

FINAL YEAR PROJECT REPORT

FOR

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Title: "Control and modelling of Kaibel distillation column"

This project work is to be carried out at the Department of Engineering Cybernetics under guidance of Professor Ole Morten Aamo as supervisor and Dr. Ivar Halvorsen and Professor Sigurd Skogestad as co-supervisors.

Trondheim, August 25, 2008.

Problem description

Title: Control and modelling of Kaibel distillation column

The project shall be based on an available model by Jens Strandberg and the laboratory column at Department of Chemical Engineering, NTNU.

Modelling:

- Extend the model to include heat loss on all stages
- Discuss briefly the issue of model parameter adjustment to fit the model to data from the laboratory column

Control:

- Linearization, linear model reduction and interaction analysis
- Decentralized control and tuning (SIMC)
- Multivariable control (MPC)

Assignment given: August 25, 2008

Supervisor: Professor Ole Morten Aamo (Department of Engineering Cybernetics)

Co-supervisors: Dr. Ivar Halvorsen (SINTEF ICT) and Professor Sigurd Skogestad (Department of Chemical Engineering)

PREFACE

This is the report for my final year project work at Norwegian University of Science and Technology (NTNU). The work is carried out at Department of Engineering Cybernetics. The project is a part of my master's degree in Engineering Cybernetics at NTNU. The assignment is given by the Department of Engineering Cybernetics in cooperation with SINTEF.

Ivar Halvorsen at SINTEF has been very helpful during the project work, giving a lot of useful tips in many different areas. I am grateful to the guys at the office in the Electro building, Bjarne, Jørgen and Morten, for exchanging useful ideas with each other.

Martin Kvernland
December 2008

Abstract

In this project assignment control and modelling of a Kaibel distillation column is taken into a closer look. The main motivation is the potential energy savings that can be achieved, but this requires good control of the column in order to achieve acceptable product purities at the same time. It is important to model the distillation column proper, such that computer simulations of the column can give results that are close to a real column.

The Kaibel distillation column is modelled in MATLAB / Simulink based on differential equations for composition, temperature and mass.

The distillation model is extended to include heat loss, which affect the product purities. A short study on the issue of parameter identification has been done. The identification problem does not have a straight forward solution, but a proposed simplification is to use a vapour bypass model together with a fixed number of stages in the column. The identification itself is not done in this work.

In the control area, some interaction analysis has been done. This analysis shows that the column can be controlled by a decentralized control strategy since no heavy interactions are present. Decentralized control is simpler and gives good results if the losses are acceptable. Multivariable control is implemented with disturbance feed-forward and thus, it gives better disturbance rejection than decentralized control. The implementation of a multivariable controller generates issues like linearization, model reduction, disturbance modelling and observability.

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INTRODUCTION

Distillation is widely used in the process industry all over the world to separate liquid mixtures. It is a highly energy-consuming technique and this give extra motivation for finding less energy-consuming methods that can be applied. According to Ognisty (1995) distillation columns alone consume around 3 % of all energy consumed in the US. This give a huge potential for energy saving. This is the background for the *Kaibel distillation column*, introduced in 1987 by Gerd Kaibel (Kaibel, 1987).

A theoretical example in Halvorsen & Skogestad (2006) shows an energy reduction of 33 % for a Kaibel distillation column compared to a conventional column setup. Even 50 % energy reduction can be achieved in special cases. These savings are not achieved without *good control* of the column, which is the main motivation for this project assignment.

The project report should give a good understanding of the distillation model and how it can be controlled to obtain high product purities. The model is based on previous work done by PhD student Jens Strandberg at Department of Chemical Engineering, NTNU. Jens Strandberg has also been involved in building a pilot plant of the Kaibel distillation column at Department of Chemical Engineering, which can be used to compare with a computer model. The given assignment was to have a look at the following points:

- Extend existing model to include heat loss
- Linearization and model reduction
- Briefly discussion on the issue of model parameter identification
- Interaction analysis
- Decentralized control and tuning
- Multivariabel control

Some theory on distillation is presented in the start to give a basic understanding of the physics involved. Further, the points listed above are presented in the same order. The control part has its main focus on stabilizing control, i.e. maintaining controlled variables at their respective setpoints.

DISTILLATION

1.1 The physics of distillation

Distillation is a physical process that separates boiling liquid mixtures based on the difference in volatility. Volatility is a measure of the tendency of a substance to vaporize (Chang, 2006). Figure 1.1 shows a distillation column with feed input and top and bottom product, D and B respectively. A column have two heat exchangers, one *reboiler* at the bottom that generates vapour V going upwards. The other one is a *condenser* at the top that cools vapour and generate top product and reflux L . Reflux is condensed liquid that goes back to the column in the top, and make the top product more pure.

The equilibrium-stage concept

A distillation column is vertically divided in a number of stages, and at each stage it is assumed vapour-liquid equilibrium (VLE). The general VLE relation is given in Equation (1.1). N_c is the number of components to be separated in the column. $y_1, y_2, \dots, y_{N_c-1}$ are mole fractions in the vapour phase for $N_c - 1$ components, similar for the liquid phase; $x_1, x_2, \dots, x_{N_c-1}$. T and P denotes temperature and pressure respectively (Halvorsen, 2001).

$$[y_1, y_2, \dots, y_{N_c-1}, T] = f(P, x_1, x_2, \dots, x_{N_c-1}) \quad (1.1)$$

The concept of equilibrium-stage is shown in Figure 1.2.

Material balance on a stage

It is easy to set up a material (mass) balance for each stage in a distillation column. Let $N_{i,k}$ denote the number of moles of component i on stage k , L_k denotes liquid molar flow from stage k to $k - 1$ and V_k is the vapour molar flow from stage k to $k + 1$. This is illustrated in Figure 1.3. The material balance will then be

$$\frac{dN_{i,k}}{dt} = (L_{k+1}x_{i,k+1} - V_k y_{i,k}) - (L_k x_{i,k} - V_{k-1} y_{i,k-1}) \quad (1.2)$$

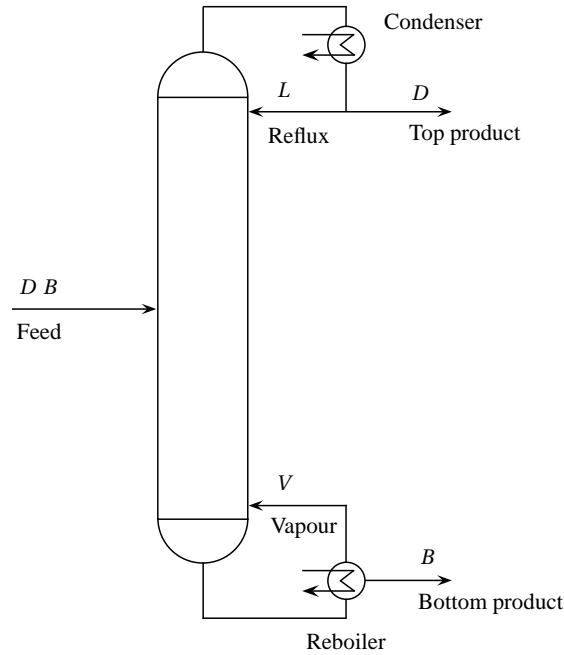


Figure 1.1: Distillation column

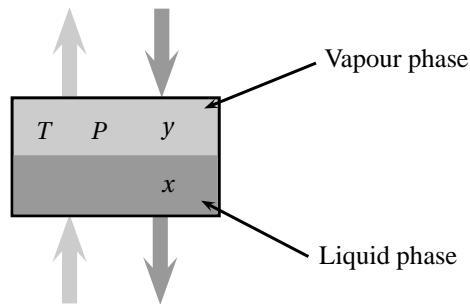


Figure 1.2: Equilibrium-stage concept

The number of moles of a component on a stage is equal to $M_k x_{i,k}$, where M_k is the total number of moles at stage k . Using this relation a differential equation for the liquid mole fraction can be derived from Equation (1.2)

$$\frac{dN_{i,k}}{dt} = \frac{d(M_k x_{i,k})}{dt} = M_k \frac{dx_{i,k}}{dt} + \frac{dM_k}{dt} x_{i,k} \quad (1.3)$$

The time derivative of M_k is

$$\frac{dM_k}{dt} = L_{k+1} - L_k + V_{k-1} - V_k \quad (1.4)$$

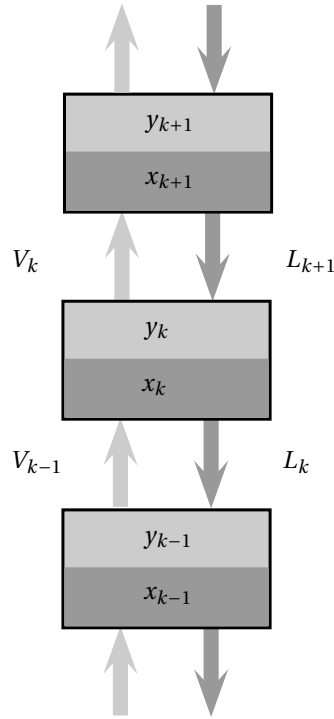


Figure 1.3: Three stages of a distillation column

The time derivative of $x_{i,k}$ can then be found

$$\frac{dx_{i,k}}{dt} = \frac{1}{M_k} \left(\frac{dN_{i,k}}{dt} - \frac{dM_k}{dt} x_{i,k} \right) \quad (1.5)$$

$$= \frac{1}{M_k} (L_{k+1}(x_{i,k+1} - x_{i,k}) - V_k(y_{i,k} - x_{i,k}) + V_{k-1}(y_{i,k-1} - x_{i,k})) \quad (1.6)$$

Stage efficiency

Murphree efficiency is a measure of the efficiency of each stage. The mathematical definition is (Murphree, 1925)

$$\eta_M = \frac{y_k - y_{k-1}}{y_{k,\text{eq}} - y_{k-1}} \quad (1.7)$$

where $y_{k,\text{eq}}$ is the vapour in equilibrium with liquid at stage k . An expression for y_k can then be obtained

$$y_k = y_{k-1} + \eta_M (y_{k,\text{eq}} - y_{k-1}) \quad (1.8)$$

Equation (1.8) show that the vapour mole fraction will increase upwards in the column, when $y_{k,\text{eq}} > y_{k-1}$. An increasing number of stages in a column will therefore result in better product purities.

Equations used in distillation

For ideal mixtures *Raoult's law* (1.9) can be used, where the partial pressure p_i of component i equals the product of the liquid mole fraction x_i and vapour pressure $p_i^0(T)$.

$$p_i = x_i p_i^0(T) \quad (1.9)$$

To determine the vapour pressure $p_i^0(T)$ Equation (1.10) is common used, where the constants a , b , c , d , e and f are found in chemical handbooks.

$$\ln p^0(T) \approx a + \frac{b}{c + T} + d \ln(T) + eT^f \quad (1.10)$$

$d = e = 0$ gives the *Antoine equation*. *Dalton's law* gives the relation between the partial pressure p_i and vapour mole fraction y_i in Equation (1.11).

$$p_i = y_i P \quad (1.11)$$

P denotes the total pressure in the vapour phase, the same as in Figure 1.2 (Halvorsen, 2001).

