

Comparison of models for 'severe slugging'

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Abstract

Slug flow is currently a problem in offshore installations. Eliminating slug flow through the use of active control is possible and has a potential for large economic gain. Simulations, development and testing of control schemes for slug flow are normally carried out in rigorous multi phase flow simulators such as OLGA, combined with experiments. For design purposes it is often helpful to study system properties. This is often hard to do in high dimensional models such as OLGA, leading to the development of a much simpler model in Matlab.

The Matlab model is a simplified nonlinear model with three states of gravity induced slug flow which is suitable for control purposes. The open loop response of the Matlab model closely follows the response in an OLGA model where the same system is studied. Closed loop response is also sufficiently accurate when P and PI control is implemented. It was found that the process gain is somewhat higher in the OLGA model compared to the Matlab model. Generally the simplified Matlab model is accurate enough to be suitable for controller design purposes.

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1 Introduction

Oil, gas and water multi phase transport in offshore installations, either from satellite platforms or sub-sea installations, to the processing platform runs through pipelines. When the oil/gas/water mix reaches the platform at seabed level it travels through a riser to reach the topside processing equipment. Multi phase flow that is forced to travel steeply uphill can give rise to a flow regime known as slug flow. In this case the pressure drop over the riser and the interphase friction is not sufficient to overcome gravity, and a liquid build-up (slug) at the low point will occur. The liquid slug will eventually block the gas flow and upstream pressure will rise. Liquid build up will continue until the upstream pressure has become high enough to overcome the weight of the slug. The slug will then be pushed up the riser into the topside processing equipment, and the process will start over.

Being able to eliminate slug flow will have large economic gains. Early solutions to the problem was to increase the pressure drop over the topside chokes, installing slug catchers topside or injecting lift gas into the riser. All of which are more or less effective, but costly. In later years, active control on the topside valves has been found to be able to eliminate slug flow. This allows for the system to run at a lower pressure without slugs being formed, thus increasing oil production. Also, it eliminates the need for costly slug catchers or lift gas injection systems.

Development of control systems for slug elimination is difficult, time consuming and therefore costly. Simulations, development and testing of control schemes are normally carried out in rigorous multi phase flow simulators such as OLGA, combined with experiments. When developing a control scheme it is very valuable for the engineer to be able to study the properties of the system. This is often hard to do in simulators such as OLGA because the models are of high complexity. With a simpler Matlab model these properties are a lot easier to study. Through studies of the system, controllability analysis and controller design in this simple model, a lot of valuable information can be obtained [1].

In this paper a comparison between a model in Matlab and one in OLGA for the same case will be performed. The goal is to validate that the Matlab model is accurate enough to be useful for controller design purposes and to unravel distinct model differences.

2 Model Descriptions

2.1 *Matlab Model*

The Matlab model developed for simulating and stabilising severe slugging is a low-dimensional nonlinear model. It was developed because a need for a simple model to study the properties of systems with slug behaviour for controller design purposes arose.

These properties are hard to study with a PDE (partial differential equation) based model, but it is relatively simple to unravel these system characteristics when using a simpler model.

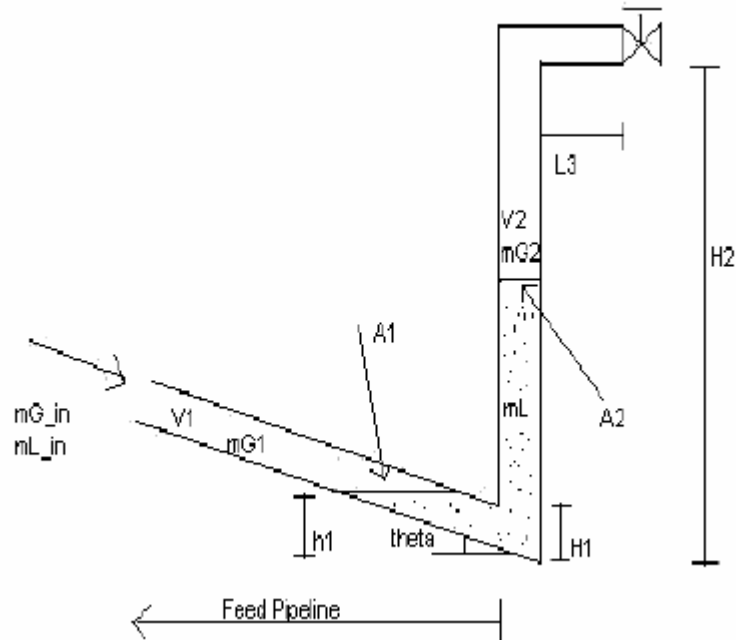


Figure 1 Geometry of the system

The model is based on the system shown in figure 1 and includes three states. The following assumptions were made in order to develop the model:

- Constant liquid velocity in “feed” pipeline allowing liquid level dynamics to be neglected. This implies:
 - Constant upstream gas volume. Any volume variations arising from liquid level variations at the lowpoint can be neglected.
 - Constant liquid feed directly into the riser.
- One liquid control volume including parts of the feed pipeline.
- Two gas control volumes that are separated by the lowpoint. These are connected through a pressure-flow relationship.
- Ideal gas law.
- Constant temperature.
- A stationary pressure balance between the riser and feed section.
- A simplified choke model for the gas and liquid leaving the riser.

Based on these assumptions a set of DAE's were developed that describes the system in figure 1. These are given in appendix 1 together with comments. The model is implemented in matlab and simulink, and can be run in both open and closed loop configurations. Using matlab tools and control theory the system characteristics can be

studied and control schemes can be developed and tested. Details on the model can be found in the model documentation [1].

2.2 OLGA Software and Model

The OLGA software is a rigorous multiphase simulation package with the dynamic capability being its most important feature. This ability gives the OLGA software an advantage over standard steady-state simulators, and an increased range of applications. Major fields of application are in pipeline design studies, operational studies and in safety analysis. The program also allows process equipment such as compressors, separators and/or controllers to be incorporated into a pipeline system that is studied. OLGA uses a numerical method that makes the software especially well suited for simulation of flow transients. With the semi-implicit time integration that is implemented in the software, relatively long time steps are allowed compared to those of an explicit method. This allows efficient simulation runs even when modelling long pipelines where simulation time is typically from hours to days.

In order to simulate a specific case in OLGA input files must be specified. The input system consists of six files where file 1 and 2 always are required. File 1 contains the data specific for any given case such as geometry, operational conditions, output data etc. File 2 contains the fluid properties given as pre-calculated tabulated values. These data are not necessarily case specific. The remaining input files are optional and are used if a three-phase flow is required (e.g. gas, oil, water), the restart option is chosen (allows for the user to continue a run done in a previous case) or when either a pump or compressor is incorporated into the simulation.

The input file to OLGA used for the simulations presented in this paper, are given in appendix 2. The geometry of the system and other system properties are all similar to those in the Matlab model.

