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SPECIALIZATION PROJECT

Validation of the SIMC PID Tuning Rules

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Abstract

The aim of this report has been to validate the SIMC PID tuning rules for second order plus time delay processes. The PID controller is the most used controller in the process industry, and the presence of simple tuning rules that can be used to tune robust and high performing controllers would be a great advantage. All calculations and simulations has been accomplished with the use of MATLAB and SIMULINK.

The trade-off between robustness and performance for the SIMC tuning rules has been investigated with the Pareto-optimal curves as a foundation. The SIMC tuning rules have been found to perform close to optimal for M_s values below two. Resulting in controllers that are less aggressive compared to the Pareto-optimal, i.e. having better setpoint performance and slightly reduced disturbance rejection.

The recommended choice of tuning parameter $\tau_c = \theta$ has been found to be too high for processes with $\frac{\tau_2}{\tau_1} < 0.5$. For such processes τ_c should be chosen smaller, e.g. $\tau_c = 0.5\theta$.

This report has tested nine different cases. The major challenge has been to get the numerical solver to converge and find the solution to the minimization problem. For future work, the numerical solver should be made more robust, or replaced, so that a wider selection of processes can be tested.

Preface

This report is the result of the Specialization Project, TKP4550, at the department of Chemical Engineering, within the group Process Systems Engineering, at NTNU, fall 2012.

The aim of the project has been to validate the SIMC PID tuning rules for a set of second order plus time delay process models. To reach the goal and achieve the results presented in this report MATLAB has been used for calculations together with simulations in SIMULINK.

The completion of this project would not have been possible without the help, guidance and support from some important people. First of all, I would like to thank my co-supervisor Chriss Grimholt for his support and guidance throughout the work with this project. Whenever I had questions he would answer to the best of his ability and get me back on the right track.

Secondly, I would like to give my thanks to my supervisor, professor Sigurd Skogestad. Though he has a busy schedule, he has directed me in the right direction when questions arose.

I would also like to give great thanks my friends, Peter Johan Bergh Lindersen and Ivar Magnus Jevne, and my partner, Pia Odden, for their moral support, inspiring discussions and proof-reading of this report. I know how hectic their schedule have been, and I am truly grateful for the help I have received.

Trondheim, December 7, 2012

Martin S. Foss

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List of Symbols

Symbol	Explanation
d	Disturbance, input
d_{out}	Disturbance, output
e	Error, controller input
g_c	Controller model (transfer function)
g_p	Process model (transfer function)
IAE	Integral absolute error
k	Process gain
K_c	Controller gain
J	Cost function
M_s	Peak in sensitivity function
ω	Frequency
s	Laplace-variable
t	Time
τ_1	1st time constant (largest)
τ_2	2nd time constant (smallest)
τ_c	Closed loop time constant
τ_D	Derivative time constant
τ_I	Integral time constant
θ	Time delay
u	Process input

1 Introduction

One of the most used controllers in the industry is the PID controller [1]. Despite the frequent use, this type of controller is often poorly tuned. Though there are only three tuning parameters in the PID controller the optimal tuning, i.e. optimal trade-off between performance and robustness, is difficult to obtain. The optimal performance can in itself also be difficult to define. The requirements of robustness and performance may need good engineering insight to be determined. It is not possible to get the best of both, so a settlement in the middle ground should be chosen.

The aim of this report is to validate the SIMC tuning rules presented by Skogestad [2]. These rules are developed to be easy to remember and to result in good closed-loop behavior. Earlier investigations that has been preformed to investigate the SIMC tuning rules with respect to PI control [3], has shown that these rules result in good trade-off between robustness and performance.

The tuning rules is tested to a set of second-order-plus-time-delay (SOPTD) processes. The performance of the PID controller tunings will be compared with the "Pareto-optimal" (PO) tunings, which can be referred to as "the best one can get". The PID tunings will also be compared with PI tunings. The latter to see if there is any need of implementing a PID controller.

2 Theory - Background

The SIMC rules presented by Skogestad [2], uses an open-loop process model to derive the controller settings for PID ond PI controllers. Dependent on the desired controller (PID or PI) the model has to be reduced to a second-order-plus-time-delay (SOPTD) model or a first-order-plus-time-delay (FOPTD), respectively. The two types are presented in Equation (2.1) and (2.2), respectively. In this project the aim has been to validate the PID tuning rules and thus, second order models has been used. However, a PI controller has been used for comparison. Thus, the SOPTD models have been reduced to FOPTD models using the "half rule" [1,2].

$$g_p = \frac{k_p}{(\tau_1 s+1)(\tau_2 s+1)} \cdot e^{-\theta s} \tag{2.1}$$

$$g_p = \frac{k_p}{(\tau_1 s+1)} \cdot e^{-\theta s} \tag{2.2}$$

Where g_p is the process transfer function, k_p is the process gain, τ_1 and τ_2 are the time constants, and θ is the time delay.

The block diagram depicted in Figure 2.1 is the conventional feedback loop, where y_s is the setpoint, e is the controller error, u is the manipulated variable (process input), d is the input disturbance, d_{out} is the output disturbance and y is the process output. g_c and g_p are the controller and process transfer functions, respectively.

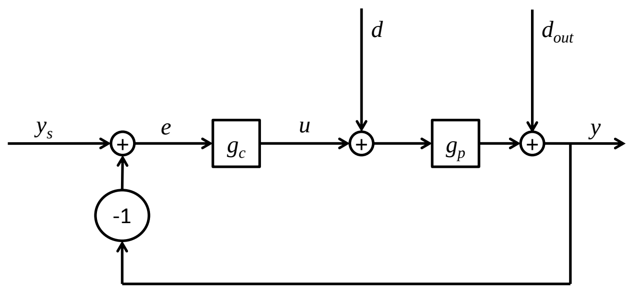


Figure 2.1: Block diagram of general feedback control system (with input and output disturbances).

The simulations performed in this project have tested the controller settings for setpoint changes and input disturbances. Output disturbances have not been investigated as they have the same effect as a change (or disturbance) in the setpoint, and thus can be treated as a special case of setpoint change. Hence, the block diagram in Figure 2.1 is slightly modified, and the block diagram used in this project can be presented as:

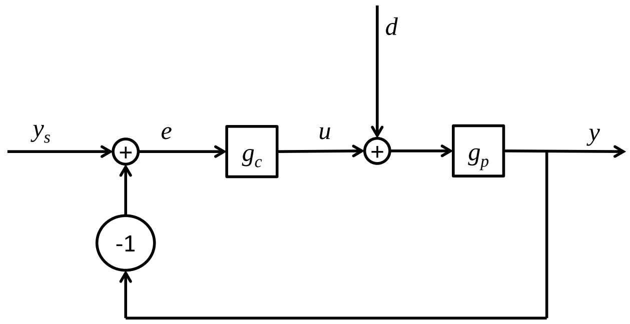


Figure 2.2: Block diagram of feedback control system used in the project.

2.1 The PID controller and SIMC tuning rules

The PID controller are often presented in its parallel form as given in Equation (2.3).

$$g_{PID} = P + \frac{I}{s} + Ds \tag{2.3}$$

Where P denotes the proportional part, I the integral part and D the derivative part. The three parts of the controller have different effects on the manipulated variable. The proportional part change the manipulated variable directly proportional to the error. The integral part change the manipulated variable proportional to the integrated error and the derivative part change the manipulated variable proportional to the derivative of the controlled variable. All in all the controller will try to minimize the error, e, in Figures 2.1 and 2.2, by adjusting the process input (u).

The three parts of the PID controller, discussed above, has their individual tuning parameters, i.e. K_c, τ_I and τ_D. To find these parameters the SIMC tuning rules uses two main steps, i.e.:

1. Obtain a FOPTD or a SOPTD model.
   - Perform open or closed loop experiments.
   - If model of higher order is known, reduce with use of the "half rule".
2. Get controller settings from the tuning rules, presented below.

The SIMC tuning rules are given for the ideal, series form, PID controller, as defined in Equation (2.4).

$$g_c(s) = K_c \cdot \left(\frac{\tau_I s + 1}{\tau_I s}\right) \cdot (\tau_D s + 1) \tag{2.4}$$

Where g_c is the controller transfer function, K_c is the controller gain, τ_I is the integral time and τ_D is the derivative time. The tuning rules [2] for a PID controller can be found from a SOPTD process, see Equation (2.1), as follows:

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta} \tag{2.5}$$

$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\} \tag{2.6}$$

$$\tau_D = \tau_2 \tag{2.7}$$

As seen from Equation (2.5) - (2.7) the SIMC tuning rules has only one independent variable, i.e. τ_c. The recommended value for this parameter is $\tau_c = \theta$, which should yield tight control with good trade-off between robustness and performance.

If a PI controller is to be tuned, the K_c and τ_I parameter are defined in the same manner. $\tau_D = 0$, as the PI controller do not have this tuning parameter.

As the PID controller often is presented in its parallell form, Equation (2.8), recalculation of the tuning parameters are required. The corresponding parallell tuning parameters can be calculated by the translation formulas presented in Equations (2.9), (2.10) and (2.11).

$$g_c'(s) = K_c'\left(1 + \frac{1}{\tau_I' s} + \tau_D' s\right) \tag{2.8}$$

$$K_c' = K_c\left(1 + \frac{\tau_D}{\tau_I}\right) \tag{2.9}$$

$$\tau_I' = \tau_I\left(1 + \frac{\tau_D}{\tau_I}\right) \tag{2.10}$$

$$\tau_D' = \frac{\tau_D}{1 + \frac{\tau_D}{\tau_I}} \tag{2.11}$$

Where g_c', K_c', τ_I' and τ_D' are the parallel controller transfer function, controller gain, integral time and derivative time, respectively.

2.2 Pareto optimization

The search after good controller tunings can be difficult without a systematic approach. A set of tuning parameters can never yield perfect performance and good robustness at the same time. A controller with good performance is normally not very robust, and vice versa. There will also be a trade-off between the response to a change in setpoint and disturbances.

To be able to find the optimal compromise between performance and robustness Pareto-optimal (PO) curves can be helpful. A PO-curve is represented in Figure 2.3. The figure depicts two conflicting objective functions plotted against each other. For each point, the optimal value of the two objective functions are plotted. The trade-off is clearly depicted. As objective function 1 is low, objective function 2 is high, and vice versa. The optimal point is somewhere in the middle (bold, red line), but exactly where is up to the individual engineer and the respective case.

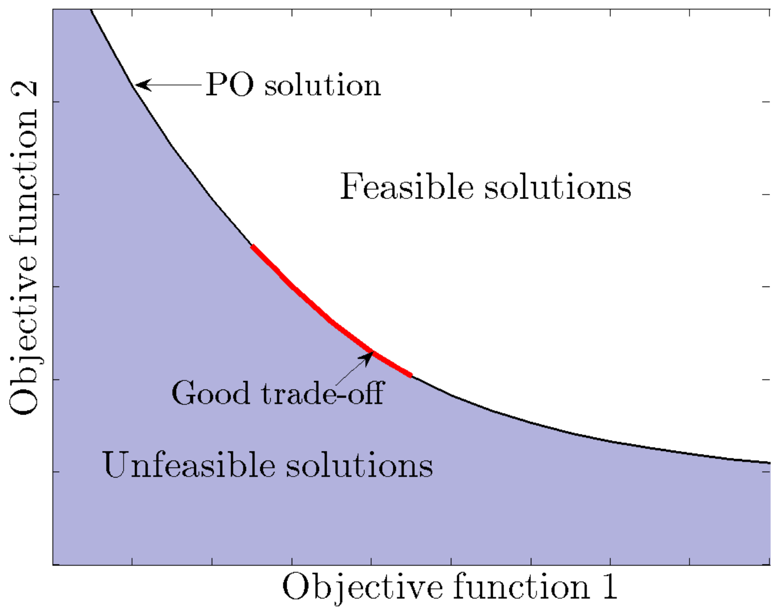


Figure 2.3: Typical Pareto-optimal curve of two conflicting objective functions.

In this project the two objective functions are, as mentioned, performance and robustness. For simplicity, the robustness has been made the independent variable, whilst the robustness is the dependent variable. That is;

$$\text{performance} = f(\text{robustness}) \tag{2.12}$$

In the following subsections the basis for the performance and robustness functions are highlighted.

2.2.1 Performance

There are several possible methods to evaluate the performance. In this project the integral absolute error (IAE) has been used. The IAE is defined in Equation (2.13) and is a good indication of the speed and precision of the controller.

$$IAE = \int_0^\infty |e| dt = \int_0^\infty |y(t) - y_s(t)| dt \tag{2.13}$$

2.2.2 Robustness

The robustness is measured by the peak in the sensitivity function, M_s. The sensitivity function, $S(j\omega)$, is defined as the closed-loop transfer function between the output disturbance, d_{out}, and the output, y, see Figure 2.1 [4]. In addition the M_s^{-1} is the closest distance to the critical point -1 in the Nyquist plot. For stability, the best thing is to be as far away from this point as possible, i.e a M_s value of 1 is desired.

$$\begin{aligned} M_s &= \max_\omega |S(j\omega)| \\ &= \max_\omega \left| \frac{1}{1+g_c(j\omega)} \right| \end{aligned} \tag{2.14}$$

For any given M_s value the following applies [4]:

$$GM \geq \frac{M_s}{M_s - 1} \qquad \text{and} \qquad PM \geq \frac{1}{M_s} \tag{2.15}$$

As the M_s-value decreases the robustness of the controller will increase. The best thing, both for stability and performance, would be to have a M_s-value close to one [4]. The M_s-value should not exceed 2[1], and the closer to 1 the more robust the controller will get. However, the cost of decreasing the M_s-value will often be to high when approaching low values, so a value between 1.6 - 1.7 is typically "good" [3].

[1] A M_s of two yields $GM \geq 2$ and $PM \geq 29.0°$, which represents the recommended upper bounds [4].

2.3 The objective function

The objective function, which should be minimized, used in this project is defined by Equation (2.16). The function has been calculated in the domain given in Equation (2.17).

$$J(c) = 0.5\left[\frac{IAE_{ys}(c)}{IAE_{ys}^{\circ}} + \frac{IAE_{d}(c)}{IAE_{d}^{\circ}}\right] \tag{2.16}$$

$$M_s = \{1.25, 1.30, \ldots, 3.00\} \tag{2.17}$$

Where IAE_{ys}° and IAE_{d}° denotes the error when there is performed an input and a disturbance step with a Pareto-optimal tuning, respectively. In this way the performance is weighted against a constant reference. As the performance is a function of the robustness, a M_s-value of 1.59 is used when the PO-curves are constructed. The resulting weights are presented in Table 3.1.

