assume that the oil flow will follow Darcy's law, while the gas flow can be expressed as the GOR times the oil rate.

$$m_g = GOR \times m_o \tag{A.1}$$

The GOR expressions used are

$$GOR_D = \frac{k_g}{k_o}(P_{wf} + P_R) \tag{A.2a}$$

$$GOR_1 = \frac{k_g}{k_o}(P_{wf}^2 - P_R^2) \tag{A.2b}$$

$$GOR_2 = \frac{k_g}{k_o}(P_{wf} - P_R) \tag{A.2c}$$

$$GOR_3 = \frac{k_g}{k_o}(P_{wf} - P_R)^2 \tag{A.2d}$$

The relative mass flow is expressed as

$$\frac{m_{tot}}{m_{tot}^{max}} \tag{A.3}$$

The maximum flow rate can be expressed as

$$m_{tot}^{max} = k_g' + k_o' \tag{A.4}$$

Where k_g' and k_o' are the maximum flow rates of gas and oil, respectively. They are found from the expression for the total maximum mass flow, which occur when the well hole pressure is at minimum (P_{wf}=0).

$$\begin{aligned} & m_{tot}^{max} = m_o^{max} + m_g^{max} && \text{(A.5a)} \\ \text{GOR}_D\text{:} \quad & m_{tot_D}^{max} = k_o P_R + k_g P_R^2 && \text{(A.5b)} \\ \text{GOR}_1\text{:} \quad & m_{tot_1}^{max} = k_o P_R + k_g P_R^3 && \text{(A.5c)} \\ \text{GOR}_2\text{:} \quad & m_{tot_2}^{max} = k_o P_R + k_g P_R^2 && \text{(A.5d)} \\ \text{GOR}_3\text{:} \quad & m_{tot_3}^{max} = k_o P_R + k_g P_R^3 && \text{(A.5e)} \end{aligned}$$

If the well only produced oil k_g is zero, and likewise if the well only produced gask_o is zero. This can be used to graph the boundary-curves for the wells in the plot, one refrence curve for only liquid flow and one for only gas flow. The graph for the two-phase wells we are investigating will be somewhere in between these two reference curves, and the well behavior can be indicated form these plots.

For the further derivation the following definitions are used:

$$\frac{k'_o}{k'_g + k'_o} = \text{a} \quad \text{and} \quad \frac{k'_g}{k'_g + k'_o} = \text{(1-a)} \tag{A.6}$$

The relative mass flow for the four well models are derived under. The pressure in the reservoir is denoted P_R and the pressure at the well head is denoted P_wf.

The relative mass flow for the model based on $\mathbf{GOR}_D$

$$\begin{aligned} m_{tot} &= m_g + m_o && \text{(A.7a)} \\ &= k_o(P_R - P_wf) + k_g(P_R^2 - P_wf^2) \\ &= k'_o(1 - \frac{P_{wf}}{P_R}) + k'_g(1 - \left(\frac{P_{wf}}{P_R}\right)^2) && \text{where } k'_o = k_o P_R \text{ and } k'_g = k_g P_R^2 \\ &= k'_o - k'_o\frac{P_{wf}}{P_R}) + k'_g - k'_g\left(\frac{P_{wf}}{P_R}\right)^2 \end{aligned}$$

Using the definitions given in Equation A.6 the behavior of this well can be expressed as:

$$\frac{m_{tot_D}}{m_{tot_D}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\frac{P_{wf}}{P_R}\right)^2 \tag{A.7b}$$

The relative mass flow for the model based on GOR$_1$

$$
\begin{aligned}
m_{tot} &= m_g + m_o && \text{(A.8a)}\\
&= k_o(P_R - P_wf) + k_g(P_R^2 - P_wf^2)(P_R - P_wf)\\
&= k_o'(1 - \frac{P_{wf}}{P_R}) + (k_g' - k_g'\frac{P_{wf}^2}{P_R^2})(1 - \frac{P_{wf}}{P_R}) \qquad \text{where } k_o' = k_oP_R \text{ and } k_g' = k_gP_R^3\\
&= k_o' - k_o'\left(\frac{P_{wf}}{P_R}\right) + k_g' + k_g\left(\left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{P_{wf}}{P_R}\right)^2 - \left(\frac{P_{wf}}{P_R}\right)\right)
\end{aligned}
$$