3 Control Theory - Stabilising Unstable Systems

Most industrial processes are open-loop stable or self-regulating. This means that the process will return to the original steady state after a transient (non-sustained) disturbance has occurred. Some systems are open-loop unstable as is the case for slug flow. These processes are extremely difficult or impossible to control without feedback control.

A feedback control system is stable if and only if all roots of the characteristic equation are negative or have negative real parts. Otherwise the system is unstable.

In this paper the following controllers were used [3]:

- Proportional(P) controller: $p(t) = \bar{p} + K_c e(t)$
- P-Integral(PI) controller: $p(t) = \bar{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* \right]$
- P-I-Derivative(PID) controller: $p(t) = \bar{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de}{dt} \right]$

Here $p(t)$ is the controller output, \bar{p} the bias (initial controller output) and $e(t)$ is the error signal. To achieve a stable system response the parameters K_c , τ_I and τ_D are tuned. A trial and error method is used to obtain the best tuning.

4 Fitting Matlab Model to OLGA Data

In order to compare the models for severe slugging developed in OLGA and Matlab it is important to tune the Matlab model to achieve a best possible fit to the OLGA model.

The tuning was done by comparing upstream pressure data from both models in a bifurcation diagram. Matlab data were obtained by performing an open-loop run of sufficient time for stable oscillations to occur at different valve openings. Max and min amplitudes were recorded and plotted. The same technique was applied to acquire the OLGA data. Stationary data are obtained in the same manner, but with a run time approximately equal to 0. The best fit achieved is shown in figure 2 on the following page.

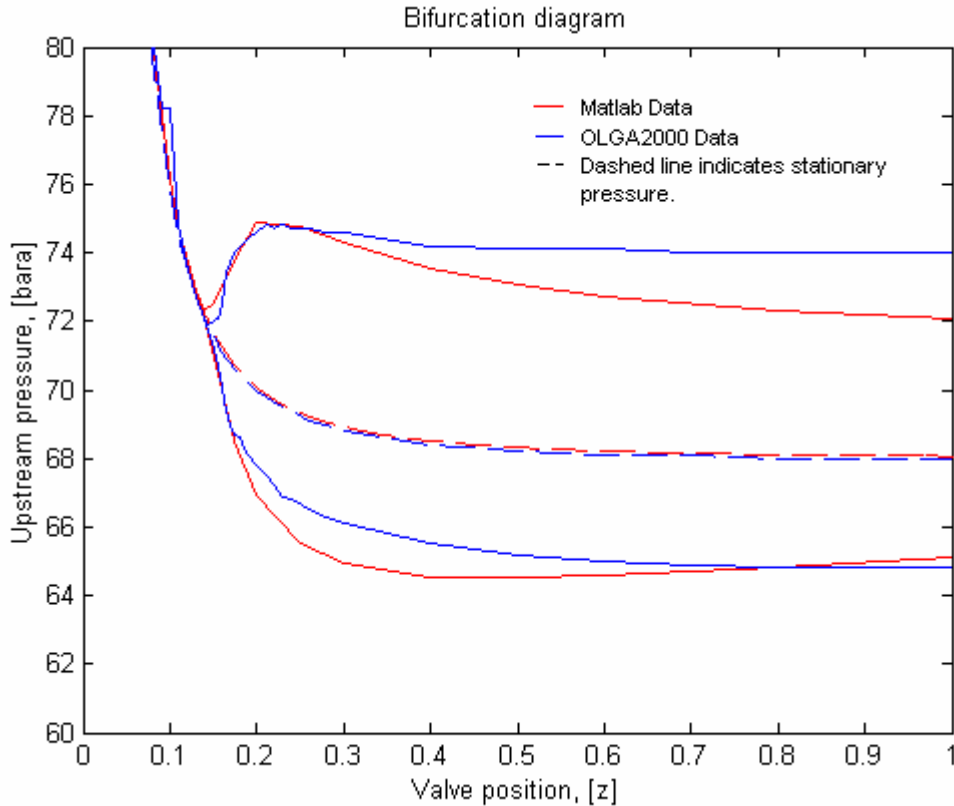


Figure 2 Bifurcation diagram

There are 4 variables available for tuning the Matlab model. These are the exponent “n” in the friction equation (equation (12) in appendix 1), K_1 (choke valve constant), K_2 (gas flow constant) and K_3 (friction parameter). When initiating the model experimental stationary values for pressures and liquid filling in riser are given. This determines the K_1 , K_2 and K_3 values. These values can be tuned by altering the upstream liquid fraction, FG, and the gas compressibility, M_G ($M_G = M_{w_g} * z$). Changing these values will alter the amplitudes in the bifurcation diagram. Changing the exponent n will alter the poles of the system and thus the instability point in the bifurcation diagram.

The best fit obtained by method of trial and error has the following values given in table 1 below:

Table 1 Tuning parameters

Variable	Value
n	2.17
FG	0.72
M_G	35

5 Comparison of open-loop response of Matlab and OLGA models

Open-loop comparison between the two models is best done through studying the model data in a bifurcation diagram and the frequency of the response. The bifurcation diagram allows comparison of max and min amplitudes at different valve openings, while the frequency study compares the pressure oscillation periods of the two models. Information on the fit of the stationary pressure data are also given in the diagram. This is sufficient to judge whether the Matlab model mimics the OLGA model in enough detail for it to be useful for controller design purposes.

A bifurcation diagram containing open-loop data both from the Matlab and OLGA models is given in figure 2. It is clear from the diagram that the two models are in good agreement for valve openings that are less than the instability point of $z = 0.15$. The OLGA curve has a small error causing it not to be totally smooth in this interval. If a small change in the choice of data points was made this error would be removed.

The Matlab model fits the OLGA data reasonably well for medium to low valve openings, $z = 0.15-0.5$, while the fit is less accurate for higher z -values. This is reasonable since a very simple choke model has been used, resulting in the model having to be tuned for a chosen valve interval, namely the low to medium range.

The current tuning will cause the Matlab model to display a better fit on the max pressure amplitudes than the min amplitudes. The fit of the stationary data are very good.

