

Optimal Temperature Control of Rooms

Specialization project 2012

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Abstract

The task of this project has been to model a room with direct heating in the floor and room using Matlab, Simulink. The heating in the floor was turned off during the simulation of this project. The aim of the project has been to keep the temperature in the room within certain, optimal bounds in order to save energy costs. The energy price is assumed constant. From this it follows that the optimal is to find the minimum energy consumption in the system. For the optimization of the system a quadratic programming method was used to find an optimal room temperature that minimizes the energy consumption.

A PI controller was implemented and the system was simulated over one day. In addition, constraints were set on the room temperature. The room was supposed to be 10°C in certain time intervals and 20°C the rest of the day. From optimization of the system it was found that the heating of the room should start a bit before in order to reach the constraint at the given time. If disturbances were introduced to the system, the time to start heating the room such that it was 20°C when it was supposed to change. The null space method was used to find a controlled variable that is independent of disturbances. The system is to achieve near optimal operation with keeping the controlled variable constant, even if the time to start the heating/cooling changes. It was found that a linear combination of the measurements did not give acceptable results. Due to lack of time other methods or adjustment of the null space method was not investigated. Improvements to the model and suggestions for further work are given in the end of the report.

Contents

1	Introduction	1
2	Process description	2
2.1	The simple model	2
2.2	Advanced model	3
2.3	Parameters and assumptions	4
3	Background	5
3.1	Heat transfer	5
3.2	Linear state space model	7
3.3	Tuning	8
3.4	Skogestad's half rule	8
3.5	Approximation of positive numerator time constants	9
3.6	Quadratic programming	10
3.7	The null space method	10
4	Calculations and results	13
4.1	Comparison of the simple and advanced model	13
4.2	Preparation to the simulation	14
4.3	Creating the simple model	15
4.4	Modelling and control in Matlab, Simulink	16
4.5	Optimal dynamical control problem	21
4.6	Energy consumption	25
4.7	Behaviour of disturbances	27
4.8	Control variable that is independent of disturbances	31
5	Discussion	43
5.1	Difference between the simple and advanced model	43
5.2	Simulation of the system	43
5.3	Null space method	43
5.4	Improvements and suggestions for further work	44
6	Conclusion	45
7	Appendix A	48
7.1	Nomenclature	48
8	Appendix B	50
8.1	How the system responds to disturbances	50
9	Appendix C	53
9.1	CD containing the Matlab and Simulink scripts	53

1 Introduction

This report is a part of the specialization project conducted during the first semester of the 5th year at NTNU. It contains the result from simulation of a room with direct heating in the floor and room. The programming tools Matlab and Simulink were used in the simulations.

This project investigates how the energy consumption in buildings and thus the energy costs can be minimized. The temperature in the room is to be varied such that the room is colder when people are not at home. The energy price can be assumed constant or varying. From this the system is to be optimized such that the energy consumption is kept at a minimum value, in order to save energy costs.

Buildings are responsible for a big part of the energy use in many countries [Kornevall]. By reducing the energy consumption less energy is needed, which result in less production of energy. The idea is to buy and store energy when the energy price is small, and use it when the energy price is high. This in addition to lower energy consumption gives lower energy costs which is favourable for the consumer. The energy could be stored in the floor or in a water heater.

The report consists of six chapters, excluded this chapter and the abstract. The first part of the report gives a description of the model that was used to simulate the room, in addition to the parameters and assumptions that were made. The second part explains the methods that were used in order to simulate the system. The third part gives a summary of how the system was implemented in Matlab, Simulink together with the obtained results. A discussion and conclusion of the results are given in the end, in addition to suggestions and improvements for further work.

2 Process description

In this project a room with heating in the floor and room is simulated using Simulink, Matlab. The aim of the project is to keep the temperature in the room within certain, optimal bounds in order to save energy cost. Two different models, a simple and a more advanced one, was used to simulate the behaviour in the room.

2.1 The simple model

A figure of the simple model of the room is given below. The model describe direct

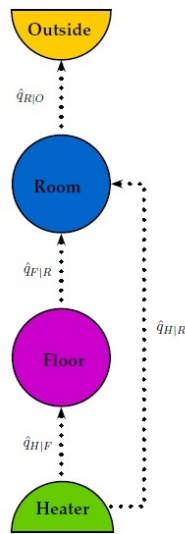


Figure 2.1.1: Overview of the system

heating from the floor and room, and q_i is the heat transfer from the heater, floor and room. The equations for the heat in the room and floor are given in equation (2.1.1) and (2.1.2).

$$\text{The floor: } (mCp)_F \dot{T}_F = -q_{F/R} + q_{H/R} \quad (2.1.1)$$

$$\text{The room: } (mCp)_R \dot{T}_R = -q_{R/O} + q_{F/R} + q_{H/R} \quad (2.1.2)$$

The equations for the heat transport between floor and room and room and outside are given below.

$$\text{Transport: } q_{i,j} = (UA)_{i,j} (T_i - T_j) \quad \forall (i,j) \in [(F,R), (R,O)] \quad (2.1.3)$$

To ease the calculation the following assumptions were made

- Constant mass
- Constant volume
- Heat transfer from four walls, not from the roof

These assumptions and a combination of equation (2.1.1), (2.1.2) and (2.1.3) gives the system of equation that is used for simulation of the simple model.

$$\dot{T}_F = \frac{-(UA)_{F/R}}{(mCp)_F}(T_F - T_R) + \frac{q_{H/F}}{(mCp)_F} \quad (2.1.4)$$

$$\dot{T}_R = \frac{-(UA)_{R/O}}{(mCp)_R}(T_R - T_O) + \frac{(UA)_{F/R}}{(mCp)_R}(T_F - T_R) + \frac{q_{H/R}}{(mCp)_R} \quad (2.1.5)$$

The inputs to the simple system are the heat from heater to floor, $q_{H/F}$, the heat from heater to room, $q_{H/R}$ and the outdoor temperature, T_O .

2.2 Advanced model

A Figure of the advanced model is given below. The difference from the simple model is

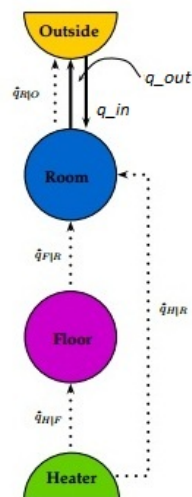


Figure 2.2.1: Overview of the system

the addition of heat transfer from the room to the outside and from the outside to the room, q_{out} and q_{in} respectively. The equations that describe the heat flow in the floor and the heat transport is the same as described in the simple model, equation (2.1.1) and (2.1.3) respectively. The equation for the room is given in equation (2.2.1) below.

The room: $(Cp)_R(m_R\dot{T}_R + T_R\dot{m}_R) = -q_{R/O} + q_{F/R} + q_{H/R} + q_{in}(CpT)_O - q_{out}(CpT)_R$ (2.2.1)

The transport of mass is given by equation (2.2.2).

$$\dot{m}_R = q_{in} - q_{out} \quad (2.2.2)$$

The heat from the room to the outside is given by equation (2.2.3) below.

$$q_{out} = k(p_R - p_O) \quad (2.2.3)$$

In the above equation P_R and P_O is the pressure in the room and outside respectively. The pressure in the room is given by ideal gas law.

$$p = \frac{NRT}{V} \quad (2.2.4)$$

The number of moles, N , in ideal gas law is replaced by $N = \frac{m}{M}$. The same assumptions as explained in the simple model were used to find the equations for the temperature in the floor and room in addition to the mass. Combining equation (2.1.1) and (2.1.3) gives the model of the heat transport in the floor. The equation of the floor temperature is the same as in the simple model. A combination of equation (2.2.1), (2.1.3), (2.2.2), (2.2.3) and (2.2.4) gives the differential equation for the room and the mass.

$$\dot{T}_F = \frac{-(UA)_{F/R}}{(mCp)_F}(T_F - T_R) + \frac{q_{H/F}}{(mCp)_F} \quad (2.2.5)$$

$$\dot{T}_R = \frac{q_{in}}{m_R}(T_O - T_R) + \frac{q_{H/R}}{(mCp)_R} + \frac{(UA)_{F/R}}{(mCp)_R}(T_F - T_R) - \frac{(UA)_{R/O}}{(mCp)_R}(T_R - T_O) \quad (2.2.6)$$

$$\dot{m}_R = q_{in} - \left(k \left(\frac{m_R R T_R}{M_R V_R} \right) - p_O \right) \quad (2.2.7)$$

The inputs to the advanced system is similar to the simple model $q_{H/F}$ and $q_{H/R}$, in addition to the heat from the outside to the room.

2.3 Parameters and assumptions

The room, both in the simple and advanced model, is assumed to have the dimensions as described in Table 2.3.1. The walls are assumed to consist of three layers; the inner

Table 2.3.1: Dimension of the room that is to be simulated

Parameter	Unit	Value
Floor area, A_{floor}	$[m^2]$	25
Floor length, l_{floor}	$[m]$	5
Wall height, h_{wall}	$[m]$	2.4
Thickness rock wool, $\Delta x_{rock\ wool}$	$[m]$	0.25 ^[MAXBO]
Thickness of the oak, Δx_{wood}	$[m]$	0.015 ^[theartofhardwood flooringBOEN]
Thickness concrete, $\Delta x_{concrete}$	$[m]$	0.1 ^[MAXBO]

and outer wall are to be made of oak, and to be isolated with rock wool in the middle. The floor is assumed to be quadratic and consist of wood and concrete. Throughout the rest of this project "wood" is used instead of oak when describing type of material. The values for the different parameters used in the project are given in Table 2.3.2.

Table 2.3.2: Values of the parameters used in the system

Parameter	Unit	Value
$\rho_{concrete}$	$[\frac{kg}{m^3}]$	2400 ^[ToolBox, d]
ρ_{wood}	$[\frac{kg}{m^3}]$	825 ^[Geankoplis, 2003]
$\rho_{rock\ wool}$	$[\frac{kg}{m^3}]$	192 ^[Geankoplis, 2003]
ρ_{air}	$[\frac{kg}{m^3}]$	1.17 ^[ToolBox, c]
$Cp_{concrete}$	$[\frac{J}{kgK}]$	630 ^[Geankoplis, 2003]
Cp_{air}	$[\frac{J}{kgK}]$	1005 ^[ToolBox, c]
$k_{concrete}$	$[\frac{J}{hmK}]$	2743.20 ^[Geankoplis, 2003]
k_{wood}	$[\frac{J}{hmK}]$	748.80 ^[Geankoplis, 2003]
$k_{rock\ wool}$	$[\frac{J}{hmK}]$	140.76 ^[Geankoplis, 2003]
h_{air}	$[\frac{J}{hm^2K}]$	54000 ^[ToolBox, a]

3 Background

This section will give an explanation of the different methods that are used in the calculations in this project.

3.1 Heat transfer

Heat transfer occur by one, or a combination of three basic mechanisms; conduction, convection or radiation [Geankoplis, 2003]. In this project heat is transferred by a combination of the first two mechanisms. Conduction is heat transferred through solids, liquids and gas because of the energy of motion between adjacent molecules [Geankoplis, 2003]. Convection means transfer of energy between an object and its environment [Geankoplis, 2003]. The heat is transferred from the gas or liquid with the highest temperature to the gas/liquid with the lower temperature [Geankoplis, 2003]. There is a difference between forced and natural/free convection. When heat is transferred from one place to another because of a pump, fan or other mechanical devices forced convection has been performed. Natural or free convection is performed when heat is transferred due to temperature differences in the fluid [Geankoplis, 2003].

The heat transfer is described by Fourier's law [Geankoplis, 2003]:

$$\frac{q_x}{A} = -k \frac{dT}{dx} \quad (3.1.1)$$

where q_x is the heat, A is the cross-sectional area and k is the thermal conductivity. The thermal conductivity is replaced by the convective coefficient, h_i , when the heat is transferred by convection. The thermal conductivity and the convective coefficient differs from one another in the units. The latter one has unit $[\frac{W}{m^2K}]$ while the first one has $[\frac{W}{mK}]$ [Geankoplis, 2003]. Heat transfer for a wall, or floor, where the area and

thermal conductivity are constant is given in equation (3.1.2) [Geankoplis, 2003].

$$\frac{q}{A} = \frac{k}{\Delta x}(T_1 - T_2) \quad (3.1.2)$$

If the wall consist of more than one solid, for example three layers, the calculations are as described in equation (3.1.3). Figure 3.1.1 gives a representation of the problem [Geankoplis, 2003]. Since the heat flow is the same in each layer q can be written as

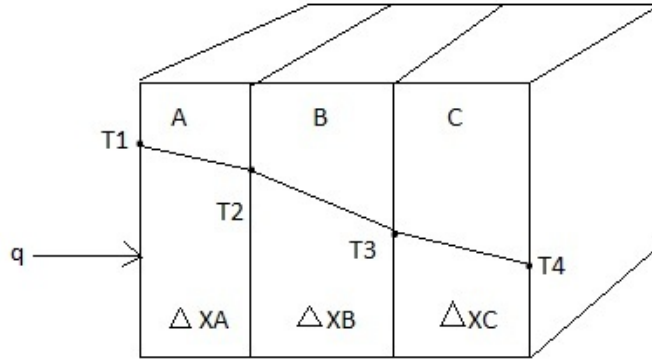


Figure 3.1.1: Heat flow through a plane wall with three layers

[Geankoplis, 2003]

$$q = \frac{k_a}{\Delta x_a}(T_1 - T_2) = \frac{k_b}{\Delta x_b}(T_2 - T_3) = \frac{k_c}{\Delta x_c}(T_3 - T_4) \quad (3.1.3)$$

T_1 and T_4 are the temperatures at the inside and outside layer respectively. Rearranging with respect to temperature and adding the equations for the different solids give the following expression for the heat flow through the wall [Geankoplis, 2003].