Usign that m_{tot}^{max} = k_g' +k_g' gives the relative mass flow expression:

$$
\frac{m_{tot}}{m_{tot}^{max}} = 1 - \frac{k_o'}{k_g' + k_o'}\left(\frac{P_{wf}}{P_R}\right) + \frac{k_g'}{k_g' + k_o'}\left(\left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{P_{wf}}{P_R}\right)^2 - \left(\frac{P_{wf}}{P_R}\right)\right)
$$

Using the definitions given in Equation A.6 the behavior of this well can be expressed as:

$$
\frac{m_{tot_1}}{m_{tot_1}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{P_{wf}}{P_R}\right)^2 - \left(\frac{P_{wf}}{P_R}\right)\right) \tag{A.8b}
$$

(A.8c)

The relative mass flow for the model based on GOR$_2$

$$
\begin{aligned}
m_{tot} &= m_g + m_o && \text{(A.9a)}\\
&= k_o(P_R - P_wf) + k_g(P_R - P_wf)^2\\
&= k_o'(1 - \frac{P_{wf}}{P_R}) + k_o'(1 - \frac{P_{wf}}{P_R})^2 \qquad \text{where } k_o' = k_oP_R \text{ and } k_g' = k_gP_R^2\\
&= k_o' - k_o'\left(\frac{P_{wf}}{P_R}\right) + k_g' + k_g'\left(-\left(\frac{2P_{wf}}{P_R}\right) + \left(\frac{P_{wf}}{P_R}\right)^2\right)
\end{aligned}
$$

Using the definitions given in Equation A.6 the behavior of this well can be expressed as:

$$
\frac{m_{tot_2}}{m_{tot_2}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{P_{wf}}{P_R}\right)^2 - \frac{2P_{wf}}{P_R}\right) \tag{A.9b}
$$

The relative mass flow for the model based on $\mathbf{GOR}_2$

$$\begin{aligned}
m_{tot} &= m_g + m_o \qquad\qquad (A.10a)\\
&= k_o(P_R - P_wf) + k_g(P_R - P_wf)^3\\
&= k'_o(1-\frac{P_{wf}}{P_R}) + k'_o(1-\frac{P_{wf}}{P_R})^3 \qquad \text{where } k'_o = k_oP_R \text{ and } k'_g = k_gP_R^3\\
&= k'_o - k'_o\left(\frac{P_{wf}}{P_R}\right) + k'_g + k'_g\left(\left(\frac{3P_{wf}}{P_R}\right)^2 + \left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{3P_{wf}}{P_R}\right)\right)
\end{aligned}$$

Using the definitions given in Equation A.6 the behavior of this well can be expressed as:

$$\frac{m_{tot_3}}{m_{tot_3}^{max}} = 1 - \text{a}\left(\frac{P_{wf}}{P_R}\right) + \text{(1-a)}\left(\left(\frac{3P_{wf}}{P_R}\right)^2 + \left(\frac{P_{wf}}{P_R}\right)^3 - \left(\frac{3P_{wf}}{P_R}\right)\right) \qquad (A.10b)$$

Appendix B

Derivation of the derivatives

To ensure optimal operation of our manifold model, the change in gas flow with respect to the oil rate in well *a* should be equal the change in well *b*. The optimization problem is defined such that the gas constraint is always active. Thus, the rates should be equal for all the gas constraints as long as the wells are operated together in the region with the non-trivial solution. Mathematically the problem can be stated:

$$\left(\frac{\delta m_g}{\delta m_o}\right)_a = \left(\frac{\delta m_g}{\delta m_o}\right)_b \tag{B.1}$$

In the derivation the well head and the reservoir pressure are denoted P_{wf} and P_R, respectively.

Since m_g is a function of P_{wf}, which is itself a function of m_o, the mass flow derivative can be expanded by applying the chain rule.

$$\boxed{\left(\frac{\delta m_g}{\delta m_o}\right) = \left(\frac{\delta m_g}{\delta P_{wf}}\right) \times \left(\frac{\delta P_{wf}}{\delta m_o}\right)} \tag{B.2a}$$