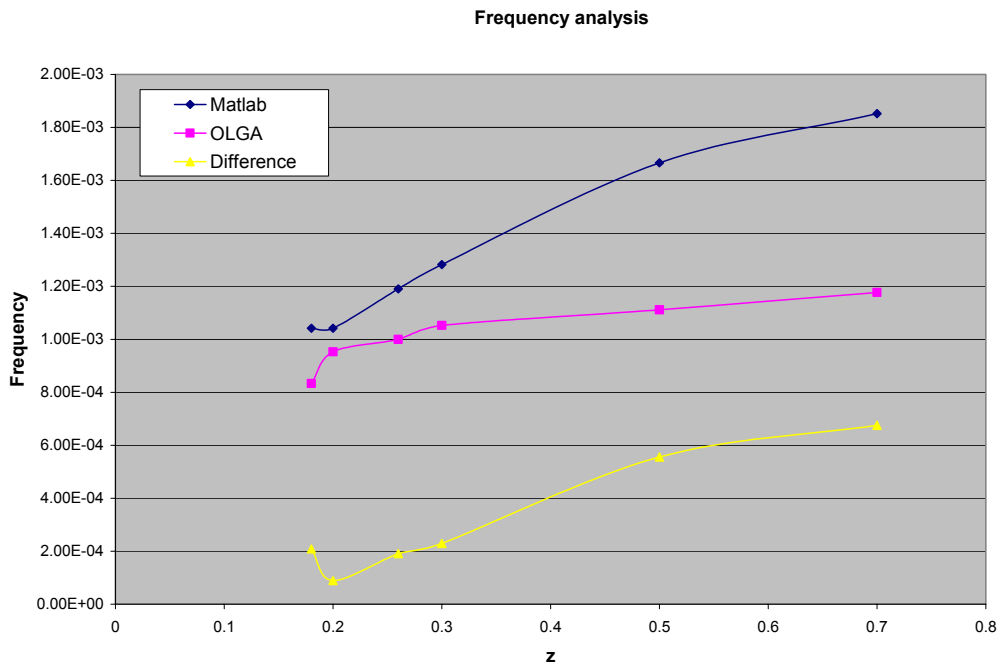


Figure 3 Frequency analysis

Table 2 Frequency analysis data

Valve opening	Frequency, Matlab		Frequency, OLGA		Deviation	% Deviation
z	delta t, min	Frequency	delta t, s	Frequency		
0.18	16	1.04E-03	1200	8.33E-04	2.08E-04	25.00
0.2	16	1.04E-03	1050	9.52E-04	8.93E-05	9.38
0.26	14	1.19E-03	1000	1.00E-03	1.90E-04	19.05
0.3	13	1.28E-03	950	1.05E-03	2.29E-04	21.79
0.5	10	1.67E-03	900	1.11E-03	5.56E-04	50.00
0.7	9	1.85E-03	850	1.18E-03	6.75E-04	57.41

In figure 3 a plot of frequency versus valve opening for both models is given. It is clear from the figure that the Matlab model predicts a lower frequency than the OLGA model. Also, the fit of the frequency data are better for low to medium valve openings as expected since the model is tuned for a best fit in this region. In general, the frequency deviation of the Matlab model is relatively low, as can be seen from table 2. The reason there is a deviation between the data is probably that upstream liquid level dynamics have been neglected in the Matlab model, as stated in the assumptions earlier. To get a good fit for the amplitude data a relatively small, constant upstream gas volume is used. In reality this volume will vary with time. These variations would reduce the frequency. If these were included into the model a better agreement of the frequency data would probably be seen.

6 Comparison of closed-loop response of Matlab and OLGA models

6.1 Basis for comparison

In order to compare the closed loop behaviour of the two models, controllers with proportional- (P), P and integral (PI) and PI and derivative (D) action were developed. Through studies of the Matlab model's properties the best variables to measure was decided. It was found that the best control scheme to stabilise slug behaviour was to have upstream pressure as controller input [1]. Through the rest of this section all controllers has upstream pressure as the measured variable unless otherwise noted.

To compare the two models, P, PI and PID controllers were implemented in OLGA and tuned to give as rapid and good response as possible. The same tuning parameters, K_c (controller gain), τ_I (integral action) and τ_D (derivative action) were implemented in the Matlab model and a comparison of the responses was performed. It is also important to take into account when the controller action is switched on. Initiation of the controller should always be when the open-loop pressure has a rising trend, preferably at max

amplitude. When initiation is done at a different point in time it is far more difficult to stabilise the system. The optimal controller tuning will be different, as will be shown. In general, it is not really of importance whether the Matlab model fit the OLGA model in this case since controller initiation is usually never done at this point.

6.2 P-Control

In figure 4 and 5 on the following page the response of the OLGA and Matlab model to P-control is shown. In both cases a controller gain of $K_c=-0.3$ is used and the loop is closed when the pressure has an upwards trend. Some differences are obvious:

1. The response of the OLGA model is generally more oscillatory in nature, giving rise to more controller usage.
2. The overshoot in the OLGA model is higher than that of the Matlab model.
3. The system is stabilised more rapidly in the Matlab model, $\sim 600s$, compared to the OLGA model, $\sim 1500s$.
4. The system is stabilised closer to the set point, e.g. less offset, with the Matlab model, compared to the OLGA model.

These differences are all related to the higher complexity of the OLGA model. Neglecting upstream liquid dynamics and reducing the number of states studied (no high order dynamics, e.g. no high frequency response) is done in the Matlab model while not in the OLGA model. This leads to a simpler less complex system and therefore less complex response. This explains that more oscillations and a higher overshoot are seen in the OLGA model. Also, the time required to stabilise the system is related to the above mentioned.