2.4 Cases

In this project nine cases have been tested. These are given in Equation (2.18) – (2.23). The first three cases, case 1 – case 3, are time delay dominated processes, whilst case 4 – case 9 are lag dominated.

Case 1:

$$g_p = \frac{1}{(s+1)(0.5s+1)} \cdot e(-s) \tag{2.18}$$

Case 2:

$$g_p = \frac{1}{(s+1)(0.8s+1)} \cdot e^{(-s)} \tag{2.19}$$

Case 3:

$$g_p = \frac{1}{(s+1)(0.3s+1)} \cdot e^{(-s)} \tag{2.20}$$

Case 4:

$$g_p = \frac{1}{(s+1)(0.5s+1)} \cdot e^{\left(-\frac{1}{3}s\right)} \tag{2.21}$$

Case 5:

$$g_p = \frac{1}{(s+1)(0.8s+1)} \cdot e^{\left(-\frac{8}{15}s\right)} \tag{2.22}$$

Case 6:

$$g_p = \frac{1}{(s+1)(0.3s+1)} \cdot e^{\left(-\frac{2}{15}s\right)} \tag{2.23}$$

Case 7:

$$g_p = \frac{1}{(s+1)(0.5s+1)} \cdot e^{(-0.25s)} \tag{2.24}$$

Case 8:

$$g_p = \frac{1}{(s+1)(0.8s+1)} \cdot e^{(-0.4s)} \tag{2.25}$$

Case 9:

$$g_p = \frac{1}{(s+1)(0.3s+1)} \cdot e^{(-0.1s)} \tag{2.26}$$

2.5 Calculations and Simulations

All calculations and simulations in this project are performed by use of MATLAB and SIMULINK. The MATLAB scripts and SIMULINK block diagram are included in Appendix A and B, respectively.

3 Results and Discussion

In the following sections all the obtained results are presented along with a discussion.

3.1 Pareto-optimal PID and PI weights

To be able to assess the performance of the controllers for the different cases the cost function in Equation (2.16) had to be solved. In order to achieve this, the IAE_{ys}° and IAE_{d}° had to be calculated. This was performed by calculating the error with only a step change in the setpoint and disturbance, respectively. These Pareto-optimal parameters was found for both the PID and a PI controllers for a M_s-value of 1.59. All the the weights are presented in Table 3.1.

As can be seen from Table 3.1 there is a consistently better performance by the PID controller, both for setpoint changes and disturbances, for all cases. This observation fits well with theory, as the PID controller has one extra tuning parameter compared with the PI controller, and should therefore perform better. The PO-controllers are also observed to perform better to disturbances than to a step in the setpoint.

Table 3.1: Comparison between the IAE-weights with a Pareto-optimal PID and PI controller for all the nine cases.

		IAE°_{ys}		IAE°_{d}	
		PID	**PI**	**PID**	**PI**
$\theta > \tau_2$	**Case 1** $\frac{1}{(s+1)(0.5s+1)} \cdot e^{(-s)}$	1.92	2.81	1.81	2.76
	Case 2 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{(-s)}$	1.98	3.07	1.86	3.00
	Case 3 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{(-s)}$	1.83	2.57	1.72	2.52
$\theta < \tau_2$	**Case 4** $\frac{1}{(s+1)(0.5s+1)} \cdot e^{\left(-\frac{1}{3}s\right)}$	0.70	1.45	0.53	1.28
	Case 7 $\frac{1}{(s+1)(0.5s+1)} \cdot e^{(-0.25s)}$	0.53	1.26	0.36	1.07
	Case 5 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{\left(-\frac{8}{15}s\right)}$	1.11	2.12	0.94	1.97
	Case 8 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{(-0.4s)}$	0.84	1.83	0.66	1.64
	Case 6 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{(-0.2s)}$	0.43	0.93	0.26	0.75
	Case 9 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{\left(-\frac{3}{20}s\right)}$	0.32	0.82	0.17	0.62

3.2 Pareto-optimal vs. SIMC tunings

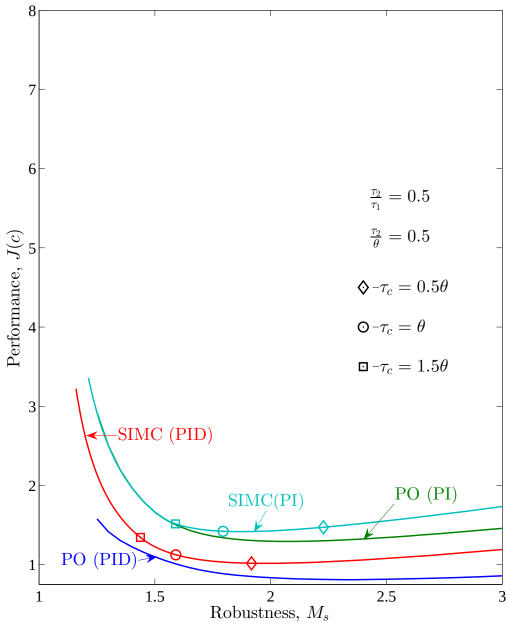
To assess the performance of the SIMC PID tuning rules the SIMC PID curve has been plotted in the same figure as the Pareto-optimal PID curve. The Pareto-optimal PI and SIMC PI curves has also been included in the same figure. The latter to investigate if there is any need and readily "profit" by implementing a PID controller, or if a PI controller will suffice.

3.2.1 Time delay dominated processes

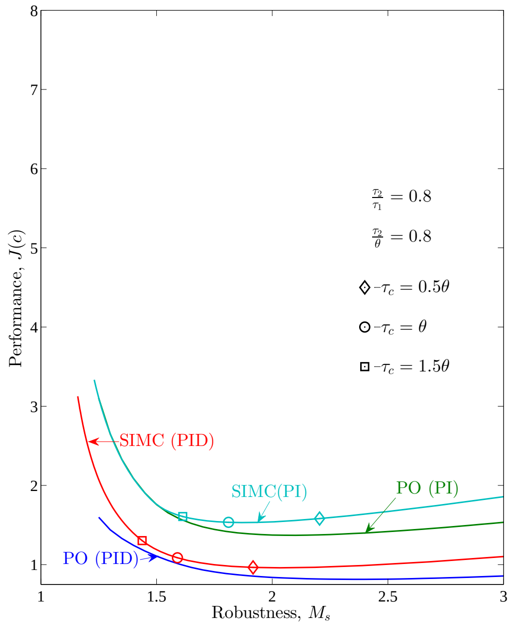
The SIMC tuning rules state that as long as the time delay is greater than the second time constant, i.e. $\theta > \tau_2$, there is no need for implementing a PID controller. This statement has been tested for three different cases, case 1 - case 3, which are depicted in Figure 3.1.

As shown in Figures 3.1a, 3.1b and 3.1c, the Pareto-optimal PID (blue) curve yields, as expected, the best performance. The Pareto-optimal PI and the SIMC PI curves (green and cyan, respectively) coincide for M_s values smaller than approximately 1.6. The SIMC PID shows better performance, compared with the PI controller, for all the three cases. In the M_s domain $[1.3 - 2.0]$, the SIMC PI controllers underperform the SIMC PID controller with approximately 35, 50 and 25 % for the three cases, respectively. Hence, the advantage of implementing a PID controller is quite small. However, the decsission to install a PID or not, is not unambiguous and it has to be investigated in more detail for each individual case. As the ratio $\frac{\tau_2}{\tau_1}$ increase, the advantage of a PID controller will also increase.

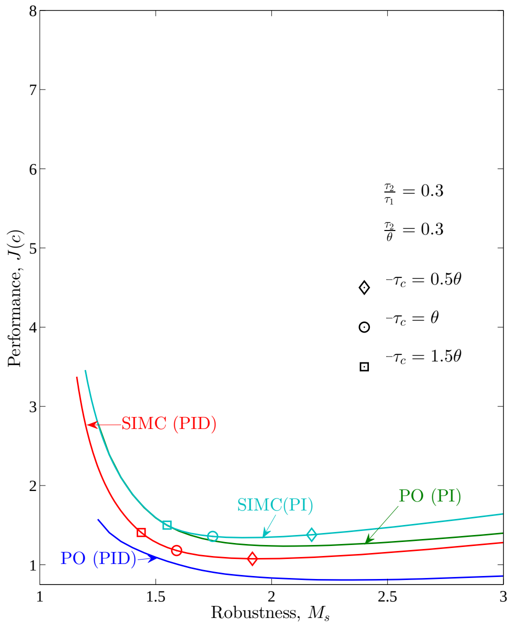
The figures show that the SIMC tuning rules give close to optimal controllers for the three cases. The recommended choice of the tuning parameter τ_c, i.e. $\tau_c = \theta$, yields a M_s value of approximately 1.6 (PID) and 1.8 (PI) for all the three cases, and thus results in a good trade-off between performance and robustness.



(a) Case 1, $\frac{\tau_2}{\tau_1} = 0.5$, $\frac{\tau_2}{\theta} = 0.5$.



(b) Case 2, $\frac{\tau_2}{\tau_1} = 0.8$, $\frac{\tau_2}{\theta} = 0.8$.



(c) Case 3, $\frac{\tau_2}{\tau_1} = 0.3$, $\frac{\tau_2}{\theta} = 0.3$.

Figure 3.1: Pareto optimal (PO) vs. SIMC tuning curves for time delay dominated second order plus time delay processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$.

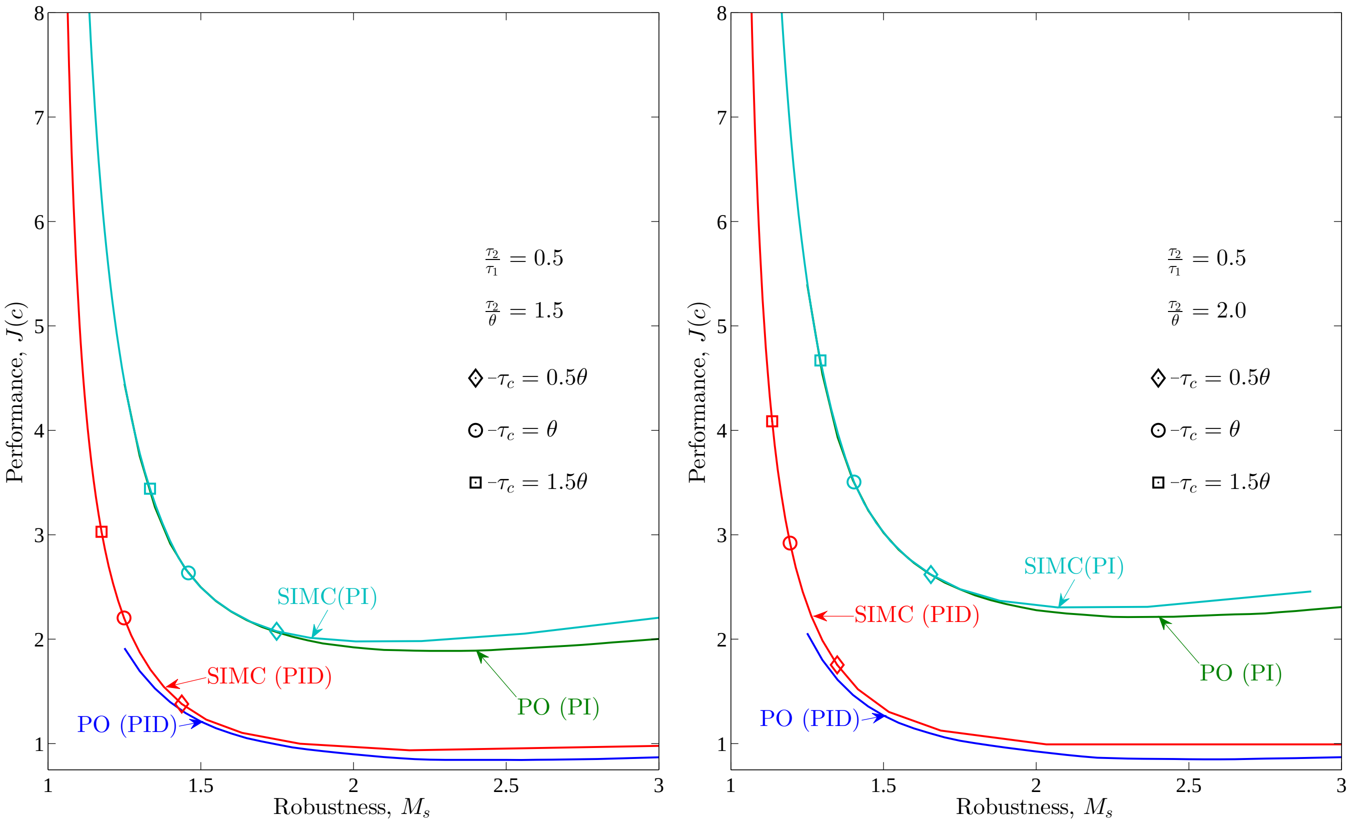
3.2.2 Lag dominated processes

The remaining cases has been plotted in the same manner as the time delay dominated processes, i.e. by comparing the Pareto-optimal PID and PI tuning curves with the SIMC PID and PI tuning curves. The results are presented in Figures 3.2 – 3.4.

Figure 3.2a and 3.2b depicts case 4 and case 7, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.5, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. For both processes the SIMC tuning rules produces controllers with almost no non-optimality loss in the preferred M_s domain, i.e. $M_s < 2.0$. As M_s decreases and approach one, the cost function increases dramatically.

In the M_s domain $[1.3 - 2.0]$ the SIMC PI underperform the SIMC PID on average with approximately 105 % for case 4 and and 125 % for case 7.

The figures show that increasing the ratio $\frac{\tau_2}{\theta}$ will result in a shift towards lower M_s values for given τ_c's. The recommended choice of $\tau_c = \theta$ is shown to result in a M_s value of 1.25 for case 4 and 1.20 for case 7. This will give a robust controller, but because of the trade-off between performance and robustness, the controller will loose performance. The steep gradients in these points indicate that by increasing the M_s value a small amount, will give a large advantage/increase in performance. A τ_c equal to $0.5 \cdot \theta$, diamond shaped point in the figures, would be a better alternative for both processes. This tuning parameter will give a M_s-value of 1.44 and 1.35 for the two cases, respectively.