The oil rate (m_o) is for all cases following Darcy's law:

$$m_o = k_o(P_{wf} - P_R)$$

And the gas rate can be expressed in terms of oil rate and the GOR as:

$$m_g = GOR \times m_o$$

The partial derivatives in Eqution B.2a are then:

$$
\begin{aligned}
&\left(\frac{\delta m_g}{\delta P_{wf}}\right) = m_o\left(\frac{\delta GOR}{\delta P_{wf}}\right) + GOR\left(\frac{\delta m_o}{\delta P_{wf}}\right)\\
&\left(\frac{\delta P_{wf}}{\delta m_o}\right) = \frac{1}{k_o}
\end{aligned}
$$

The partial derivative of the GOR with respect to P_{wf} varies for each of the models, and must be derivative for all the four GOR expressions. The GOR expressions and their derivatives are given next:

GOR_D:

$$
\begin{aligned}
GOR_D &= \frac{k_g}{k_o}(P_{wf} + P_R)\\
\left(\frac{\delta GOR_D}{\delta P_{wf}}\right) &= \frac{k_g}{k_o}
\end{aligned}
$$

GOR_1:

$$
\begin{aligned}
GOR_1 &= \frac{k_g}{k_o}(P_{wf}^2 - P_R^2)\\
\left(\frac{\delta GOR_1}{\delta P_{wf}}\right) &= 2\frac{k_g}{k_o}P_{wf}
\end{aligned}
$$

GOR_2:

$$
\begin{aligned}
GOR_2 &= \frac{k_g}{k_o}(P_{wf} - P_R)\\
\left(\frac{\delta GOR_2}{\delta P_{wf}}\right) &= \frac{k_g}{k_o}
\end{aligned}
$$

GOR_3:

$$
\begin{aligned}
GOR_3 &= \frac{k_g}{k_o}(P_{wf} - P_R)^2\\
\left(\frac{\delta GOR}{\delta P_{wf}}\right) &= 2\frac{k_g}{k_o}(P_{wf} - P_R)
\end{aligned}
\tag{B.4a}
$$

The change in gas flow rate with respect to oil rate for each of the well models are found to be:

In general:

$$\left(\frac{\delta m_g}{\delta m_o}\right) = \left(m_o\left(\frac{\delta GOR}{\delta P_{wf}}\right) + GOR\left(\frac{\delta m_o}{\delta P_{wf}}\right)\right)\left(\frac{\delta P_{wf}}{\delta m_o}\right)$$

$$\left(\frac{\delta m_g}{\delta m_o}\right) = \left(m_o\left(\frac{\delta GOR}{\delta P_{wf}}\right) + GORk_o\right)\left(\frac{1}{k_o}\right)$$

Inserting the espression for m$_o = k_o(P_{wf} - P_R)$:

$$\left(\frac{\delta m_g}{\delta m_o}\right) = \left(k_o(P_{wf} - P_R)\left(\frac{\delta GOR}{\delta P_{wf}}\right) + GORk_o\right)\left(\frac{1}{k_o}\right)$$

For each spesific well model the expression becomes:

GOR$_D$:

$$\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_D} = \frac{k_g}{k_o}(P_{wf} - P_R) + GOR_D$$

GOR$_1$:

$$\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_1} = 2\frac{k_g}{k_o}P_{wf}(P_{wf} - P_R) + GOR_1$$

GOR$_2$:

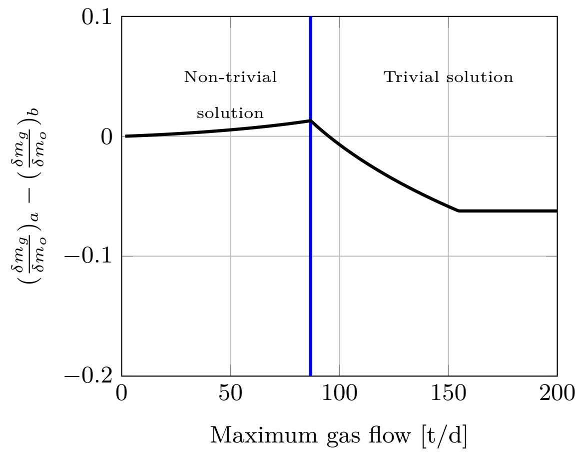
$$\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_2} = \frac{k_g}{k_o}(P_{wf} - P_R) + GOR_2 = 2GOR_2$$

GOR$_3$:

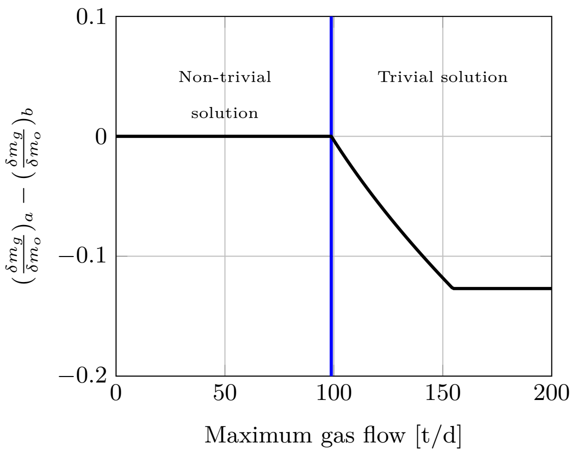
$$\left(\frac{\delta m_g}{\delta m_o}\right)_{GOR_3} = 2\frac{k_g}{k_o}(P_{wf} - P_R)^2 + GOR_3 = 3GOR_3$$

B.1 The difference between the GOR for the wells

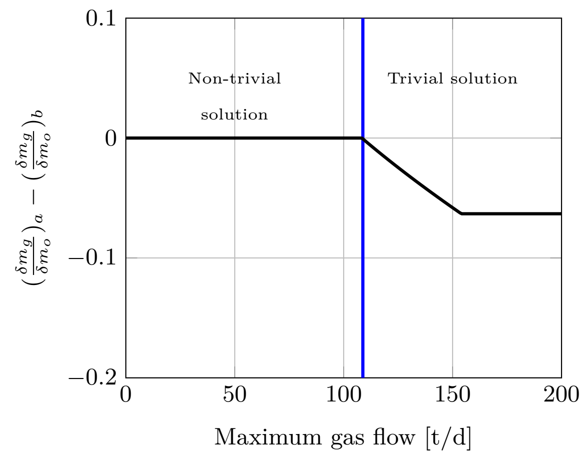
The difference between the optimal GOR values are for well a and well b at optimal solution is shown in this section.



a) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR$_1$**

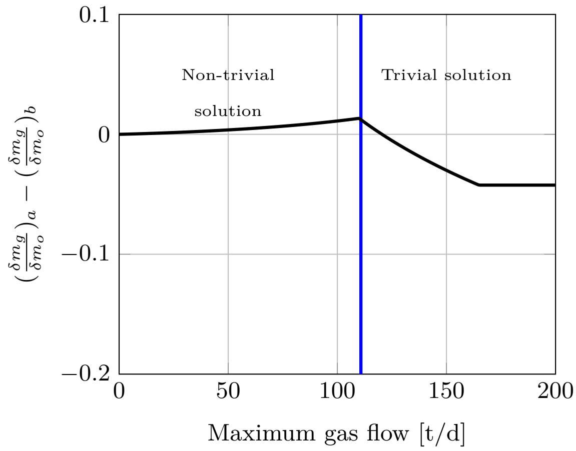


c) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR$_2$**

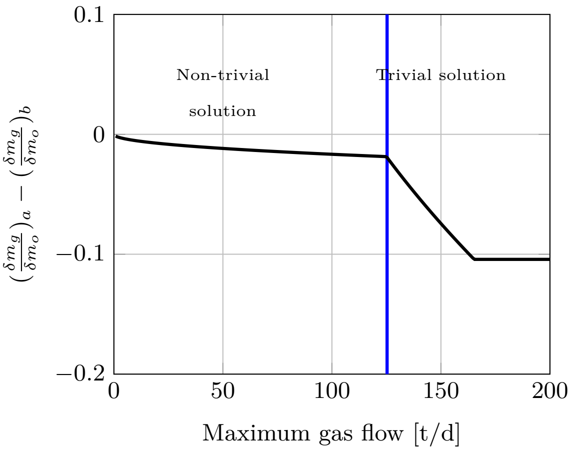


d) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR$_3$**

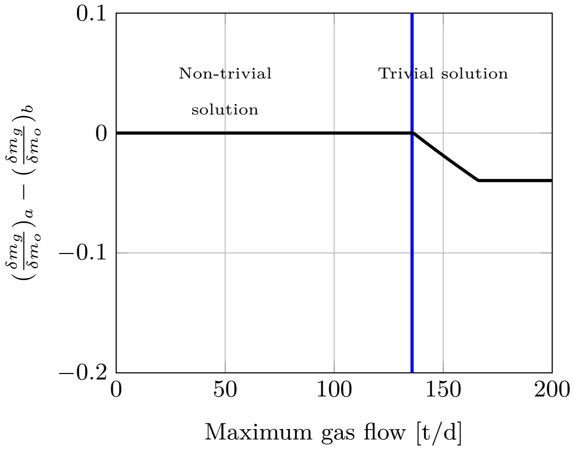
Figure B.1: The difference between the derivative of the gas rate with respect to the oil rate for well a and well b. Where the difference is zero the derivatives are equal, the the solution found is truly optimal.