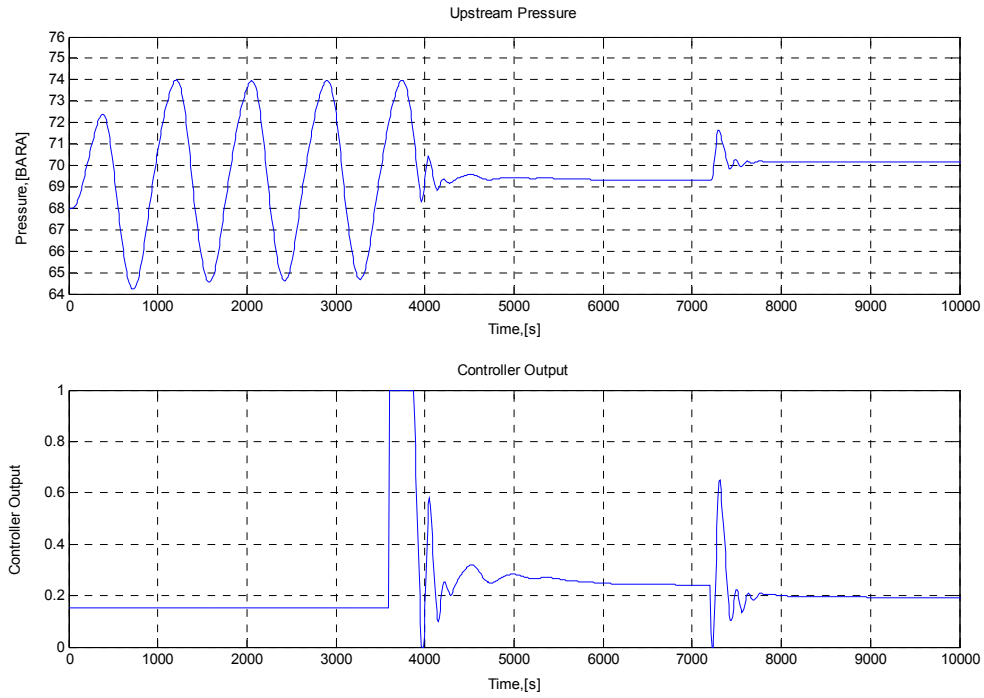


Figure 4 OLGA model response to P-control, $K_c = -0.3$

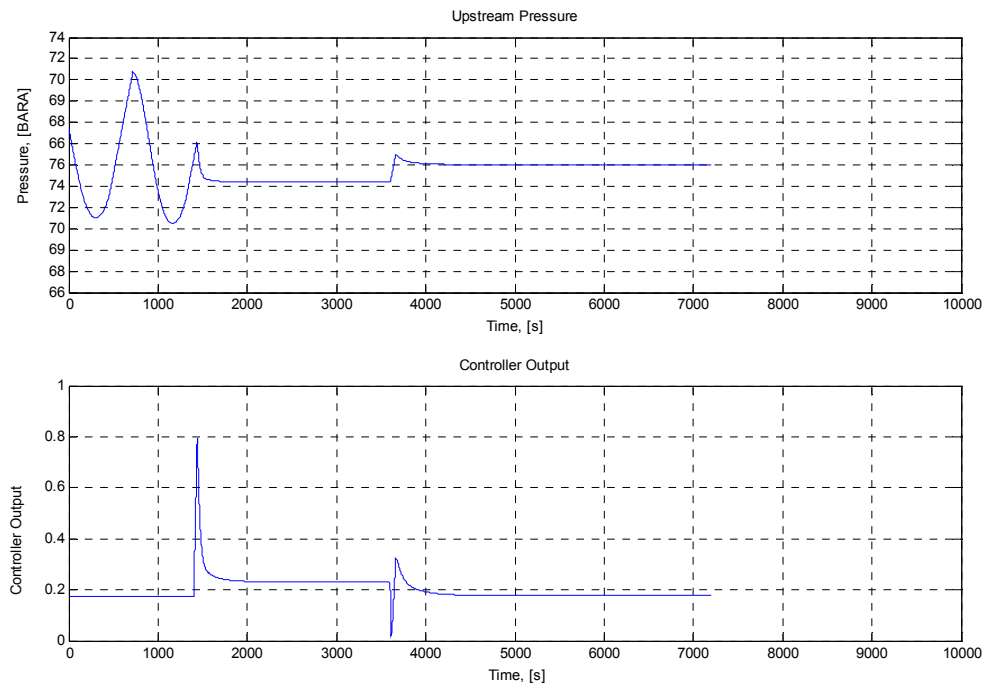


Figure 5 Matlab model response to P-control, $K_c = -0.3$

6.3 PI-control

Figure 6 below and 7 on the following page shows the response of the models to PI-controllers. Initiation of the controllers is done when the open loop response has an upwards trend. Again some differences between the models are obvious:

1. Points 1 and 2 regarding oscillations in the response and overshoot in the previous section, also apply for PI-control.
2. In both models introducing integral action in the controller brings the system down to the set point. However, this is done much faster and with no oscillations in the Matlab model compared to the results from the OLGA model.

The same reasons that explain the model differences in the previous section applies here.

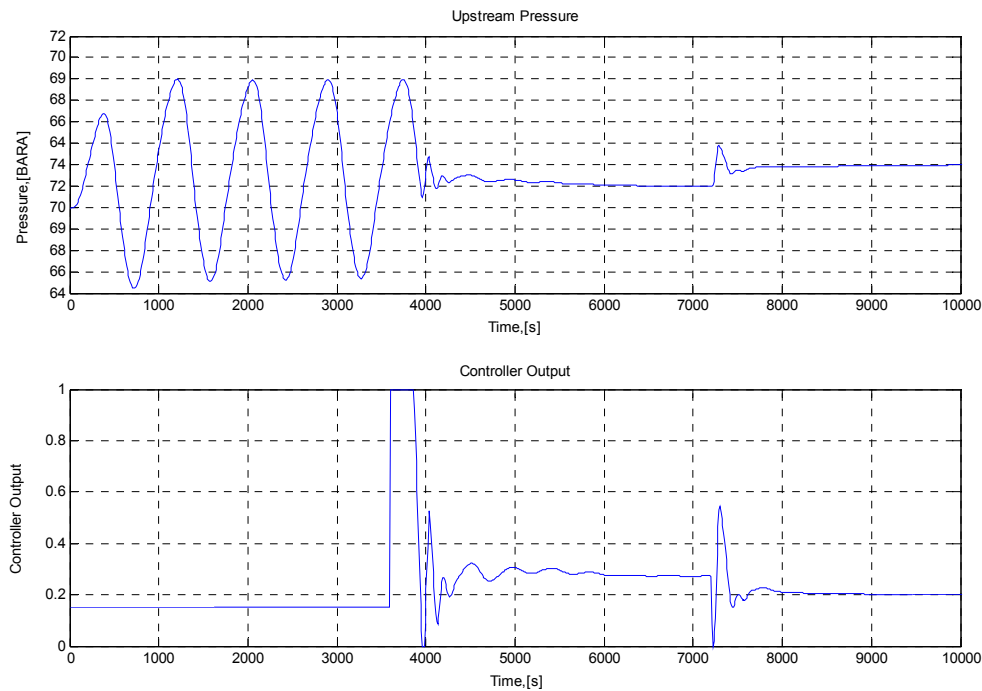


Figure 6 OLGA response to PI-control, $K_c=-0.3$ and $T_{au}=300$.

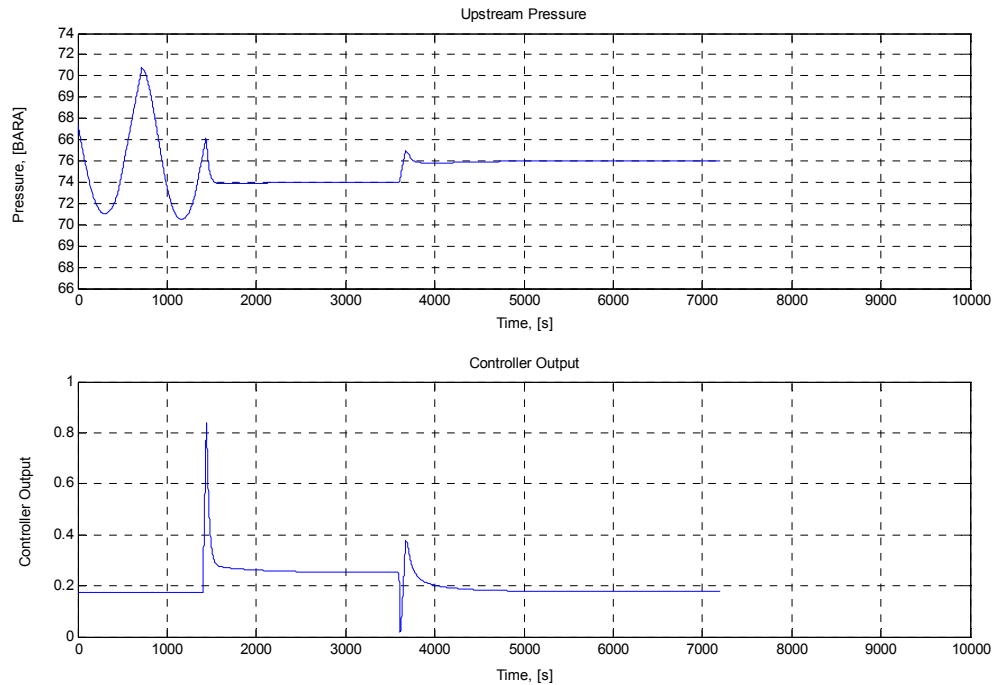


Figure 7 Matlab model response to PI-control, $K_c=-0.3$ and $Tau_I=300$.