$$q = \frac{T_1 - T_4}{\left(\frac{\Delta x_a}{k_a A}\right) + \left(\frac{\Delta x_b}{k_b A}\right) + \left(\frac{\Delta x_c}{k_c A}\right)} \quad (3.1.4)$$

Consider a wall with fluid on both sides of the solid surfaces as given in Figure 3.1.2 [Geankoplis, 2003]. A hot fluid with temperature T_1 on the inside of the surface and a colder fluid on the outside surface with temperature T_2 [Geankoplis, 2003]. The convective coefficients for the inside and outside are h_i and h_o respectively. As explained before the heat flow is written as described in equation (3.1.5).

$$q = h_i A(T_1 - T_2) = \frac{k_a A}{\Delta x_a}(T_2 - T_3) = h_o A(T_3 - T_4) \quad (3.1.5)$$

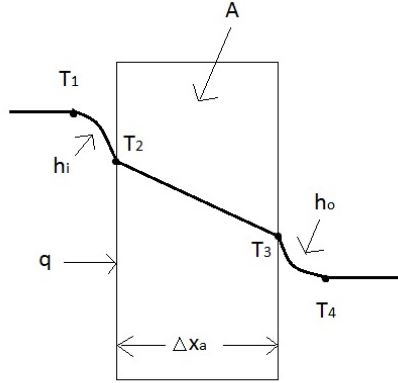


Figure 3.1.2: Heat flow through a plane wall with convective boundaries

Combining the expression in the above equations as described earlier gives the overall heat transfer by combined convection and conduction [Geankoplis, 2003].

$$q = \frac{T_1 - T_4}{\frac{1}{h_i A} + \frac{\Delta x_a}{k_a A} + \frac{1}{h_o A}} \quad (3.1.6)$$

The overall heat transfer is usually expressed in the following way

$$q = U A \Delta T \quad (3.1.7)$$

In equation (3.1.7) U represents the overall heat transfer coefficient. The overall heat transfer coefficient for the above example is given in equation (3.1.8) below.

$$U = \frac{1}{\frac{1}{h_i A} + \frac{\Delta x_a}{k_a A} + \frac{1}{h_o A}} \quad (3.1.8)$$

3.2 Linear state space model

A state space model gives a simple representation of systems of ordinary differential equations, ODEs. The linear state space model is given in equation (3.2.1) and (3.2.2) below [Seborg, Dale E. , Edgar et al., 2011]

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{d} \quad (3.2.1)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (3.2.2)$$

In the above equations \mathbf{x} represents the state vector, \mathbf{u} the input vector, \mathbf{y} the vector with the output variables, and \mathbf{d} the disturbances. The matrices \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} are matrices with constant values [Seborg, Dale E. , Edgar et al., 2011].

3.3 Tuning

A proportional-integral-derivative, PID, controller is given in equation (3.3.1) below [Seborg, Dale E. , Edgar et al., 2011]

$$G_c = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \quad (3.3.1)$$

In the above equation K_c is the controller gain while τ_I and τ_D is the integral and derivative time respectively. It is not easy to find good values for this parameters without using systematic procedures [Skogestad, 2003]. The SIMC rules, Skogestad's IMC rules, [Skogestad, 2003] is such a procedure. In contrast to the IMC rules, the SIMC rules propose only one tuning rule for tuning of PI controllers. The tuning rule is given in equation (3.3.2) and (3.3.3) [Skogestad, 2003].

$$K_c = \frac{1}{k'} \frac{1}{\theta + \tau_c} \quad \text{where } k' = \frac{K}{\tau_I} \quad (3.3.2)$$

$$\tau_I = \min(\tau_I, 4(\tau_c + \theta)) \quad (3.3.3)$$

The tuning parameter, τ_c , must be in the range $-\theta < \tau_c < \infty$ to get a positive and non-zero controller gain [Skogestad, 2003]. There are two main possibilities for the optimal value of τ_c [Skogestad, 2003]:

1. Tight control
 - Fast response with good robustness
 - $\tau_c = \theta$
2. Smooth control
 - Slow control with acceptable disturbance rejection
 - $\tau_c > \theta$

It depends on the system if tight or smooth control is the best choice.

3.4 Skogestad's half rule

One way to find the values of the parameters in equation (3.3.1) is to represent the system by a first or second-order-plus-time-delay, FOPTD or SOPTD, transfer function [Seborg, Dale E. , Edgar et al., 2011].

$$G = \frac{K e^{-\theta s}}{\tau s + 1} \quad (3.4.1)$$

$$G = \frac{K e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (3.4.2)$$

A FOPTD gives a PI controller while a SOPTD gives a PID controller [Seborg, Dale E. , Edgar et al., 2011]. In case the system is not originally on the form (3.4.1) or (3.4.2) different methods have to be used to get the equations on the desired form.

Skogestad's half rule is an approximation method to get higher order transfer functions with multiple time constants to a lower order transfer function. The time delay, $e^{-\theta s}$, in a transfer function can be expressed as a Taylor series as given in equation (3.4.3) [Seborg, Dale E. , Edgar et al., 2011].

$$e^{-\theta s} = 1 - \theta s + \frac{\theta^2 s^2}{2!} - \frac{\theta^3 s^3}{3!} + \frac{\theta^4 s^4}{4!} - \frac{\theta^5 s^5}{5!} + \dots \quad (3.4.3)$$

A first order approximation of the time delay is used for the approximation of higher order transfer functions [Skogestad, 2003]:

$$e^{-\theta s} \approx 1 - \theta s \quad (3.4.4)$$

or alternatively

$$e^{-\theta s} = \frac{1}{e^{\theta s}} \approx \frac{1}{1 + \theta s} \quad (3.4.5)$$

The half rule is as follows [Skogestad, 2003],[Seborg, Dale E. , Edgar et al., 2011]:

Approximate the largest neglected time constant by adding one-half of its value to the existing time delay, and the other half is added to the smallest retained time constant

The rule is best illustrated by an example. Consider a system as described below

$$G = \frac{1}{(s + 1)(0.2s + 1)} \quad (3.4.6)$$

The idea is to use Skogestad's half rule to approximate the system to a first order plus time delay model. The largest neglected time constant is 0.2 This gives $\theta = 0.2/2$ and $\tau = 1+0.2/2$. The result is given in equation (3.4.7).

$$G = \frac{1e^{-0.1s}}{1.1s + 1} \quad (3.4.7)$$

3.5 Approximation of positive numerator time constants

Skogestad's half rule cannot be used if the transfer function contain positive numerator time constants. The positive numerator is to be cancelled out by a denominator term by one of the following rules [Skogestad, 2003].

$$\frac{T_0 s + 1}{\tau_0 s + 1} \approx \begin{cases} T_0/\tau_0 & \text{for } T_0 \geq \tau_0 \geq \theta & T1 \\ T_0/\theta & \text{for } T_0 \geq \theta \geq \tau_0 & T1a \\ 1 & \text{for } \theta \geq T_0 \geq \tau_0 & T1b \\ T_0/\tau_0 & \text{for } \tau_0 \geq T_0 \geq 5\theta & T2 \\ \frac{\tilde{\tau}_0/\tau_0}{(\tilde{\tau}_0 - T_0)s + 1} & \text{for } \tilde{\tau}_0 = \min(\tau_0, 5\theta) \geq T_0 & T3 \end{cases} \quad (3.5.1)$$

Selection of the denominator term depends on a few rules. τ_0 is normally selected as the closest larger denominator time constant [Skogestad, 2003]. If so, rule *T2* or *T3* is to be used. In case there exist no larger τ_0 then rule *T1*, *T1a* or *T1b* is to be used. The first three rules are also to be used if there exists smaller τ_0 that are close to T_0 . The smaller τ_0 is selected if $T_0/\tau_0 < \tau_{0a}/T_0$ and $T_0/\tau_{0b} < 1.6$. Here τ_{0a} and τ_{0b} represent the large and small τ_0 respectively. Both conditions have to be satisfied if the smallest τ_0 is to be used [Skogestad, 2003].

In equation (3.5.1) θ represent the effective delay. The value of the effective delay is the value of θ calculated from using Skogestad's half rule [Skogestad, 2003]. In case there is no delay, then θ is replaced by τ_c [Skogestad, 2003]. If there exist more than one positive numerator time constant one should approximate one T_0 at a time, and start with the largest one [Skogestad, 2003].

If there is no delay, then replace θ by τ_c where τ_c is the desired closed-loop response time.

3.6 Quadratic programming

A quadratic programming, QP, problem was used to optimize the system in this project. The general equation that describes such a programming system is given in equation (3.6.1) below [Jensen and Bard].

$$\min_x f(x) = 0.5x^T Hx + f^T x \quad \text{subject to} \quad \mathbf{A}x \leq b \quad (3.6.1)$$

The quadratic objective function, $f(x)$, is to be minimized with respect to x , where $x \in R^{n \times n}$ [Jensen and Bard]. The vector f in the above equation describe the coefficients that is to be optimized in the objective function. The $n \times n$ symmetric matrix, H , describe the coefficients in the quadratic term [Jensen and Bard]. If there is no quadratic term, then H is a zero-matrix with dimension $n \times n$. The constraint A is a $m \times n$ matrix while b is a vector with dimension m [Jensen and Bard]. The constraints in a QP problem is linear. Additional constraints that can be included and which will limit the solution is equality constraints, $A_{eq}x = b_{eq}$ and upper and lower bounds for the coefficients.

3.7 The null space method

The idea of the null space method is to find a controlled variable, c , that is independent of disturbances [Skogestad and Alstad, 2006]. In this project the set point for the controlled variable changes when new disturbances are introduced. This is to be avoided when using the null space method. The objective is to select controlled variables that achieve near optimal operation with constant set point, and which do not change with new disturbances [Skogestad and Alstad, 2006]. The optimal controlled variables, according to the null space method, is

$$c = Hy \quad (3.7.1)$$

It can be seen from equation (3.7.1) that the controlled variables are a linear combination of the measurements, y . For the case when the active constraints are controlled, the

following optimization problem is to be solved [Skogestad and Alstad, 2006]:

$$\min_u = J(\mathbf{u}, \mathbf{d}) \quad (3.7.2)$$

In the above expression \mathbf{u} and \mathbf{d} is the inputs and disturbances respectively. The cost function, J , includes both the model equations and the active constraints [Skogestad and Alstad, 2006]. The loss is the difference between the actual cost and the optimal cost.

$$L = J(\mathbf{c}, \mathbf{d}) - J(\mathbf{c}^{opt}(\mathbf{d}), \mathbf{d}) \quad (3.7.3)$$

The null space method for such a optimization problem is given below [Skogestad and Alstad, 2006]:

Given a sufficient number of measurements ($ny \geq nu + nd$) and no measurement noise ($ny = 0$), select the optimal measurement matrix, H , such that

$$HF = 0 \quad (3.7.4)$$

where the optimal sensitivity matrix, F , is

$$F = \frac{\partial y^{opt}}{\partial d} \quad (3.7.5)$$

The number of independent unconstrained free variables, independent disturbances and independent measurements are denoted nu , nd and ny respectively. [Skogestad and Alstad, 2006]. The assumptions that are made is listed below [Skogestad and Alstad, 2006]:

- Only steady state behaviour is considered and that the control system can quickly bring the system to its new steady state.
- Only disturbances that affect steady state operation are included.
- The active constraints are active and controlled for all values of the disturbances.
- No implementation error. The implementation error is the sum of the control error and the effect of the measurement error.

This section will give a proof that given that the optimal value of c is independent of disturbances this will result in equation 3.7.4 [Skogestad and Alstad, 2006].

$$\frac{dc_{opt}}{dd} = 0 \quad \text{or} \quad \Delta c_{opt} = 0\Delta d \quad (3.7.6)$$

The expression for Δc_{opt} is

$$\Delta c_{opt} = H\Delta y_{opt} \quad (3.7.7)$$

For a given disturbance, Δd ,

$$\Delta y_{opt} = F\Delta d \quad (3.7.8)$$

Combining equation (3.7.7) and (3.7.8) and substituting into equation (3.7.6) gives

$$H\Delta y_{opt} = HF\Delta d = 0\Delta d \quad (3.7.9)$$

Finally, rearranging gives

$$HF = 0 \quad (3.7.10)$$

The above result is always correct given that $ny \geq nu + nd$.

4 Calculations and results

The first task of this project was to develop a Matlab/Simulink model of the room. This project will focus on the simple model. A simple control strategy, using a PI controller, was implemented and the behaviour of the various disturbances was simulated. A quadratic programming problem formulation was chosen for the optimization. The solution of the optimization problem depends on the constraints. From simulation of the system it was found that the set point of the controlled variable changes when new disturbances are introduced to the system. The null space method was used to find a controlled variable that is independent of disturbances. The Matlab and Simulink scripts are found in Appendix C.