1.2 The Kaibel distillation column

The Kaibel distillation column was introduced in 1987 (Kaibel, 1987). Normally a distillation column separates the input mixture (feed) into two products, which is a binary distillation column (See Figure 1.1). Generally if you want to separate a mixture into n products, you will need $n - 1$ binary columns. This requires $n - 1$ reboilers. The Kaibel column can separate the feed into four products in the same column shell, using only a single reboiler. These are the main reasons for using a Kaibel column, it require less investment capital and it is more energy efficient. An extra advantage is that the space required in a process plant is less for a Kaibel column than for conventional binary columns.

To obtain good product specifications; high product purities, the Kaibel column require good control. Figure 1.4 show a Kaibel column with the four product streams D , S_1 , S_2 and B and where the input feed is a mixture of the four products. D is the lightest product and B is the heaviest. The left part of the column is the *prefractionator* and the right is the *main column* where the products are drained. Another version of the Kaibel column is shown in Figure 1.5. This *dividing wall column* is built only of a single column shell. Figure 1.6 show a Kaibel column similar to the laboratory column at the Department of Chemical Engineering, mentioned in the introduction. This column is built to do more research on the Kaibel column because of its potential energy savings.

The four products D , S_1 , S_2 and B and the purities x_D , x_{S_1} , x_{S_2} and x_B respectively. F denotes feed stream in at the middle of the column. The feed has a mole fraction z_F and a liquid fraction q . When the liquid fraction equals 1, all feed is liquid. The feed mole fraction z_F is a vector for three of the four components in the feed, thus

$$z_F = [z_D \quad z_{S_1} \quad z_{S_2}]^T \quad (1.12)$$

Since the mole fractions always sum to 1, it is only needed to include three of them in the model.

The column has valves for controlling reflux and vapour to each middle part of the column. The liquid split, R_L , controls the reflux and the vapour split, R_V , controls vapour.

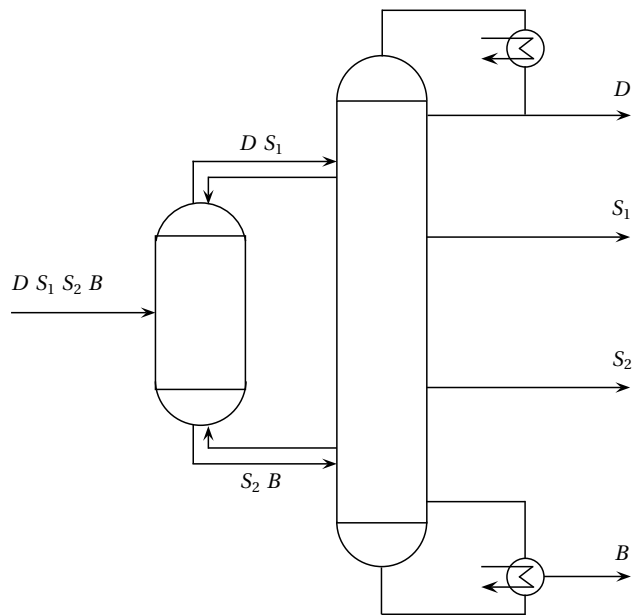


Figure 1.4: The Kaibel column, prefractionator to the left

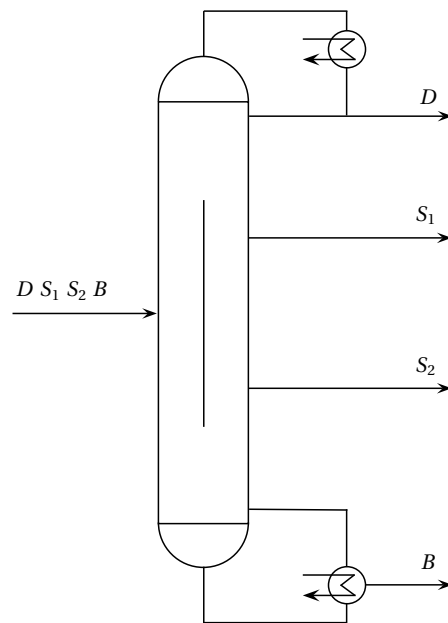


Figure 1.5: Dividing wall Kaibel column

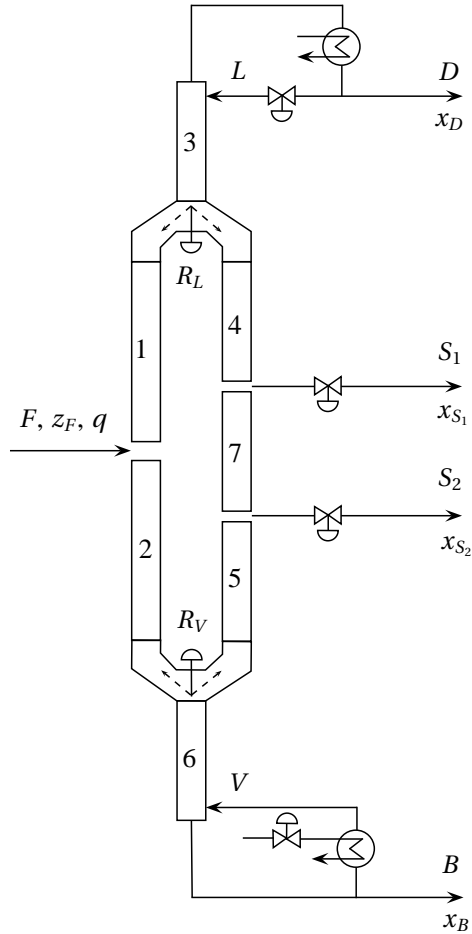


Figure 1.6: The Kaibel column at Department of Chemical Engineering, NTNU

Laboratory arrangement

Figure 1.6 show that the column consists of seven sections (numbered). Each section consists of a number of stages as discussed in Section 1.1. The laboratory column uses the alcohols methanol, ethanol, propanol and butanol as feed. The lightest product is methanol and is therefore the top product denoted D . The heaviest alcohol used is butanol, thus the bottom product, B . Side stream 1 (S_1) is ethanol and side stream 2 (S_2) is propanol.

The column has 7 degrees of freedom. These degrees of freedom are the four output product streams, reflux and vapour and the liquid split R_L . The laboratory column have 24 temperature sensors that can be used by controllers (Strandberg & Skogestad, 2006). Temperature measurements are used in distillation because it gives a good indication of the product composition.

The condenser has an open vent, i.e. pressure inside the column will always go to the atmospheric pressure (Strandberg & Skogestad, 2006).

MODELLING

2.1 Model

The Kaibel column from Figure 1.6 is implemented in MATLAB / Simulink by Jens Strandberg at Department of Chemical Engineering and is further developed in this project work. The CD attached to this report (Appendix B) includes MATLAB and Simulink files that are used in the work. The column model have 13 inputs, shown in Table 2.1. The table also show values for nominal operating points when totally 64 stages in the distillation column are used. Some of the inputs from Table 2.1 can not be controlled and must be considered as disturbances in the model. These inputs are the the feed stream, feed mole fractions and liquid fraction in the feed stream. One of the main challenges in control of the Kaibel column is to control the vapour split, R_V . This split is supposed to control the amount of vapour that should go into the main column and the prefractionator. This has shown to be very

Variable	Explanation	Nominal value	Unit
L	Reflux	2.8492	mol/min
V	Vapour boilup	3.0000	mol/min
S_1	Sidestream 1	0.2494	mol/min
S_2	Sidestream 2	0.2497	mol/min
R_L	Liquid split	0.2572	<ratio>
R_V	Vapour split	0.3770	<ratio>
F	Feed stream	1.0000	mol/min
z_D	Mole fraction of D in feed stream	0.2500	<ratio>
z_{S_1}	Mole fraction of S_1 in feed stream	0.2500	<ratio>
z_{S_2}	Mole fraction of S_2 in feed stream	0.2500	<ratio>
q	Liquid fraction of feed stream	0.9000	<ratio>
D	Top product	0.2508	mol/min
B	Bottom product	0.2503	mol/min

Table 2.1: Inputs for column model, see Figure 1.6 for further explanation

difficult to do in practice, therefore R_V is also considered as a disturbance in this assignment. Vectors for controllable inputs, u , and disturbances, d , are then

$$u = [R_L \quad L \quad V \quad S_1 \quad S_2 \quad D \quad B]^T \quad (2.1a)$$

$$d = [R_V \quad F \quad z_D \quad z_{S_1} \quad z_{S_2} \quad q]^T \quad (2.1b)$$

As mentioned above, the nominal operating points given in Table 2.1 is when 64 stages are used as the total number of stages in the column. Column section 1 and 2 from Figure 1.6 are longer than the other sections and have some more stages in the model. These sections are modelled with 12 stages and the other ones have 8, which give totally 64 stages. Notice that these numbers can be adjusted in the model.

All stages are numbered in the same order as the sections, i.e. T_1 is the temperature in the top stage of section 1. If section 1 have 12 stages, T_{12} will be the temperature in the bottom stage of section 1 and next; T_{13} will be the temperature in the top stage of section 2.

2.2 Existing model

A material balance is set up for each stage in the column. To find the vapour composition used in the material balance, the Antoine equation from Section 1.1 is used. This equation computes the vapour pressure and then an extended version of Raoult's law (2.2) is used to find mole fractions in vapour phase.

$$P y_i = x_i \gamma_i p_i^0(T) \quad (2.2)$$

γ_i is the activity coefficient, to find it *Wilson's model*¹ is used. Wilson's model is a nonlinear model. When vapour mole fraction is computed the whole material balance can be computed since liquid and vapour flows and liquid mole fractions are known from initial conditions.

Equation (2.2) can be summed over for all components and the pressure can be computed (the sum of all mole fractions is 1).

$$P \sum_{i=1}^{N_c} y_i = \sum_{i=1}^{N_c} x_i \gamma_i p_i^0(T) \quad (2.3)$$

$$P(x, T) = \sum_{i=1}^{N_c} x_i \gamma_i p_i^0(T) \quad (2.4)$$

The pressure from Equation (2.4) can be different from the initial pressure in the column P_0 . The pressure in the vapour phase on a stage does not change while the column is in operation, therefore the real pressure on each stage equals P_0 (1 atm) all the time. Therefore we want to drive $P(x, T)$ from Equation (2.4) to the correct pressure P_0 . The pressure is changed by temperature, and this gives a simple update law for the temperature in Equation (2.5).

$$\frac{dT}{dt} = k(P_0 - P(x, T)) \quad (2.5)$$

¹For further information about Wilson's model, see Gmehling & Onken (1977)

Where k is a positive constant that makes the units on each side of the equation to be the same. In the implementation this constant equals one. It is important to notice that Equation (2.5) does not come from any physical laws, it is just a numerical update law that assures the pressure computed using Raoult's law and Wilson's model to converge to the correct pressure P_0 .