6.4 PID-control and cascade control

When derivative action is introduced in the controller, the response from the OLGA model is smoothed to a certain degree. This can be seen in figure 8 on the next page. Ideally, modified derivative action should be used [1]. This is not possible to simulate in OLGA so standard derivative action is used. Because the system does not really tolerate normal derivative action, the value and range available for τ_D are both very low. Having a derivative action with $\tau_D=0.005$ does not have a visual effect in the Matlab model, and the response is similar to that in figure 7. In the OLGA model the response is slightly smoother, but the system is highly unstable and would not tolerate disturbances of significant size. In general, given that modified derivative action cannot be applied, a comparison of the response of the models to PID-control is not possible.

In some cases it is not possible to have upstream pressure as the measured variable for various reasons. Earlier studies have shown that it is possible to stabilise the system with cascade control [1]. In this case liquid volume flow and pressure measured in the vertical pipe section downstream from the riser is used as controller input for the controller input. Implementation of a cascade controller in OLGA is difficult, and a functioning control scheme could not be achieved. Therefore, a comparison of model responses to cascade control has not been done.

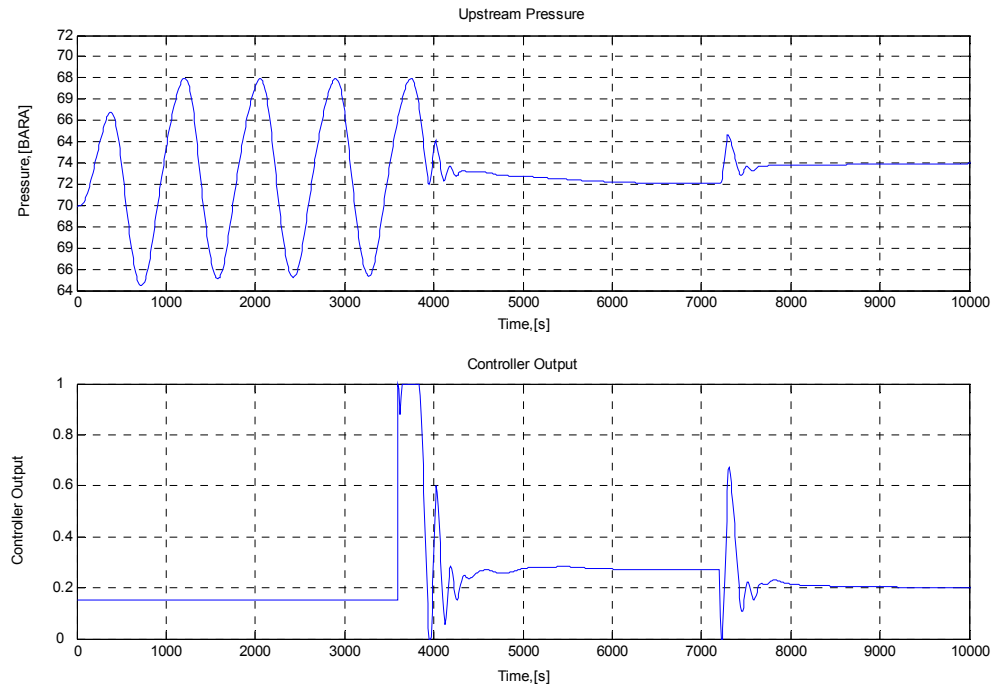


Figure 8 OLGA model response to PID-control, $K_c=-0.3$, $\tau_I=350$ and $\tau_D=0.005$

6.5 Model responses to P- PI- and PID control when initiation of the controller is on a downwards trend of the open loop response.

Generally speaking, the same points made in the three previous sections with regards to model differences also apply when the initiation is at a different point in time. Some differences are however obvious, and are discussed below.

The nature of the system makes it harder to stabilise when controllers are turned on when the open loop response has a downwards trend [1]. The system behaves in a more complex manner which gives rise to more oscillations in the controller response. This is demonstrated clearly in figure 9 on the next page. As can be seen from figure 10, the Matlab model does not show this oscillatory nature at all.

The OLGA model has a very low value for $\tau_I=40$. This is necessary to reach the set point. From the plot, figure 9, one can see that a lot of controller action is required to stabilise the system, and that it takes relatively long time to reach the set point. This trend is not obvious in Matlab at all, which stabilises very rapidly with no oscillations. Again, this is caused by the higher complexity of the OLGA model.

The relatively low similarity of the models in this case is not of high importance since initiation when the open loop pressure is decreasing will never be done in reality.