4.1 Comparison of the simple and advanced model

The equation for the temperature in the floor is the same for both the simple and advanced model. The room temperature is defined as follows for the simple and advanced model

$$\text{Simple model } \dot{T}_R = \frac{-(UA)_{R/O}}{(mCp)_R}(T_R - T_O) + \frac{(UA)_{F/R}}{(mCp)_R}(T_F - T_R) + \frac{q_{H/R}}{(mCp)_R} \quad (4.1.1)$$

$$\text{Advanced model } \dot{T}_R = \frac{q_{in}}{m_R}(T_O - T_R) + \frac{q_{H/R}}{(mCp)_R} + \frac{(UA)_{F/R}}{(mCp)_R}(T_F - T_R) - \frac{(UA)_{R/O}}{(mCp)_R}(T_R - T_O) \quad (4.1.2)$$

Rearranging equation (4.1.2) gives

$$\begin{aligned} \dot{T}_R &= \left(\frac{q_{in}}{m_R} + \frac{(UA)_{R/O}}{(mCp)_R} \right) (T_O - T_R) + \frac{q_{H/R}}{(mCp)_R} + \frac{(UA)_{F/R}}{(mCp)_R} (T_F - T_R) \\ &= (UA)_{effective} (T_O - T_R) + \frac{q_{H/R}}{(mCp)_R} + \frac{(UA)_{F/R}}{(mCp)_R} (T_F - T_R) \end{aligned} \quad (4.1.3)$$

The equations (4.1.3) and (4.1.1) is the same except for the term $(UA)_{effective}$. To compensate for the heat flow to the room, q_{in} , the heat transfer in the room, $UA_{R/O}$, is multiplied with a constant number in the simple model. The number that is to be multiplied with has to be chosen in a way such that the term $(UA)_{R/O}$ will be similar to $(UA)_{effective}$. This will make the behaviour of the simple model similar to the advanced one. By the method trial and error it was found that $UA_{R/O}$ should be multiplied with 10 to get the best behaviour of the system. The advanced model has an extra equation compared to the simple one. Table 4.1.1 and 4.1.2 give the parameters (which is not already given in Table 2.3.2) necessary to calculate the mass. It is assumed atmospheric pressure outside the room, p_O . To ease the calculations, the value of k in Table 4.1.2 is assumed to be 1. It is assumed that air contains 78% N_2 , 21% O_2 and 1% Ar . From the values given in the tables above it can be seen that the mass is approximately equal to q_{in} . The value of q_{in} is assumed to be small. From this section it can be seen that the simple and advanced model are not that different. This project will focus on the simple model, and the rest of this report will therefore describe the simple system.

Table 4.1.1: Parameters of air that is used in the advanced system

Parameter	Unit	Value
$C_{p_{air}}$	$[\frac{J}{kgK}]$	1005 ^[ToolBox, c]
M_r	$[\frac{kg}{mol}]$	0.02897 ^[ToolBox, b]
ρ_{air}	$[\frac{kg}{m^3}]$	1.166 ^[ToolBox, c]

Table 4.1.2: Other parameters that is used in the advanced system

Parameter	Unit	Value
k		1
p_o	[bar]	1.01
R	$[\frac{J}{Kmol}]$	8.314×10^5 ^[AylwardandFindlay, 2008]

4.2 Preparation to the simulation

According to equation (2.1.4) and (2.1.5) the overall heat transfer coefficients in the floor and room have to be calculated in order to simulate the system. The overall heat transfer coefficients, U_i , were found using the same procedure as explained previously in the theory concerning heat transfer. Equation (4.2.1) and (4.2.2) gives the expression for the overall heat transfer coefficient of the room and floor.

$$\text{Room: } U_{R/O} = \frac{1}{\frac{1}{h_{air}} + \frac{\Delta x_{wood}}{k_{wood}} + \frac{\Delta x_{rock\ wool}}{k_{rock\ wool}} + \frac{\Delta x_{wood}}{k_{wood}} + \frac{1}{h_{air}}} \quad (4.2.1)$$

$$\text{Floor: } U_{F/R} = \frac{1}{\frac{\Delta x_{wood}}{k_{wood}}} \quad (4.2.2)$$

The heat from the room to the outside is described with a combination of convective and conductive heat transfer. The heat is transferred through three solids, wood which is isolated with rock wool in the middle, and with air at different temperature on the outer walls. The heat from the floor to the room is a conductive heat transfer through the floor. The obtained values for the overall heat transfer coefficients are given in Table 4.2.1. In addition to the overall heat transfer the mass of the wall and floor has to be

Table 4.2.1: Values for the overall transfer coefficients

Parameter	Unit	Value
$U_{R/O}$	$[\frac{J}{hm^2K}]$	539.61
$U_{F/R}$	$[\frac{J}{hm^2K}]$	49 920

calculated. The general formula to calculate the mass is a follows

$$m = \rho V \quad (4.2.3)$$

The room contains air, the mass is calculated as described in equation (4.2.4).

$$m_{room} = \rho_{air}(A_{floor}h_{wall}) \quad (4.2.4)$$

To find the mass of the floor it is necessary to have the density of concrete and volume of the floor.

$$m_{floor} = \rho_{concrete}(A_{floor}\Delta x_{concrete}) \quad (4.2.5)$$

The values of the different parameters used to find the mass of the floor and room are given in Table 2.3.1 and 2.3.2. The calculated values for the mass are given in Table 4.2.2. Last but not least, the area of the floor and room has to be calculated. The area of

Table 4.2.2: Values for the mass of the floor and room

Parameter	Unit	Value
m_R	[kg]	69.96
m_F	[kg]	6000

the floor is given in Table 2.3.1. The area of the room is $A_{room} = 4(h_{room}l_{floor}) = 48m^2$. As mentioned earlier in the process description, it is assumed that the heat transfer in the room is from four walls.

4.3 Creating the simple model

The system of transfer functions that describes the model is given in equation (2.1.4) and (2.1.5). From these equations the following state space model is obtained.

$$\begin{bmatrix} \frac{T_F}{dt} \\ \frac{T_R}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-(UA)_{F/R}}{(mCp)_F} & \frac{(UA)_{F/R}}{(mCp)_F} \\ \frac{(UA)_{F/R}}{(mCp)_R} & \left(\frac{-(UA)_{R/O}}{(mCp)_R} - \frac{(UA)_{F/R}}{(mCp)_R} \right) \end{bmatrix} \begin{bmatrix} T_F \\ T_R \end{bmatrix} + \begin{bmatrix} \frac{1}{(mCp)_F} & 0 & 0 \\ 0 & \frac{1}{(mCp)_R} & \frac{(UA)_{R/O}}{(mCp)_R} \end{bmatrix} \begin{bmatrix} q_{H/F} \\ q_{H/R} \\ T_o \end{bmatrix} \quad (4.3.1)$$

$$\begin{bmatrix} T_F \\ T_R \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_F \\ T_R \end{bmatrix} \quad (4.3.2)$$

In the above expression the state vector is a vector with T_F and T_R , while the input vector contains the inputs $q_{H/F}$, $q_{H/R}$ and T_o . The \mathbf{A} and \mathbf{B} matrices are both constant. A state space block with the vectors and matrices as described in equation (4.3.1) and 4.3.2 was implemented in Simulink. A figure of the Simulink model is given in Figure 4.3.1. The system was simulated over one day, with no heating in the floor, and max heating in the room. These parameters will be kept constant throughout the rest of this project. The value of the inputs used in the simulation is given in Table 4.3.1. The values of the initial conditions are given in Table 4.3.2. The initial conditions are not varied throughout the rest of this project. The system was simulated with no disturbances. Figure 4.3.2 gives the result from the simulation. The parameter $UA_{R/O}$ is not multiplied with any constant at this moment. It can be seen from Figure 4.3.2 that the temperature in the floor increases linearly with time. The temperature in the room increases linearly with time after a 20 minutes as is shown in Figure 4.3.3. The room temperature increases faster the first 20 minutes because of the difference between T_o and T_F is small. The heat transfer to the floor is small. Little by little will this temperature difference increase, together with the heat transfer to the floor. This result

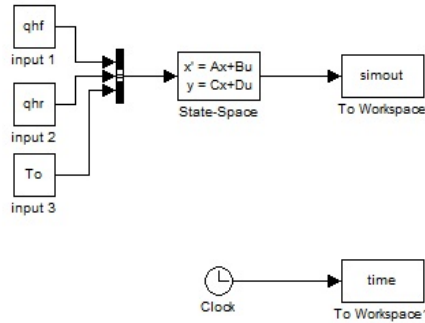


Figure 4.3.1: Overview of the simple model in Simulink

Table 4.3.1: Values for the inputs that is used in the simulation

Parameter	Unit	Value
$q_{H/R}$	$[\frac{J}{h}]$	no heat on: 0 max: $4000 \times 60 \times 60^{[radiatorWorks]}$
$q_{H/F}$	$[\frac{J}{h}]$	no heat on: 0 max: $575 \times 60 \times 60^{[Hunton]}$
T_o	$[K]$	273

in a less steeper curve in the room temperature. Figure 4.3.4 gives the result from the simulation if $UA_{R/O}$ is multiplied with 10. Throughout the rest of this project this is the value of $UA_{R/O}$ that is used if anything else is not mentioned. The temperatures in Figure 4.3.4 is curved compare to Figure 4.3.2, where the temperature is linear after the first 20 min. A curved behaviour is more realistic. The temperature in the room will decrease and starts to flatten out after a while if no extra heat is turned on.

4.4 Modelling and control in Matlab, Simulink

The second step in the project is to implement a control structure to the system. A PI controller was implemented, where the room temperature is measured by regulating the heat in the room, $q_{H/R}$. A picture of the control structure is given in figure 4.4.1. From Figure 4.4.1 it can be seen that it was implemented some constraints to the temperature set point. The room temperature is supposed to be 10°C in some parts of the day, and 20°C the rest of the day. In this project five time intervals was simulated. In the first time interval the room temperature is 10°C , in the next time interval it is 20°C in the room, in the third one it is 10°C again and so on. The time each intervals starts and ends will vary. In the beginning it was assumed that the temperature in the room should be 20°C when people are assumed to be home, between 7 and 8 am and between 4 to

Table 4.3.2: Initial conditions used in the simulation

Parameter	Unit	Value
T_F	[K]	283
T_R	[K]	283

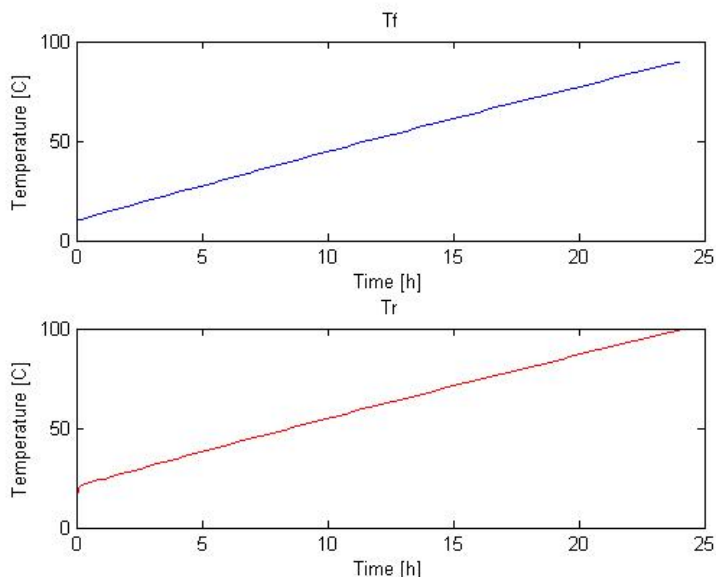


Figure 4.3.2: Temperature in the room and floor of the simple model

11 pm respectively. The rest of the day should the temperature be 10°C . The time to switch between one interval to another was implemented in the switch block in Simulink.

To find the tuning parameters of the PI controller the SIMC rules were used. The transfer functions from the state space model were found in Matlab by using the command tf . The following transfer function with input $q_{H/R}$ and output T_R was obtained.

$$\begin{aligned}
 G &= \frac{1.442 \times 10^{-5}s + 7.826 \times 10^{-8}}{s^2 + 0.3075s + 3.379 \times 10^{-5}} \\
 &= \frac{2.3145 \times 10^{-3}(181.702s + 1)}{(9090.909s + 1)(3.253s + 1)}
 \end{aligned} \tag{4.4.1}$$

In order to get the transfer function on FOPTD form, equation (3.4.1), the method that approximate positive numerator time constants has to be used. When approximating the positive numerator term the numerator is to be cancelled out with a denominator term, by using the rules as explained in section 3.5. The first step is to check if the smallest or largest time constant is to be used. The value of the different time constants are given below.

$$\tau_{0,a} \text{ (large)} = 9090.909$$

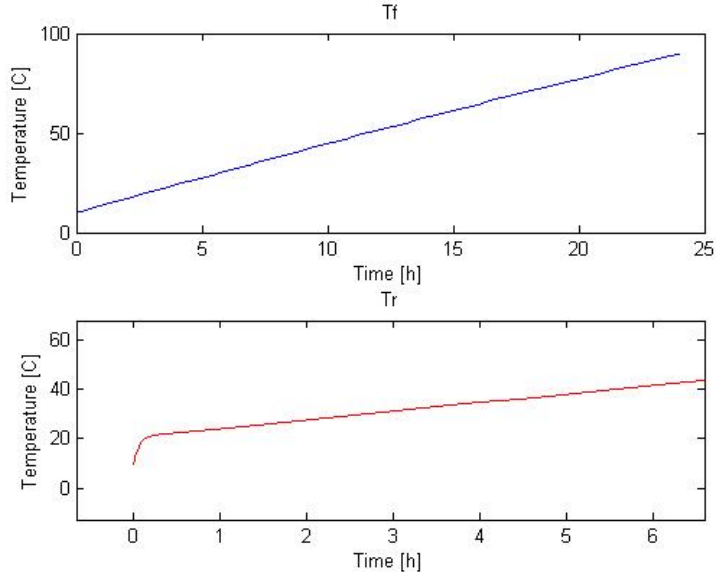


Figure 4.3.3: Temperature in the room and floor of the simple model

$$\tau_{0,b} \text{ (small)} = 3.253$$

$$T_o = 181.702$$

The smallest denominator time constant is to be chosen if

$$1) \frac{T_0}{\tau_{0,b}} < \frac{\tau_{0,a}}{T_o} \quad : \quad 55.86 > 50.03 \quad (4.4.2)$$

$$2) \frac{T_0}{\tau_{0,b}} < 1.6 \quad : \quad 55.86 > 1.6 \quad (4.4.3)$$

As can be seen from equation (4.4.2) and (4.4.3) none of this requirements are satisfied, and the largest time constant is to be used in the approximation. From the above results it can be seen that rule T2 in equation (3.5.1) is to be used. The value of τ_c that is used in the simulation is 0.01. This value is chosen because the transfer function does not have any time delay, so a small value is chosen for τ_c . The approximation of the positive numerator time constant is calculated in equation (4.4.4).