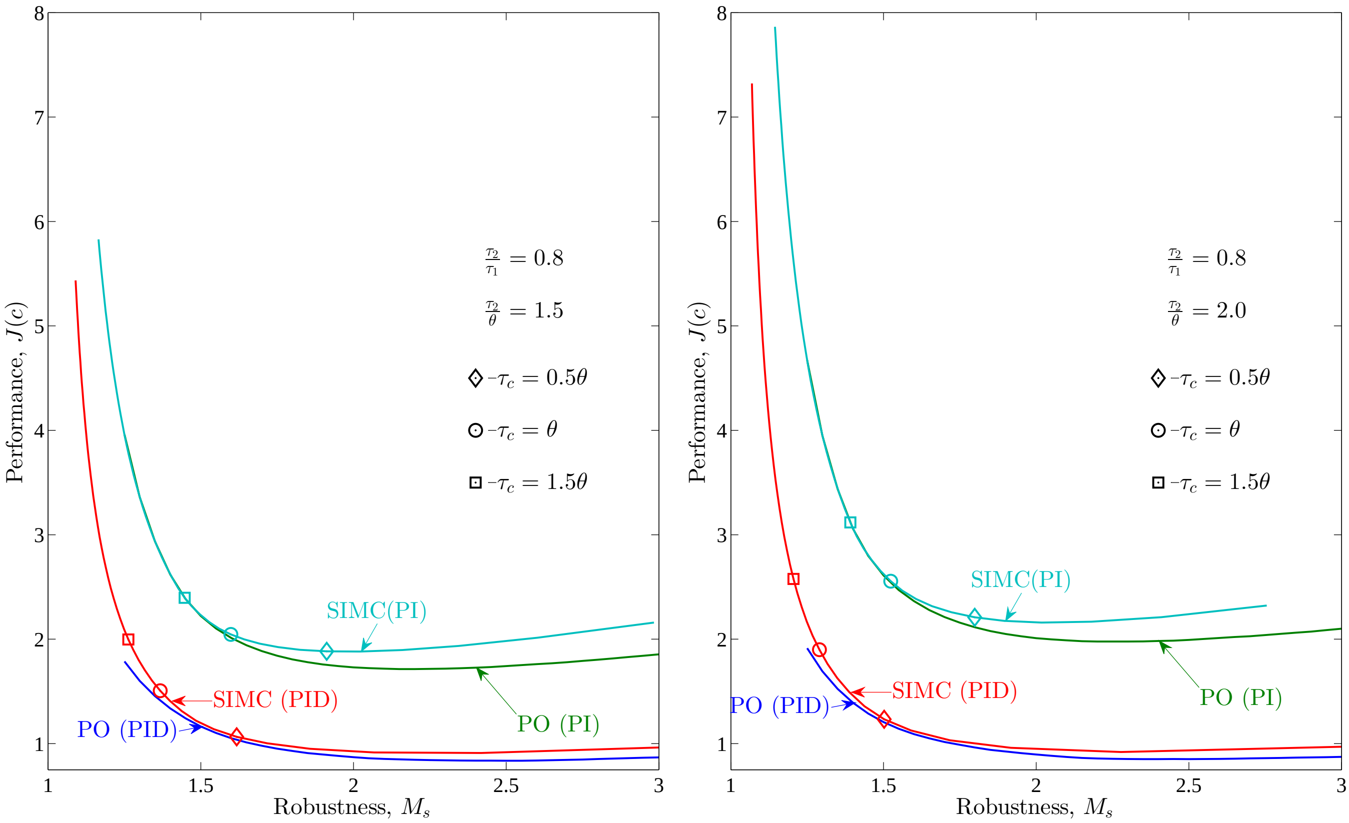
(a) Case 4, $\frac{\tau_2}{\theta} = 1.5$. **(b)** Case 7, $\frac{\tau_2}{\theta} = 2.0$.

Figure 3.2: Pareto optimal (PO) vs. SIMC tuning curves for second order plus time delay processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$, with $\frac{\tau_2}{\tau_1} = 0.5$.

Figure 3.3a and 3.3b depicts case 5 and case 8, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.8, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. For both processes the SIMC tuning rules produces controllers with almost no non-optimality loss in the preferred M_s domain, i.e. $M_s < 2.0$. As M_s decreases and approach one, the cost function increases dramatically.

In the M_s domain $[1.3 - 2.0]$ the SIMC PI underperform the SIMC PID on average with approximately 95 % for case 5 and 120 % for case 8.

The figures show that increasing the ratio $\frac{\tau_2}{\theta}$ will result in a shift towards lower M_s values for given τ_c's. The recommended choice of $\tau_c = \theta$ is shown to result in a M_s value of 1.37 for case 5 and 1.29 for case 8. These tunings give a good trade-off between performance and robustness. If the tuning parameter, τ_c, was selected to $0.5 \cdot \theta$, diamond shaped point in the figures, the resulting M_s values would be 1.62 and 1.50 for the two cases, respectively. These values will also result in a decent trade-off.



(a) Case 5, $\frac{\tau_2}{\theta} = 1.5$.

(b) Case 8, $\frac{\tau_2}{\theta} = 2.0$.

Figure 3.3: Pareto optimal (PO) vs. SIMC tuning curves for second order plus time delay processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$, with $\frac{\tau_2}{\tau_1} = 0.8$.

Figure 3.4a and 3.4b depicts case 6 and case 9, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.3, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. For both processes the SIMC tuning rules produces controllers with almost no non-optimality loss in the preferred M_s domain, i.e. $M_s < 2.0$. As M_s decreases and approach one, the cost function increases dramatically.

In the M_s domain $[1.3 - 2.0]$ the SIMC PI underperform the SIMC PID on average with approximately 100 % for case 6 and 130 % for case 9.

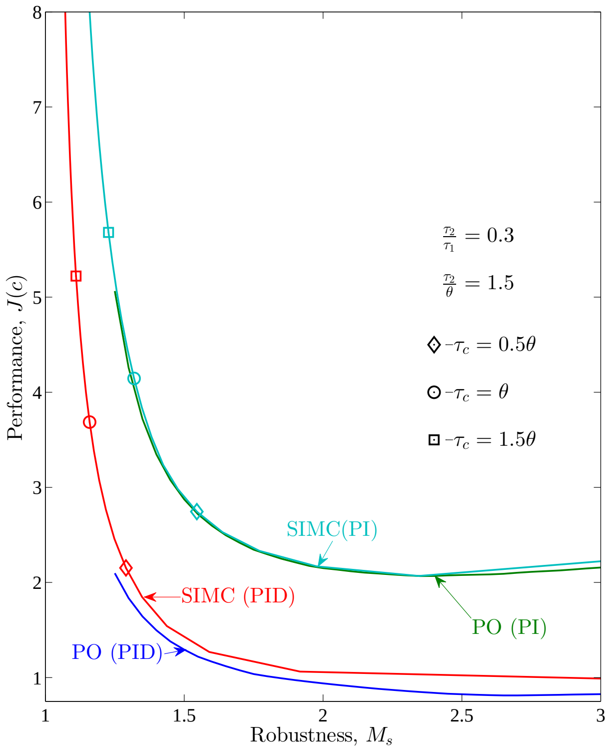
The figures show that increasing the ratio $\frac{\tau_2}{\theta}$ will result in a shift towards lower M_s values for given τ_c's. The recommended choice of $\tau_c = \theta$ is shown to result in a M_s value of 1.16 for case 6 and 1.12 for case 8. These tunings will give a robust controller, but the robustness/performance trade-off yields quite poor performance. The steep gradients in these points indicate that a slight increasing in the M_s-value lead to a large advantage/increase in performance. A τ_c equal to $0.5 \cdot \theta$, diamond shaped point in the figures would be a better alternative for both processes. This tuning parameter result in a M_s-value of 1.29 and 1.23 for the two cases, respectively. For these two cases the tuning parameter could even be chosen smaller than $0.5 \cdot \theta$.

3.2.3 Summary of Pareto-optimal vs. SIMC tunings

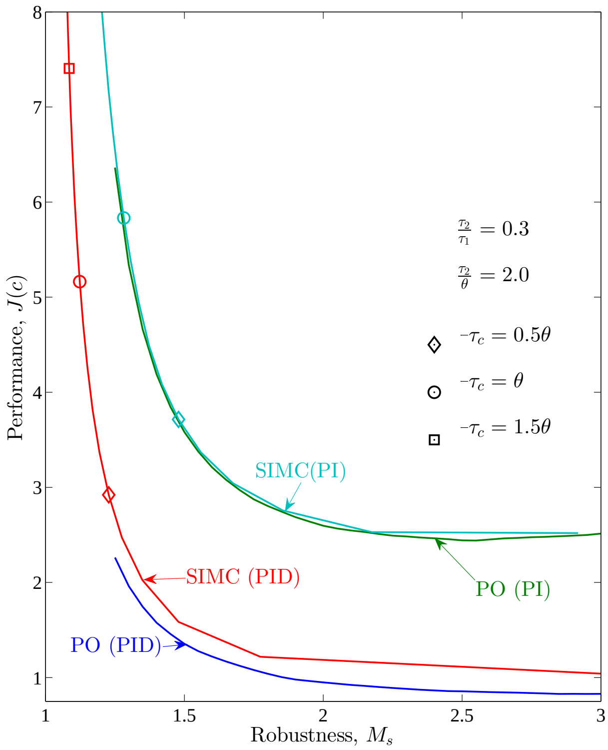
As depicted in Figures 3.2, 3.3 and 3.4 the SIMC tuning rules result in controllers with close to zero non-optimality loss for M_s values in the domain $[1.3 - 2.0]$. The SIMC PI tunings is shown to follow the Pareto-optimal surprisingly well over the this M_s-domain. The Pareto-optimal curves are only calculated in the domain $M_s = [1.25 - 3]$, as the MATLAB-solved did not converge for smaller M_s-values.

All the simulations show that there will be a profit from implementing a PID controller instead of a PI controller, as the latter underperform on average approximately 100 %.

There is a clear shift towards lower M_s values for the tuning parameter τ_c as the ratio $\frac{\tau_2}{\tau_1}$ decreases and the ratio $\frac{\tau_2}{\theta}$ increases.



(a) Case 6, $\frac{\tau_2}{\theta} = 1.5$.



(b) Case 9, $\frac{\tau_2}{\theta} = 2.0$.

Figure 3.4: Pareto optimal (PO) vs. SIMC tuning curves for second order plus time delay processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$, with $\frac{\tau_2}{\tau_1} = 0.3$.

3.3 Step responses

The Pareto-optimal and SIMC tunings has been tested by performing step changes in setpoint and input disturbance. The SIMULINK block diagram is included in Appendix A. The controller tunings used corresponds to a M_s value of 1.7. All the tuning variables, in parallel form, used for the step response experiments are presented in Table 3.2.

In the subsequent sections the step response experiments for the different cases are presented by the use of plots of the output, y, and input, u, as functions of time, t.

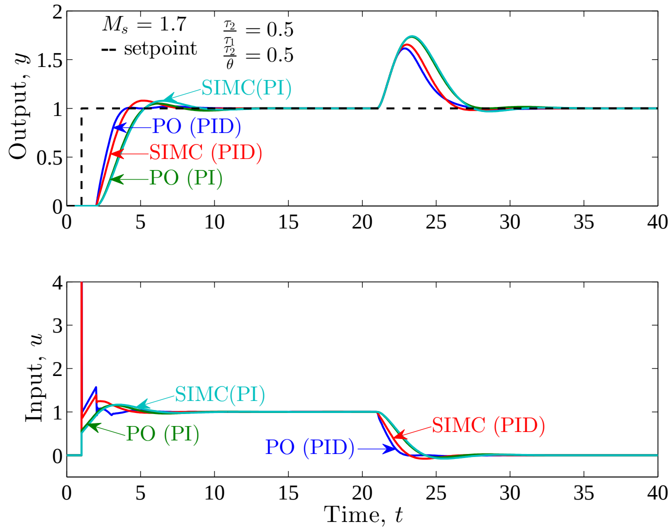
Table 3.2: Tuning variables in parallel form for PID and PI controllers, for both Pareto-optimal and SIMC tunings. $M_s = 1.7$

				Pareto-optimal PID	Pareto-optimal PI	SIMC PID	SIMC PI
$\theta > \tau_2$	**Case 1** $\frac{1}{(s+1)(0.5s+1)} \cdot e^{(-s)}$	$\frac{\tau_2}{\tau_1} = 0.5$, $\frac{\tau_2}{\theta} = 0.5$	K_c	0.97	0.57	0.83	0.51
			τ_I	1.59	1.42	1.50	1.25
			τ_D	0.53	–	0.33	–
	Case 2 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{(-s)}$	$\frac{\tau_2}{\tau_1} = 0.8$, $\frac{\tau_2}{\theta} = 0.8$	K_c	1.09	0.61	1.00	0.52
			τ_I	1.85	1.67	1.80	1.40
			τ_D	0.62	–	0.44	–
	Case 3 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{(-s)}$	$\frac{\tau_2}{\tau_1} = 0.3$, $\frac{\tau_2}{\theta} = 0.3$	K_c	0.91	0.56	0.72	0.51
			τ_I	1.42	1.29	1.30	1.15
			τ_D	0.45	–	0.23	–
$\theta < \tau_2$	**Case 4** $\frac{1}{(s+1)(0.5s+1)} \cdot e^{\left(-\frac{1}{3}s\right)}$	$\frac{\tau_2}{\tau_1} = 0.5$, $\frac{\tau_2}{\theta} = 1.5$	K_c	2.49	1.13	2.37	1.06
			τ_I	1.27	1.30	1.50	1.25
			τ_D	0.37	–	0.33	–
	Case 7 $\frac{1}{(s+1)(0.5s+1)} \cdot e^{(-0.25s)}$	$\frac{\tau_2}{\tau_1} = 0.5$, $\frac{\tau_2}{\theta} = 2.0$	K_c	3.41	1.36	3.33	1.25
			τ_I	1.13	1.32	1.50	1.25
			τ_D	0.33	–	0.33	–
	Case 5 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{\left(-\frac{8}{15}s\right)}$	$\frac{\tau_2}{\tau_1} = 0.8$, $\frac{\tau_2}{\theta} = 1.5$	K_c	1.87	0.90	1.74	0.81
			τ_I	1.71	1.60	1.80	1.40
			τ_D	0.53	–	0.44	–
	Case 8 $\frac{1}{(s+1)(0.8s+1)} \cdot e^{(-0.4s)}$	$\frac{\tau_2}{\tau_1} = 0.8$, $\frac{\tau_2}{\theta} = 2.0$	K_c	2.49	1.08	2.25	0.93
			τ_I	1.59	1.60	1.80	1.40
			τ_D	0.48	–	0.44	–
	Case 6 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{(-0.2s)}$	$\frac{\tau_2}{\tau_1} = 0.3$, $\frac{\tau_2}{\theta} = 1.5$	K_c	3.66	1.57	3.25	1.53
			τ_I	0.87	1.06	1.30	1.15
			τ_D	0.24	–	0.23	–
	Case 9 $\frac{1}{(s+1)(0.3s+1)} \cdot e^{\left(-\frac{3}{20}s\right)}$	$\frac{\tau_2}{\tau_1} = 0.3$, $\frac{\tau_2}{\theta} = 2.0$	K_c	5.05	1.85	3.71	1.92
			τ_I	0.79	1.03	1.3	1.15
			τ_D	0.21	–	0.23	–