a) $\Delta \frac{dm_g}{dm_o}$ **for well model GOR**$_1$

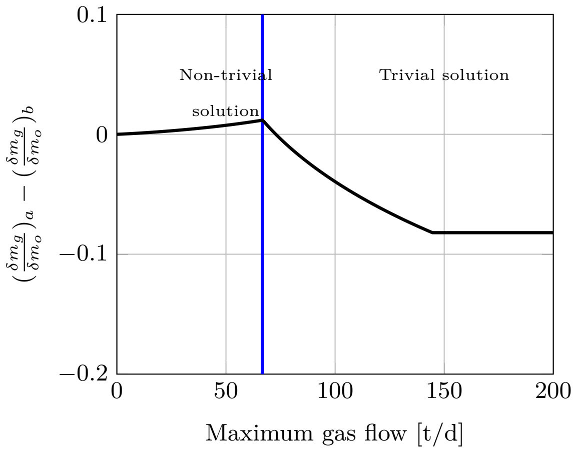


c) $\Delta \frac{dm_g}{dm_o}$ **for well model GOR**$_2$

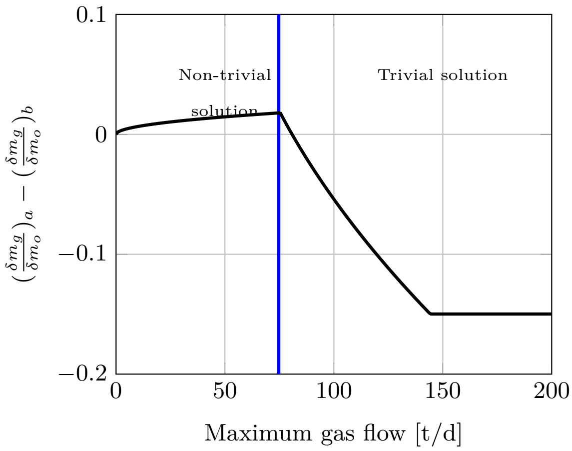


d) $\Delta \frac{dm_g}{dm_o}$ **for well model GOR**$_3$

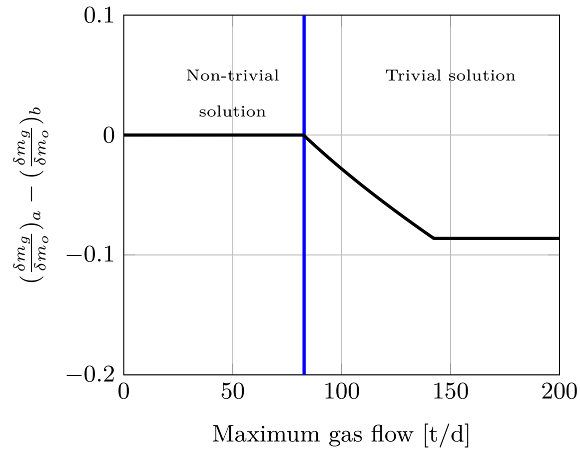
Figure B.2: The difference between the derivative of the gas rate with respect to the oil rate for well a and well b for a disturbance +15% in the GOR. Where the difference is zero the derivatives are equal, the the solution found is truly optimal.



a) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR**$_1$



c) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR**$_2$



d) $\Delta\frac{dm_g}{dm_o}$ **for well model GOR**$_3$

Figure B.3: The difference between the derivative of the gas rate with respect to the oil rate for well a and well b for a disturbance +15% in the GOR. Where the difference is zero the derivatives are equal, the the solution found is truly optimal.