2.3 Extension with heat loss

One of the objectives of this project was to extend the model to include heat loss. Heat loss is caused by the temperature difference inside and outside the column wall and can make a considerable difference in the product purities. Heat loss can be found by applying *Newton's law of cooling*:

$$\frac{dQ}{dt} = UA(T - T_s) \quad (2.6)$$

where T_s is the temperature in the surroundings. For a distillation column the heat loss for stage k with temperature T_k can be written as

$$\Delta Q_{k,\text{loss}} = U_k A_k (T_k - T_s) \quad (2.7)$$

A_k is the surface area for the stage. If the column shell is circular with diameter d and the height of one stage is h , area equals πdh . U_k ($\text{W}/\text{m}^2\text{K}$) is the heat transfer coefficient.

Considering a single stage some of the vapour condense into liquid because of the heat loss. This is why heat loss should be included in the model; some reflux is added at each stage downwards the column. Taking heat loss into account the material balance introduced in Section 1.1 must be modified. Assume the extra reflux caused by the heat loss at each stage is ΔL_k , and that this extra reflux is condensed from ΔV_k . The new material balance will then be

$$\begin{aligned} \frac{dN_{i,k}}{dt} = & ((L_{k+1} + \Delta L_{k+1})x_{i,k+1} - (V_k - \Delta V_k)y_{i,k}) \\ & - ((L_k + \Delta L_k)x_{i,k} - (V_{k-1} - \Delta V_{k-1})y_{i,k-1}) \end{aligned} \quad (2.8)$$

Since L_k and V_k are molar flows, ΔV_k equals $-\Delta L_k$. To obtain how much of the vapour inside a stage that is condensed using the known heat loss, we need to use the molar enthalpy of vapourization, $\Delta_{vap}H(T)$. The molar enthalpy of vapourization is the enthalpy that is required to transfer 1 mol of liquid into vapour for a given temperature T (Aylward & Findlay, 2008). The relationship between heat loss and amount of condensed vapour is given in Equation (2.9).

$$\Delta V_k = \frac{\Delta Q_{k,\text{loss}}}{\Delta_{vap}H(T)} \quad (2.9)$$

$\Delta_{vap}H(T)$ is given in chemical handbooks² for each component.

Simulation with heat loss

Figure 2.1 shows the product compositions of a MATLAB simulation with heat loss included in the model. The simulation starts with nominal values for the original

²E.g. Aylward & Findlay (2008)

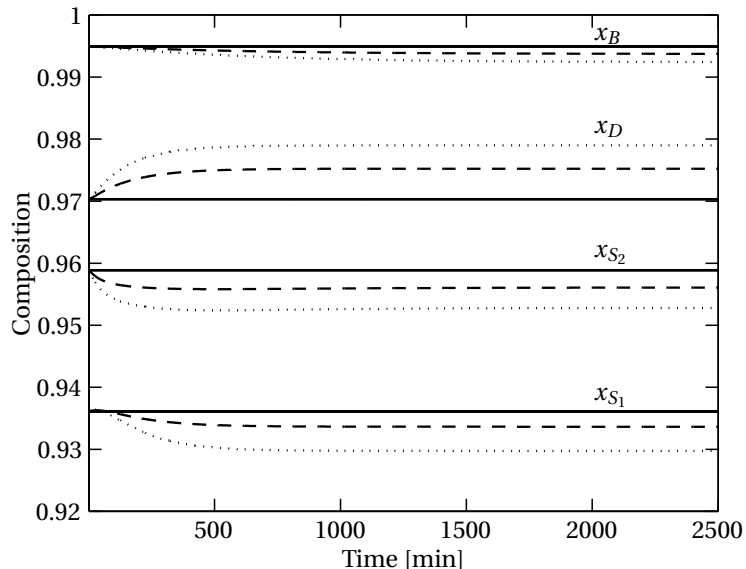


Figure 2.1: Effect of heat loss, — $U = 0\text{W/m}^2\text{K}$, - - - $U = 1.05\text{W/m}^2\text{K}$, ... $U = 2.10\text{W/m}^2\text{K}$

model. These nominal values are 0.9703, 0.9361, 0.9589 and 0.9949 for x_D , x_{S_1} , x_{S_2} and x_B respectively. The heat transfer coefficient was chosen for different values as shown in the figure. These values were based on heat transfer coefficient for glass, which is the construction material for the laboratory column at Department of Chemical Engineering. An average of molar enthalpy of vapourisation for the four components is used in the implementation, i.e.

$$\begin{aligned}\Delta_{vap}H_{used} &= \frac{\Delta_{vap}H_{meth} + \Delta_{vap}H_{eth} + \Delta_{vap}H_{prop} + \Delta_{vap}H_{but}}{4} \\ &= \frac{38 + 43 + 47 + 52}{4} \text{ kJ/mol} = \underline{45 \text{ kJ/mol}}\end{aligned}$$

The same heat transfer coefficient was used for all sections. The figure shows that less is vapourized when the column gets cooler and the top product get a higher mole fraction. The products beneath will get an extra amount of the products that are above and their compositions goes down. The overall composition will get worse when heat loss is included.

2.4 Adjustment of model parameters

An important thing is to compare laboratory tests at the real column at Department of Chemical Engineering with MATLAB simulations. Section 1.1 introduces the equilibrium-stage concept for distillation columns. Most of industrial distillation towers have such physical stages or trays, like Figure 2.2a. The Kaibel column at Department of Chemical Engineering is a packed column; it is filled with small glass rings (Raschig rings) or small cylinders to have much surface area (Strandberg & Skogestad, 2006). This concept is shown in Figure 2.2b. To simulate a packed column on a computer it is desirable to approximate it to an ordinary stage column in

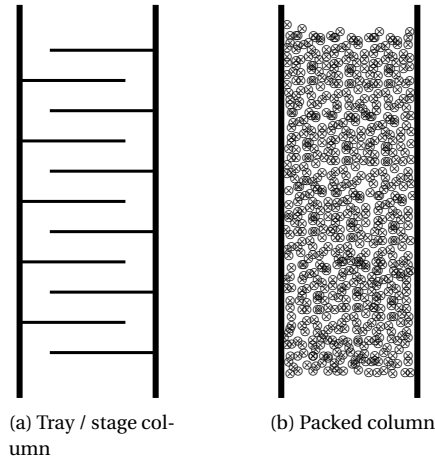
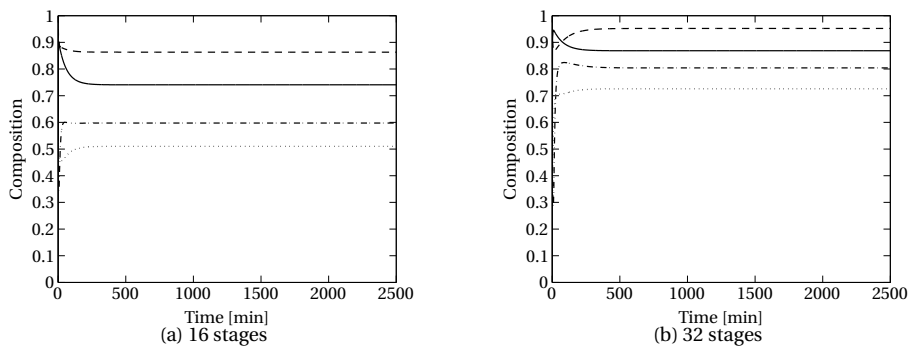


Figure 2.2: Different column content

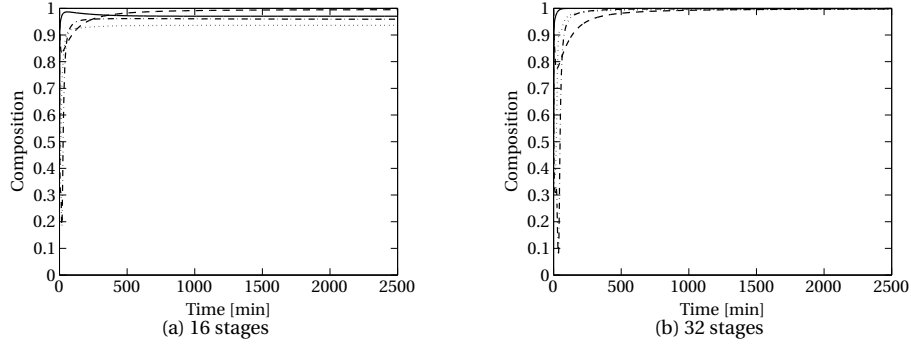
order to use the different equations from Chapter 1. Figure 2.3 and 2.4 show some plots from MATLAB simulations of the product compositions for different number of stages in the Kaibel model. The figures clearly show that a higher number of stages give higher purities, which corresponds to the Murphree efficiency introduced in Section 1.1. In the simulations initial purities were 0.25 on all stages, initial temperature was 340K. The simulations were performed without heat loss. Steady state values were achieved after some time. The final purities are shown in Table 2.2. Table 2.2 also show measured mole fractions from the laboratory column during steady state operation. These measurements are done by Jens Strandberg at Department of Chemical Engineering. The idea is to find the number of

Figure 2.3: Different number of stages, — x_D , \cdots x_{S_1} , \cdots x_{S_2} , $-\cdot-$ x_B

stages together with a heat transfer coefficient that make the laboratory test result fit the computer simulations as good as possible. The identification problem can be mathematically be written as a least square problem

$$\min_{\theta} (y - \hat{y}(\theta))^2 \quad (2.10)$$

where y is measured data from laboratory tests and $\hat{y}(\theta)$ is simulation data. θ is the parameters that is supposed to be adjusted. These parameters are number of

Figure 2.4: Different number of stages, — x_D , \cdots x_{S_1} , $-\cdot-$ x_{S_2} , $---$ x_B

	16 stages	32 stages	64 stages	128 stages	Lab. column
x_D	0.7408	0.8685	0.9703	0.9993	0.80 - 0.85
x_{S_1}	0.5102	0.7258	0.9361	0.9960	0.60 - 0.70
x_{S_2}	0.5975	0.8042	0.9591	0.9982	0.60 - 0.65
x_B	0.8633	0.9522	0.9947	0.9974	0.90 - 0.95

Table 2.2: Steady state purities for different number of stages

stages for the 7 sections and a heat transfer coefficient for the whole column, hence

$$\theta = [N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6 \ N_7 \ U]^\top \quad (2.11)$$

The N 's are integer variables and they also affect the number of states (See next section). Since Wilson's model is used the full distillation model becomes nonlinear and together with the integer number of stages, the identification problem will be hard to solve. Standard methods like least square estimation presented in Ljung (1999) can not be used here.