$$\frac{T_0 s + 1}{\tau_0 s + 1} \approx \frac{T_0}{\tau_0} = \frac{181.702}{9090.902} = 0.019987 \quad (4.4.4)$$

The expression of the transfer function after the approximation of the positive numerator time constant is calculated in equation (4.4.5).

$$\begin{aligned} G &= \frac{2.3145 \times 10^{-3} \times 0.019987}{3.253s + 1} \\ &= \frac{4.6599 \times 10^{-5}}{3.253s + 1} \end{aligned} \quad (4.4.5)$$

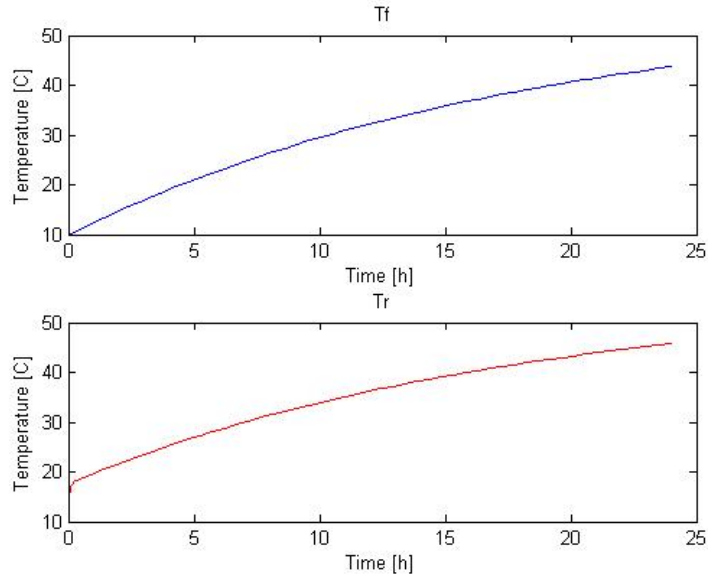


Figure 4.3.4: Temperature in the room and floor of the simple model with $UA_{R/O} \times 10$

From this expression the controller gain and integral time was calculated using the SIMC rules. A tight control was used, and the calculated values are given below.

$$Kc = \frac{1}{4.6599 \times 10^{-5}} \frac{3.253}{0.01} = 7032008.28 \quad (4.4.6)$$

$$\tau_I = \min(3.253, 0.04) = 0.04 \quad (4.4.7)$$

In Matlab the proportional part, P, and integral part, I, of equation (3.3.1) is to be implemented. From this equation it can be seen that $P = Kc$ and $I = \frac{Kc}{\tau_I}$. The calculated values of P and I that was used in the simulation is given below.

$$P = 7032008.28 \quad (4.4.8)$$

$$I = \frac{7032008.28}{0.04} = 17580007.1 \quad (4.4.9)$$

The system was simulated with the same inputs as described in Table 4.3.1, and without disturbances. Figure 4.4.1 show a saturation block on $q_{H/R}$. In the beginning this block was used to set the maximum and minimum value on the heat flow. The maximum and minimum values of $q_{H/R}$ are given in Table 4.3.1. Later this block was not necessary because the saturation limits was set in the PI-controller, by using the anti-windup method clamping. The anti-windup method turn off the integral part in the PI controller when the upper or lower limits of the saturation is reached. In the rest of this project a PI controller with anti-windup is used. Figure 4.4.2 gives the result of the simulation. It can be seen from Figure 4.4.2 that the temperature in the room do not reach 20°C in

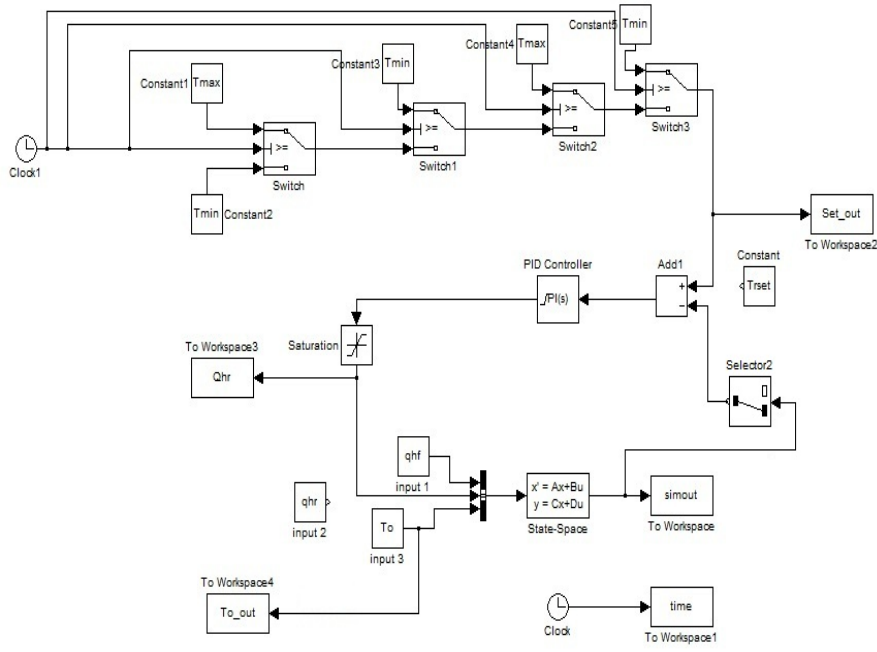


Figure 4.4.1: Representation of the simple control structure

the first time interval. This shows that it takes some time, more than one our, to heat the room from 10 °C to 20 °C when the outdoor temperature is 0 °C. If the temperature is to be 20 °C at 7am, Figure 4.4.2 indicate that the heating of the room should start a bit earlier. From the last time interval it can be seen that the temperature in the room do not reach 10 °C before a new day begin. In addition to room temperature, Figure 4.4.2 gives a representation of the heat flow in the room. The heat is constant in the first interval. The heat that is used is needed to maintain a temperature of 10 °C. When the temperature increases, more heat is used. From Figure 4.4.2 it can be seen that the upper saturation limit is reached in the second time interval. The heat decreases when the temperature decreases. For a short period of time no heat is necessary, but to have 10 °C in the room the heat increases again. Again, the heat increases when the temperature increases. The heat reaches it saturation limit. It do not need max heat to maintain 20 °C, this explains the decrease of heat in the fourth time interval.

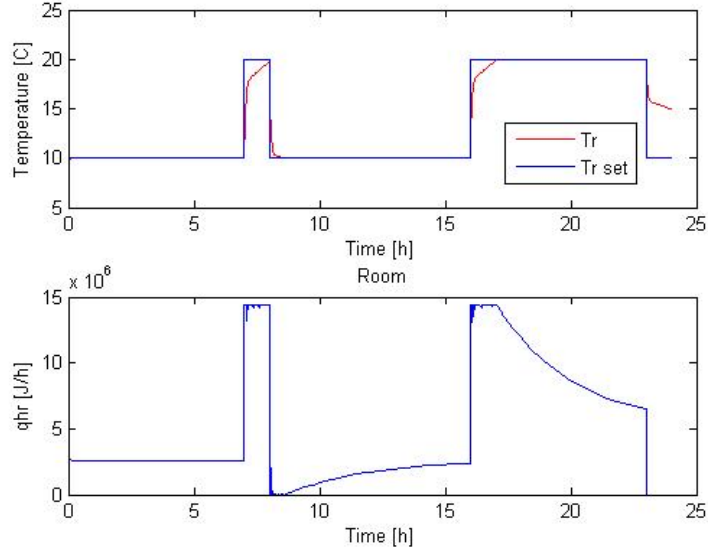


Figure 4.4.2: Room temperature and heat versus time for the model when the control structure is implemented to the model

4.5 Optimal dynamical control problem

The objective of the optimization is to minimize the energy consumption in the room with regards to room temperature.

$$\begin{aligned}
 & \min_{T_R} \sum_{k=0}^{n-1} q_{H/R} \\
 & \text{subject to} \\
 & q_{H/R,k} = q_{max} \\
 & T_{R,k} \geq T_{min}
 \end{aligned} \tag{4.5.1}$$

As can be seen from equation (4.5.1) the energy consumption is found by adding the values of $q_{H/R}$ at all the time steps, k . A quadratic programming method was used to solve the optimization problem. The function *quadprog* in Matlab was used to solve the problem. The symmetric matrix, H , is zero because the problem is linear. The feasible region of a linear programming problem is convex [Siyavula (Technology-powered-learning)]. This means that the minimum that is found is a global minimum. Since the H -matrix is zero, a linear programming method could have been used instead.

In order to find a model for the equality constraints a discrete model were obtained from the the state space model representation. Assume h represent the distance between each time step, and k is a vector equal to $[0, \dots, n]$ where $n = \frac{t_{simulation\ end} - t_{simulation\ start}}{h}$. The method trial and error was used the best value of h , 0.02. A derivation of the discrete

model is given below. From the state space model the following expression was obtained

$$\dot{x}_k = \mathbf{A}x_k + \mathbf{B}u_k \quad \text{where} \quad \dot{x}_k = \frac{x_{k+1} - x_k}{h} \quad (4.5.2)$$

Combination of both expressions in equation (4.5.2) gives the discrete representation of the state vector, x.

$$\begin{aligned} x_{k+1} - x_k &= h\mathbf{A}x_k + h\mathbf{B}u_k \\ x_{k+1} &= (h\mathbf{A} + 1)x_k + h\mathbf{B}u_k \\ &= \mathbf{A}_d x_k + \mathbf{B}_d u_k \end{aligned} \quad (4.5.3)$$

In the same way the discrete representation of the output vector, y, was found.

$$\begin{aligned} y_{k+1} &= \mathbf{C}x_k \\ &= \mathbf{C}_d x_k \end{aligned} \quad (4.5.4)$$

To be able to solve the problem the discrete model has to be on the form $\mathbf{A}\mathbf{z} = \mathbf{b}$. This expression can easily be derived if the discrete model is investigated for the first values of k.

$$\begin{aligned} x_1 &= \mathbf{A}_d x_0 + \mathbf{B}_d u_0 \quad \rightarrow x_1 - \mathbf{B}_d u_0 = \mathbf{A}_d x_0 \\ x_2 &= \mathbf{A}_d x_1 + \mathbf{B}_d u_1 \quad \rightarrow x_2 - \mathbf{B}_d u_1 - \mathbf{A}_d x_1 = 0 \\ x_3 &= \mathbf{A}_d x_2 + \mathbf{B}_d u_2 \quad \rightarrow x_3 - \mathbf{B}_d u_2 - \mathbf{A}_d x_2 = 0 \end{aligned}$$

If the equation are written in matrix form the following expression is obtained for the first three k's.

$$\begin{bmatrix} 1 & 0 & 0 & -\mathbf{B}_d & 0 & 0 \\ -\mathbf{A}_d & 1 & 0 & 0 & -\mathbf{B}_d & 0 \\ 0 & -\mathbf{A}_d & 1 & 0 & 0 & -\mathbf{B}_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u_0 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_d x_0 \\ 0 \\ 0 \end{bmatrix} \quad (4.5.5)$$

The above equation can easily be expanded to apply for all values of k. The matrix \mathbf{A} and vector \mathbf{z} and \mathbf{b} for the general case are given below.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -\mathbf{B}_d & 0 & 0 \\ -\mathbf{A}_d & 1 & 0 & 0 & -\mathbf{B}_d & 0 \\ 0 & -\mathbf{A}_d & 1 & 0 & 0 & -\mathbf{B}_d \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (4.5.6)$$

$$\mathbf{b} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ u_0 \\ u_1 \\ u_2 \\ \cdot \\ \cdot \end{bmatrix} \quad (4.5.7)$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{A}_d x_0 \\ 0 \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \quad (4.5.8)$$

In addition to the equality constraints above, the constraints to the room temperature was included. To ease the calculations the day were divided in five time intervals of equal length, $\frac{n}{5}$. In the first interval the room temperature is 10°C , in the second it is 20°C , in the next one the temperature is 10°C again and so on. Additional constraints as upper and lower bounds on the input and output was included to narrow the optimization problem. The values of the upper and lower bounds that were used in this project is given in Table 4.5.1. The result from the simulation of the optimized system

Table 4.5.1: Values of the upper and lower bounds used in the optimization problem

Parameter	Unit	Value
Upper bounds		
T_R	[K]	400
T_F	[K]	400
T_o	[K]	Value of T_o given in the input
$q_{H/R}$	[J/h]	$4000 \times 60 \times 60$
$q_{H/F}$	[J/h]	0
Lower bounds		
T_R	[K]	277
T_F	[K]	277
T_o	[K]	Value of T_o given in the input
$q_{H/R}$	[J/h]	0
$q_{H/F}$	[J/h]	0

is given in Figure 4.5.1. The system was simulated without disturbances. From Figure 4.5.1 it can be seen that the heating of the room begins approximately 1.5 hour before the constraint in the first time interval. It can be seen from the third time interval that

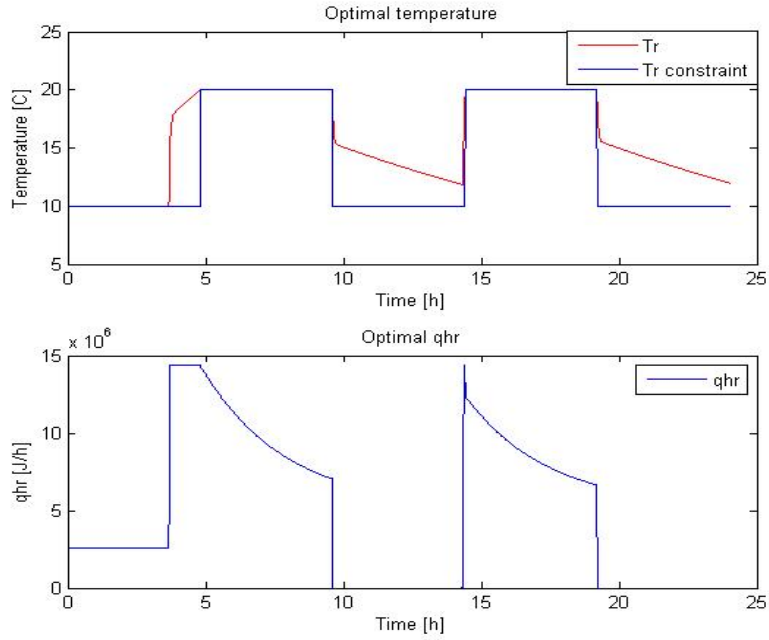


Figure 4.5.1: Optimal room temperature and optimal heat flow versus time

the temperature do not reach 10°C before it begins to heat up again. The same is true for the last interval. The tuning parameters in the PI controller has to be retuned in such a way that Figure 4.5.1 get the same behaviour as the optimized figure. To retune the tuning parameters the method trial-and-error was used. The same method was used to get the time to switch between the different intervals such that they are equal to the intervals for heating and cooling in the optimized problem. The new tuning parameters are given in Table 4.5.2 and the new switching times are given in Table 4.5.3. The sys-

Table 4.5.2: New tuning parameters

Parameter	Value
P	7032008.28×4
I	175800207.1

tem was simulated with the new tuning parameters and no disturbances. Figure 4.5.2 gives the results from the control system. In addition to the room temperature, the set point and constraints are included in the figure. Figure 4.5.2 demonstrate how the temperature begin to heat approximately 1.5 hour before the constraint. A comparison between Figure 4.5.1 and 4.5.2 show similar behaviour of the two graphs.