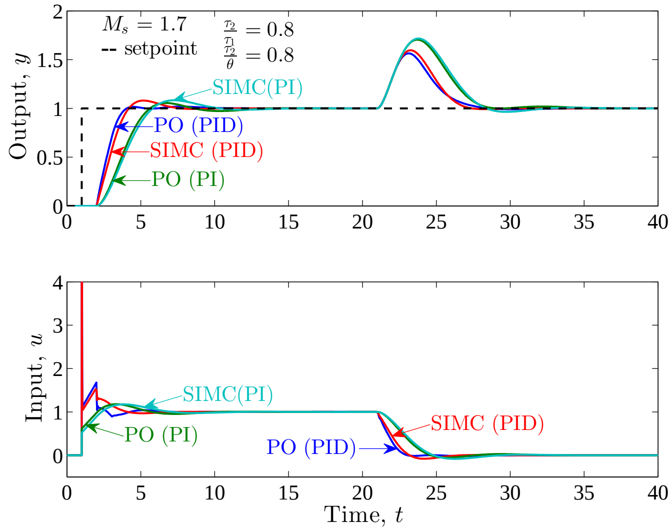
3.3.1 Time delay dominated processes

Figure 3.5a, 3.5b and 3.5c depicts the step responses for the time delay dominated processes, i.e. case 1, 2 and 3, respectively. For the time delay dominated processes the SIMC PID tunings have a bit slower response and a higher overshoot compared with the Pareto-optimal PID in the output, y. Compared to the PI controllers, both the Pareto-optimal and the SIMC, the PID controllers have a faster response with approximately the same overshoot for a setpoint change. The response to a disturbance is both faster and with less overshoot.

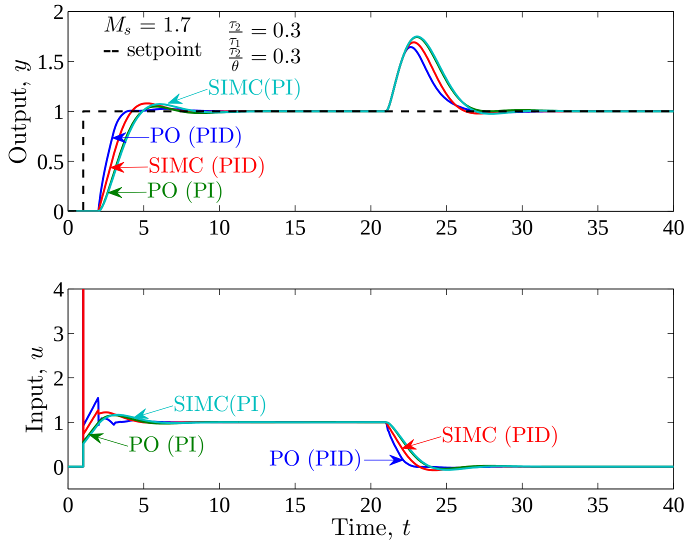
As can be seen from the figures, the Pareto-optimal and SIMC PID controllers have a spike in the input, u, when they are subjected to a setpoint change. This is due to the derivative action in the controller. The derivative part change the manipulated variable, i.e. the input, proportional to the derivative of the controlled variable. As a setpoint change is performed, this becomes infinity, and the spike in the input function is observed. This could be avoided by reconstructing the controller so that the derivative part is only dependent on the feedback and not on the input. This has not been implemented in this project as the problem would only be transferred to a step in the output disturbance. As stated earlier, the input step is treated as a special case of output disturbance. As the ratio $\frac{\tau_2}{\tau_1}$ decrease, the difference between the PID and PI controllers gets smaller. This correspond the the fact that in the limit $\tau_2 \to 0$, the PID controller should be equal to the PI controller.



(a) Case 1, $\frac{\tau_2}{\tau_1} = 0.5$, $\frac{\tau_2}{\theta} = 0.5$.



(b) Case 2, $\frac{\tau_2}{\tau_1} = 0.8$, $\frac{\tau_2}{\theta} = 0.8$.



(c) Case 3, $\frac{\tau_2}{\tau_1} = 0.3$, $\frac{\tau_2}{\theta} = 0.3$.

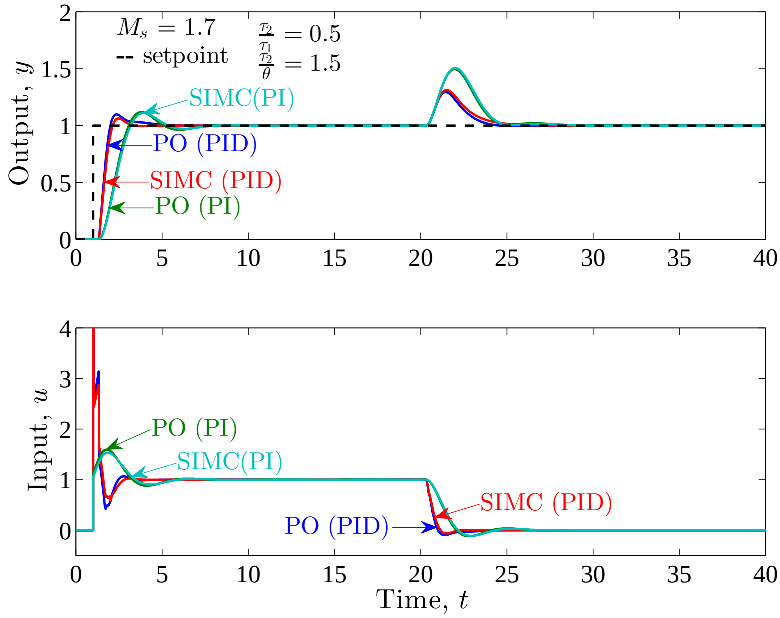
Figure 3.5: Step responses for time delay dominated second order plus time delay processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot exp(-\theta s)$.

3.3.2 Lag dominated processes

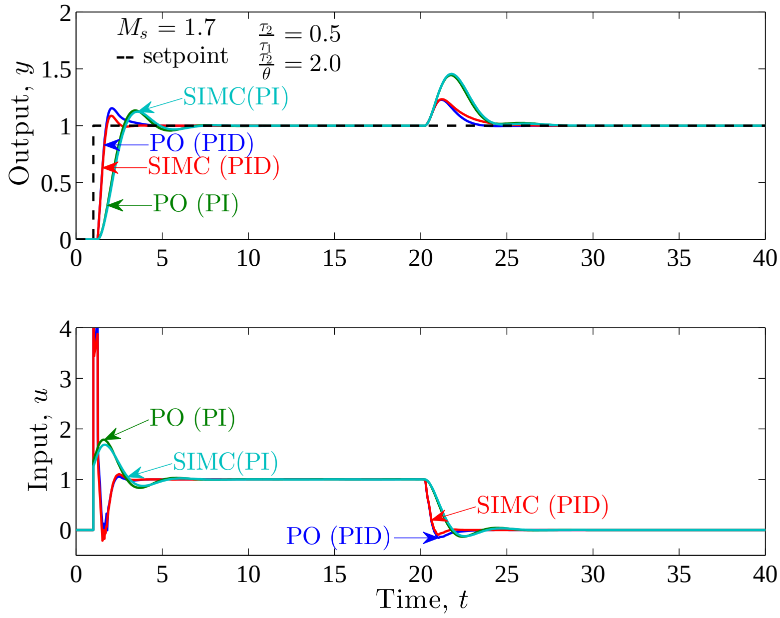
Figure 3.6a and 3.6b depicts case 4 and case 7, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.5, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. The output performance, y, can be seen to be nearly optimal for both the PID and PI controllers. The SIMC PID controller has a little slower response than the corresponding Pareto-optimal PID controller, but result in less overshoot for both cases. It is also observed that the SIMC PID controller reaches the new setpoint in less time than the Pareto-optimal PID controller. The faster response and higher overshoot points to a more aggressive controller, which is confirmed by the values given in Table 3.2.

Both the Pareto-optimal and SIMC PI controllers have a slower response, higher overshoot and uses more time to stabilize at the new setpoint. This slower, smoother control is confirmed by evaluating the input, u, as it is much more aggressive for the PID controllers than for the PI controllers. Again a derivative spike is observed for the PID controllers at the setpoint step. The controllers gets more aggressive as the ratio $\frac{\tau_2}{\theta}$ increases, which follows from the SIMC rules.

For the disturbance rejection some of the same observations can be made. The respective SIMC controllers behaves almost close to optimal with only a little higher overshoot and the need of some extra time before stabilizing, compared to the Pareto-optimal controllers. This observation decrease as the $\frac{\tau_2}{\theta}$ ratio increases as expected for the more aggressive controllers.



(a) Case 4, $\frac{\tau_2}{\theta} = 1.5$.

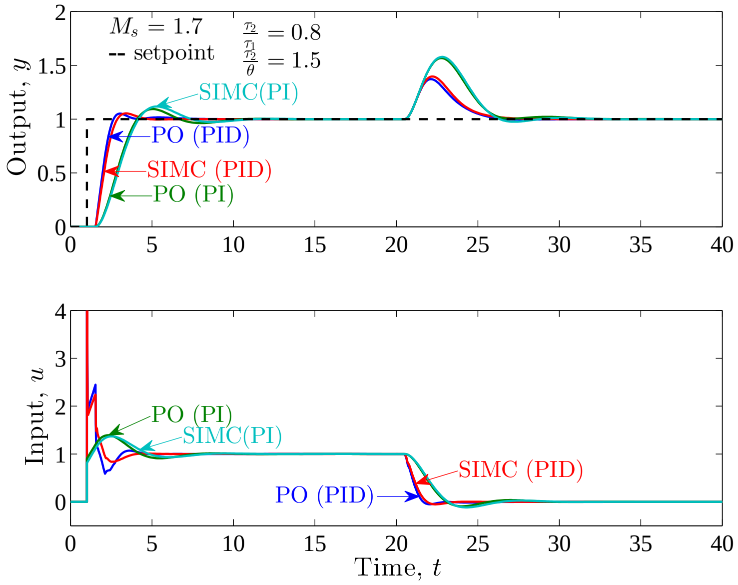


(b) Case 7, $\frac{\tau_2}{\theta} = 2.0$.

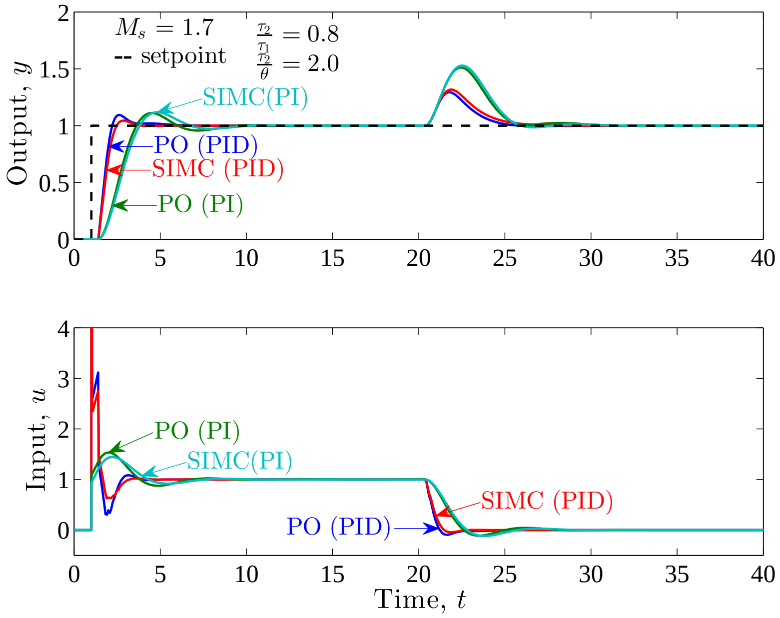
Figure 3.6: Step responses for Pareto-optimal PID and PI, and SIMC PID and PI for second order plus time dealy processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$. $M_s = 1.7$ and $\frac{\tau_2}{\tau_1} = 0.5$.

Figure 3.7a and 3.7b depicts case 5 and case 8, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.8, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. The output response, y, on a step change in setpoint can be seen to be nearly optimal for both the PID and PI controllers. The SIMC PID controller has a little slower response than the corresponding Pareto-optimal PID controller, but result in less overshoot for case 8. It is also observed that the SIMC PID controller reaches the new setpoint in less time than the Pareto-optimal PID controller. Both the Pareto-optimal and SIMC PI controllers have a slower response, higher overshoot and uses more time to stabilize at the new setpoint. This slower and smoother control is confirmed by evaluating the input, u, as it is much more aggressive for the PID controllers than for the PI controllers, or by examination of Table 3.2. Again a derivative spike is observed for the PID controllers at the setpoint step. The controllers gets more aggressive as the ratio $\frac{\tau_2}{\theta}$ increases.

For the disturbance rejection some of the same observations can be made. The respective SIMC controllers behaves almost close to optimal with only a little higher overshoot and the need of some extra time before stabilizing, compared to the Pareto-optimal controllers. This observation decrease as the $\frac{\tau_2}{\theta}$ ratio increases as expected for the more aggressive controllers.



(a) Case 5, $\frac{\tau_2}{\theta} = 1.5$.