One approach to this problem is to use a brute force identification strategy to identify the best fitting integer parameters, and afterwards optimize this solution further by adjusting the heat transfer coefficient that is a continuously parameter. The problem with this solution is that the identification is divided into two parts. I.e. the model output can be a good fit to the real data, but it can return parameter values that are wrong. A brute force strategy will also be very time consuming when the simulations take long time to finish.

Another approach is to fix a number of stages in the column based on physical analysis, computer simulations or laboratory experiments and use a modified model for the vapour flow. This modified model is referred to as a *vapour bypass* model, where the vapour coming in at stage k for component i is written as

$$V_k y_{i,k} = (1 - \alpha) V_{k-1} y_{i,k-1} + \alpha V_{k-2} y_{i,k-2} \quad (2.12)$$

where α ($0 \leq \alpha \leq 1$) is a continuous parameter that decide how much vapour that is bypassed from the second stage below and how much that comes directly from the first stage below. When the number of stages is fixed, the only parameters that needs to be found are the bypass parameter α and the heat transfer coefficient U . Hence

$$\theta = [\alpha \ U]^\top \quad (2.13)$$

This bypassing of the vapour simplifies the identification to some extent when we are assuming a fixed number of stages, since it will be only two parameters that are needed to be found.

The identification itself is not done in this project since it requires a considerable amount of work. The laboratory column at Department of Chemical Engineering have not been operative during the project work, thus, no measurement data could have been collected. Therefore, a fixed number of stages are used in the simulations in the rest of this report. This number is 64 stages and is frequently used in earlier work done by Jens Strandberg.

2.5 State space model

Three differential equations give the state space model for the whole column. These are differential equations for composition, temperature and mass, given in Equations (1.2), (1.4) and (2.5) respectively. The nonlinear state space model can be written as:

$$\dot{x} = f(x, u, d) \quad (2.14a)$$

$$y = g(x, u, d) \quad (2.14b)$$

In the nonlinear system for the Kaibel column the inputs does not directly affect the measurements, i.e. $y = g(x, u, d) = h(x)$. This property is used in the rest of this assignment.

The nonlinear model from Equation (2.14) can easily be linearized in MATLAB by using the command `linmod`. This command compute a linear model from the nonlinear model around a specified operating point by doing small perturbations on the inputs. This gives

$$\frac{d}{dt}\Delta x = A\Delta x + B\Delta u + E\Delta d \quad (2.15a)$$

$$\Delta y = C\Delta x \quad (2.15b)$$

where the Δ -variables are deviations from nominal (steady state) values that are those points where the linearization were performed around. Linearization is important for development of a multivariable controller later in Chapter 3.

2.6 Model reduction

When simulating with 64 stages in the Kaibel column model, the number of states becomes 341. The model can be reduced quite a lot without affecting its dynamic behaviour for relevant frequencies. The motivation behind model reduction is to get a smaller optimization problem for a multivariable controller. A reduction from 341 to 10 states reduces the size of the Hessian matrix³ from the optimization problem with a factor of almost 20, which is essentially for the running time of the optimization solver. The singular values for the systems gives information about the gains of the plant (Skogestad & Postlethwaite, 2005). Therefore by plotting the singular values around closed loop frequency for different models gives a good indication of how good approximation the reduced models are. Figure 2.5 shows that a

³The Hessian matrix is what you are left with when you take the second-order partial derivative of a function f . In this context f is the objective function of the optimization problem. Hence, for a quadratic problem with objective function $f = \frac{1}{2}x^T Hx + f^T x$, the Hessian matrix is H

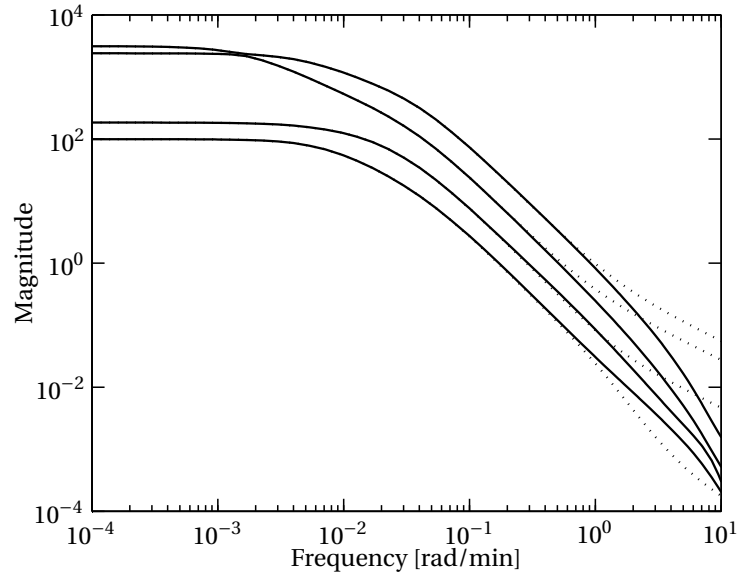


Figure 2.5: Singular values, — original linearized model, ··· reduced linear model (order = 10)

model with order 10 can be a good enough approximation for relevant frequencies. In MATLAB `balred` can be used to reduce models. The model was reduced using the *truncation* method. It consists of two steps (Green & Limebeer, 1994):

1. Transform the state-space model to

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (2.16)$$

where $x_2(t)$ are the states that is discarded. The corresponding *Hankel singular values*⁴ and the specified order of the reduced system decides which states that are discarded and not. When the states have changed their order, the state space matrices becomes

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} \quad (2.17a)$$

$$\bar{C} = [\bar{C}_1 \quad \bar{C}_2], \quad \bar{D} = D \quad (2.17b)$$

2. The reduced model is then given by

$$A_r = \bar{A}_{11}, \quad B_r = \bar{B}_1 \quad (2.18a)$$

$$C_r = \bar{C}_1, \quad D_r = \bar{D} \quad (2.18b)$$

⁴Hankel singular values is a measure of energy for each state in the system. It is closer described in Skogestad & Postlethwaite (2005)

CONTROL

As pointed out in the introduction, good control of the Kaibel column is essential to achieve the potential energy savings. It was also mentioned in the introduction that the control part of this assignment focuses on stabilization of the column. Previous work done in Strandberg & Skogestad (2006) uses this approach and is continued in this project.

3.1 Stabilization

The degrees of freedom are L , V , S_1 , S_2 , R_L , D and B as discussed in Chapter 2. To avoid saturation in the condenser and reboiler levels, control is needed for those. D is therefore used to control condenser level and B for reboiler level. These levels are controlled by PI-controllers and are assumed to be working. Since the pressure inside the column equals the atmospheric pressure (1 atm) it is not necessary to control the pressure. The vapour V is fixed to its nominal value like done in previous work (Strandberg & Skogestad, 2006). The degrees of freedom left to decentralized and multivariable control are then

$$u = [R_L \quad L \quad S_1 \quad S_2]^T \quad (3.1)$$

Column profile

In order to get high product purities it is necessary to stabilize the column profile¹ (Skogestad, 2007). In an ordinary distillation column condenser and reboiler levels and column pressure are first stabilized by some degrees of freedom and the rest are used for composition control, i.e. stabilization of the column profile. If the degrees of freedom that are left for composition control are reflux L and vapour V (most common), the column is said to have a LV -configuration. The degrees of freedom that are not used for level stabilization in this case are L , V , S_1 , S_2 and R_L . To stabilize the column profile, four of these have to be used (Strandberg & Skogestad,

¹The column profile for a product shows how the composition is for each stage in the column. It is desirable to have the product outlet where its composition profile has its maximum value.

2006). In Strandberg & Skogestad (2006), L , S_1 , S_2 and R_L are chosen to control the composition, since V is assumed to be fixed. The same manipulating variables are chosen in this project work, thus, the input vector is given by Equation (3.1).

Measurements

Temperature measurements are most used in distillation to control composition. An online *gas chromatograph* can be used to measure exact mole fraction of the products, but it is a very expensive instrument and the measurements will arrive to the controller with quite much delay. Temperature measurements are therefore taken along the column.

Self-optimizing control

The following definition is taken from Skogestad & Postlethwaite (2005)

Definition 1 *Self-optimizing control is when we can achieve an acceptable loss with constant setpoint values for the controlled variables without the need to reoptimize when disturbances occurs.*

In Strandberg & Skogestad (2006), self-optimizing control is used to control the Kaibel column using constant setpoints on some of the temperatures measured. These setpoints are found by minimization of an overall objective function that weighs the impurity flows:

$$J = D(1 - x_D) + S_1(1 - x_{S_1}) + S_2(1 - x_{S_2}) + B(1 - x_B) \quad (3.2)$$

This objective function is the loss as described in the definition above. By minimization of this objective function it is possible to find the values for the setpoints used for the self-optimizing control. Then, by holding the controlled variables at these setpoints by adjusting the degrees of freedom from Equation (3.1), the column should operate optimal. The controlled variables are found in earlier work (Strandberg & Skogestad, 2006) and are further used throughout this assignment:

$$y = [T_{17} \quad T_{30} \quad T_{59} \quad T_{49}]^T \quad (3.3)$$

The control configuration is shown in Figure 3.3.

3.2 Analysis of interaction

When controlling a system with multiple inputs and multiple outputs it is important to look closer to interactions in the system. E.g. what effect does side stream 1 have on the outputs that is supposed to control other inputs? From a physical point of view the Kaibel column must be an interactive process because of the different vapour and reflux connections that arise from the connection of the prefractionator and the main column. These interactions can give challenges in achieving good control of the column. A useful tool to analyze interactions is to use the *Relative Gain Array* (RGA).