Appendix C

MATLAB codes

In this appendix the most important Matlab-codes are given together with a short description of what they are used for. They are only given for the model derived by first principles and only for well$_a$. They look the same for the three other models and for the two wells in the manifold. The only parameters that are changed are the k-values, the expression for m$_g$, and the valve characteristic in cases where both a linear and logarithmic valve are tested.

C.1 Finding the Constants in Darcy's law

In this file the k-values applied in Darcy's law are calculated. The data for the base-wells are used in the calculations. In order to use the script you need a base well with known flow rates of oil, gas and water at a certain well reservoir pressure and well head pressure.

```
%% well_1

Pwf=150;
Pr=300;

mo=461997;
mg=56429;
mw=144599;

%GOR D
disp('Darcy')
kg=mg/(Pwf^2-Pr^2)
ko=mo/(Pwf-Pr)
kw=mw/(Pwf-Pr)

%GOR 1
disp('GOR 1')
```

```
kg=mg/((Pwf-Pr)*(Pwf^2-Pr^2))

%GOR 2
disp('GOR 2')
kg=mg/(Pwf-Pr)^2

%GOR 3
disp('GOR 3')
kg=mg/(Pwf-Pr)^3
```

C.2 The change in the gas-oil-ratio as a function of production

This generates the plot which shows the change in the gas-oil-ratio when the production is increased ($P_{wf} \rightarrow 0$). Both the GOR on volumetric and mass basis can be generated.

```
%Estimaton the slope by using a well model where the massflows are assumed
%known, when the pressure in 135 bar, and 0 at reservoir pressure
%Dencity at stp
rhoo=0.8;                     %ton/m3
rhow=1;                       %ton/m3
rhog=0.656e-3;                %ton/m3

%Pressure in bar:
Presb=300;             %bar
Pknownb=150;           %bar

kob=-3080;      %slope oil
kwb=-964;     %slope water
kgb=-1.551;  %ton/bar3 slope gas

Pb=Pknownb:1:Presb;
mob=kob*(Pb-Presb);              %ton/day
mwb=kwb*(Pb-Presb);              %ton/day ton/bar
mgb=kgb*(Pb.^2-Presb^2);         %ton/day ton/bar2
figure(1)
hold on
plot(Pb,mob,'b',Pb,mwb,'r',Pb,mgb,'y')
 xlabel('well hole pressure bar')
 ylabel('mass flow ton/day')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Gas - Oil - Ratio

%mass flow basis
GORm = mgb./mob;
%GORm = kgb/kob*(Pb.^2-Presb^2)./(Pb-Presb);
%flowrate basis
```

```
qob=mob./rhoo;
qgb=mgb./rhog;
GORq = qgb./qob;

figure(2)
hold on
plot(Pb,GORm)
 xlabel('well hole pressure bar')
 ylabel('GOR (mg/mo)')

% figure(4)
% plot(Pb,GORq)
%  xlabel('well hole pressure bar')
%  ylabel('GOR (qg/qo)')

%
```

C.3 Identifying well types

This code is used to generate the well-type plots, with the boundaries for a pure oil or pure gas phase well. The k-values can be changed, and the expression for m_g.

```
%% Flow equations %%
ko =-3080;          % Slope found from darcy's law ton/bar
kw =-964;        % Slope found from darcy's law
kg =-0.836;
Pr=300;
Pwf=0:10:300;
Prel=Pwf/Pr;



mo=ko*(Pr-Pwf);
momax=ko*Pr;
mg=kg*(Pr^(2)-Pwf.^(2));
mgmax=kg*Pr^2;
mw=kw*(Pr-Pwf);
mwmax=kw*Pr;

morel=1-(Pwf/Pr);
mwrel=1-(Pwf/Pr);
mgrel=1-(Pwf.^2/Pr^2);

%Oil and gas only
a=mgmax/(mgmax+momax);
mtotrel=a*mgrel+(1-a)*morel;

figure(1)
plot(mgrel,Prel,'r',morel,Prel,'b',mtotrel,Prel,'g')
```

```
    xlabel('m/m_{max}')
    ylabel('Pwf/Pr')

%Oil, water and gas
mtotm=momax+mgmax+mwmax;
mtrel=((momax/mtotm)*morel)+((mgmax/mtotm)*mgrel)+((mwmax/mtotm)*mwrel);
%Mixed liquid phase water + oil
mlmax=momax+mwmax;
mlrel=((momax/mlmax)*morel)+((mwmax/mlmax)*mwrel);

% figure(2)
% plot(mgrel,Prel,'r',mlrel,Prel,'b', mtrel,Prel,'g')
%     xlabel('m/m_{max}')
%     ylabel('Pwf/Pr')
```