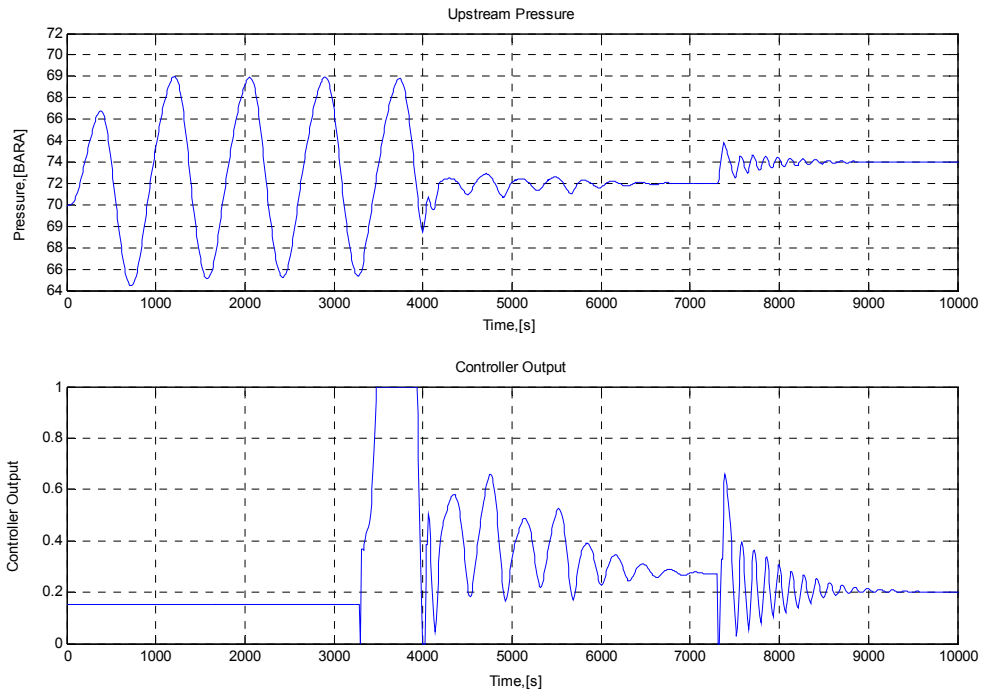


Figure 9 OLGA model response to PI-control, $K_c=-0.4$ and $Tau_I=40$

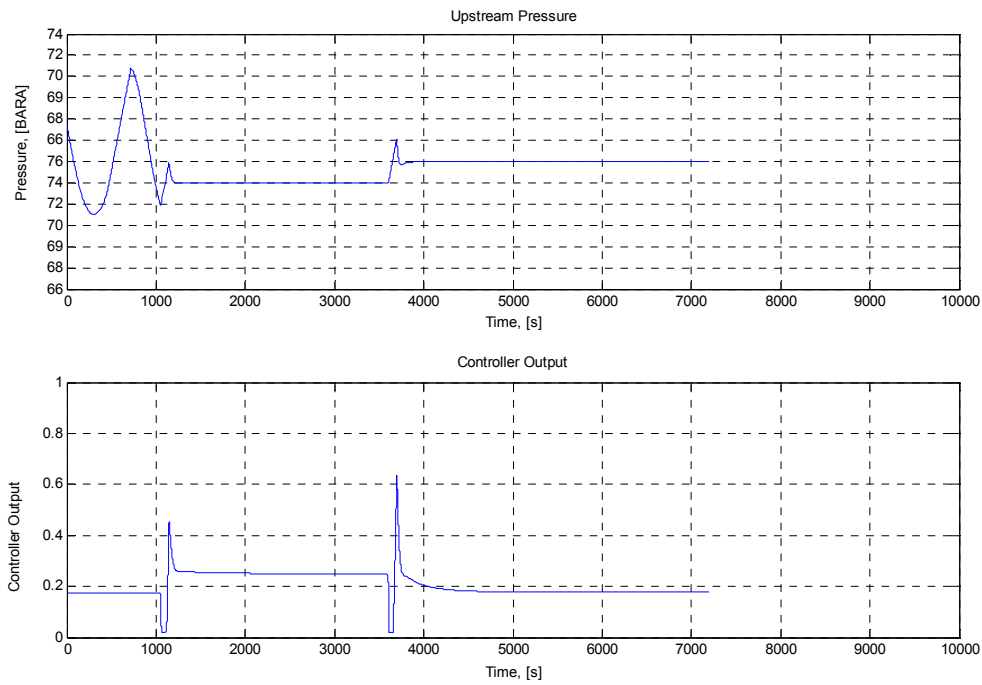


Figure 10 Matlab model response to PI-control, $K_c=-0.4$ and $Tau_I=40$

6.6 General considerations and summary

In the process of devising the controllers used in the comparison above several runs were made with different controller tunings than those given above. From this information and the earlier discussion some general points can be made:

1. It seems that the process gain is somewhat larger in the OLGA model compared to the Matlab model. This results in the need for using a slightly higher value for K_c when a controller developed in Matlab is transferred to OLGA.
2. The K_c range that stabilises the system is wider in Matlab. The lower boundary in OLGA is slightly higher due to point 1. The upper boundary is considerably lower because higher order dynamics neglected in the Matlab model makes the system unstable.
3. When integral action is included in the controller in Matlab, no significant oscillation occurs in the response. This is not the case in OLGA, where oscillations in the response can be high.
4. The OLGA model takes significantly longer time and much more valve action to stabilise at a new set point compared to the Matlab model, especially for PI-control.
5. The figures earlier shows that the controller output in the OLGA model often goes to saturation. This indicates that the gain margin of a controller is smaller in the OLGA model than it is in Matlab.

The reasons for the deviations demonstrated in point 3-5 are all related to the higher complexity of the OLGA model compared to the Matlab model.

7 Conclusion

The open loop response of both models are similar and in good agreement. Both stationary pressure and pressure amplitudes in the models follow each other closely for the valve opening interval the models are tuned for. Pressure oscillation frequency is not in especially good agreement, but this does not prevent the Matlab model from being useful. Care should be taken when tuning the model to obtain the best results.

The closed loop response is generally also in sufficiently good agreement between the models. The Matlab model relatively accurately follows the OLGA model response when P- and PI-control is implemented. It was found that the process gain is somewhat larger in the OLGA model.

Controller initiation should always be done when the open loop pressure trend is rising. The Matlab model does not accurately predict the complex response experienced in the OLGA model for this case.

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PID- and cascade control could not be studied because of difficulties with implementing these controller schemes in the OLGA software. However, given the Matlab model's good agreement for PI- and P-control one would expect a relatively reasonable similarity of the model responses.

Gloeshaugen, 25. November 2002

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Wiley & sons Inc, New York, USA, 1989.

Appendix 1 – Matlab Model Equations

The equations given on the following pages are the ones used to describe and implement the dynamic slug model in Matlab. Symbols used in the equations are defined below.

Symbol	Description	Unit	Remarks
m_{Gi}	Mass of gas in volume i	Kg	State variable
m_L	Mass of liquid	Kg	State variable
V_{Gi}	Gas volume i	m^3	$V_{G1} = const$
V_L	Volume occupied by liquid	m^3	
V_{LR}	Volume of liquid in riser	m^3	
V_T	Total volume in riser	m^3	
P_i	Pressure in volume i	$\frac{N}{M^2}$	
ρ_{Gi}	Gas density in volume i	$\frac{kg}{m^3}$	
ρ_L	Liquid density	$\frac{kg}{m^3}$	Constant
$\bar{\rho}$	Average density in riser	$\frac{kg}{m^3}$	
ρ_T	Density upstreams valve	$\frac{kg}{m^3}$	
v_{G1}	Gas velocity at lowpoint	$\frac{m}{s}$	
$v_{mix,out}$	Liquid velocity through choke valve	$\frac{m}{s}$	
\dot{m}_{G1}	Internal gas mass flowrate	$\frac{Kg}{s}$	
$\dot{m}_{G,out}$	Gas mass flowrate through choke valve	$\frac{Kg}{s}$	
$\dot{m}_{L,out}$	Liquid mass flowrate through choke valve	$\frac{Kg}{s}$	
α_L	Average liquid fraction in riser, volume basis	-	
α_{LT}	Liquid fraction upstreams valve, volume basis	-	
α_L^m	Liquid fraction upstreams valve, mass basis	-	
h_1	Liquid level upstreams the dip	m	
H_1	Critical liquid level	m	Constant
H_2	Height of riser	m	Constant
r	Radius of pipe	m	Constant
A_1	Area in horizontal plane, V_1	m^2	Constant
A_2	Cross section area, V_2	m^2	Constant
\dot{A}	Gas flow area at lowpoint	m^2	
L_3	Length of horizontal top section	m	Constant
θ	Feed pipe inclination	rad	Constant
R	Gas constant	$8314 \frac{J}{K \cdot Kmol}$	Constant
g	Specific gravity	$9.81 \frac{m}{s^2}$	Constant
T	System temperature	K	Constant
M_G	Molecular weight of gas	$\frac{Kg}{Kmol}$	Constant
$\dot{m}_{G,in}$	Mass rate of gas into system	$\frac{Kg}{s}$	Disturbance
$\dot{m}_{L,in}$	Mass rate of liquid into system	$\frac{Kg}{s}$	Disturbance
P_0	Pressure after choke valve	$\frac{N}{M^2}$	Disturbance
z	Valve position	-	Input
K_1	Choke valve constant	-	Tuning parameter
K_2	Gas flow constant	-	Tuning parameter
K_3	Friction parameter	-	Tuning parameter
n	w^n in the friction expression	-	Tuning parameter