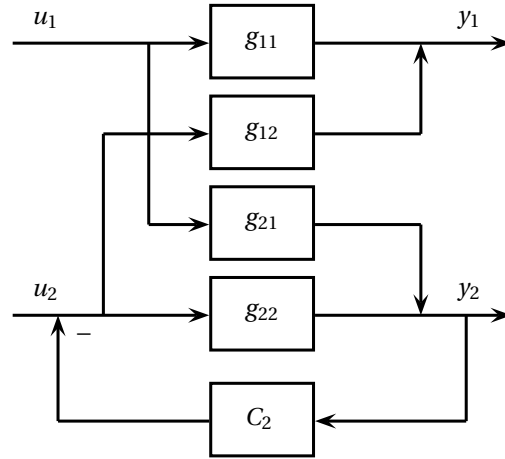


Figure 3.1: System with interaction

Relative Gain Array

Consider a 2×2 system shown in Figure 3.1. When u_2 equals zero, the open loop transfer function from u_1 to y_1 is

$$\left(\frac{y_1}{u_1} \right)_{\text{open loop, } u_2=0} = g_{11} \quad (3.4)$$

If y_2 is controlled "tight", it equals zero and u_2 can be computed

$$u_2 = -\frac{g_{21}}{g_{22}} u_1 \quad (3.5)$$

y_1 then becomes

$$y_1 = g_{11} u_1 + g_{12} u_2 = g_{11} \left(1 - \frac{g_{12} g_{21}}{g_{11} g_{22}} \right) u_1 \quad (3.6)$$

and the closed loop transfer function from u_1 to y_1 becomes

$$\left(\frac{y_1}{u_1} \right)_{\text{closed loop, } y_2=0} = g_{11} \left(1 - \frac{g_{12} g_{21}}{g_{11} g_{22}} \right) \quad (3.7)$$

The *relative gain* (Bristol, 1966) is defined as

$$\lambda_{11} = \frac{\left(\frac{y_1}{u_1} \right)_{\text{open loop, } u_2=0}}{\left(\frac{y_1}{u_1} \right)_{\text{closed loop, } y_2=0}} \quad (3.8)$$

It is desirable to have the relative gain close to 1 when u_1 is controlling y_1 . When it is close to 1 y_1 is not affected much by u_2 . A general definition of the RGA for a $n_y \times n_u$ system is

$$\begin{aligned} RGA(G) &= \Lambda(G) \equiv G \times (G^{-1})^T \\ &= \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1n_u} \\ \vdots & \ddots & \vdots \\ \lambda_{n_y 1} & \cdots & \lambda_{n_y n_u} \end{bmatrix} \end{aligned} \quad (3.9)$$

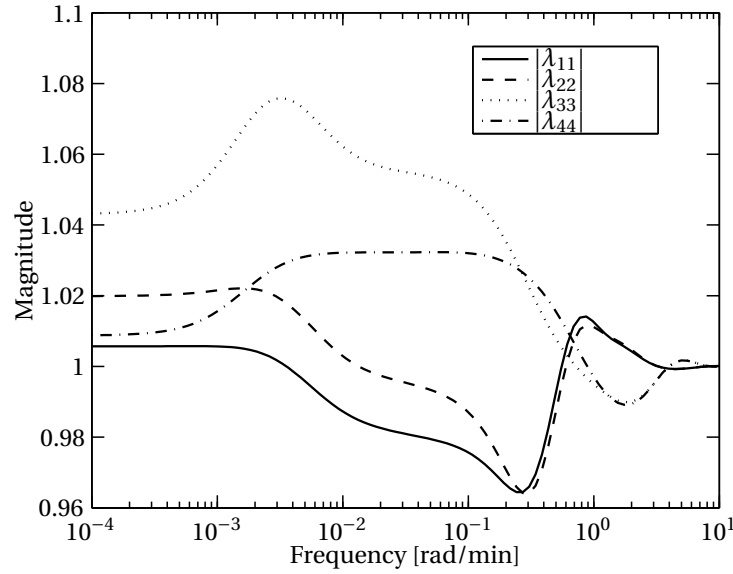


Figure 3.2: Magnitude of diagonal RGA-elements

where G is the transfer function matrix from the input to output. n_y is number of outputs and n_u is number of inputs. Skogestad & Postlethwaite (2005), states that inputs should control outputs where the corresponding RGA elements are close to 1 when using decentralized control.²

Interaction

Use of RGA as an analysis tool requires the model to be linearized, then for the Kaibel model, the linearized model given by Equation (2.15) must be used. The transfer function matrix G can easily be computed

$$G = C(sI - A)^{-1}B \quad (3.10)$$

The RGA is frequency dependent and the elements can be plotted as a function of frequency. Figure 3.2 shows the absolute value of relevant RGA-elements around closed loop frequencies for the control configuration showed in Figure 3.3. The RGA-plot in figure 3.2 is for the pairings³ done in Figure 3.3. They seem to be the correct ones since the corresponding RGA-elements are close to 1.

3.3 Decentralized control

Decentralized control is the simplest multivariable control that can be done. It requires the interactions to be small, i.e. diagonal elements of RGA-matrix to be close

²Decentralized control is when a plant $G(s)$ is controlled by a diagonal controller $K(s)$, i.e. the matrix $K(s)$ consists of only diagonal elements. The corresponding open loop transfer function matrix is $G(s)K(s)$ (Skogestad & Postlethwaite, 2005).

³Pairing tells which inputs that are chosen to control which measurements in a decentralized controller, i.e. each input is paired with a measurement

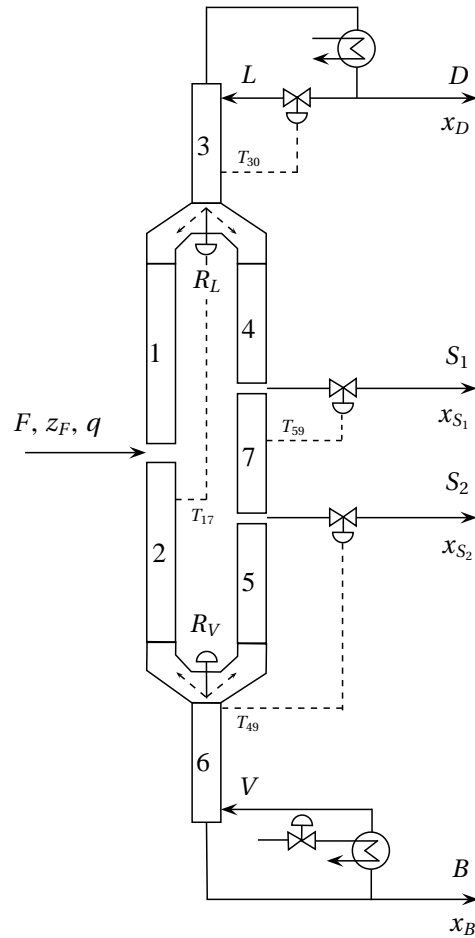


Figure 3.3: Example of decentralized control structure

to 1. Strandberg & Skogestad (2006), have proposed a decentralized control structure similar to the one in Figure 3.3. Thus, four PI-controllers have been used to control the composition in the column. These controllers need to be tuned. A simple and good way to find the controller parameters is to use the Simple / Skogestad internal model tuning (SIMC) rules from Skogestad (2003).

SIMC tuning rules

SIMC-PID tuning rules from Skogestad (2003) for first- and second-order processes are given in Table 3.1. k is process gain, θ denotes its delay and τ_1 and τ_2 are time constants. τ_c is the closed loop time constant, and is the only tuning parameter. Skogestad (2003), suggests $\tau_c = \theta$ to obtain a fast response with good robustness.

Tuning of decentralized controllers

Tuning of a decentralized controlled plant can be done in three different ways (Skogestad & Postlethwaite, 2005):

Process	$g(s)$	PID-gains		
		K_c	τ_I	τ_D
First-order	$k \frac{e^{-\theta s}}{(\tau_1 s + 1)}$	$\frac{1}{k} \frac{\tau_1}{\tau_c + \theta}$	$\min(\tau_1, 4(\tau_c + \theta))$	-
Second-order	$k \frac{e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{1}{k} \frac{\tau_1}{\tau_c + \theta}$	$\min(\tau_1, 4(\tau_c + \theta))$	τ_2

Table 3.1: SIMC-PID tuning rules

- Fully coordinated design
- Independent design
- Sequential design

A fully coordinated design requires the controllers to be designed simultaneously based on the complete model $G(s)$. This requires the full model, and a multivariable controller can be used instead. In the sequential design each controller is designed and putted in operation when the next one is designed. The simplest design method is to design each controller based on the corresponding element in $G(s)$, this is called independent design. In this assignment independent design is chosen.

SIMC tuning

From the RGA-analysis we know that the pairings are along the diagonal, i.e. u_1 controls y_1 , u_2 controls y_2 etc. This gives a controller matrix $K(s)$ that is diagonal. Each diagonal element $g_{ii}(s)$ in the transfer function matrix can be approximated by simple step-testing in the simulator. Table 3.2 shows transfer function coefficients for first order approximations and the corresponding controller gain and integral time. Closed loop time constant τ_c is chosen to be the same as time delay. Figure 3.4 show the response of a positive step on R_L and its approximation.

Step on variable	Δu	k	τ_1	θ	K_c	τ_I
R_L	+0.01	-694.6	37.2	20.7	-0.0013	37.2
	-0.01	-459.6	33.4	17.5	-0.0021	33.4
L	+0.01	-114.0	114.2	25.5	-0.0196	114.2
	-0.01	-178.9	186.2	25.5	-0.0205	186.2
S_1	+0.01	201.7	34.8	22.2	0.0039	34.8
	-0.01	145.6	38.8	17.3	0.0077	38.8
S_2	+0.01	694.7	121.8	48.8	0.0018	121.8
	-0.01	488.1	79.6	23.9	0.0077	179.6

Table 3.2: SIMC tuning rules applied on step test results

Tuning results

In the simulation an average of the different values for K_c and τ_I from positive and negative step were used. I.e. for the liquid split controller, controller gain was set to -0.0017 and integral time to 35.3. The figures 3.5 and 3.6 shows different disturbance responses for the 4 controlled temperatures before and after tuning. The

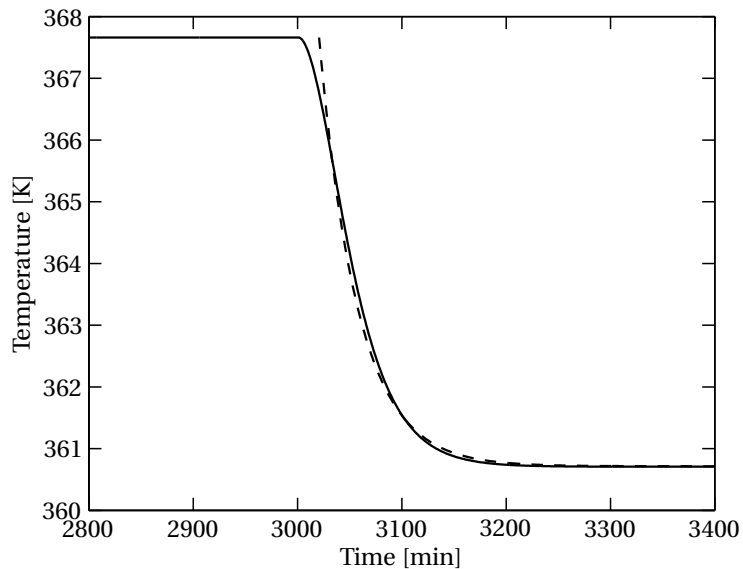


Figure 3.4: Response when stepping on R_L , — T_{17} , - - - first order approximation

Disturbance variable	Change from nominal value
Feed flow (F)	+ 10 %
Ethanol composition in feed (z_{S_1})	+ 20 %
Vapour split (R_V)	+ 10 % and + 50 %

Table 3.3: Disturbance tests

response for the original tunings from previous work are plotted to have something to compare the new tunings with. The control configuration is the same as shown in Figure 3.3. The different disturbances (shown in Table 3.3) tested are based on earlier work done by Jens Strandberg, which he have used for comparison between different control configurations. The figures show a good improvement in the control compared to old tuning parameters; less oscillatory behaviour and less settling time.