Table 4.5.3: New switching parameters

Parameter	Value
Switch block 1	3.5
Switch block 2	9.6
Switch block 3	13.94
Switch block 4	19.2

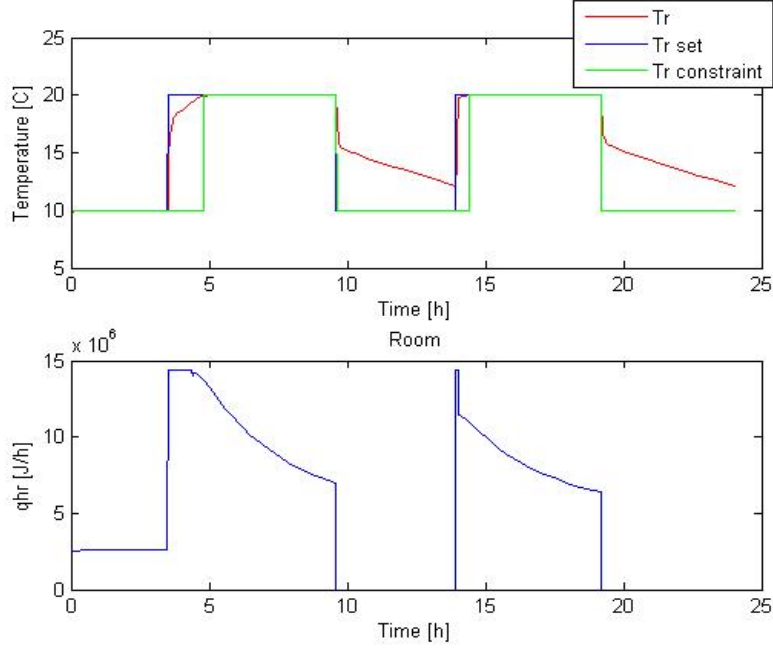


Figure 4.5.2: Room temperature heat flow versus time for the model with new tuning parameters

4.6 Energy consumption

As described in section 4.5, the objective of the optimization was to minimize the energy consumption. The optimal cost function is defined as $J_{opt} = \int_0^t (q_{H/F} + q_{H/R})$. In this project we do not have any heating in the floor and because of this we do not include $q_{H/F}$ in the cost function. The energy consumption, denoted E in this project, was included as an extra element in the state vector in the state space model.

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{-(UA)_{F/R}}{(mCp)_F} & \frac{(UA)_{F/R}}{(mCp)_F} & 0 \\ \frac{(UA)_{F/R}}{(mCp)_R} & \left(\frac{-(UA)_{R/O}}{(mCp)_R} - \frac{(UA)_{F/R}}{(mCp)_R} \right) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_F \\ T_R \\ E \end{bmatrix} + \begin{bmatrix} \frac{1}{(mCp)_F} & 0 & 0 \\ 0 & \frac{1}{(mCp)_R} & \frac{(UA)_{R/O}}{(mCp)_R} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_{H/F} \\ q_{H/R} \\ T_o \end{bmatrix} \quad (4.6.1)$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_F \\ T_R \\ E \end{bmatrix} \quad (4.6.2)$$

From the state space model, the energy consumption is defined as $E = \int \dot{x} = x$. The energy consumption was calculated in Simulink by adding a integrator block to $q_{H/R}$ as shown in Figure 4.6.1. In addition to the modification on the Simulink model and the

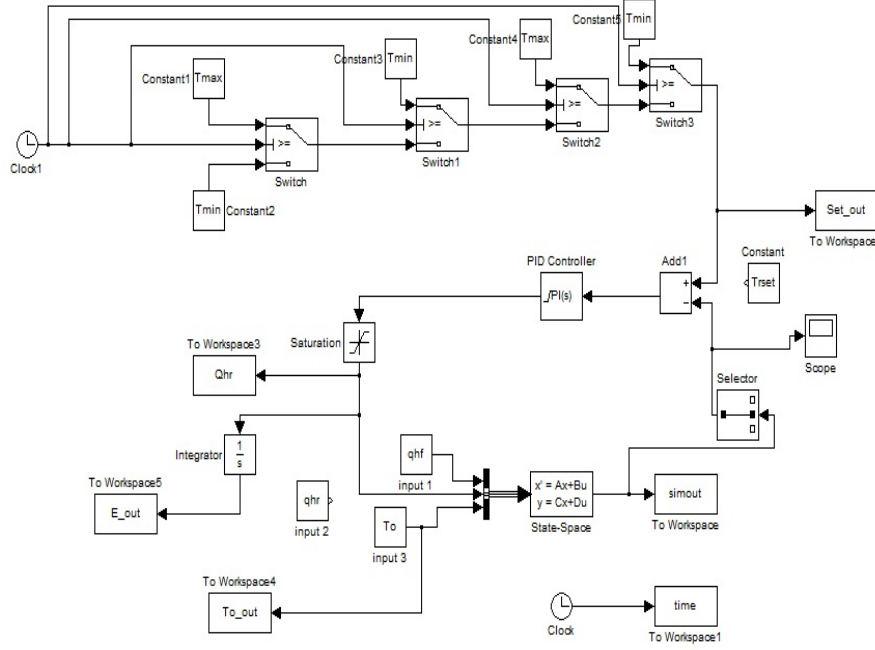


Figure 4.6.1: Simulink model of the simple system where the computation of energy consumption is included

state space model, the upper and lower bounds of E has to be included in the optimization problem. The upper and lower bounds of E are given in Table 4.6.1. The result from the

Table 4.6.1: Upper and lower bounds for the energy consumption

Parameter	Value
E_{upper}	∞
E_{lower}	0

simulation is given in Figure 4.6.2. The optimal energy consumption is included in the figure together with the energy consumption calculated from the model with the control structure. The optimal energy consumption is a bit lower than the calculated energy consumption. This can be explained by the small differences in the amount of heat used in the optimal system and in the other as demonstrated in Figure 4.5.1 and 4.5.2. The

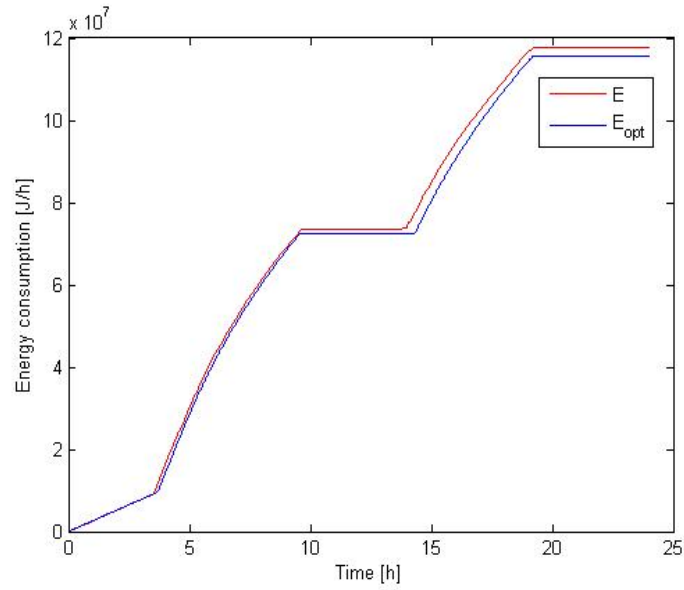


Figure 4.6.2: Energy consumption versus time

energy consumption is constant between 9 am and 2 pm, and between 9 pm and out the rest of the day because no heat is used at that time, as can be seen from Figure 4.5.2. Naturally, the energy consumption is highest when heating the room and keeping it at that temperature.

4.7 Behaviour of disturbances

So far the system has been simulated with no disturbances. This is not realistic in practise. The system was simulated with different outdoor temperatures as described below. The temperature is given in °C.

$$T_o = \begin{cases} -14 \\ -10 \\ -4 \\ -2 \\ 0 \\ 2 \\ 4 \\ 10 \\ 14 \end{cases} \quad (4.7.1)$$

$$T_o = 10 \sin\left(\frac{2\pi}{24}t - \frac{\pi}{2}\right) + 273$$

Figure 4.7.1 to 4.7.5 gives the result from the simulations with outdoor temperature equal to -10°C , $+2^\circ\text{C}$ and $+10^\circ\text{C}$, in addition to the result when the outdoor temperature is

represented by a sinus wave. The rest of the result from the simulation can be found in Appendix B. From Figure 4.7.1 to 4.7.5 it can be seen that the point to start heating

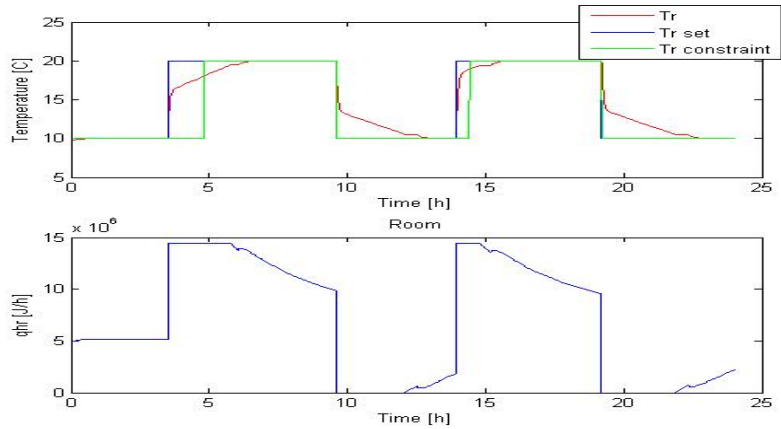


Figure 4.7.1: Room temperature and heat versus time for the system with constant outdoor temperature equal to -10°C

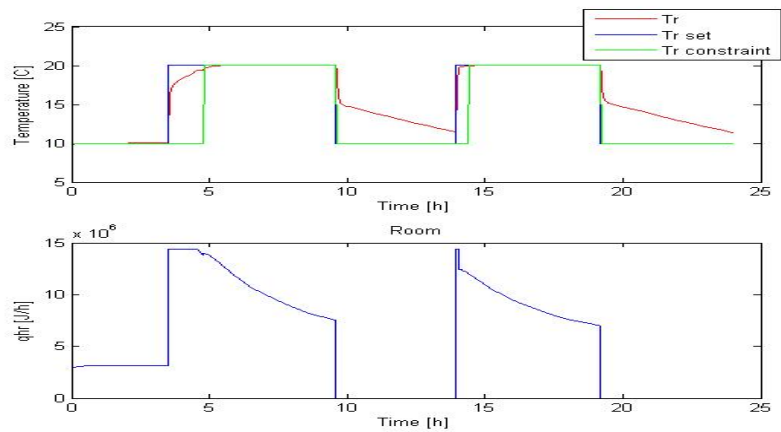


Figure 4.7.2: Room temperature and heat versus time for the system with constant outdoor temperature equal to -2°C

the room such that it is 20°C when it is supposed to be changes when disturbances are introduced to the system. If it is colder than 0°C outside the heating of the room begins to late. Naturally, the colder the temperature the earlier the heating of the room should begin. The opposite is true if the temperature is above 0°C . Then the room temperature reach 20°C before it should, and this is not optimal with respect to energy consumption. Figure 4.7.1 to 4.7.5 indicate that the system behaves worse the colder the temperature is. This because the difference from where the constraint start and the

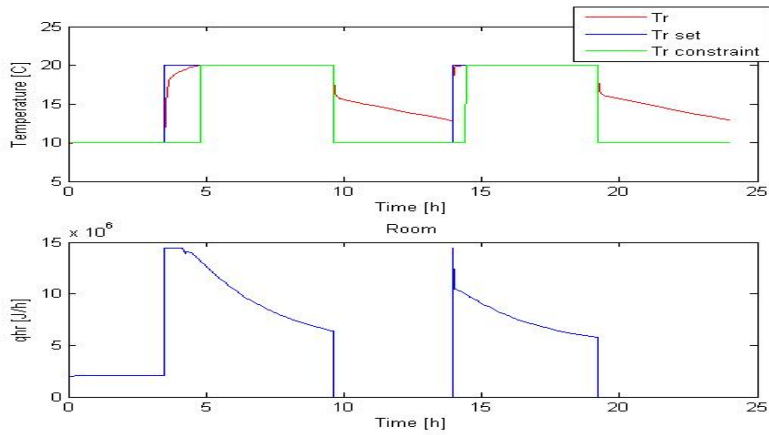


Figure 4.7.3: Room temperature and heat versus time for the system with constant outdoor temperature equal to 2°C

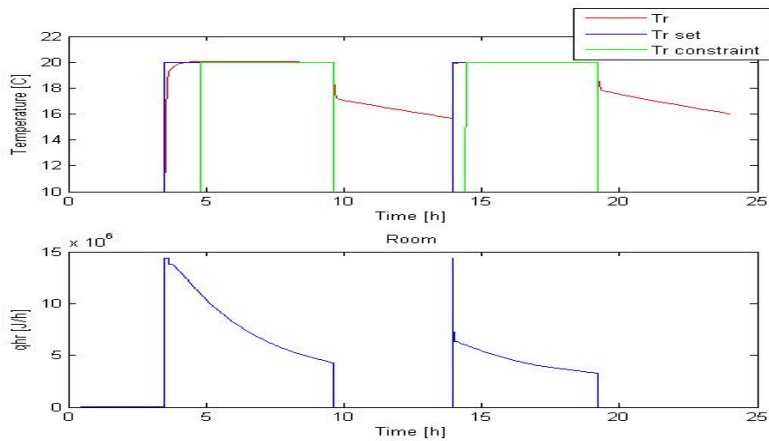


Figure 4.7.4: Room temperature and heat versus time for the system with constant outdoor temperature equal to 10°C

point where the room temperature is 20°C is larger for colder temperatures compared to warmer outdoor temperatures. When the outdoor temperature varies as a sine wave, Figure 4.7.5, the room temperature acts like in the warmer region. The heating of the room starts to early.