Internal Equations

$$P_1 = \frac{m_{G1}RT}{V_{G1}M_G} \quad (1)$$

$$\rho_{G1} = \frac{m_{G1}}{V_{G1}} \quad (2)$$

$$V_L = \frac{m_L}{\rho_L} \quad (3)$$

$$h_1 A_1 + V_{LR} = V_L \quad (4)$$

$$V_T = A_2 (H_2 + L_3) \quad (5)$$

$$V_{G2} = V_T - V_{LR} \quad (6)$$

$$\rho_{G2} = \frac{m_{G2}}{V_{G2}} \quad (7)$$

$$\alpha_L = \frac{V_{LR}}{V_T} \quad (8)$$

$$P_2 = \frac{m_{G2}RT}{V_{G2}M_G} \quad (9)$$

$$\bar{\rho} = \frac{m_{G2} + V_{LR}\rho_L}{V_T} \quad (10)$$

$$\bar{\rho}g(H_2 + H_3) - \rho_L g h_1 = P_1 - P_2 \quad (11)$$

$$\alpha_{LT} = (V_{LR} > H_2 A_2) \frac{V_{LR} - A_2 H_2}{A_3 H_3}$$

$$+ \frac{w^n}{1 + w^n} \left(\alpha_L - (V_{LR} > H_2 A_2) \frac{V_{LR} - A_2 H_2}{A_3 H_3} \right), w = \frac{K_3 \rho_{G1} v_{G1}^2}{\rho_L - \rho_{G1}} \quad (12)$$

$$\rho_T = \alpha_{LT} \rho_L + (1 - \alpha_{LT}) \rho_{G2} \quad (13)$$

$$\alpha_L^m = \frac{\alpha_{LT} \rho_L}{\alpha_{LT} \rho_L + (1 - \alpha_{LT}) \rho_{G2}} \quad (14)$$

Transport Equations

$$v_{G1} = (h_1 < H_1) K_2 \frac{H_1 - h_1}{H_1} \sqrt{\frac{P_1 - P_2 - \rho_L g \alpha_L H_2}{\rho_{G1}}} \quad (15)$$

$$\dot{m}_{G1} = v_{G1} \rho_{G1} \hat{A} \quad (16)$$

$$m_{mix,out} = K_1 z \sqrt{\rho_T (P_2 - P_0)} \quad (17)$$

$$\dot{m}_{G,out} = (1 - \alpha_L^m) m_{mix,out} \quad (18)$$

$$\dot{m}_{L,out} = \alpha_L^m m_{mix,out} \quad (19)$$

Geometric Equations

$$\begin{aligned}
 H_1 &= \frac{2r}{\cos(\theta)} \\
 A_1 &= \frac{A_2}{\sin(\theta)} \\
 \phi &= ((H_1 - h_1) \cos(\theta) < r) \left(\pi - \arccos \left(1 - \frac{(H_1 - h_1) \cos(\theta)}{r} \right) \right) + \\
 &\quad ((H_1 - h_1) \cos(\theta) > r) \left(\arccos \left(\frac{(H_1 - h_1) \cos(\theta)}{r} - 1 \right) \right) \\
 \dot{A} &= r^2 (\pi - \phi - \cos(\pi - \phi) \sin(\pi - \phi)) \quad (20)
 \end{aligned}$$

Conservation Equations

$$\frac{d}{dt} m_L = \dot{m}_{L,in} - \dot{m}_{L,out} \quad (21)$$

$$\frac{d}{dt} m_{G1} = \dot{m}_{G,in} - \dot{m}_{G1} \quad (22)$$

$$\frac{d}{dt} m_{G2} = \dot{m}_{G1} - \dot{m}_{G,out} \quad (23)$$

Equations 1 through 10 plus 13, 14, 18 and 19 are normal straight forward equations describing pressures, densities, volumes etc. Equation 11 is a stationary pressure balance connecting the two gas control volumes. It neglects dynamic effects, acceleration and wall friction.

Equation 12 is an entrainment equation. It describes the liquid fraction in the gas filled part of the riser related to the gas velocity. Obviously it is not valid when the riser is completely filled with liquid. Therefore an equality constraint is incorporated into the equation.

Equation 15 describes the gas velocity at the low point. Stating that the pressure drop between the two gas control volumes has a quadratic dependency on the relative opening at the low point equation 15 can be developed. Clearly, there can be no gas flow if the low point is liquid filled, resulting in the equality constraint. This equation then gives rise to equation 16 which describes the mass flow.

Equation 17 is a simple valve equation assuming constant mass fraction through the valve. The remaining equations are also straight forward equations regarding geometric calculations and mass conservation. Further details on the equations used in the Matlab model can be found in reference [1].