C.4 The Model

These two codes are the well model. The first one is the equations used to model the system. The other takes in specified parameters (like the k-values), in this file the valve characteristic and the expression for m_g is specified.

```
function [ error ] = newmodel(P2,param)
    z=param.z;
    P1=param.P1;
    P4=param.P4;
    ko=param.ko;
    kw=param.kw;
    kg=param.kg;
    g=param.g;
    Cd=param.Cd;
    A=param.A;
    R=param.R;
    T=param.T;
    Mg=param.Mg;
    rhoo=param.rhoo;
    rhow=param.rhow;
    z_valve = param.z_valve;
    fz=param.fz;
    mg=param.mg(P2);

%System equations:
    mo=ko*(P2-P1);
    mw=kw*(P2-P1);
    %mg=kg*(P2-P1)*(P2^(2)-P1^(2));
    mtot=mo+mw+mg;
    D=(mg*R*T)/Mg;
    B=(mo/rhoo)+(mw/rhow);
```

```
    if z_valve <= 1e−20;
        P3=P4;
        mtot_vl = 0;
    else
        P3=P2−((mtot*g*P2*z)/(D+B*P2));
        rho_mix3=(mtot)/((D/P3)+B);
        rho_mix4=(mtot)/((D/P4)+B);

        if (P3−P4)>=0
            mtot_vl=fz(z_valve)*Cd*A*sqrt(((rho_mix3+rho_mix4)/2)*(P3−P4));
        elseif (P3−P4)<0
            mtot_vl=−fz(z_valve)*Cd*A*sqrt(((rho_mix3+rho_mix4)/2)*(−P3+P4));
        end
    end




    error=mtot−mtot_vl;
%MODEL Summary of this function goes here
%   Detailed explanation goes here


end
```

```
function output = newwell_1(z_valve)

    param.z_valve = z_valve;

%%%%Giving the parameters vaules:
    param.z  =1000;             % Lenght of pipeline m
    param.P1 =300e5;        % Pressure in reservoir Pa
    param.P4 =100e5;          % Sressure in reservoir Pa
    param.ko =−3.08e3/1e5;          % Slope found from darcy's law
    param.kw =−964/1e5;        % Slope found from darcy's law
    param.kg =−0.836*0.85/(1e5)^2;    % Slope found from darcy's law
    param.g  =9.8;              % Gravity constant m/s2
    param.Cd =1*84600;               % Valve coeffficient scaled to ...
        give m in /d
    param.A  = 5.56*10^(−4);     % Valve cross section area m2
    param.R  =8.314;            % Gas constant J/Kmol
    param.T  =373;              % Temerature assumed constant K
    param.Mg =16.04*10^(−3);  % Molar mass gas kg/mol
    param.rhoo=800;           % Density of oil kg/m3
    param.rhow=1000;          % Density of water kg/m3
    param.fz = @(z) z;
    %param.fz = @(z) log10(9*z+1);
    param.mg = @(P2) param.kg*(P2^2−param.P1^2);

   if z_valve <= 1e−10
       P2=param.P1;
       output=[0,0,0,param.P1*1e−5,P2*1e−5,NaN,param.P4*1e−5];
```

```
   else
    P20=250e5;
    P2 = fzero(@(P2) newmodel(P2,param), P20);


    z=param.z;
    P1=param.P1;
    P4=param.P4;
    ko=param.ko;
    kw=param.kw;
    kg=param.kg;
    g=param.g;
    Cd=param.Cd;
    A=param.A;
    R=param.R;
    T=param.T;
    Mg=param.Mg;
    rhoo=param.rhoo;
    rhow=param.rhow;
    z_valve = param.z_valve;
    fz=param.fz;

%System equations:
    mo=ko*(P2-P1);
    mw=kw*(P2-P1);
    mg=param.mg(P2);
    mtot=mo+mw+mg;
    D=(mg*R*T)/Mg;
    B=(mo/rhoo)+(mw/rhow);
    P3=P2-((mtot*g*P2*z)/(D+B*P2));
    rho_mix3=(mtot)/((D/P3)+B);
    rho_mix4=(mtot)/((D/P4)+B);
    mtot_vl=fz(z_valve)*Cd*A*sqrt(((rho_mix3+rho_mix4)/2)*(P3-P4));
    error=mtot-mtot_vl;


%     fprintf('param.z_valve: %.2f\n',param.z_valve)
%     fprintf('mo: %.2f, mg: %.2f, mw: %.2f, mtot: %.2f\n',mo,mg,mw,mtot)
%     fprintf('P1: %.2f, P2: %.2f, P3: %.2f, P4: ...
    %.2f\n',P1*1e-5,P2*1e-5,P3*1e-5,P4*1e-5)


   output=[mo,mg,mw,P1*1e-5,P2*1e-5,P3*1e-5,P4*1e-5];
    end
end
```