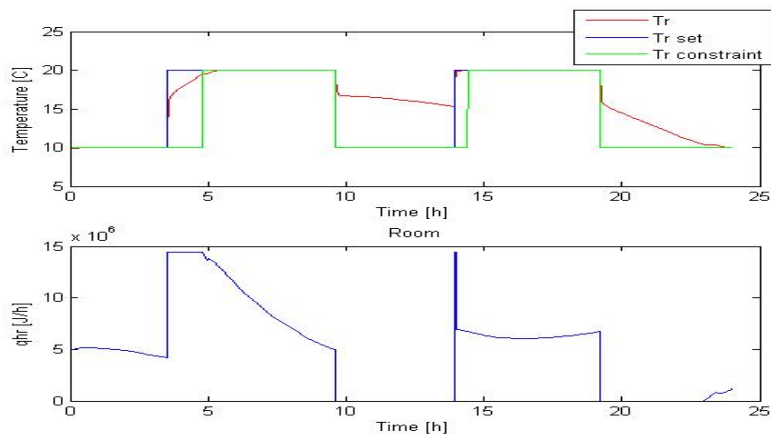


Figure 4.7.5: Room temperature and heat versus time for the system with varying outdoor temperature equal to a sine wave with amplitude equal to 10, frequency equal to $\frac{2\pi}{24}$ and phase equal to $\frac{\pi}{2}$

4.8 Control variable that is independent of disturbances

The null space method was used to find a controlled variable, c , that is independent of disturbances. Due to lack of time only the first time of switch, from 10°C to 20°C , was examined. There were some problems with finding a good controlled variable. When checking the response of the system with the different values of c it responds well around the nominal case, or well to outdoor temperatures below 0°C , but not for temperature above 0°C . The same is true for the opposite case. This section will give a summary of the controlled variables that was calculated and how the system behave when these controlled variables where implemented. The controlled variables were calculated using different combinations of disturbances and measurements. The two main disturbances is the outdoor temperature and $UA_{R/O}$. The values of the disturbances in the nominal case, in addition to the three disturbances that were used in the simulation of the null space method is given in Table 4.8.1 The difference between the nominal case and the

Table 4.8.1: The disturbances that was used in the calculation of the null space method

Parameter	Unit	Value
Nominal value		
T_o	$[K]$	273
$UA_{R/O}$	$[\frac{J}{hK}]$	$UA_{R/O} \times 10$
Disturbance 1		
T_o	$[K]$	268
$UA_{R/O}$	$[\frac{J}{hK}]$	$UA_{R/O} \times 10$
Disturbance 2		
T_o	$[K]$	273
$UA_{R/O}$	$[\frac{J}{hK}]$	$UA_{R/O} \times 8$
Disturbance 3		
T_o	$[K]$	277
$UA_{R/O}$	$[\frac{J}{hK}]$	$UA_{R/O} \times 10$

new disturbances is defined as $\Delta d = d_i - d_{nominal}$. This values was found from Table 4.8.1, and are given in Table 4.8.2. The measurements that was checked is given in

Table 4.8.2: Difference in disturbance from the nominal case

Parameter	Unit	Δd
Nominal case	$[K]$	0
Disturbance 1	$[K]$	5
Disturbance 2	$[\frac{J}{hK}]$	52 000
Disturbance 3	$[K]$	4

equation (4.8.1).

$$y = \begin{bmatrix} T_o \\ t_{s,opt} \\ \Delta T = T_R - T_O \\ Distance \\ q_{H/R} \\ tsqrt = \sqrt{|t_c - t_s|} \\ tsq = \sqrt{t_s} \end{bmatrix} \quad (4.8.1)$$

In equation (4.8.1) $t_{s,opt}$ is defined as the time where the optimal temperature starts to increase. $q_{H/R}$ is the value at switch, i.e. at 10°C. The distance is given as described in equation (4.8.2)

$$Distance = \left\| \left[\begin{array}{c} \frac{T_c - T_s}{(T_c - T_s)_n} \\ \frac{t_c - t_s}{(t_c - t_s)_n} \end{array} \right] \right\| \quad (4.8.2)$$

where T_c and T_s represent the temperature at the constraint and at the switch respectively. The time at the constraint and at the switch is t_c and t_s respectively. Figure 4.8.1 gives a representation of the values that is used to calculate the distance. To find the norm of the disturbance vector it has to be scaled. This because it contains values with different units. In order to delete the units the values was scaled as described in equation (4.8.2). In order to calculate a good optimal sensitivity and optimal measure-

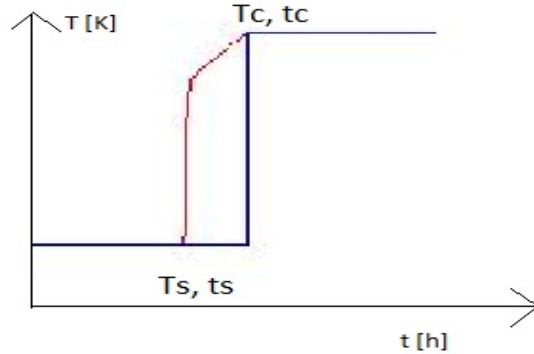


Figure 4.8.1: Representation of the optimal temperature at the first time of switch

ment matrix the disturbances and measurements has to be scaled as well. If the values are not scaled some of the measurements or disturbances will be weighted more than others because they are bigger. For example T_o will have a large value compared to $t_{s,opt}$ since the unit of temperature are Kelvin and the unit of time are hours. The scaling of the disturbances and measurements was calculated by the following general formula

$$x = \frac{x - x_{nominal}}{\Delta x_{max}} \quad (4.8.3)$$

The Δx_{max} values for the disturbances and measurements are given in Table 4.8.3. The values were found by taking the largest difference from the nominal value. The values of

Table 4.8.3: Δx_{max} values

Parameter	Unit	Δx_{max}
Measurements		
ΔT_{omax}	[K]	5
$\Delta t_{s,opt_{max}}$	[h]	0.4
$\Delta/DeltaT_{max}$	[K]	15
$\Delta q_{H/R_{max}}$	$[\frac{J}{h}]$	1.44×10^7
$\Delta distance_{max}$		0.4396
$\Delta tsqrt_{max}$	[h]	0.2664
Δtsq_{max}	[h]	0.1018
Disturbances		
ΔT_{omax}	[K]	5
$\Delta UA_{R/O_{max}}$	$[\frac{J}{hK}]$	$0.5 \times UA_{R/O,n}$

the measurements for the nominal case and for the cases with the different disturbances are given in Table 4.8.4. The optimal sensitivity matrix was calculated as described below.

$$F = \begin{bmatrix} y_{di} - y_n \\ \Delta di \end{bmatrix} \quad \text{where } i = [1, 2, 3] \quad (4.8.4)$$

The optimal measurement matrix, H, was calculated from equation (3.7.4). The measurements and H was implemented in Simulink, and the control variable was calculated as described in equation (3.7.1). A representation of the Simulink model is given in Figure 4.8.2. A time delay block was implemented in the Simulink model in order for the system to run properly. The time delay was set to a small value, 0.001, such that it do not affect the simulations. Table 4.8.5 gives a representation of the different combination of measurements and disturbances that was used to calculate the different controlled variables. In addition, Table 4.8.5 gives the controlled variable that gave the best respond in each case, its value at switch and the combination of measurements that contribute in the calculations. How this values are found are given in an example below. The controlled variable that is written first is the value that is emphasized most in the calculations.

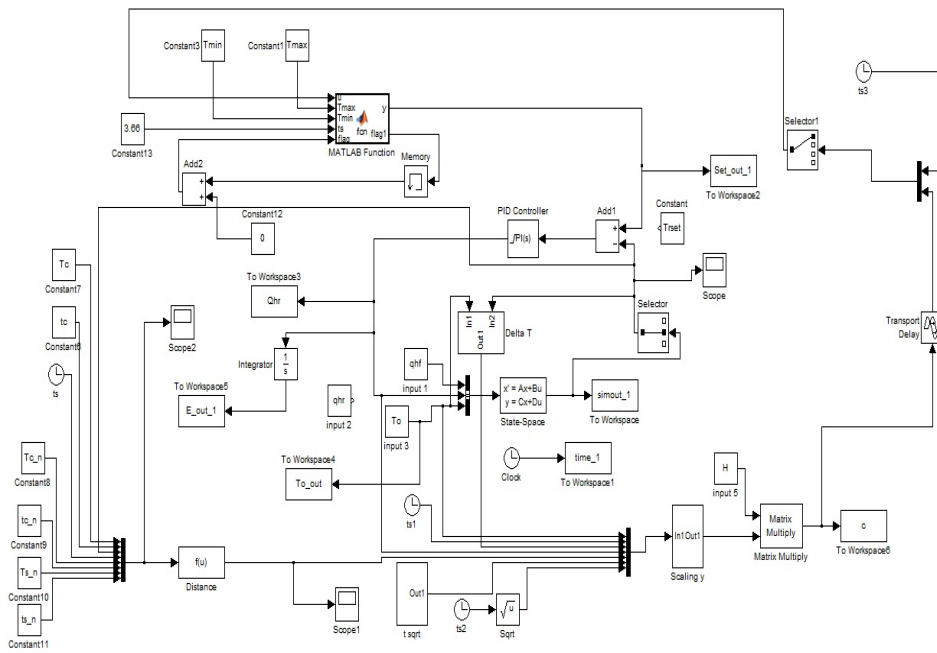


Figure 4.8.2: Representation of the Simulink model which was used to calculate the control variable, c

Table 4.8.4: Disturbance used in the null space method

Parameter	Unit	value
Nominal		
T_o	[K]	273
$t_{s,opt}$	[h]	3.66
ΔT	[K]	10
Distance		$\sqrt{2}$
$q_{H/R}$	$[\frac{J}{h}]$	2.59×10^6
$tsqrt$	[h]	1.0677
tsq	[h]	1.9131
Disturbance 1		
T_o	[K]	268
$t_{s,opt}$	[h]	3.04
ΔT	[K]	15
Distance		1.854
$q_{H/R}$	$[\frac{J}{h}]$	3.885×10^6
$tsqrt$	[h]	1.3342
tsq	[h]	1.7436
Disturbance 2		
T_o	[K]	273
$t_{s,opt}$	[h]	4.06
ΔT	[K]	10
Distance		1.202
$q_{H/R}$	$[\frac{J}{h}]$	2.072×10^6
$tsqrt$	[h]	0.8718
tsq	[h]	2.0149
Disturbance 3		
T_o	[K]	277
$t_{s,opt}$	[h]	4.06
ΔT	[K]	6
Distance		1.733
$q_{H/R}$	$[\frac{J}{h}]$	1.544×10^6
$tsqrt$	[h]	0.8718
tsq	[h]	2.0149

Table 4.8.5: Combination of measurements and disturbances that was used to calculate the controlled variables, in addition to the best controlled variable for each case

Case	Disturbances	Measurements	Optimal controlled variable	Value
1	$d1 + d2$	$T_o + \Delta T + t_{s,opt} + q_{H/R} + \text{distance}$	A combination of ΔT and $t_{s,opt}$	-0.000126
2		$T_o + \Delta T + t_{s,opt} + q_{H/R} + \text{distance} + \text{t sqrt}$	A combination of T_o and $t_{s,opt}$	-0.000387
3		$T_o + \Delta T + t_{s,opt} + q_{H/R} + \text{distance} + \text{t sqrt} + \text{tsq}$	A combination of tsq , $t_{s,opt}$, distance and t sqrt	9.5×10^{-5}
4		$T_o + \Delta T + t_{s,opt} + \text{distance} + \text{t sqrt} + \text{tsq}$	A combination of tsq and $t_{s,opt}$	-0.000145
5		$T_o + \Delta T + \text{distance} + \text{t sqrt} + \text{tsq}$	A combination of ΔT , distance , t sqrt and tsq	-3.3×10^{-5}
6	$d1 + d2 + d3$	$T_o + \Delta T + t_{s,opt} + q_{H/R} + \text{distance} + \text{t sqrt} + \text{tsq}$	A combination of ΔT , T_o , $t_{s,opt}$ and distance	-0.000238
7		$T_o + \Delta T + t_{s,opt} + q_{H/R} + \text{distance}$	A combination of ΔT and $t_{s,opt}$	-0.000159
8	$d1 + d3$	$T_o + \Delta T + \text{distance} + \text{tsq}$	A combination of ΔT and tsq	-0.000188
9		$\Delta T + \text{distance} + \text{tsq}$	distance	0.000384

From Table 4.8.5 it can be seen that the time of switch, $t_{s,opt}$, t sqrt or tsq contribute in the calculation of the controlled variable in all cases, except for the last one. In addition do ΔT and distance contribute in about 50% of the cases. $q_{H/R}$ do not contribute in any of the cases.