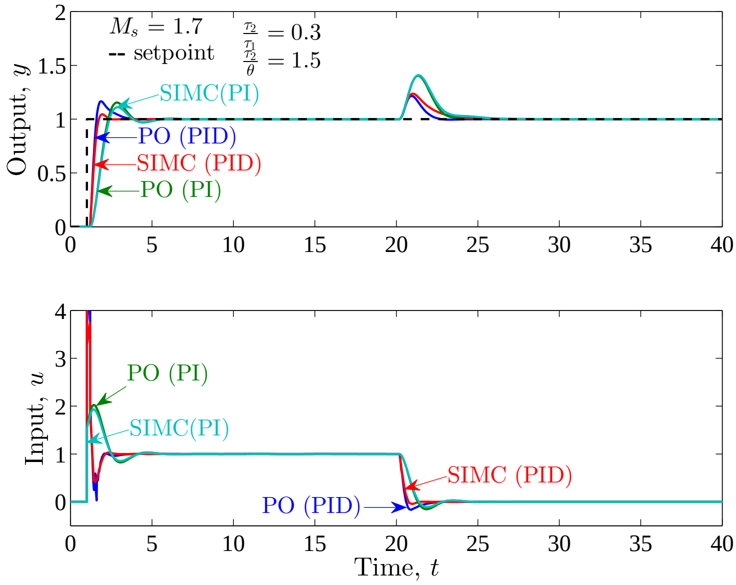


(b) Case 8, $\frac{\tau_2}{\theta} = 2.0$.

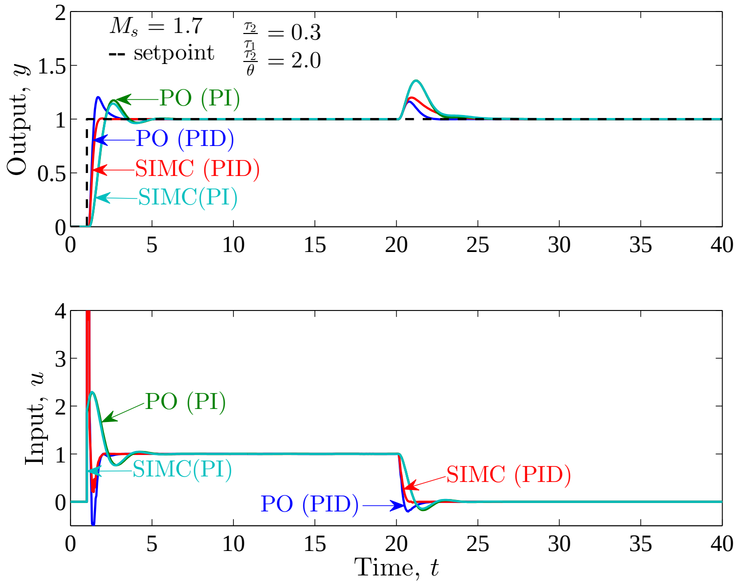
Figure 3.7: Step responses for Pareto-optimal PID and PI, and SIMC PID and PI for second order plus time dealy processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$. $M_s = 1.7$ and $\frac{\tau_2}{\tau_1} = 0.8$.

Figure 3.8a and 3.8b depicts case 6 and case 9, respectively. In both these cases the ratio $\frac{\tau_2}{\tau_1}$ is kept constant at 0.3, whilst the ratio $\frac{\tau_2}{\theta}$ is 1.5 and 2.0, respectively. The output performance, y, on a step change in setpoint can be seen to be nearly optimal for both the PID and PI controllers. The SIMC PID controller has a less aggressive response than the corresponding Pareto-optimal PID controller, and thus result in less overshoot for both cases. It is also observed that the SIMC PID controller reaches the new setpoint in less time than the Pareto-optimal PID controller. Both the Pareto-optimal and SIMC PI controllers have a slower response, higher overshoot and uses more time to stabilize at the new setpoint. This slower, smoother control is confirmed by evaluating the input, u, as it is much more aggressive for the PID controllers than for the PI controllers. Again a derivative spike is observed for the PID controllers at the setpoint step. The controllers gets more aggressive as the ratio $\frac{\tau_2}{\theta}$ increases.

For the disturbance rejection some of the same observations can be made. The respective SIMC controllers behaves almost close to optimal with only a little higher overshoot and the need of some extra time before stabilizing, compared to the Pareto-optimal controllers. This observation decrease as the $\frac{\tau_2}{\theta}$ ratio increases as expected for the more aggressive controllers.



(a) Case 6, $\frac{\tau_2}{\theta} = 1.5$.



(b) Case 9, $\frac{\tau_2}{\theta} = 2.0$.

Figure 3.8: Step responses for Pareto-optimal PID and PI, and SIMC PID and PI for second order plus time dealy processes on the form $g_p = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \cdot \exp(-\theta s)$. $M_s = 1.7$ and $\frac{\tau_2}{\tau_1} = 0.3$.

3.3.3 Summary step resonses

As depicted in Figures 3.6, 3.7 and 3.8 the SIMC tuning rules result in controllers with a good trade-off between setpoint performance and disturbance rejection, if tunings corresponding to $M_s = 1.7$ is used. As can be seen from both the figures and the values in Table 3.2 the Pareto-optimal controllers behave more aggressive than the corresponding SIMC controllers. This result in enhanced disturbance rejection, for five out of six cases, while the performance of setpoint response is reduced.

The difference between the SIMC PID and PI controllers are seen to decrease as the ratio $\frac{\tau_2}{\tau_1}$ decreases. As τ_2 tends to zero, the PID and PI controllers should be the same. Hence, this observation fits well with theory.

3.4 Challenges and future work

Throughout the work with this project the major challenge has been to obtain solutions from the numeric solver in MATLAB. The problem at hand, is a minimization problem to a convex function and to find the right solution has not been easy. Many more cases have been attempted, without been able to make them converge.

The cases tested in this project cannot be used as a satisfactory basis for any conclusions regarding the SIMC tuning rules, but they can be used as a starting point. So far the SIMC tuning rules seems to perform close to optimal, and give good trade-off between performance and robustness. For future work the solution algorithm may have to be improved. In addition to be able to solve for $M_s < 1.25$ a broader specter of processes can be examined and more accurate conclusions can be drawn. Pareto-optimal tunings for a FOPTD process on the form: $g_p = \frac{1}{s+1} \cdot e\left(-\theta s\right)$, should also be calculated to show how much there is to profit from implementing a PID instead of a PI controller in the limit as τ_2 tend to zero.

4 Conclusions

The rather small selection of cases tested in the project does not give a solid foundation to build any conclusions on, however it can be used as a starting point. The performed calculations and simulations show that the SIMC PID tuning rules give close to optimal performance when M_s is used as a measure of robustness and a weighted function of the absolute integral error is used as a measure for performance. The investigations show that the recommended choice of not to implement a PID controller for time delay dominated process will be dependent on the process. For the three cases tested in this project the SIMC PI controller underperformed, on average, approximately 35 %. The performance reward by implementing a PID controller for these processes must be compared with the extra price and complexity a PID controller introduces. For the lag dominated cases tested in this project the PID controller is shown to significantly overperform the PI controller, and thus the reward of implementing a PID controller is much greater than for the time delay dominated processes.

The recommended choice of the SIMC tuning parameter, $\tau_c = \theta$ is shown to hold for processes with a $\frac{\tau_2}{\tau_1}$ ratio greater than 0.5. As the ratio decreases the corresponding M_s values decreases, and the controller get more and more robust. As the trade-off between robustness and performance always prevails, the controller performance is decreased. If the τ_c value is decreased, the controller will lose some robustness, but gain performance. A $\tau_c = 0.5\theta$, or lower, is thus recommended to increase the performance.

Compared to the Pareto-optimal tunings, the SIMC tuning rules result in less aggressive controllers. These controllers have better setpoint performance, and slightly poorer disturbance rejection.

References

[1] S. Skogestad, "Simple analytic rules for model reduction and PID controller tuning," J.Process Control, vol. 13, no. 4, pp. 291–309, 2003.

[2] S. Skogestad, "Probably the best simple PID tuning rules in the world," 2001.

[3] C. Grimholdt and S. Skogestad, "Optimal PI-Control and Verification of the SIMC Tuning Rule," 2012.

[4] S. Skogestad and I. Postlethwaite, Multivariable Feedback Control Analysis and Design. Chichester: John Wiley & Sons, 2012.

A MATLAB Scripts

In the following sections the different MATLAB scripts, used in this project, are presented. They are presented in the order which they need to be executed, that is:

1. Pareto optimal PID tunings (mainOptimalTuningPID.m).
2. Pareto optimal PI tunings (mainOptimalTuningPI.m).
3. Pareto optimal vs. SIMC tunings (mainPoVsSimcPlot.m).
4. Step response (mainStepResponsePlotPo.m).
5. Parallel tunings (tuningParmParallel.m).

A.1 Obtain pareto optimal tunings

A.1.1 Optimal PID tunings, main file

```
% Script for generating the PO-PID curve for a given process gp
% The controller used is an ideal PID controller
% Written by: Martin S. Foss, fall 2012

%clc
clear all

global gp msEq iaeWeights manWeights iaeTuning

%Adding "sharedFiles" to MatLab search directory
curDir = pwd;
mainDir = fileparts(curDir);
sharedDir = fullfile(mainDir,'sharedFiles');
addpath(sharedDir);

%%
tic
modelId = 9;
```

```
gp = model(modelId);     %getting the model

%% Finding the iaeWeights
    fprintf('Finding Optimal IAE Weights \n')
    fprintf('Case: %g\n',modelId)
    fprintf('***************************\n')
    fprintf('\n')
    fprintf('costFun(iae) \t minTuning \t\t Ms \t exitFlag \n')
    fprintf('--------------------------------------------------\n')

msEq = 1.59;    %Ms for the iaeWeights
opt = optimset('algorithm','active-set','Display','off','TolCon',1e-4);
               %'active-set', 'trust-region-reflective', 'interior-point',
               %'interior-point-convex', 'levenberg-marquardt',
               %'trust-region-dogleg', 'lm-line-search', or 'sqp'.

%Initial solution guesses
X0 = [0.9 0.5 0.4    %case 1
      1.0 0.6 0.6    %case 2
      0.8 0.7 0.3    %case 3
      2.2 1.5 0.8    %case 4
      1.7 0.9 0.8    %case 5
      3.2 2.4 0.8    %case 6
      2.9 1.9 1.0    %case 7
      2.2 1.2 1.0    %case 8
      4.2 3.2 1.0]; %case 9
x0 = X0(modelId,:)';

iaeWeights = [1; 1];     %cost function weigths
manWeights = [1 0];

[minTuningSp,iaeSp,exitFlagSp] = fmincon(@costFun,x0,[],[],[],[],...
                                          [0;0;0],[],@conFun,opt);

    fprintf('%0.2f \t \t %.2f %.2f %.2f \t %.2f\t %i \t \n',iaeSp,...
            minTuningSp, msEq, exitFlagSp)

%%
%Initial solution guesses
```

```
X0 = [0.9 0.7 0.5   %case 1
      1.0 0.6 0.6   %case 2
      0.8 0.7 0.35  %case 3
      2.2 2.3 0.8   %case 4
      1.7 1.3 0.8   %case 5
      3.3 5.0 0.8   %case 6
      3.0 3.4 1.0   %case 7
      2.2 1.8 1.0   %case 8
      4.5 7.6 1.0]; %case 9
x0 = X0(modelId,:)';

manWeights = [0 1];

[minTuningD,iaeD,exitFlagD] = fmincon(@costFun,x0,[],[],[],[],....
                                       [0;0;0],[],@conFun,opt);

    fprintf('%0.2f \t \t %.2f %.2f %.2f \t %.2f\t %i \t \n',iaeD,...
            minTuningD, msEq, exitFlagD)
    fprintf('\n');
    fprintf('\n');
    fprintf('\n');
    fprintf('\n');

iaeWeights = [iaeSp; iaeD];

%% Generating curve
i = 1;              %iteration counter for command window printout
minTuning = [];     %matrices for storing results
costTuning = [];
iaeOptTun = [];

msSpace = 1.25:0.05:3;  %Ms search range

%Initial solution guesses
X0 = [0.7 0.5 0.4   %case 1
      0.8 0.6 0.5   %case 2
      0.4 0.4 0.3   %case 3
      1.1 1.1 0.4   %case 4
      0.9 0.7 0.5   %case 5
```

```
        1.6 1.9 0.4   %case 6
        1.5 1.4 0.5   %case 7
        1.1 0.9 0.6   %case 8
        2.0 3.0 0.6]; %case 9
x0 = X0(modelId,:)';

manWeights = [.5, .5];

    fprintf('Generating the PO Curve\n')
    fprintf('***********************\n')
    fprintf('\n')
    fprintf('Number of iterations: %i \n', length(msSpace))
    fprintf('\n')
    fprintf('costFun(%.1f, %.1f) \t minTuning \t \t Ms ',manWeights)
    fprintf('\t exitFlag \t iterations left \n')
    fprintf('------------------------------------------------------------')
    fprintf('--------------------\n')

opt = optimset('algorithm','active-set','Display','off','TolCon',1e-4);
               %'active-set', 'trust-region-reflective', 'interior-point',
               %'interior-point-convex', 'levenberg-marquardt',
               %'trust-region-dogleg', 'lm-line-search', or 'sqp'.

%Optimizing
for msEq = msSpace;

    [minTuningTemp, iaeTuningTemp, exitFlagTuningTemp] = ...
                    fmincon(@costFun,x0,[],[],[],[],[0;0;0],[],@conFun,opt);

    minTuning(:,i) = minTuningTemp;          %storing resutlts
    costTuning(i) = iaeTuningTemp;
    exitFlagTuning(i) = exitFlagTuningTemp;
    iaeOptTun(i,:) = iaeTuning;

    if modelId == 2 && i == 1 || modelId == 3 && i == 1 ||...
       modelId == 5 && i == 1
        x0 = 1.1*minTuningTemp;
    elseif modelId == 6 && i == 7 || modelId == 9 && i == 32
        x0 = 0.9*minTuningTemp;
```

```
    elseif modelId == 9 && i == 17
        x0 = 0.9*minTuningTemp;
    elseif modelId == 9 && i == 16 || modelId == 9 && i >= 34
        x0 = 0.91*minTuningTemp;
    else
        x0 = minTuningTemp;
    end

    fprintf('%0.2f \t \t \t %.2f  %.2f %.2f \t %.2f\t %i \t \t\t %i \n',...
            iaeTuningTemp, minTuningTemp, msEq, exitFlagTuningTemp,...
            length(msSpace)-i)

    i = i + 1; %updating iteration counter
end

    fprintf('\n')
    fprintf('Calculation Finished!\n')
    fprintf('=====================\n')
toc

%% Ploting the results
%Cost function, J vs. Ms
figure(modelId)
clf
h(1) = plot(msSpace,costTuning);
axis([1.2 3 0.8 1.3])
xlabel('Robustness, $M_s$', 'interpreter','latex','FontSize',14)
ylabel('Performance, $J$', 'interpreter','latex','FontSize',14)
titleName = {'Case 1','Case 2','Case 3','Case 4','Case 5',...
             'Case 6','Case 7','Case 8','Case 9'};

title(titleName{modelId},'interpreter','latex','FontSize',14)

%% Storing results
modelName = {'case1','case2','case3','case4','case5','case6',...
             'case7','case8','case9'};

res.case = num2str(modelName{modelId});