C.5 Optimizing the manifold

This is the code where *fmincon* is run.

```

%initial guess
z0=[1, 1]';

%constraints
lb=[0,0]';
ub=[1,1]';
opt_well_1=[];
opt_well_2=[];

mgmax_space= linspace(200e3,0,250);
z1=[];
z2=[];
opt_mo=[];
opt=optimset('algorithm','active-set','display','off');
for  i = 1:length(mgmax_space)
     mgmax = mgmax_space(i);
    [zopt,fval,exitflag]=fmincon(@newcost,z0,[],[],[],[],lb,ub,@(z)newnonlincon(z,mgmax),opt);

    z0=zopt;



 z1(i)=zopt(1);
 z2(i)=zopt(2);
 opt_well_1(i,:)=newwell_1(zopt(1));
 opt_well_2(i,:)=newwell_2(zopt(2));
 fprintf('%.2f    %.2f\n', zopt)
 opt_mo(i)=fval;
end
% hold on
% figure(1)
% plot(mgmax_space/10^3,z1,'b',mgmax_space/10^3,z2,'r')
% xlabel('Maximim gas flow allowed through the system')
% ylabel('valve opening')
%
% figure(2)
% hold on
% plot(mgmax_space/10^3,opt_well_1(:,6),'b', ...
    mgmax_space/10^3,opt_well_2(:,6), 'r')
% xlabel('Maximim gas flow allowed through the system')
% ylabel('P3')
%
% figure(3)
% hold on
% plot(mgmax_space/10^3,opt_well_1(:,5),'b', ...
    mgmax_space/10^3,opt_well_2(:,5), 'r')
% xlabel('Maximim gas flow allowed through the system')
% ylabel('P2')
%
% figure(4)
% %hold on
% plot(mgmax_space/10^3,-opt_mo/10^3)
```

```
% xlabel('Maximim gas flow allowed through the system')
% ylabel('Opt oil flow')
%
% figure(5)
% plot(mgmax_space/10^3,opt_well_1(:,5)/opt_well_2(:,5))
% xlabel('Maximim gas flow allowed through the system')
% ylabel('P3a/P3b')
```

C.6 Comparing the optimal solution to the optimization criteria

After finding the different derivative expressions for the models to be tested as the optimization criteria this code can be used to generate the plots that visualize the change in the derivatives for the two wells.

```
%output=[mo,mg,mw,P1*1e-5,P2*1e-5,P3*1e-5,P4*1e-5];

GOR_a = opt_well_1(:,2)./opt_well_1(:,1); %mg/mo
kg_a =-0.836;
ko_a =-3080;
mo_a = opt_well_1(:,1);
p2_a = opt_well_1(:,5);

GOR_b = opt_well_2(:,2)./opt_well_2(:,1);
kg_b =-1.551;
ko_b =-3854;
mo_b = opt_well_2(:,1);
p2_b = opt_well_2(:,5);

p1=300;

dmg_dmo_a= ((kg_a/ko_a)*(p2_a-p1))+GOR_a;
dmg_dmo_b= ((kg_b/ko_b)*(p2_b-p1))+GOR_b;

figure(5)
hold on
plot(mgmax_space*10^(-3),dmg_dmo_a,mgmax_space*10^(-3),dmg_dmo_b )
xlabel('Maximim gas flow allowed through the system')
ylabel('GOR for well a and b')
%ylim([0 1000])


figure(6)
hold on
plot(mgmax_space*10^(-3),dmg_dmo_a-dmg_dmo_b )
xlabel('Maximim gas flow allowed through the system')
ylabel('GOR for well a and b')
%ylim([0 1000])
```