From the simulation several combinations of H was calculated for each case. For example the different combinations of H for case 8 was:

$$H = \begin{bmatrix} 0.001053 & 0 & -0.101503 & 0 & 0.991704 & 0 & 0.078849 \\ -0.037406 & 0 & 0.964192 & 0 & 0.078814 & 0 & 0.250444 \end{bmatrix}$$

The values in matrix H above are round down from the one given in Matlab. The values close to the absolute value of 1 is the one that contribute the most in the calculations. From this and the order of the measurements in equation (4.8.1) it can be found that the two possible choices of different controlled variables was

C1 = A combination of distance and ΔT

C2 = A combination of ΔT and tsq

The first measurement is the one that contribute the most in the calculations. The controlled variables versus time is given in Figure 4.8.3. The optimal time to switch was

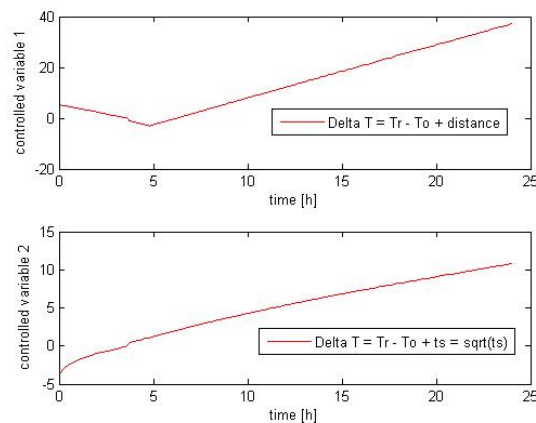


Figure 4.8.3: Value of the controlled variable versus time

found from Figure 4.5.1 The value of the controlled variable at the optimal time at switch, 3.66 h, was found from Table 4.8.3 and plotted in the switch block in Simulink. From Figure 4.8.3 it can be seen that C2 is monotonically increasing around the time at switch. The controlled variable that was the most independent of disturbances was used for the summary in this project. In case 8 this was C2.

The difference between case 1, 2 and 3 in Table 4.8.5 is the addition of t sqrt and

tsq. This parameters was not originally included in the measurements. After simulation of case 1 it could be seen that the controlled variable gave better results for temperatures below 0°C than for temperatures above. The result from the simulation for $+10^{\circ}\text{C}$ and -10°C is given in Figure 4.8.4 and 4.8.5. The time at switch was calculated for

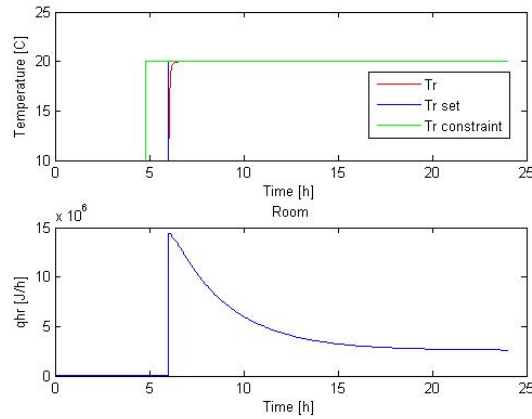


Figure 4.8.4: Simulation of case 1 with a outdoor temperature of $+10^{\circ}\text{C}$

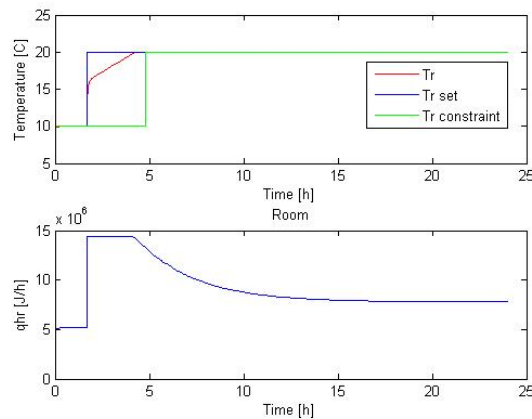


Figure 4.8.5: Simulation of case 1 with a outdoor temperature of -10°C

different outdoor temperatures and then plotted. From the plot it could be seen the plot of time is not linear. The case with tsq and t sqrt implemented gives a more linear behaviour. A plot of time and tsq is given in Figure 4.8.6 Since the controlled variables is a linear combination of the measurements a linear behaviour of the measurements may give better results.

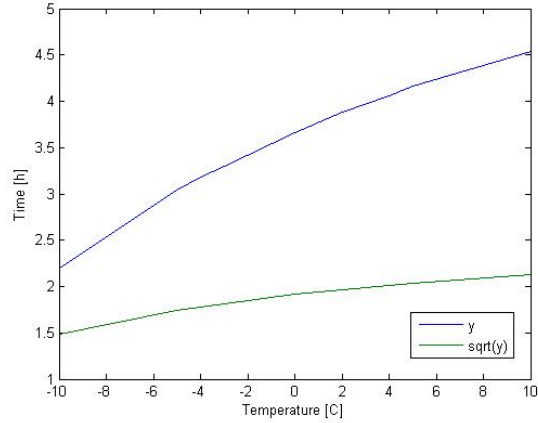


Figure 4.8.6: Temperature versus time

The loss is defined as

$$L = \frac{J_{\text{case } i} - J_{\text{opt}}}{J_{\text{opt}}} \quad (4.8.5)$$

In the above equation $J_{\text{case } i}$ and J_{opt} represent the energy consumption in *case i* and in *the optimal case* respectively. As explained earlier, only the first time of switch is considered in this project, this means that the energy consumption has to be calculated from time 0 to the time where the room temperature decrease from 20 °C to 10 °C the first time. This value is found from Figure 4.5.1, and the time is 9.6 h. The loss was calculated for different cases using the controlled variable that gave the best respond, as given in Table 4.8.5. The calculated values are summarized in Table 4.8.6. The same table gives a summary of which case the temperature reach the constraint when it supposed to and when it do not. The system was simulated with different values of the disturbances $UA_{R/O}$ and T_o . The value of $UA_{R/O}$ equals the value at the nominal case when changing T_o , and when changing $UA_{R/O}$ the outdoor temperature is kept at the nominal value.

Table 4.8.6: Calculated loss for each case

Disturbance	Unit	Value	Loss [%]								
			Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
T_o	[°C]	-10	43.135	43.520	37.243	43.064	39.593*	45.167	42.231	39.133*	31.934
	[°C]	-5	22.134	22.256	19.917*	22.163	21.014*	22.997	22.172	20.148†	16.364
	[°C]	-2	9.039*	9.058*	8.688*	9.050*	8.898*	9.182*	9.101*	8.224†	6.656*
	Nominal case [°C]	0	0.072†	0.072†	0.067†	0.072†	0.078†	0.078†	0.072†	0.077†	0.077†
	[°C]	2	-9.133*	-9.067*	-15.973	-9.099*	-14.594	-8.788*	-8.978*	-8.208†	-6.570*
	[°C]	5	-23.588	-22.995	-24.462	-23.117	-24.927	-21.570*	-22.188*	-20.848*	-16.646
	[°C]	10	-50.661	-56.093	-44.339	-53.554	-42.577	-42.549	-43.296	-42.277	-33.673
$UA_{R/O}$	$\left[\frac{J}{hK}\right]$	×8	2.567	2.567	2.565	2.567	2.567	2.574	2.567	2.574	2.573
	$\left[\frac{J}{hK}\right]$	×11	-1.292*	-1.292*	-1.294*	-1.292*	-1.292*	-1.285*	-1.292*	-1.285*	-1.286*

† means that the room temperature hit the constraint when it is supposed to

* means that the room temperature is close to hit the constraint

From Table 4.8.6 it can be seen that case 8 hit the constraints or is close to for most of the disturbances, except for when T_o is $+10^\circ\text{C}$ and $UA_{R/O}$ is multiplied with 8. This means that even if the time at switch changes with disturbances the controlled variable is kept constant. Figure 4.8.7 and 4.8.8 gives a representation of how the system behaves with outdoor temperatures equal to $+5^\circ\text{C}$ and -5°C respectively. From the Figure

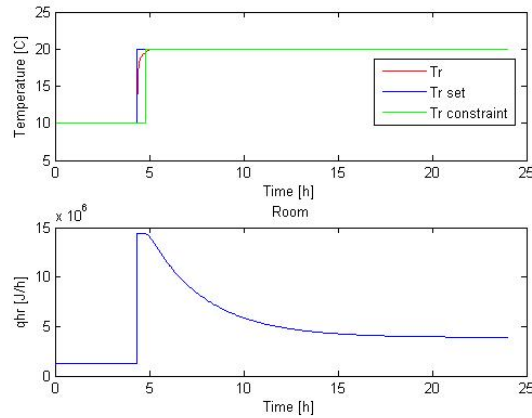


Figure 4.8.7: Simulation of case 8 with a outdoor temperature of $+5^\circ\text{C}$

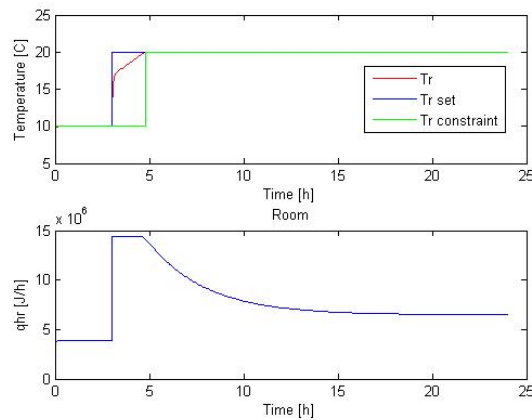


Figure 4.8.8: Simulation of case 8 with a outdoor temperature of -5°C

4.8.7 and 4.8.8 it can be seen that the temperature hit the constraint at $T_o = -5^\circ\text{C}$, and almost when $T_o = +5^\circ\text{C}$. Case 1, 2 4 and 9 is just working around 0°C . Case 6 and 7 works fine for outdoor temperatures below 0°C , but not for temperatures above 0°C . The opposite is true for case 3 and 5. The system responds fine with a larger $UA_{R/O}$ than in the nominal case, but not for smaller values. This is true for all the cases.

Table 4.8.6 gives a representation of the calculated loss for each case. It can be seen that the loss is small at the nominal case, which is reasonable, since no disturbances are introduced. For outdoor temperatures below 0°C a negative loss is obtained. This means that the cost function calculated for the different cases is smaller than the cost function in the optimal case. The reason to the negative values is that the heating starts after the constraint, as given in Figure 4.8.4, or that the heating of the room continues after the constraint, as given in Figure 4.8.7. Both of these cases will give a smaller energy consumption compared to the optimal case. The room will not be 20°C as long as in the optimal case, and less heat is used. Even if the energy consumption is smaller in the cases explained above they are not optimal. The temperature in the room do not reach the temperature from the constraint at the correct time. It can be seen that the loss increases, get more negative, the further away from the nominal case.

For temperatures above 0°C , a positive loss is obtained. The heating of the room starts to early such that it reach the constraint, 20°C , before it is meant to. Naturally, this will increase the energy consumption. The room is 20°C for a longer period of time compared to the optimal case and more heat is used. Figure 4.8.5 gives a representation of a room where the room temperature reach 20°C before the constraint.

The two last columns in Table 4.8.6 gives the loss when the value of $UA_{R/O}$ is changed. A positive loss is obtained when a lower value of $UA_{R/O}$ is used, and a negative loss is calculated with a larger value of $UA_{R/O}$. The loss is almost constant for all the nine cases, both for $UA_{R/O}$ time 8 and 11. It seems like the outdoor temperature has bigger impact on when to start heat the room than $UA_{R/O}$. The respond of the system with different disturbances was checked in section 4.7. In this section only different outdoor temperatures was used, not different values of $UA_{R/O}$.

5 Discussion

5.1 Difference between the simple and advanced model

The difference between the simple and advanced model is the heat flow to and from the outside, q_{out} and q_{in} respectively, in addition to the transport of mass in the advanced model. As explained in section 4.1, the extra information given in the advanced model do not change the model significantly. A comparison of the equations for the room temperature show that just one term differs between the simple and advanced model. Adding a factor 10 to this term in the simple model adjust for this difference and the simple model behaves similar to the advanced one. The equation for the temperature in the floor is the same in both models. The transport of mass in the advance model is closed to q_{in} , which is one of the inputs to the model. This value is assumed to be small. These comparisons explains why the behaviour of the two models are similar if small adjustments were made to the simple model.

5.2 Simulation of the system

From simulation of the optimal room temperature and the room temperature when the PI controller was implemented to the system, similar behaviour is observed. Only small differences, due to inaccuracy in the retuning of the switching times and tuning parameters, is observed when comparing the two figures. This inaccuracies is the reason to the difference in energy consumption for the two systems.