3.4 Multivariable control

The design of a model predictive controller (MPC) can be done in a "simple" way using software that imports the model with constraints and generate a controller to the user. Such software can be *MPC toolbox* for MATLAB or *Unisim Profit Design Studio* made by Honeywell. These software packages can be very useful for fast design of an MPC controller, but it does not give much insight in the interesting theory behind MPC. The author has therefore chosen to make the MPC controller "from scratch" to obtain more knowledge on MPC theory and meet the different issues that arises in the design of an MPC controller.

The self-optimizing control way of thinking is also used here, by using the MPC to control the four temperatures to their setpoints. The MPC could have been used to optimize the overall objective function given in Equation (3.2) and then give

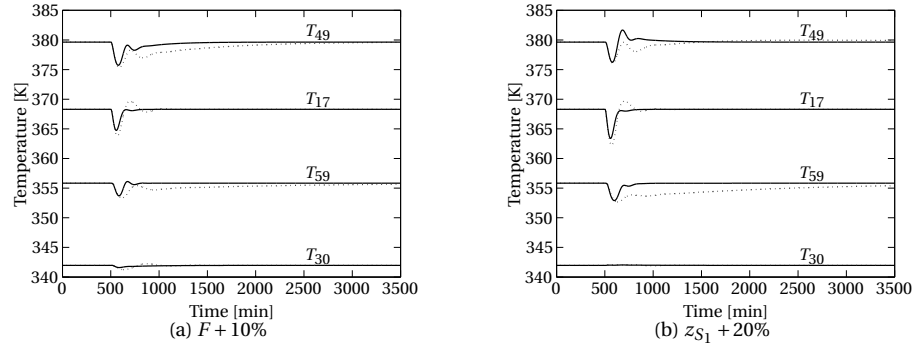


Figure 3.5: Disturbance in feed flow and feed composition, \cdots old tuning, $—$ new tuning

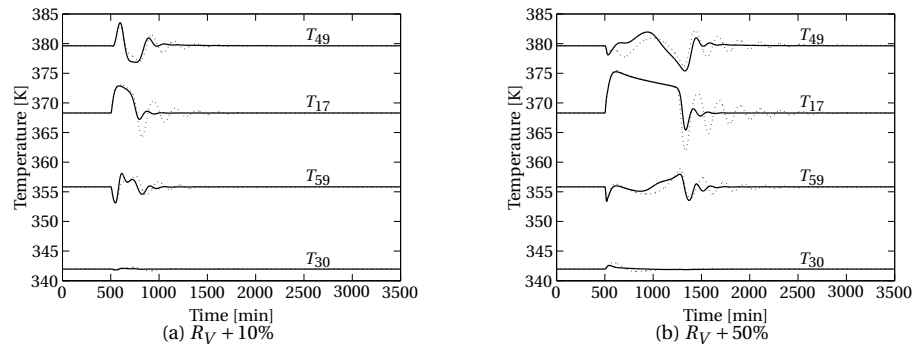


Figure 3.6: Disturbance in vapour split, \cdots old tuning, $—$ new tuning

setpoints to the original PI-controllers for optimal disturbance rejection. This approach would give a layered control structure (Skogestad & Postlethwaite, 2005) with the MPC on the top. The MPC made in this project does *not* use this structure, it minimizes the deviation from the given setpoints and optimizes the control inputs (u) directly.

MPC introduction

A model predictive controller is a controller which (Imslund, 2007)

1. Uses a multivariable process model to predict future behaviour
2. Optimizes predicted future performance with mathematical programming
3. Handles constraints on inputs and states

MPC is known as the advanced process control method in chemical process industry and has been used in chemical plants and oil refineries since the 1980s. The basic idea of predictive control is shown in Figure 3.7. The MPC optimizes at every time instant and calculates optimal inputs and states for H_p time samples into the future. H_p is called prediction horizon. To save computations when optimizing it is possible to have a separate prediction horizon for the inputs, H_u , that is less

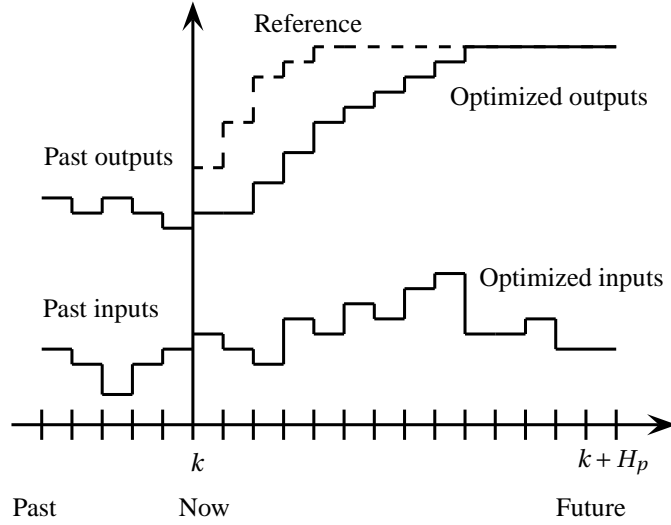


Figure 3.7: Basic idea of predictive control

than the state prediction horizon (Maciejowski, 2002). This approach is not taken in consideration here. The optimization problem done at each time step k in the MPC can be formulated as

$$\min \sum_{i=0}^{H_p-1} \left(\|x_{k+i} - x_{k+i}^{\text{ref}}\|_Q^2 + \|u_{k+i} - u_{k+i}^{\text{ref}}\|_R^2 \right) + \|x_{k+H_p} - x_{k+H_p}^{\text{ref}}\|_S^2 \quad (3.11)$$

subject to constraints on inputs and states

where Q , R and S are weight matrices. S is the terminal weight (Muske & Rawlings, 1993) for the state deviations after the prediction horizon and must be chosen sufficiently large. It can be found by solving a discrete Lyapunov equation (Hovd, 2008):

$$S - (A + BK)^T S (A + BK) = Q \quad (3.12)$$

where K is a the stabilizing feedback. K and S can be calculated in MATLAB using `dlqr`.

Another optimization formulation that is more preferred in practice according to Borrelli & Morari (2007) is

$$\min \sum_{i=0}^{H_p-1} \left(\|x_{k+i} - x_{k+i}^{\text{ref}}\|_Q^2 + \|\delta u_{k+i}\|_R^2 \right) + \|x_{k+H_p} - x_{k+H_p}^{\text{ref}}\|_S^2 \quad (3.13)$$

subject to constraints on inputs and states

where δu_k is the change in the input at time step k , i.e. $u_{k+1} = u_k + \delta u_k$. Using this formulation avoids computation of u_k^{ref} from Equation (3.11) at each time step. u_k^{ref} will change when disturbances occurs and therefore the formulation in Equation (3.13) is preferable. The MPC will use the following sequence at each time step k :

1. Solve the optimization problem and find an optimal sequence of inputs

2. Apply the first input of the sequence to the process
3. Set $k = k + 1$ and go to step 1

Linear MPC

A linear MPC uses a linear model and the optimization is a quadratic problem. The linear model is the discrete version of the state space model given in Equation (2.15).

$$\Delta x_{k+1} = A_d \Delta x_k + B_d \Delta u_k + E_d \Delta d_k \quad (3.14a)$$

$$\Delta y_k = C_d \Delta x_k \quad (3.14b)$$

Model reduction was presented in Chapter 2 to reduce the MPC optimization problem without removing too much dynamic behaviour. States that are controlled by the level controllers are removed before the reduction.

The Δ -notation is now dropped and the reduced state space model is written as

$$x_{k+1} = Ax_k + Bu_k + Ed_k \quad (3.15a)$$

$$y_k = Cx_k \quad (3.15b)$$

The formulation from Equation (3.13) is used and the quadratic optimization problem for step k can now be written as

$$\min \sum_{i=0}^{H_p-1} (x_{k+i}^\top Q x_{k+i} + \delta u_{k+i}^\top R \delta u_{k+i}) + x_{k+H_p}^\top S x_{k+H_p} \quad (3.16a)$$

subject to

$$x_{k+i+1} = Ax_{k+i} + Bu_{k+i} + Ed_{k+i}, \forall i = 0 \dots H_p - 1 \quad (3.16b)$$

$$u_{k+i+1} = u_{k+i} + \delta u_{k+i}, \forall i = 0 \dots H_p - 2 \quad (3.16c)$$

$$x_{\min, k+i} \leq x_{k+i} \leq x_{\max, k+i}, \forall i = 0 \dots H_p \quad (3.16d)$$

$$u_{\min, k+i} \leq u_{k+i} \leq u_{\max, k+i}, \forall i = 0 \dots H_p - 1 \quad (3.16e)$$

$$\delta u_{\min, k+i} \leq \delta u_{k+i} \leq \delta u_{\max, k+i}, \forall i = 0 \dots H_p - 1 \quad (3.16f)$$

The constraint (3.16f) arises from maximal change in the valves that are the physical inputs.

Implementation

Using `quadprog` in MATLAB, it is desirable to have the optimization problem in the following form

$$\min \frac{1}{2} v^\top H v \quad (3.17a)$$

subject to

$$A_{\text{eq}} v = b_{\text{eq}} \quad (3.17b)$$

$$v_{\min} \leq v \leq v_{\max} \quad (3.17c)$$

where v is a vector containing all input and state predictions. Derivations of these matrices are left to Appendix A.

Integral action

Hovd (2008) describes two approaches to add integral action to the MPC. One approach is to use the *input change* as a free variable in the optimization, using formulation given by Equation (3.13), this is already added. To test whether this would work or not, a simple linear MPC was made in MATLAB to test it. The process simulated on was the reduced model given in Equation (3.15). The simulations show that when applying steps in the disturbances the MPC give a stationary deviation. To avoid this stationary deviation it is possible to add integral action by using the other approach described by Hovd (2008). This method is to include the disturbance model in the MPC predictions, i.e. by using the constraint given in Equation (3.16b). This adds *feed-forward* from the disturbances to the MPC. This constraint requires that the disturbances and states are known (measured) or estimated. In a real column arrangement like the laboratory column at Department of Chemical Engineering the states and disturbances have to be estimated. In a real column arrangement it is possible to use the linearized and *nonreduced* model and estimate the real states for the column, but if the number of states are large it is difficult to estimate all the states without adding a lot of new measurements. Therefore we want to use the reduced model in the observer.