```

```
res.POpid.minSp.tuning = minTuningSp;
res.POpid.minSp.iae = iaeSp;
res.POpid.minSp.exitFlag = exitFlagSp;

res.POpid.minD.tuning = minTuningD;
res.POpid.minD.iae = iaeD;
res.POpid.minD.exitFlag = exitFlagD;

res.POpid.minTuning.ms = msSpace;
res.POpid.minTuning.tuning = minTuning;
res.POpid.minTuning.costFun = costTuning;
res.POpid.minTuning.exitFlag = exitFlagTuning;
res.POpid.minTuning.iae = iaeOptTun;

% pause
% saving struct
save([mainDir,'\dataFiles\','resPOpid_',...
      num2str(modelName{modelId}),'.mat'],'res')    %saving "globaly"
save(['poPIDresults\','resPOpid_',...
      num2str(modelName{modelId}),'.mat'],'res')    %saving "localy"

% saving figures
saveas(h(1),['figures\','POpidCurve_',num2str(modelName{modelId}),'.fig'])

restoredefaultpath
```

A.1.2 Optimal PI tunings, main file

```
% Script for generating the PO-PI curve for a given process gp
% The controller used is an ideal PI controller
% Written by: Martin S. Foss, fall 2012

%clc
clear all

global gp msEq iaeWeights manWeights iaeTuning

%Adding "sharedFiles" to MatLab search directory
```

```matlab
curDir = pwd;
mainDir = fileparts(curDir);
sharedDir = fullfile(mainDir,'sharedFiles');
addpath(sharedDir);

%%
tic
modelId = 9;
modelName = {'case1','case2','case3','case4','case5','case6',...
             'case7','case8','case9','case10'};
gp = model(modelId);    % getting the model

%% Finding the iaeWeights for the PI-controller
    fprintf('Finding Optimal IAE Weights (PI-controller) \n')
    fprintf('Case: %g\n',modelId)
    fprintf('***************************\n')
    fprintf('\n')
    fprintf('costFun(iae) \t minTuning \t\t Ms \t exitFlag \n')
    fprintf('--------------------------------------------------\n')

msEq = 1.59;   %Ms for the iaeWeights
opt = optimset('algorithm','active-set','Display','off','TolCon',1e-4);
               %'active-set', 'trust-region-reflective', 'interior-point',
               %'interior-point-convex', 'levenberg-marquardt',
               %'trust-region-dogleg', 'lm-line-search', or 'sqp'.

%Initial solution guesses
X0 = [0.5 0.4 0   %case 1
      0.5 0.3 0   %case 2
      0.5 0.4 0   %case 3
      1.1 0.7 0   %case 4
      0.8 0.5 0   %case 5
      1.5 1.2 0   %case 6
      1.3 0.9 0   %case 7
      1.0 0.6 0   %case 8
      1.9 1.4 0]; %case 9
x0 = X0(modelId,:)';

iaeWeights = [1; 1];    %cost function weigths
```

```
manWeights = [1 0];

Aeq = [0 0 1];          %constraints
Beq = 0;

[minTuningSp,iaeSp,exitFlagSp] =  fmincon(@costFun,x0,[],[],Aeq,Beq,...
                                          [0;0;0],[],@conFun,opt);

    fprintf('%0.2f \t \t %.2f %.2f %.2f \t %.2f\t %i \t \n',iaeSp,...
            minTuningSp, msEq, exitFlagSp)

%%
%Initial solution guesses
X0 = [0.5 0.4 0   %case 1
      0.5 0.3 0   %case 2
      0.5 0.4 0   %case 3
      0.9 0.8 0   %case 4
      0.7 0.5 0   %case 5
      1.3 1.3 0   %case 6
      1.1 0.9 0   %case 7
      0.9 0.6 0   %case 8
      1.5 1.6 0]; %case 9
x0 = X0(modelId,:)';

manWeights = [0 1];

[minTuningD,iaeD,exitFlagD] = fmincon(@costFun,x0,[],[],Aeq,Beq,...
                                      [0;0;0],[],@conFun,opt);

    fprintf('%0.2f \t \t %.2f %.2f %.2f \t %.2f\t %i \t \n',iaeD,...
            minTuningD, msEq, exitFlagD)
    fprintf('\n');
    fprintf('\n');
    fprintf('\n');
    fprintf('\n');

%% Generating curve
try
    load(fullfile(mainDir,'dataFiles',['resPOpid_',...
```

```
                    num2str(modelName{modelId}),'.mat']))
catch me
    ME = MException(me.identifier,...
           'could not open file, check corrrect modelId and data folder!');
    throw(ME)
end
minSp = res.POpid.minSp.iae;
minD = res.POpid.minD.iae;
iaeWeights = [minSp; minD];

i = 1;                  %iteration counter for command window printout
minTuning = [];     %matrices for storing results
costTuning = [];
iaeOptTun = [];

msSpace = 1.25:0.05:3; %the Ms search range

%Initial solution guesses
X0 = [0.2 0.2 0    %case 1
      0.3 0.2 0    %case 2
      0.2 0.2 0    %case 3
      0.4 0.4 0    %case 4
      0.5 0.4 0    %case 5
      0.6 0.6 0    %case 6
      0.5 0.4 0    %case 7
      0.4 0.3 0    %case 8
      1.2 1.1 0]; %case 9
x0 = X0(modelId,:)';

manWeights = [.5, .5];

    fprintf('Generating the PO Curve\n')
    fprintf('***********************\n')
    fprintf('\n')
    fprintf('Number of iterations: %i \n', length(msSpace))
    fprintf('\n')
    fprintf('costFun(%.1f, %.1f) \t minTuning \t \t Ms',manWeights)
    fprintf(' \t exitFlag \t iterations left \n')
    fprintf('------------------------------------------------------------')
```

```
    fprintf('--------------------\n')

opt = optimset('algorithm','active-set','Display','off','TolCon',1e-4);
               %'active-set', 'trust-region-reflective', 'interior-point',
               %'interior-point-convex', 'levenberg-marquardt',
               %'trust-region-dogleg', 'lm-line-search', or 'sqp'.

%Optimizing
for msEq = msSpace;

    [minTuningTemp, iaeTuningTemp, exitFlagTuningTemp] = ...
            fmincon(@costFun,x0,[],[],Aeq,Beq,[0;0;0],[],@conFun,opt);

    minTuning(:,i) = minTuningTemp;         %storing resutlts
    costTuning(i) = iaeTuningTemp;
    exitFlagTuning(i) = exitFlagTuningTemp;
    iaeOptTun(i,:) = iaeTuning;

    if modelId == 1 && i == 22 || modelId == 1 && i == 32 || ...
       modelId == 3 && i == 22 || modelId == 9 && i == 29 || ...
       modelId == 9 && i >= 32
        x0 = 0.9*minTuningTemp;
    elseif modelId == 5 && i == 1 || modelId == 5 && i == 2
        x0 = 1.2*minTuningTemp;
    else
        x0 = minTuningTemp;
    end

    fprintf('%0.2f \t \t \t %.2f  %.2f %.2f \t %.2f\t %i \t \t\t %i \n',...
            iaeTuningTemp, minTuningTemp, msEq, exitFlagTuningTemp,...
            length(msSpace)-i)

    i = i + 1;  %updating iteration counter
end

    fprintf('\n')
    fprintf('Calculation Finished!\n')
    fprintf('=====================\n')
toc
```

```

%% Plotting the results
% Cost function, J, vs. Ms
figure(modelId)
clf
h(1) = plot(msSpace,costTuning);
xlabel('Robustness, $M_s$', 'interpreter','latex','FontSize',14)
ylabel('Performance, $J$', 'interpreter','latex','FontSize',14)
titleName = {'Case 1','Case 2','Case 3','Case 4','Case 5',...
             'Case 6','Case 7','Case 8','Case 9',};

title(titleName{modelId},'interpreter','latex','FontSize',14)

%% Storing results (in the same struct as the PO (PID) tunings)
res.POpi.minSp.tuning = minTuningSp;
res.POpi.minSp.iae = iaeSp;
res.POpi.minSp.exitFlag = exitFlagSp;

res.POpi.minD.tuning = minTuningD;
res.POpi.minD.iae = iaeD;
res.POpi.minD.exitFlag = exitFlagD;

res.POpi.minTuning.ms = msSpace;
res.POpi.minTuning.tuning = minTuning;
res.POpi.minTuning.costFun = costTuning;
res.POpi.minTuning.exitFlag = exitFlagTuning;
res.POpi.minTuning.iae = iaeOptTun;

pause
%saving struct
save([mainDir,'\dataFiles\','resPO_',...
      num2str(modelName{modelId}),'.mat'],'res')
save(['poPIresults\','resPO_',...
      num2str(modelName{modelId}),'.mat'],'res')

%saving figures
saveas(h(1),['figures\','POpiCurve_',num2str(modelName{modelId}),'.fig'])

restoredefaultpath
```

A.1.3 Cost function

```
function J = costFun(x)

global gp iaeWeights manWeights iaeTuning

%Controller
gc = controller(x(1),x(2),x(3));

%Feedback loops
gey = feedback(1,gc*gp);
ged = feedback(gp*-1,gc,1);

%Output response to input and output disturbance
sys = [gey;ged];
[e,t]=step(sys,100);

iaeTuning = iae(t,e);

J = manWeights*(iaeTuning./iaeWeights);
return
```

A.1.4 Constraints

```
function [c, ceq] = conFun(x0)

global gp msEq

c   = [];
ceq = msEq - ms(gp, controller(x0(1),x0(2),x0(3)));
return
```

A.2 Obtain PO vs. SIMC tuning plots

A.2.1 Main file

```
% Script for creating plots comparing SIMC tunings with the PO curve
% Written by: Martin S. Foss, fall 2012

clear all

%Adding "sharedFiles" to MatLab search directory
curDir = pwd;
mainDir = fileparts(curDir);
sharedDir = fullfile(mainDir,'sharedFiles');
addpath(sharedDir);

%%
% for m = 1:8
modelId = 9;
modelName = {'case1','case2','case3','case4','case5','case6',...
             'case7','case8','case9','case10'};

    fprintf('Finding SIMC tunings \n')
    fprintf('Case: %g\n',modelId)
    fprintf('***************************\n')
    fprintf('\n')

try
    load(fullfile(mainDir,'dataFiles',['resPO_',...
                  num2str(modelName{modelId}),'.mat'])) %loading datafiles
catch me
    ME = MException(me.identifier,...
                    'could not open file, check correct modelId!');
    throw(ME)
end

minSp = res.POpid.minSp.iae;    %loading iaeWeights;
minD  = res.POpid.minD.iae;
minWeights = [minSp;minD];

manWeights = [.5 .5];   %setting manWeights

tcSpace = 0:0.1:5;      %closed loop time constant search rang for simc