5.3 Null space method

The time to switch from one temperature to another changes when disturbances are introduced to the system. The null space method is used to find a controlled variable that is constant with disturbances even if the time at switch changes. It was found that a linear combination of the measurements did not give acceptable respond. The system works well around the nominal value, or good at temperatures above the nominal value, but not for temperatures below. The same is true for the opposite case. Different combinations of disturbances and measurements was tried in order to find the best controlled variable. The system has two main disturbances, $UA_{R/O}$ and T_o . This project has been focusing on the outdoor temperature when changing the disturbances. This may explain why case 8 gave the best respond. In this case only outdoor temperature is used as disturbances. In the same case all the measurements are used except for $q_{H/R}$, $t_{s,opt}$ and t_{sqrt} . The time at switch that gives the most linear response is used. From the measurements ΔT and tsq is the one that is taken the most into consideration when calculating c . The time at switch, $t_{s,opt}$, t_{sqrt} or tsq , is important measurements. They contributes in the calculation of c in all the cases. In addition to the time at switch, ΔT and distance are important measurements. They are emphasized in about 50% of the cases. On the other hand, $q_{H/R}$ do not contribute in any of the cases. This indicate that the outdoor temperature, room temperature, temperature and time at the constraint, and time of switch is good candidates for the measurements when using the null space

method.

From calculations of the loss it can be seen that it increases the further away from the nominal value. For outdoor temperatures equal to $\pm 10^\circ\text{C}$ the loss is around 40-50% in most of the cases. This indicates that the null space method does not give acceptable results for outdoor temperatures too far away from the nominal value.

5.4 Improvements and suggestions for further work

In section only changes in outdoor temperature is considered when checking how the model reacts to disturbances. An improvement to the system would have been to find out how the model reacts to changes in $UA_{R/O}$ as well. This because $UA_{R/O}$ and T_o are the two most important disturbances in the system.

Due to lack of time only the first switch was investigated. For further work, controlled variables that is independent of disturbances should be found for the other switching times as well. The same method that was used for the first time at switch can easily be used for the other switching times.

Before finding the controlled variables for the other switching times, the null space method should be improved. The heating in the room should start to heat/cool when it is supposed to for all disturbances. A suggestion to such an improvement is to calculate two H-matrices. The first one is to be valid for temperature below the nominal value and the second one is to be used for temperatures above 0°C . In other words, if the temperature is below 0°C , the first matrix should be used in the calculation of c , and when the temperature is above the nominal value the other matrix should be used in the calculations.

6 Conclusion

The system gave satisfactory response when implementing a PI controller to the system. From Figure 4.5.1 in section 4.5 it could be seen that the heating of the room has to start approximately 1.5 hour before in order for the room temperature to be 20°C at the constraint for the first switch. The cooling of the room happens immediately. From Figure 4.5.1 it can be seen that in the third time interval, the room do not reach 10°C , before it begin to heat up again. The same is true for the last interval. The control structure was retuned such that it has the same behaviour as the optimized system.

The behaviour of the model turned out satisfactory when the system was simulated without disturbances. On the the other hand when the system was introduced to different outdoor temperatures the time to start heating the room, in order to reach the constraint at correct time, changes. When the outdoor temperature is above 0°C the room temperature reach 20°C before the constraint. With T_o below 0°C do the room temperature reach 20°C after the constraint. When the outdoor temperature was simulated as a sine wave as given in equation (4.7.2) the room temperature behaves as if the temperature was above 0°C . The further away from 0°C the outdoor temperature is, the worse was the behaviour of the system.

The null space method was used to find a controlled variable, c , that is independent of disturbances. Even if the time at switch changes with disturbances, the value of c should be constant and the system should achieve near optimal behaviour. From simulation it was found that a linear combination of the measurements do not give acceptable results for all disturbances. Cases with different combinations of measurements and disturbances was used in the calculation of the controlled variable. The two disturbances that was checked was changes in T_o and $UA_{R/O}$. The measurements that was used is given in equation (4.8.1). The behaviour of the system was acceptable with T_o around 0°C for all the cases. On the other hand, if the system gave good response for temperatures below 0°C , it did not give satisfactory response for temperatures above 0°C .

The best combination was found if the disturbance did not take $UA_{R/O}$ into consideration. The measurements that contributed the most in all the cases was the different variations of the switching time, ΔT and the distance. From Table 4.8.6 it can be seen that the loss, calculated from equation (4.8.5), is small at the nominal case. For outdoors temperature below 0°C a negative loss is obtained, while for temperatures above 0°C positive loss is obtained. From the same table it can be seen that the loss is larger the further away from the nominal value. If the value of $UA_{R/O}$ was changed, small values of the loss was calculated.

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7 Appendix A

7.1 Nomenclature

Parameter	Description	Unit
T_F	Temperature in the floor	$[K]$
T_R	Temperature in the room	$[K]$
T_o	Outdoor temperature	$[K]$
T_s	Temperature at switch	$[K]$
T_c	Temperature at constraint	$[K]$
$q_{F/R}$	Heat flow from floor to room	$[\frac{J}{h}]$
$q_{H/F}$	Heat flow from heater to floor	$[\frac{J}{h}]$
$q_{H/R}$	Heat flow from heater to room	$[\frac{J}{h}]$
$q_{R/O}$	Heat flow from room to outside	$[\frac{J}{h}]$
q_{in}	Heat flow from outside to the room	$[\frac{J}{h}]$
q_{out}	Heat flow from room to the outside	$[\frac{J}{h}]$
E	Energy consumption	$[\frac{J}{h}]$
m	Mass	$[kg]$
N	Number of moles	$[mol]$
M	Molar mass	$[\frac{kg}{mol}]$
A	Heat transfer area	$[m^2]$
V	Volume	$[m^3]$
Cp	Heat capacity	$[\frac{J}{kgK}]$
ρ	Density	$[\frac{kg}{m^3}]$
P	Pressure	$[bar]$
R	Gas constant	$[\frac{J}{Kmol}]$
U	Overall heat transfer coefficient	$[\frac{J}{hm^2K}]$
k	Thermal conductivity	$[\frac{J}{hmK}]$
h	Convective coefficient	$[\frac{J}{hm^2K}]$
t	Time	$[h]$
$t_{s,opt}$	Optimal time at switch	$[h]$
t_c	Time at constraint	$[h]$

Parameter	Description
Other parameters	
K_c	Controller gain
τ_I	Integral time constant
τ_D	Derivative time constant
τ_c	Tuning parameter
θ	Time delay
c	Controlled variable
t sqrt	$\sqrt{ tc - ts }$
tsq	\sqrt{ts}
L	Loss
J	Cost function

8 Appendix B

8.1 How the system responds to disturbances

This Appendix give the result from the simulation in section 4.7 where different disturbances are introduced to the system. Figure 8.1.1 to 8.1.5 gives the result with outdoor temperature equal to -14°C , -6°C , 0°C , $+6^{\circ}\text{C}$ and $+14^{\circ}\text{C}$.

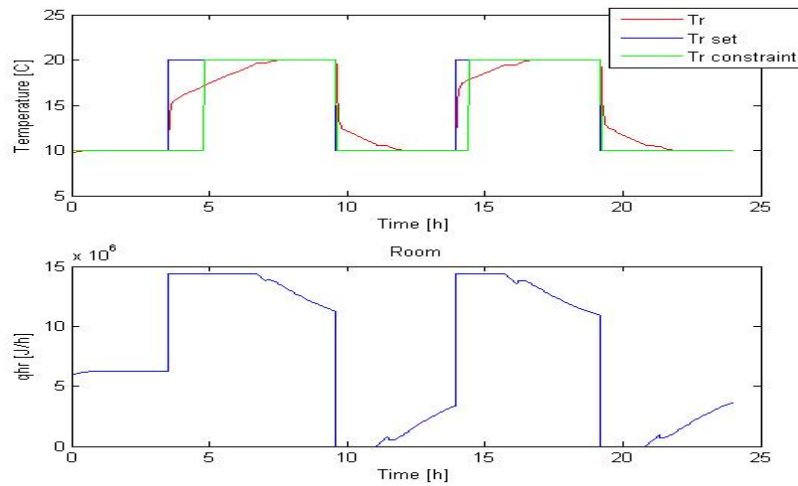


Figure 8.1.1: Room temperature and heat versus time for the system with constant outdoor temperature equal to -14°C

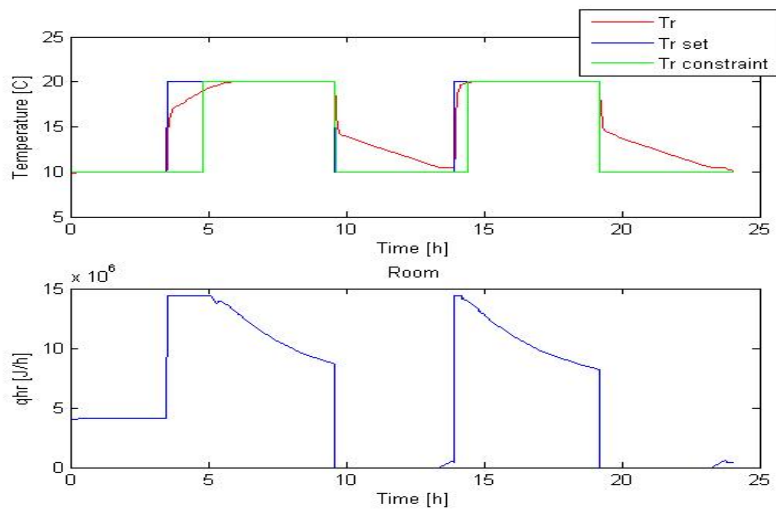


Figure 8.1.2: Room temperature and heat versus time for the system with constant outdoor temperature equal to -6°C

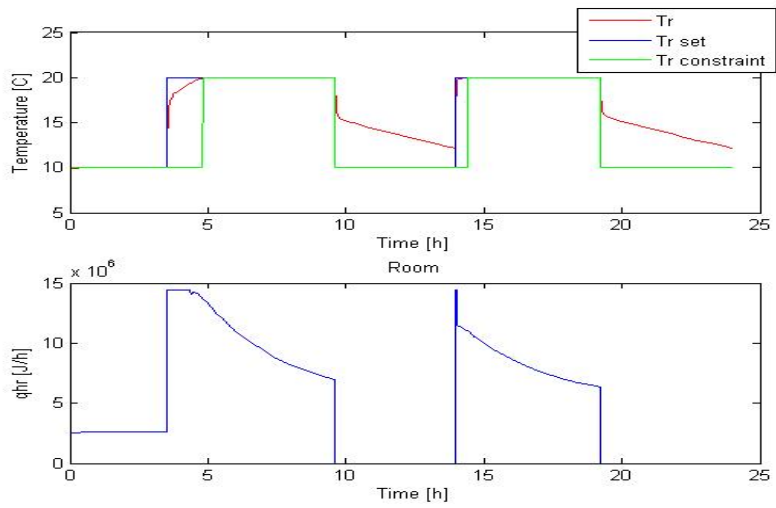


Figure 8.1.3: Room temperature and heat versus time for the system with constant outdoor temperature equal to 0°C

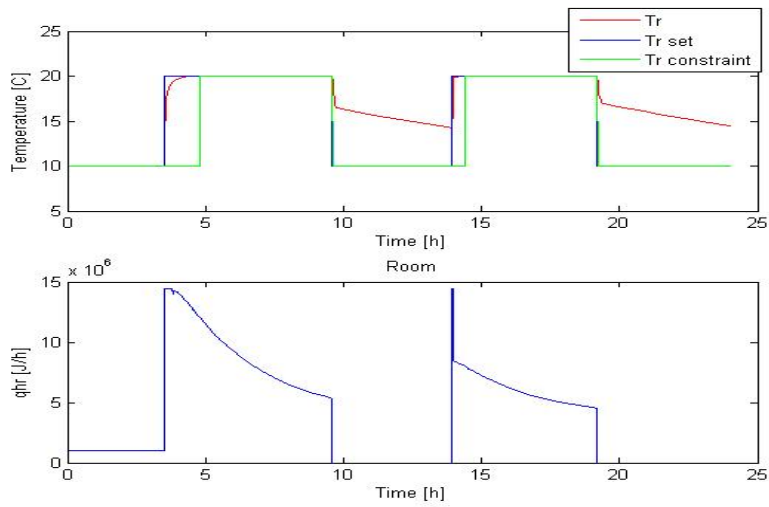


Figure 8.1.4: Room temperature and heat versus time for the system with constant outdoor temperature equal to 6°C

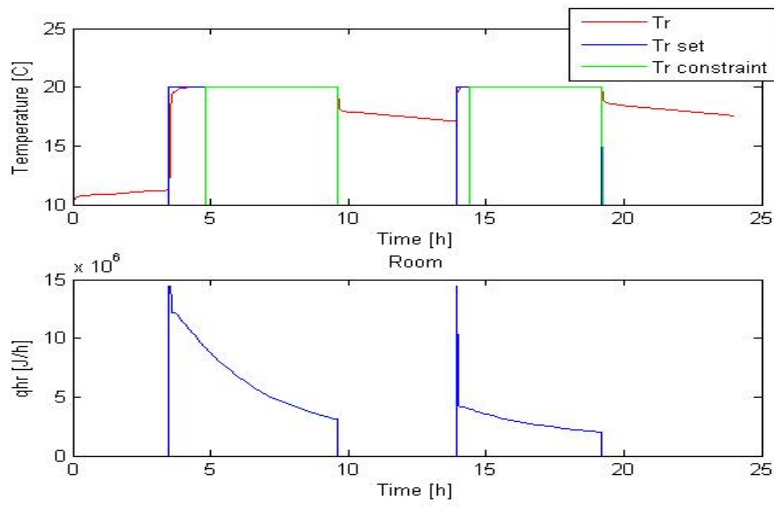


Figure 8.1.5: Room temperature and heat versus time for the system with constant outdoor temperature equal to 14°C

9 Appendix C

9.1 CD containing the Matlab and Simulink scripts