Observability

The following definition of observability is taken from Chen (1999):

Definition 2 *The state space model $\dot{x} = Ax + Bu$, $y = Cx + Du$ is said to be observable if for any unknown initial state $x(0)$, there exists a finite $t_1 > 0$ such that the knowledge of the input u and the output y over $[0, t_1]$ suffices to determine uniquely the initial state $x(0)$. Otherwise, the equation is said to be unobservable.*

To check whether the system is observable or not the rank of the observability matrix (given in Equation (3.18)) can be computed. If it has full rank, the $n \times n$ state space system is observable and the states can be estimated (Chen, 1999).

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (3.18)$$

The rank of the observability matrix can easily be checked in MATLAB using the commands `rank` and `obsv`. (A, C) from the linearized model (3.15) is observable. As mentioned above, if both states and disturbances are needed to be estimated because of the integral action, it is necessary for this system to be observable. To check that, the disturbances can be augmented into the states:

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & E \\ 0 & I \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \quad (3.19a)$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} x_k \\ d_k \end{bmatrix} \quad (3.19b)$$

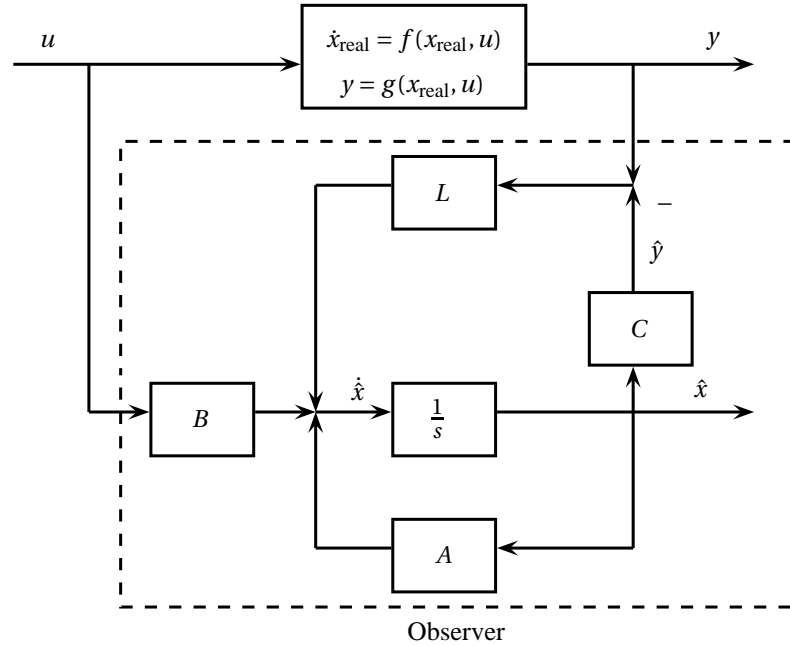


Figure 3.8: Continuous closed loop observer

Notice that the property, $d_k = d_{k+1} = d_{k+2} = \dots$, is used in the augmentation. (\tilde{A}, \tilde{C}) must now be observable, but this is unfortunately not the case for the Kaibel model. Borrelli & Morari (2007) remarks that for the system (3.19) the number of disturbances needs to be smaller or equal to the number of measurements for the system to be observable. 6 disturbances are modelled into the linear Kaibel model (See Section 2.1) and only 4 measurements. Hence, at least 2 measurements must be added to make it observable. In the simulations performed in this chapter the disturbances are assumed to be known to simplify the problem, i.e. only the states are estimated.

Observer dynamics

Figure 3.8 show a continuous closed loop observer similar to the one used in the MPC implementation. The observer is based on the linear and reduced model and the process is the nonlinear distillation process with different states. The real / physical states are denoted x_{real} . L denotes the observer gain and should be chosen such that the error dynamics, $(A - LC)$, is stable. In the implementation this gain was chosen using pole placement, such that the poles of $(A - LC)$ were putted inside the unit circle. The equations for the observer are

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \quad (3.20a)$$

$$\hat{y} = C\hat{x} \quad (3.20b)$$

Logarithmic transformation

Distillation columns are known to have nonlinear dynamics (Skogestad, 1997) and the linearized model used in an MPC controller can be a poor approximation to

the nonlinear model. A common trick to deal with this problem is to make the measurements *more linear* with respect to the inputs. Mejdell & Skogestad (1991) define the logarithmic temperature as

$$T^{\log} = \ln \left(\frac{T_{H,\text{ref}} - T}{T - T_{L,\text{ref}}} \right) \quad (3.21)$$

where $T_{H,\text{ref}}$ is the boiling point of the heavy component and $T_{L,\text{ref}}$ for the light component. This transformation is used for the temperature feedback in the MPC implementation. I.e. the temperature measurements "seen" by the MPC are transformed via the logarithmic function f_{\log} , such that $y_{\text{MPC}} = f_{\log}(y)$. The MPC tries to control y_{MPC} to the transformed setpoints $y_{\text{MPC}}^s = f_{\log}(y^s)$. To visualize the difference between the linear and nonlinear responses, a step in the feed flow (F) was applied. Figure 3.9 show the response of y_4 (T_{49}) for the two different models. The linear response is a poor approximation to the nonlinear response and the nonlinear dynamics of the column can be seen pretty clear. The use of logarithmic transformation on the linear response gives a better approximation for use in the MPC.

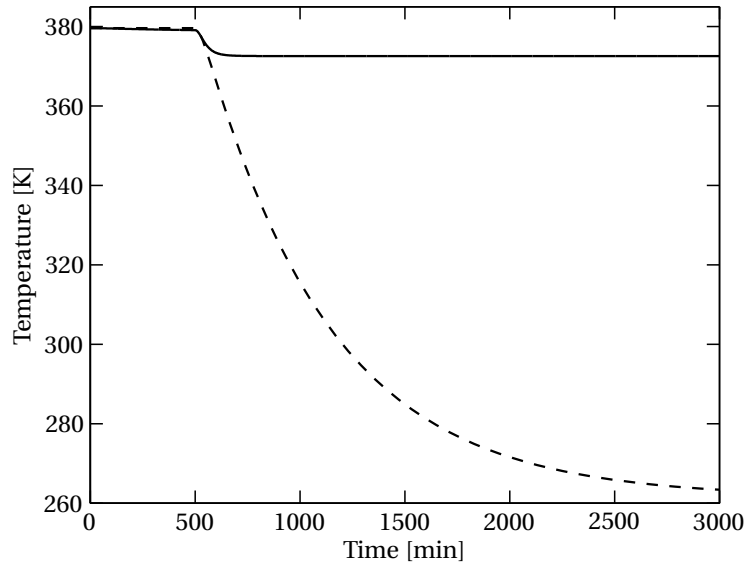


Figure 3.9: Response of y_4 (T_{49}) after a step in feed flow ($F + 10\%$), — nonlinear model response, - - - linear model response

Tuning of MPC

Maciejowski (2002) presents different parameters that can be used to tune the responses of an MPC:

- Weights
- Prediction horizon
- Disturbance model and observer dynamics

Variable	Value
Input weight matrix (R)	$1000 \cdot I$
State weight matrix (Q)	$C^T \cdot 100 \cdot I \cdot C$
Prediction horizon (H_p)	5
Sampling time of discrete model used in the MPC and observer	10 min
Minimum input	$u_{\min} = [0 \ 0 \ 0 \ 0]^T$
Maximum input	$u_{\max} = [1 \ 3 \ 1 \ 1]^T$
Minimum change in input	$\delta u_{\min} = [-10 \ -10 \ -10 \ -10]^T$
Maximum change in input	$\delta u_{\max} = [10 \ 10 \ 10 \ 10]^T$

Table 3.4: Simulation parameters used for MPC simulation (I is the identity matrix). Minimum and maximum constraint for the states x were simply set to negative infinity and infinity respectively.

The weights are the input change weight matrix R and state deviation weight matrix Q . These matrices are chosen by trial and error in the simulations and their values are shown in Table 3.4. The choice of prediction horizon is based on the closed-loop response time constant. The prediction horizon should be chosen sufficiently large, but an increasing value increases the size of the optimization problem quite much.

Simulation of the MPC

To compare decentralized control with multivariable control the disturbances from Table 3.3 was tested using MPC and plotted in the same figures. These plots are shown in Figure 3.10 and 3.11. The responses are significantly better for the MPC, but it is important to notice that the included disturbance model given by the constraint (3.16b) in the optimization problem, give a feed-forward from the disturbances. There is no feed-forward from the disturbances in the decentralized controller. It should also be noticed again that the disturbances are assumed to be known (measured) in the MPC simulations. In a real application it is likely that they must be estimated. It is reason to believe that the disturbance responses will get some worse than the responses in the figures when the disturbances must be estimated caused by limited observer dynamics.

3.5 Comparison between decentralized and multivariable control

The results shown from simulations using decentralized and multivariable control have focused on the temperatures and not the real objective; the product purities. To repeat, the overall objective function that we want to minimize is the total impurity flow:

$$J = D(1 - x_D) + S_1(1 - x_{S_1}) + S_2(1 - x_{S_2}) + B(1 - x_B) \quad (3.22)$$

The impurity flow can be plotted as a function of time for decentralized and multivariable control, see Figure 3.12 and 3.13. It is difficult to compare the two control approaches to much since the implemented MPC have a clear benefit with the disturbance feed-forward and not the decentralized controller. A feed-forward for the

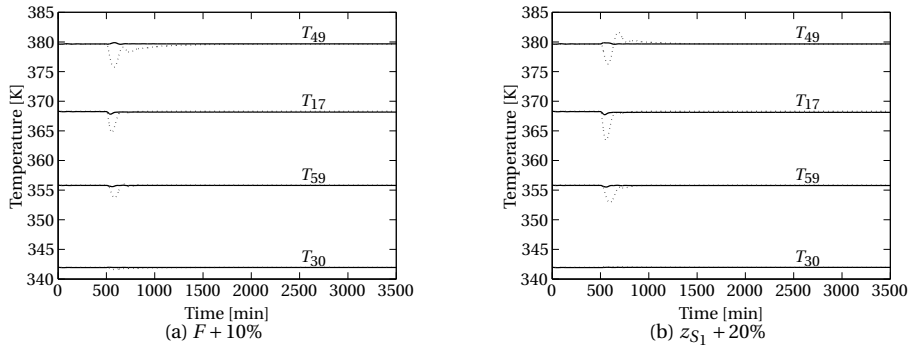


Figure 3.10: Disturbance in feed flow and feed composition, \cdots Tuned decentralized PI-controllers, $—$ MPC

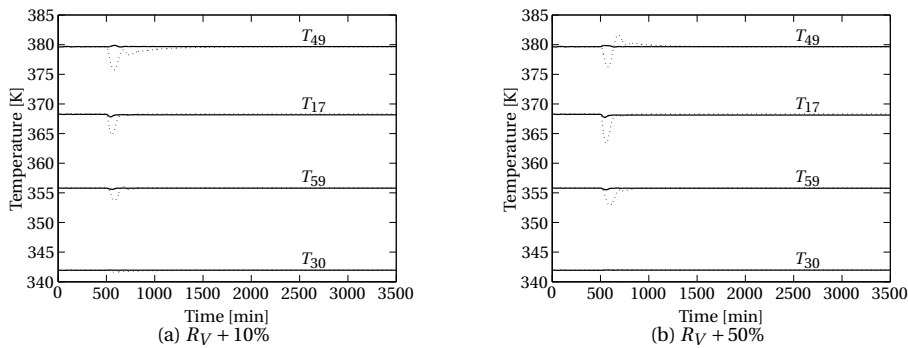


Figure 3.11: Disturbance in vapour split, \cdots Tuned decentralized PI-controllers, $—$ MPC

decentralized control approach would probably result in much less peaks for the impurities after the occurring disturbance as seen in the figures. Considering the amount of work to implement the two different control approaches, decentralized control is implemented quite fast and the tuning is straight forward when using SIMC tuning. It does not require any model since the tuning is based only on step tests. The decentralized controller need to know which manipulated variables that shall control which controlled variables, in an MPC it is no need to think of this. One of the main motivations for using MPC compared to a decentralized controller is the ability to handle constraints. The only constraints that were used in these simulations were input constraints.