```

```
%% SIMC PI-controller
gp = model(modelId);     %get the model

simcJ  = [];             %setting up result matrices
simcMs = [];
simcTuning = [];
i = 1;                   %setting iteration counter

for tc = tcSpace
    [simcTuningTemp, simcGc] = simcPID(gp,tc); %finding the simc PID tuning
    jTemp = costFun(gp,simcGc,minWeights,manWeights);      %finding the cost
    msTemp = ms(gp,simcGc);                            %finding simc ms value

    simcJ(i)  = jTemp;           %storing results
    simcMs(i) = msTemp;
    simcTuning(:,i) = simcTuningTemp;
    i = i+1;                     %updating iteration counter
end

%Finding refrence dots for SIMC PID-tuning;
simcRefJ  = [];      %setting up result matrices
simcRefMs = [];

for tc = [.5 1 1.5]
    [simcTuningTemp simcGc] = simcPID(gp,tc);  %finding the simc PID tuning
    jTemp = costFun(gp,simcGc,minWeights,manWeights);      %finding the cost
    msTemp = ms(gp,simcGc);                            %finding simc ms value

    simcRefJ  = [simcRefJ jTemp];        %storing results
    simcRefMs = [simcRefMs msTemp];
end

%% SIMC PI-controller
gpPI = modelPI(modelId);     %get the model

simcJPI  = [];               %setting up result matrices
simcMsPI = [];
simcTuningPI = [];
i = 1;                       %setting iteration counter
```

```

for tc = tcSpace
    [simcTuningTemp, simcGc] = simcPI(gpPI,tc); %finding the simc PI-tuning
    jTemp = costFun(gp,simcGc,minWeights,manWeights);     %finding the cost
    msTemp = ms(gp,simcGc);                              %finding simc ms value

    simcJPI(i)   = jTemp;     %storing results
    simcMsPI(i) = msTemp;
    simcTuningPI(:,i) = simcTuningTemp;
    i = i+1;                  %update the iteration counter
end

%Finding refrence dots for SIMC PI-tuning;
simcRefJPI   = [];            %setting up results matrices
simcRefMsPI = [];

for tc = [.5 1 1.5]
    [simcTuningTemp simcGc] = simcPI(gpPI,tc);  %finding the simc PI-tuning
    jTemp = costFun(gp,simcGc,minWeights,manWeights);     %finding the cost
    msTemp = ms(gp,simcGc);                              %finding simc ms value

    simcRefJPI   = [simcRefJPI jTemp];       %storing results
    simcRefMsPI = [simcRefMsPI msTemp];
end

%% Plotting the results
colorSet = colormap('lines');
figure(modelId)
clf

h = plot(res.POpid.minTuning.ms,res.POpid.minTuning.costFun,...  %PO(PID)
         res.POpi.minTuning.ms,res.POpi.minTuning.costFun,...    %PO(PI)
         simcMs,simcJ,...                                        %SIMC(PID)
         simcMsPI,simcJPI);                                      %SIMC(PI)
set(h,'LineWidth',1.5)

markerStyles = cellstr(char('d','o','s'));

%Points for SIMC (PID)
```

```
hold on
for i = 1:length(simcRefMs)
    h(i) = plot(simcRefMs(i),simcRefJ(i));
    set(h(i),'color',colorSet(3,:),'LineWidth',1.5,'Marker',...
        markerStyles{i},'MarkerSize',10);
end

axis([1 3 0.75 8]);

%Points for SIMC (PI)
for i = 1:length(simcRefMsPI)
    h(i) = plot(simcRefMsPI(i),simcRefJPI(i));
    set(h(i),'color',colorSet(4,:),'linewidth',1.5,'Marker',...
        markerStyles{i},'MarkerSize',10);
end

%% Printing info
tau2tau1Info = {'$\frac{\tau_2}{\tau_1}=0.5$',...
                '$\frac{\tau_2}{\tau_1}=0.8$',...
                '$\frac{\tau_2}{\tau_1}=0.3$',...
                '$\frac{\tau_2}{\tau_1}=0.5$',...
                '$\frac{\tau_2}{\tau_1}=0.8$',...
                '$\frac{\tau_2}{\tau_1}=0.3$',...
                '$\frac{\tau_2}{\tau_1}=0.5$',...
                '$\frac{\tau_2}{\tau_1}=0.8$',...
                '$\frac{\tau_2}{\tau_1}=0.3$'};
tau2thetaInfo = {'$\frac{\tau_2}{\theta}=0.5$',...
                 '$\frac{\tau_2}{\theta}=0.8$',...
                 '$\frac{\tau_2}{\theta}=0.3$',...
                 '$\frac{\tau_2}{\theta}=1.5$',...
                 '$\frac{\tau_2}{\theta}=1.5$',...
                 '$\frac{\tau_2}{\theta}=1.5$',...
                 '$\frac{\tau_2}{\theta}=2.0$',...
                 '$\frac{\tau_2}{\theta}=2.0$',...
                 '$\frac{\tau_2}{\theta}=2.0$'};
curveInfo = {'PO (PID)','PO (PI)','SIMC (PID)','SIMC(PI)'};
pointInfo = {'$\tau_c=0.5\theta$','$\tau_c=\theta$','$\tau_c=1.5\theta$'};

infoFontSize = 18;
```

```
xlab = xlabel('Robustness, $M_s$');
ylab = ylabel('Performance, $J(c)$');
set(xlab,'interpreter','latex','fontsize',infoFontSize)
set(ylab,'interpreter','latex','fontsize',infoFontSize)

set(gca,'fontsize',16,'FontName','Times New Roman')

%Model info
[figx figy] = dsxy2figxy(gca,2.4,5.5);
textBoxTau1Tau2Info = annotation('textbox',[figx figy .07 .03],...
        'string',tau2tau1Info{modelId},'interpreter','latex',...
        'fontsize',infoFontSize,'color',[0 0 0],'FitBoxToText','on',...
        'LineStyle','none');

[figx figy] = dsxy2figxy(gca,2.4,5); % (gca,3,3)
textBoxTau2ThetaInfo = annotation('textbox',[figx figy .07 .03],...
        'string',tau2thetaInfo{modelId},'interpreter','latex',...
        'fontsize',infoFontSize,'color',[0 0 0],'FitBoxToText','on',...
        'LineStyle','none');

%Markers for different tau_c
h = plot(2.4,4.5,'marker',markerStyles{1},'markerSize',10,...
         'linewidth',1.5,'color',[0 0 0]);
[figx figy] = dsxy2figxy(gca,2.4,4.5);
point1 = annotation('textarrow',[figx+0.025 figx+0.015],[figy figy],...
                    'string',pointInfo{1},'interpreter','latex',...
                    'fontsize',infoFontSize,'headstyle','none');

h = plot(2.4,4,'marker',markerStyles{2},'markerSize',10,...
         'linewidth',1.5,'color',[0 0 0]);
[figx figy] = dsxy2figxy(gca,2.4,4);
point2 = annotation('textarrow',[figx+0.025 figx+0.015],[figy figy],...
                    'string',pointInfo{2},'interpreter','latex',...
                    'fontsize',infoFontSize,'headstyle','none');

h = plot(2.4,3.5,'marker',markerStyles{3},'markerSize',10,...
         'linewidth',1.5,'color',[0 0 0]);
[figx figy] = dsxy2figxy(gca,2.4,3.5);
```

```
point3 = annotation('textarrow',[figx+0.025 figx+0.015],[figy figy],...
                    'string',pointInfo{3},'interpreter','latex',...
                    'fontsize',infoFontSize,'headstyle','none');

%PO (PID)
if modelId == 8
    x1 = find(res.POpid.minTuning.ms >= 1.4);
else
    x1 = find(res.POpid.minTuning.ms >= 1.5);
end
[figx figy] = dsxy2figxy(gca,res.POpid.minTuning.ms(x1(1)),...
                         res.POpid.minTuning.costFun(x1(1)));
curve1 = annotation('textarrow',[figx-0.03 figx],[figy-0.005 figy],...
                    'string',curveInfo{1},'interpreter','latex',...
                    'fontsize',infoFontSize,'color',colorSet(1,:));

%PO (PI)
x2 = find(res.POpi.minTuning.ms >= 2.4);
[figx figy] = dsxy2figxy(gca,res.POpi.minTuning.ms(x2(1)),...
                         res.POpi.minTuning.costFun(x2(1)));
if modelId < 4
    curve2 = annotation('textarrow',[figx+0.05 figx],[figy+0.05 figy],...
                        'string',curveInfo{2},'interpreter','latex',...
                        'fontsize',infoFontSize,'color',colorSet(2,:));
else
    curve2 = annotation('textarrow',[figx+0.05 figx],[figy-0.05 figy],...
                        'string',curveInfo{2},'interpreter','latex',...
                        'fontsize',infoFontSize,'color',colorSet(2,:));
end

%SIMC (PID)
if modelId == 1 || modelId == 2 || modelId == 3
    x3 = find(simcMs <= 1.20);
elseif modelId == 7
    x3 = find(simcMs <= 1.3);
else
    x3 = find(simcMs <= 1.4);
end
[figx figy] = dsxy2figxy(gca,simcMs(x3(1)),simcJ(x3(1)));
```

```
if modelId == 4
    curve3 = annotation('textarrow',[figx+0.05 figx],[figy+0.01 figy],...
                        'string',curveInfo{3},'interpreter','latex',...
                        'fontsize',infoFontSize,'color',colorSet(3,:));
else
    curve3 = annotation('textarrow',[figx+0.05 figx],[figy figy],...
                        'string',curveInfo{3},'interpreter','latex',...
                        'fontsize',infoFontSize,'color',colorSet(3,:));
end

%SIMC (PI)
if modelId == 5 || modelId == 7
    x4 = find(simcMsPI <= 2.1);
else
    x4 = find(simcMsPI <= 2);
end
[figx figy] = dsxy2figxy(gca,simcMsPI(x4(1)),simcJPI(x4(1)));
curve4 = annotation('textarrow',[figx+0.02 figx],[figy+0.03 figy],...
                    'string',curveInfo{4},'interpreter','latex',...
                    'fontsize',infoFontSize,'color',colorSet(4,:));

set(gca,'Layer','top','Box','on')
set(gcf,'paperpositionmode','auto')

%% Storing results and figures
res.simcPID.simcJ = simcJ;
res.simcPID.simcMs = simcMs;
res.simcPID.simcTuning = simcTuning;

res.simcPI.simcJ = simcJPI;
res.simcPI.simcMs = simcMsPI;
res.simcPI.simcTuning = simcTuningPI;

pause
save([mainDir,'\dataFiles\','resSimc_',...
      num2str(modelName{modelId}),'.mat'],'res')
save(['simcResults\','resSimc_',num2str(modelName{modelId}),'.mat'],'res')

saveas(h,['SIMCfigures\','simcRes_',...
```

```
            num2str(modelName{modelId}),'.eps'],'psc2')
saveas(h,['SIMCfigures\','simcRes_',num2str(modelName{modelId}),'.fig'])
% end

restoredefaultpath
```

A.2.2 Cost function

```
function J = costFun(gp,gc,iaeWeights,manWeights)

%Feedback loops
gey = feedback(1,gc*gp);
ged = feedback(gp*-1,gc,1);

%Output response to input and output disturbance
sys = [gey;ged];
[e,t] = step(sys,100);

iaeTuning = iae(t,e);

J = manWeights*(iaeTuning./iaeWeights);
return
```

A.2.3 SIMC PI-tunings

```
% function for simc PI tuning
% input: 2nd order model + tuning parameter
% returns: simc tuning paramer for a PI controller on the form
% gc = K + I/s
% Written by: Martin S. Foss, fall 2012

function [tuning gc] = simcPI(gp,tc)

%Determining the model
t = gp.den{1}(1);            %time constant
d = totaldelay(gp);          %time delay
g = gp.num{1}(end);          %gain
```

```
z = zero(gp);                 %zeros
p = pole(gp);                 %poles

if length(gp.den{1}) > 2
    disp('model order to high')
    return
elseif isempty(z) == 0
    disp('model cannot contain zeros')
    return
end

K = t/(g*(tc+d));
I = K/min(t,4*(tc+d));

tuning = [K I]';
[gcPID gc] = controller(K,I);
return
```

A.2.4 SIMC PID-tunings

```
% function for simc PID tuning
% input: 2nd order model + tuning parameter
% returns: simc tuning paramer for a PID controller on the form
% gc = K + I/s + D*s
% Written by: Martin S. Foss, fall 2012

function [tuning, gc] = simcPID(gp,tc)

%Determining the model
d = totaldelay(gp);           %time delay
g = gp.num{1}(end);           %gain
z = zero(gp);                 %zeros
p = pole(gp);                 %poles
t1 = abs(p(2));               %time constant 1 (largest)
t2 = abs(p(2)/p(1));          %time constant 2 (smallest)

if nargin == 1
    tc = d;
```

```
end

if length(gp.den{1}) > 3
    disp('model order to high')
    return
elseif isempty(z) == 0
    disp('model cannot contain zeros')
    return
end

%SIMC-tuning (series)
Kc = 1/g*(t1/(tc+d));
taui = min(t1,4*(tc+d));
taud = t2;

%SIMC-tuning (parallel)
K_merk = Kc*(1+taud/taui);
I_merk = taui*(1+taud/taui);
D_merk = taud/(1+taud/taui);

%SIMC-tuning (given parameterization)
K = K_merk;
I = K_merk/I_merk;
D = K_merk*D_merk;

tuning = [K I D]';
gc = controller(K,I,D);
return
```