A probably better way to build the multivariable controller would be to use the MPC on top of the PI-controllers already implemented in the decentralized control. As mentioned in the introduction of Section 3.4 this would give a layered control structure that give the ability for the MPC to optimize the real loss function; the impurity flows in this case. The MPC would then give optimal setpoints to the PI-controllers in the layer beneath with respect to this optimization objective. The MPC should then be more easy to tune and it would not be necessary to find the self-optimizing control variables (Skogestad & Postlethwaite, 2005).

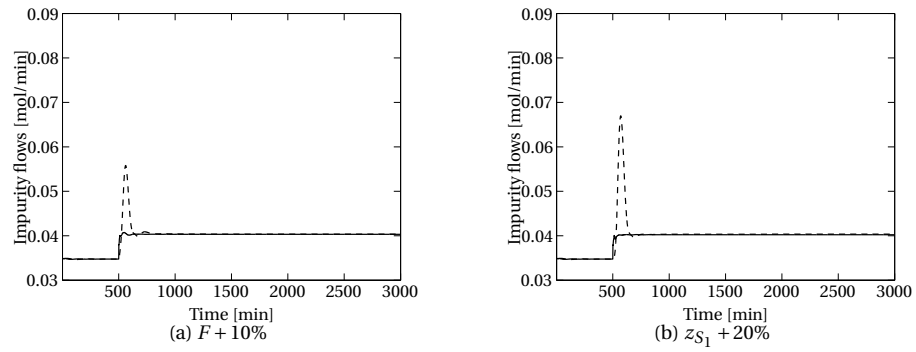


Figure 3.12: Disturbance in feed flow and feed composition, \cdots decentralized control, $—$ multivariable control

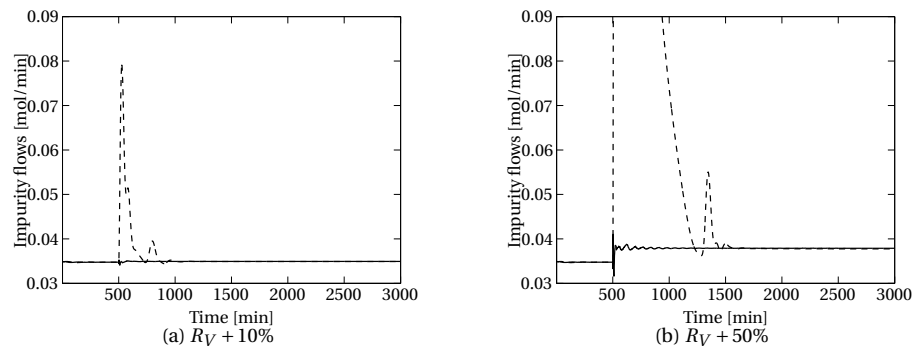


Figure 3.13: Disturbance in vapour split, \cdots decentralized control, $—$ multivariable control

Conclusion and further work

3.6 Conclusion

The main objective of this project work was to do some modelling of the Kaibel distillation column based on earlier work and do a study regarding control of the column. The Kaibel distillation column has potential energy savings compared to conventional distillation column arrangements, which is the motivation of this work. The control objective is to maintain acceptable product purities during disturbances. The distillation model is implemented in MATLAB / Simulink.

Including heat loss in the distillation model can be essential to make the results generated by the model to be more correct when comparing with a real distillation column. Heat loss make the top product more pure because less is vapourized, but the products beneath will be less pure. The overall purity get worse with heat loss included.

Adjusting the model parameters to make the model output fit to real plant measurements is a difficult problem because it involves a nonlinear model of which the number of states is dependent of the model parameters, and in addition, some of these model parameters are required to be integer numbers. By introducing a bypass vapour model and assuming a fixed number of stages in the distillation column, the identification problem can be simplified.

The Kaibel column is an interactive process and this may result in difficulties regarding control, but the RGA-analysis does not indicate heavy interactions and decentralized control is therefore possible. The PI-controllers in the decentralized control approach are tuned using Simple / Skogestad internal model tuning (Skogestad, 2003) and this give a fair disturbance rejection for the disturbances tested. Implementation of a multivariable controller involves several issues. A linear MPC require a linear model and the nonlinear model has to be linearized. Because of the large number of states it is an advantage to reduce this linear model into a smaller one so the size of the optimization problem gets reduced. A quite significant reduction can be done for the Kaibel model without removing too much of the dynamic behaviour. The MPC is made with feed-forward from the disturbances in order to avoid stationary error. This feed-forward require a disturbance model for the column. An observer is also needed to estimate the states in the reduced model. Because of its advantage with feed-forward, the MPC give much better disturbance rejection compared to conventional decentralized control.

3.7 Further work

The model parameter identification problem is presented in this report, but the identification itself has not been done. Therefore, laboratory experiments should

be done to collect measurement data for use in the identification. Unfortunately, the laboratory column at Department of Chemical Engineering has not been operative during this work.

The proposed controllers presented in the report should also be tested on the real column in order to determine if it really would work or not.

A more sophisticated way to implement a multivariable would be to put it on top of the regulatory layer which consists of PI-controllers as discussed in the report. This will result in easier tuning of the MPC and ability to optimize over the real objective.

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Appendix A

Vectors and matrices from MPC optimization problem

The different vectors and matrices from Equation (3.17) are derived here. The optimization variable v contains all input and state predictions:

$$v = \begin{bmatrix} x_{k+1} - x_{k+1,\text{ref}} \\ x_{k+2} - x_{k+2,\text{ref}} \\ \vdots \\ x_{k+H_p} - x_{k+H_p,\text{ref}} \\ u_k \\ u_{k+1} \\ \vdots \\ u_{k+H_p-1} \\ \delta u_k \\ \delta u_{k+1} \\ \vdots \\ \delta u_{k+H_p-1} \end{bmatrix} \quad (\text{A.1})$$

$x_{k+i,\text{ref}}$ is the desired value of the state x_{k+i} in time step k . The H -matrix becomes

$$H = \text{diag}(Q, \dots, Q, S, 0, \dots, 0, R, \dots, R) \quad (\text{A.2})$$

The weight matrices Q , S and R are positive definite. It is difficult to know how much each state should be weighted, therefore a weight matrix, \tilde{Q} , is introduced so the weighing is performed on the measurements. Q is then given by

$$Q = C^\top \tilde{Q} C \quad (\text{A.3})$$

The equality constraints arises from the linearized state space model (Equation (3.15)):

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ed_k \\ x_{k+2} &= Ax_{k+1} + Bu_{k+1} + Ed_{k+1} \\ &\vdots \\ x_{k+H_p} &= Ax_{k+H_p-1} + Bu_{k+H_p-1} + Ed_{k+H_p-1} \end{aligned}$$

and the constraint on δu :

$$\begin{aligned}
 u_{k+1} &= u_k + \delta u_k \\
 u_{k+2} &= u_{k+1} + \delta u_{k+1} \\
 &\vdots \\
 u_{k+H_p-1} &= u_{k+H_p-2} + \delta u_{k+H_p-2}
 \end{aligned}$$

In matrix form this becomes

$$A_{\text{eq}} = \begin{bmatrix} A_{\text{eq},11} & A_{\text{eq},12} & A_{\text{eq},13} \\ A_{\text{eq},21} & A_{\text{eq},22} & A_{\text{eq},23} \end{bmatrix} \quad (\text{A.4})$$

where these submatrices are given by

$$A_{\text{eq},11} = \begin{bmatrix} -I & 0 & 0 & \cdots & 0 \\ A & -I & 0 & \cdots & 0 \\ 0 & A & -I & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & A & -I \end{bmatrix} \quad (\text{A.5a})$$

$$A_{\text{eq},12} = \text{diag}(B) \quad (\text{A.5b})$$

$$A_{\text{eq},13} = 0 \quad (\text{A.5c})$$

$$A_{\text{eq},21} = 0 \quad (\text{A.5d})$$

$$A_{\text{eq},22} = \begin{bmatrix} -I & 0 & 0 & \cdots & 0 \\ A & -I & 0 & \cdots & 0 \\ 0 & A & -I & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & A & -I \end{bmatrix} \quad (\text{A.5e})$$

$$A_{\text{eq},23} = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \quad (\text{A.5f})$$

The optimization variable v contains the state references $x_{k+i,\text{ref}}$ and must be added to b_{eq} to get the correct state space equations

$$b_{\text{eq}} = - \begin{bmatrix} -I & 0 & 0 & \cdots & 0 \\ A & -I & 0 & \cdots & 0 \\ 0 & A & -I & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & A & -I \\ 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_{k+1,\text{ref}} \\ x_{k+2,\text{ref}} \\ \vdots \\ x_{k+H_p,\text{ref}} \end{bmatrix} \quad (\text{A.6})$$

$$- \begin{bmatrix} E & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} d_k \\ d_{k+1} \\ \vdots \\ d_{k+H_p-1} \end{bmatrix} - \begin{bmatrix} Ax_k \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (\text{A.7})$$

The inequality constraints are just minimal and maximal constraints and are placed in the vectors v_{min} and v_{max} . For further understanding of these matrices and vectors it is recommended to have a look at the MATLAB files included in the CD attached to this report.

Appendix B

CD

The CD attached to this report contains the MATLAB and Simulink files that are used in the project work, together with some saved data files. A file description named `description.pdf` and an electronic version of this report is on the CD as well.