A.3 Obtain step responses

A.3.1 Main file

```
% Script evaluating step response for PID controller using both PO and SIMC
% controller tunings
% Written by: Martin S. Foss, fall 2012

clear all
% close all
clc

%Adding "sharedFiles" to MatLab search directory
curDir = pwd;
mainDir = fileparts(curDir);
sharedDir = fullfile(mainDir,'sharedFiles');
addpath(sharedDir);

% for m = 1:8
modelId = 9; %m;
modelName = {'case1','case2','case3','case4','case5','case6',...
             'case7','case8','case9','case10'};
msSet = 1.7;    %define the Ms value

load(fullfile(mainDir,'dataFiles',['resSimc_',...
              num2str(modelName{modelId}),'.mat'])) %loading datafiles

%Collecting PO and SIMC tunings
index = find(res.POpid.minTuning.ms == msSet);
tuning{1} = res.POpid.minTuning.tuning(:,index(1));

index2 = find(res.POpi.minTuning.ms == msSet);
tuning{2} = res.POpi.minTuning.tuning(:,index2(1));

index3 = find(res.simcPID.simcMs <= msSet);
tuning{3} = res.simcPID.simcTuning(:,index3(1));

index4 = find(res.simcPI.simcMs <= msSet);
```

```
tuning{4} = res.simcPI.simcTuning(:,index4(1));

gp = model(modelId);          %loading the model

%Extracting parameters from the model
p = pole(gp);                 %poles
t1 = abs(p(2));               %time constant 1 (largest)
t2 = abs(p(2)/p(1));          %time constant 2 (smallest) = taud

%% Plotting PO (PID) and SIMC (PID)
ysTime = 1;           %define simaulation parameters
dTime = 20;
stopTime = 40;

figure(modelId)
clf
colorSet = colormap('lines');

Y = [];       %setting up result matrices
U = [];
T = [];

for i = [1 3]
    subplot(2,1,1)
    gc = controller(tuning{i}(1),tuning{i}(2),tuning{i}(3),t2);
    sim('simModel')           %running Simulink

    r = plot(t,y);            %plotting output vs. time
    set(r,'Color',colorSet(i,:),'LineWidth',1.5);
    axis([0 stopTime 0 2])
    hold on

    subplot(2,1,2)
    r = plot(t,u);            %plotting input vs. time
    set(r,'Color',colorSet(i,:),'LineWidth',1.5);
    axis([0 40 -.5 4])
    hold on

    Y{i} = y;                 %storing results
```

```
    U{i} = u;
    T{i} = t;
end

%% Plotting PO (PI) and SIMC (PI)
for k = [2 4]
    subplot(2,1,1)
    [a gc] = controller(tuning{k}(1),tuning{k}(2));
    sim('simModel');
    r = plot(t,y);          %plotting output vs. time
    set(r,'Color',colorSet(k,:),'LineWidth',1.5);

    subplot(2,1,2)
    r = plot(t,u);          %plotting inputs vs. time
    set(r,'Color',colorSet(k,:),'LineWidth',1.5);

    Y{k} = y;               %storing results
    U{k} = u;
    T{k} = t;
end

subplot(211)
r = plot(t,ys,'--k','linewidth',1.5);       %plotting setpoint
set(r,'Color',[0 0 0],'LineWidth',1.5,'LineStyle','--');

%% Printing info
caseInfo = {'$\frac{\tau_2}{\tau_1}=0.5$ $\frac{\tau_2}{\theta}=0.5$',...
            '$\frac{\tau_2}{\tau_1}=0.8$ $\frac{\tau_2}{\theta}=0.8$',...
            '$\frac{\tau_2}{\tau_1}=0.3$ $\frac{\tau_2}{\theta}=0.3$',...
            '$\frac{\tau_2}{\tau_1}=0.5$ $\frac{\tau_2}{\theta}=1.5$',...
            '$\frac{\tau_2}{\tau_1}=0.8$ $\frac{\tau_2}{\theta}=1.5$',...
            '$\frac{\tau_2}{\tau_1}=0.3$ $\frac{\tau_2}{\theta}=1.5$',...
            '$\frac{\tau_2}{\tau_1}=0.5$ $\frac{\tau_2}{\theta}=2.0$',...
            '$\frac{\tau_2}{\tau_1}=0.8$ $\frac{\tau_2}{\theta}=2.0$',...
            '$\frac{\tau_2}{\tau_1}=0.3$ $\frac{\tau_2}{\theta}=2.0$'};
curveInfo = {'PO (PID)','PO (PI)','SIMC (PID)','SIMC(PI)'};
msString={'$M_s$ $=$ $1.7$'};

stdFont = 16;
```

```
infoFontSize = 18;

subplot(211)
ylab = ylabel('Output, $y$');
set(ylab,'interpreter','latex','fontsize',infoFontSize)
set(gca,'fontsize',stdFont,'FontName','Times New Roman')

subplot(212)
ylab = ylabel('Input, $u$');
xlab = xlabel('Time, $t$');
set(xlab,'interpreter','latex','fontsize',infoFontSize)
set(ylab,'interpreter','latex','fontsize',infoFontSize)
set(gca,'fontsize',stdFont,'FontName','Times New Roman')

subplot(211)
[figx figy] = dsxy2figxy(gca,1.9,1.8); % (gca,3,3)
textBoxMs = annotation('textbox',[figx figy .07 .03],'string',...
            msString{1},'interpreter','latex','fontsize',stdFont,...
            'color',[0 0 0],'FitBoxToText','on','LineStyle','none');

[figx figy] = dsxy2figxy(gca,10,1.75); % (gca,3,3)
textBoxCaseInfo = annotation('textbox',[figx figy .07 .03],'string',...
                  caseInfo{modelId},'interpreter','latex','fontsize',...
                  stdFont,'color',[0 0 0],'FitBoxToText','off',...
                  'LineStyle','none');

[figx figy] = dsxy2figxy(gca,1.9,1.55); % (gca,3,3)
textBoxSetPoint = annotation('textbox',[figx, figy, .07 .03],'string',...
                  '{\bf-}{\bf-} setpoint','interpreter','latex',...
                  'fontsize',stdFont,'color',[0 0 0],'FitBoxToText',...
                  'on','LineStyle','none');

%PO (PID)
y1 = find(Y{1}>=0.8);
[figx figy] = dsxy2figxy(gca,T{1}(y1(1)),Y{1}(y1(1)));
curve1 = annotation('textarrow',[figx+0.05 figx],[figy figy],...
         'string',curveInfo{1},'interpreter','latex','fontsize',stdFont,...
         'color',colorSet(1,:));
```

```
%PO (PI)
y2 = find(Y{2}>=0.25);
[figx figy] = dsxy2figxy(gca,T{2}(y2(1)),Y{2}(y2(1)));
curve2 = annotation('textarrow',[figx+0.05 figx],[figy figy],...
         'string',curveInfo{2},'interpreter','latex','fontsize',stdFont,...
         'color',colorSet(2,:));

%SIMC (PID)
y3 = find(Y{3}>=0.5);
[figx figy] = dsxy2figxy(gca,T{3}(y3(1)),Y{3}(y3(1)));
curve3 = annotation('textarrow',[figx+0.05 figx],[figy figy],...
         'string',curveInfo{3},'interpreter','latex','fontsize',stdFont,...
         'color',colorSet(3,:));

%SIMC (PI)
[y4val y4ind] = max(Y{4}(1:150));
[figx figy] = dsxy2figxy(gca,T{4}(y4ind),Y{4}(y4ind));
curve4 = annotation('textarrow',[figx+0.05 figx],[figy+0.02 figy],...
         'string',curveInfo{4},'interpreter','latex','fontsize',stdFont,...
         'color',colorSet(4,:));

subplot(212)
%PO (PID)
if modelId == 7
    [u1val u1ind] = min(U{1});
    [figx figy] = dsxy2figxy(gca,T{1}(u1ind),U{1}(u1ind));
    curve1 = annotation('textarrow',[figx-0.05 figx],[figy figy],...
             'string',curveInfo{1},'interpreter','latex',...
             'fontsize',stdFont,'color',colorSet(1,:));
else
    u1 = find(U{1}(100:end)<=0.2);
    [figx figy] = dsxy2figxy(gca,T{1}(u1(1)+100),U{1}(u1(1)+100));
    curve1 = annotation('textarrow',[figx-0.05 figx],[figy figy],...
             'string',curveInfo{1},'interpreter','latex',...
             'fontsize',stdFont,'color',colorSet(1,:));
end

%PO (PI)
if modelId == 4 || modelId == 5 || modelId == 7 || modelId == 8 || modelId == 9
```

```
    [u2val u2ind] = max(U{2});
    [figx figy] = dsxy2figxy(gca,T{2}(u2ind),U{2}(u2ind));
    curve2 = annotation('textarrow',[figx+0.05 figx],[figy+0.03 figy],...
             'string',curveInfo{2},'interpreter','latex',...
             'fontsize',stdFont,'color',colorSet(2,:));
else
    u2 = find(U{2}>=0.70);
    [figx figy] = dsxy2figxy(gca,T{2}(u2(1)),U{2}(u2(1)));
    curve2 = annotation('textarrow',[figx+0.05 figx],[figy-0.02 figy],...
             'string',curveInfo{2},'interpreter','latex','fontsize',...
             stdFont,'color',colorSet(2,:));
end

%SIMC (PID)
u3 = find(U{3}(100:end)<=0.5);
[figx figy] = dsxy2figxy(gca,T{3}(u3(1)+100),U{3}(u3(1)+100));
curve3 = annotation('textarrow',[figx+0.05 figx],[figy+0.02 figy],...
         'string',curveInfo{3},'interpreter','latex',...
         'fontsize',stdFont,'color',colorSet(3,:));

%SIMC (PI)
[u4maxVal u4maxInd] = max(U{4}(1:150));
u4 = find(U{4}(u4maxInd:end)<=1.12);
[figx figy] = dsxy2figxy(gca,T{4}(u4(1)+u4maxInd),U{4}(u4(1)+u4maxInd));
curve4 = annotation('textarrow',[figx+0.05 figx],[figy+0.02 figy],...
         'string',curveInfo{4},'interpreter','latex','fontsize',stdFont,...
         'color',colorSet(4,:));

%% Saving figures
pause
saveas(r,['stepFigures\','stepResponse_',...
          num2str(modelName{modelId}),'.eps'],'psc2')
saveas(r,['stepFigures\','stepResponse_',...
          num2str(modelName{modelId}),'.fig'])
% end

restoredefaultpath
```

A.4 Obtain parallel tuning parameters

A.4.1 Main file

```
% Script calculating tuning parameters (parallel form) for PO(PID),
% PO(PI), SIMC-PID and SIMC-PI
% Written by: Martin S. Foss, fall 2012

% clc
clear all

%Adding "sharedFiles" to MatLab search directory
curDir = pwd;
mainDir = fileparts(curDir);
sharedDir = fullfile(mainDir,'sharedFiles');
addpath(sharedDir);

%%
% for m = 1:9
modelId = 9; %m

    fprintf('Finding tuningparamaters (parallel form) \n')
    fprintf('Case: %g\n',modelId)
    fprintf('***************************\n')
    fprintf('\n')

msSet = 1.7;        %define Ms value

modelName = {'case1','case2','case3',...
             'case4','case5','case6',...
             'case7','case8','case9'};

load(fullfile(mainDir,'dataFiles',['resSimc_',...
                num2str(modelName{modelId}),'.mat'])) %loading datafiles

%Collecting PO and SIMC tunings
index = find(res.POpid.minTuning.ms == msSet);
tuning{1} = res.POpid.minTuning.tuning(:,index(1));
```

```
index2 = find(res.POpi.minTuning.ms == msSet);
tuning{2} = res.POpi.minTuning.tuning(:,index2(1));

index3 = find(res.simcPID.simcMs <= msSet);
tuning{3} = res.simcPID.simcTuning(:,index3(1));

index4 = find(res.simcPI.simcMs <= msSet);
tuning{4} = res.simcPI.simcTuning(:,index4(1));

x = [1 2 3 4];

tuningParallel = [];          %setting up result matrix
for i=1:length(tuning)
    Kc = tuning{i}(1);        %finding Kc
    taui = Kc/tuning{i}(2);   %calculating tau_i

    if i == 2 || i == 4       %calculating tau_d
        taud = NaN;
    else
        taud = tuning{i}(3)/Kc;
    end

    tuningParallel(:,i) = [Kc; taui; taud];     %storing results
end

res.tuningPara = tuningParallel;

%% Saving results
save([mainDir,'\dataFiles\','resTunPar_',num2str(modelName{modelId}),...
        '.mat'],'res');
save(['tuningResults\','resTunPar_',num2str(modelName{modelId}),...
        '.mat'],'res');
% end

restoredefaultpath
```

A.5 Shared files

The following m.files are used of several of the other files.

A.5.1 Cases - second order models

```
% Function calculating second order transfer functions for the different
% cases
% Written by: Martin S. Foss, fall 2012
function gp = model(caseNumber)

s=tf('s');
tau1 = 1;

switch caseNumber
    case 1
        tau2 = 0.5*tau1;
        theta = 1;

    case 2
        tau2 = 0.8*tau1;
        theta = 1;

    case 3
        tau2 = 0.3*tau1;
        theta = 1;

    case 4
        tau2 = 0.5*tau1;
        theta = tau2/1.5;

    case 5
        tau2 = 0.8*tau1;
        theta = tau2/1.5;

    case 6
        tau2 = 0.3*tau1;
```

```
            theta = tau2/1.5;

        case 7
            tau2 = 0.5*tau1;
            theta = tau2/2;

        case 8
            tau2 = 0.8*tau1;
            theta = tau2/2;

        case 9
            tau2 = 0.3*tau1;
            theta = tau2/2;
end

gp = 1/((tau1*s+1)*(tau2*s+1));
gp.outputd = theta;
```

A.5.2 Cases - first order models

```
% Function calculating first order transfer functions, using the half rule,
% for the different cases
% Written by: Martin S. Foss, fall 2012
function gp = modelPI(caseNumber)

s=tf('s');
tau1 = 1;

switch caseNumber
    case 1
        tau2 = 0.5*tau1;
        theta = 1;

    case 2
        tau2 = 0.8*tau1;
        theta = 1;

    case 3
```

```
        tau2 = 0.3*tau1;
        theta = 1;

    case 4
        tau2 = 0.5*tau1;
        theta = tau2/1.5;

    case 5
        tau2 = 0.8*tau1;
        theta = tau2/1.5;

    case 6
        tau2 = 0.3*tau1;
        theta = tau2/1.5;

    case 7
        tau2 = 0.5*tau1;
        theta = tau2/2;

    case 8
        tau2 = 0.8*tau1;
        theta = tau2/2;

    case 9
        tau2 = 0.3*tau1;
        theta = tau2/2;
end

Tau1 = tau1 + 0.5*tau2;
Theta = theta + 0.5*tau2;

gp = 1/(Tau1*s+1);
gp.outputd = Theta;
```

A.5.3 Controller

```
% function for generating the controller transferfunction
% on the form gc = K + I/s
```

```
% Written by: Martin S. Foss, fall 2012

function [gc gcPI varargout] = controller(K,I,D,taud,varargin)

s = tf('s');

if nargin == 3
    gc = K + I/s + D*s;      %PID Controller
elseif nargin == 2
    gcPI = K + I/s;          %PI controller
    gc = NaN;
else
    gc = K + I/s + (D*s)/(0.01*taud*s+1);     %PID controller with
                                              %derivative filter
end
return
```

A.5.4 Integral absolute error

```
function iae = iae(t,y)
% This is the function iae.m
% Simple integration routine made for computing IAE of time signal using
% trapez integration
% y - time signal vector
% t - time signal vector

% Initialize
i = 1;
npoints = length(t);
iae = zeros(1,size(y,2));

% Integrate
while i  < npoints,
  yavg = (y(i,:) + y(i+1,:))/2;
  dt   = t(i+1) - t(i);
  int = abs(yavg)*dt;
  iae = iae + int;
  i = i + 1;
```

```
end

iae=iae';
return
```

A.5.5 Ms

```
% Function for calculating the Ms value
% written by: Chriss Grimholt 11 jan. 2012

function ms = ms(gp,gc)

ms = max(abs(freqresp(feedback(1,gp*gc),logspace(-4,4,40000))));

return
```

B SIMULINK Model

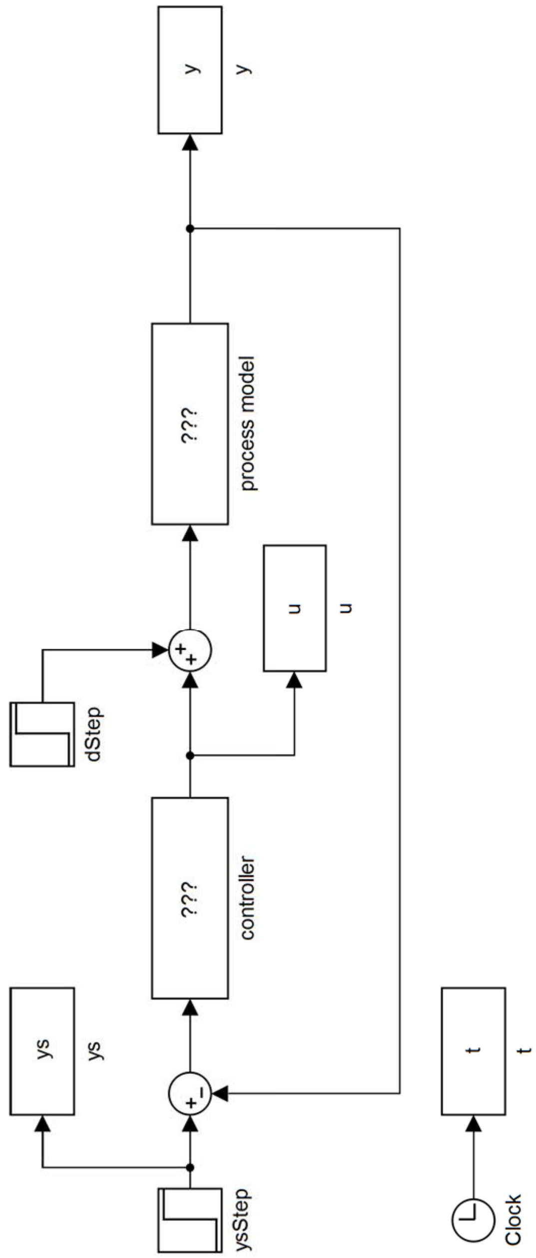


Figure B.1: SIMULINK model