Appendix 2 – OLGA Model Parameters

```
!*****
!- CASE Definition
!-----
CASE AUTHOR="Scandpower A/S", \
  PROJECT="OLGA Sample cases", \
  TITLE="Verification of severe slugging"

!
!*****
!- OPTIONS Definition
!-----
OPTIONS COMPOSITIONAL=OFF, DEBUG=OFF, PHASE=TWO, POSTPROCESSOR=OFF,
SLUGVOID=SINTEF, STEADYSTATE=ON, TEMPERATURE=WALL, \
  WAXDEPOSITION=OFF

!
!*****
!- FILES Definition
!-----
FILES PVTFILE="ol-terslug-1.tab"

!
!*****
!- INTEGRATION Definition
!-----
INTEGRATION CPULIMIT=1000 s, DTSTART=0.01 s, ENDTIME=18000 s, MAXDT=5 s,
MAXTIME=0 s, MINDT=0.01 s, MINTIME=0 s, \
  NSIMINFO=10, STARTTIME=0 s

!
!*****
!- MATERIAL Definition
!-----
MATERIAL LABEL=MATER-1, CAPACITY=500 J/kg-C, CONDUCTIVITY=50 W/m-K,
DENSITY=7850 kg/m3, TYPE=SOLID
MATERIAL LABEL=MATER-2, CAPACITY=880 J/kg-C, CONDUCTIVITY=1 W/m-K,
DENSITY=2500 kg/m3, TYPE=SOLID

!
!*****
!- WALL Definition
!-----
WALL LABEL=WALL-1, ELECTRICHEAT=OFF, MATERIAL=( MATER-1, MATER-2, MATER-2 ),
POWERCONTROL=OFF, THICKNESS=( 0.009, \
  2:0.02 ) m
WALL LABEL=WALL-2, ELECTRICHEAT=OFF, MATERIAL=( MATER-1, MATER-2, MATER-2 ),
POWERCONTROL=OFF, THICKNESS=( 0.0075, \
  2:0.02 ) m
```

```
!*****
*****
!   GEOMETRY Definition
!*****
*****
GEOMETRY LABEL=GEOM-1, XSTART=0 m, YSTART=0 m, ZSTART=0 m

PIPE LABEL=PIPE-1, DIAMETER=0.12 m, ELEVATION=0 m, LENGTH=2000 m, NSEGMENTS=5,
ROUGHNESS=2.8e-005 m, WALL=WALL-1
PIPE LABEL=PIPE-2, DIAMETER=0.12 m, ELEVATION=-34.9 m, LENGTH=2000 m,
NSEGMENTS=5, ROUGHNESS=2.8e-005 m, \
    WALL=WALL-1
PIPE LABEL=PIPE-3, DIAMETER=0.12 m, ELEVATION=-5.24 m, LENGTH=300 m, LSEGMENT=(
200, 100 ) m, NSEGMENTS=2, \
    ROUGHNESS=2.8e-005 m, WALL=WALL-1
PIPE LABEL=PIPE-4, DIAMETER=0.1 m, ELEVATION=300 m, LENGTH=300 m, NSEGMENTS=4,
ROUGHNESS=2.8e-005 m, WALL=WALL-2
PIPE LABEL=PIPE-5, DIAMETER=0.1 m, ELEVATION=0 m, LENGTH=100 m, NSEGMENTS=2,
ROUGHNESS=2.8e-005 m, WALL=WALL-2

!
!*****
!-  NODE Definition
!-----
NODE LABEL=INLET, TYPE=TERMINAL, X=0 m, Y=0 m, Z=0 m
NODE LABEL=OUTLET, TYPE=TERMINAL, X=4399.65 m, Y=259.86 m, Z=0 m

!*****
*****
!   BRANCH Definition
!*****
*****
BRANCH LABEL=BRAN-1, FLOAT=ON, FLUID="1", FROM=INLET, GEOMETRY=GEOM-1,
TO=OUTLET

!
!*****
!-  BOUNDARY Definition
!-----
BOUNDARY NODE=INLET, TYPE=CLOSED
BOUNDARY GASFRACTION=2:1 -, NODE=OUTLET, PRESSURE=2:5000000 Pa,
TEMPERATURE=2:22 C, TIME=( 0, 1000000 ) s, \
    TYPE=PRESSURE, WATERFRACTION=2:0 -

!
!*****
!-  HEATTRANSFER Definition
!-----
HEATTRANSFER BRANCH=BRAN-1, HAMBIENT=5:6.5 W/m2-C, HMININNERWALL=500 W/m2-
C, HOUTEROPTION=HGIVEN, PIPE=PIPE-1, \
    TAMBIENT=5:6 C
HEATTRANSFER BRANCH=BRAN-1, HAMBIENT=5:6.5 W/m2-C, HMININNERWALL=500 W/m2-
C, HOUTEROPTION=HGIVEN, PIPE=PIPE-2, \
    TAMBIENT=5:6 C
```


HEATTRANSFER BRANCH=BRAN-1, HAMBIENT=2:6.5 W/m²-C, HMININNERWALL=500 W/m²-C, HOUSEROPTION=HGIVEN, PIPE=PIPE-3, \
TAMBIENT=2:6 C
HEATTRANSFER BRANCH=BRAN-1, HAMBIENT=4:6.5 W/m²-C, HMININNERWALL=500 W/m²-C, HOUSEROPTION=HGIVEN, PIPE=PIPE-4, \
TAMBIENT=4:6 C
HEATTRANSFER BRANCH=BRAN-1, HAMBIENT=2:6.5 W/m²-C, HMININNERWALL=500 W/m²-C, HOUSEROPTION=HGIVEN, PIPE=PIPE-5, \
TAMBIENT=2:6 C

```
*****
! CONTROLLER Definition
*****
CONTROLLER LABEL=CONTROLLER-1, AMPLIFICATION=0.5 , BIAS=0.175 ,
BRANCH=BRAN-1, COMBINEVARIABLES=OFF, DERIVATIVECONST=0 s, \  
EXTENDED=OFF, INTEGRALCONST=500 s, MAXCHANGE=0.2 , MAXSIGNAL=1 ,
MINSIGNAL=0 , NORMRANGE=1 , \  
PIPE=PIPE-1, REFCONDITION=IN-SITU, SAMPLETIME=10 s, SECTIONBOUNDARY=1,
SETPOINT=( 70, 69, 70 ) , \  
STROKETIME=60 s, TIME=( 0, 1, 2 ) h, TYPE=PID, VARIABLE=PT BARA

!
*****
!- SOURCE Definition
!-----
SOURCE LABEL=SOUR-1-1, BRANCH=BRAN-1, CRITFLOWMODEL=FROZEN,
GASFRACTION=4:-1 -, MASSFLOW=4:9 kg/s, PIPE=PIPE-1, \  
SECTION=1, TEMPERATURE=4:63 C, TIME=4:0 s, WATERFRACTION=4:0 -

!
*****
!- VALVE Definition
!-----
VALVE LABEL=CHOKE-1-1, BRANCH=BRAN-1, CD=0.75 , CRITFLOWMODEL=FROZEN,
DIAMETER=0.075 m, OPENING=0.15 , \  
PIPE=PIPE-5, SECTIONBOUNDARY=2

!
*****
!- PRINTINPUT Definition
!-----
PRINTINPUT KEYWORD=GEOMETRY

!
*****
!- OUTPUT Definition
!-----
OUTPUT COLUMNS=4, DELETETPREVIOUS=OFF, DTOUT=18000 s
OUTPUT BRANCH=BRAN-1, COLUMNS=4, DELETETPREVIOUS=OFF
OUTPUT COLUMNS=4, DELETETPREVIOUS=OFF, VARIABLE=( UL, UG, UD, USL, USG, USD,
AL, PT, DPT, BE, GA, PSI, ID, \  
RMTOT, BOU, MG, ML, MD, TM, VOL )

!
*****
```

```
!- TREND Definition
```

```
!-----  
TREND DELETEPREVIOUS=OFF, DTPLOT=10 s, TIME=0 s  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-1, SECTION=1, TIME=0 s,  
VARIABLE=PT BARA  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-1, SECTION=1, TIME=0 s,  
VARIABLE=GT  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-5, SECTION=3, TIME=0 s,  
VARIABLE=( QLT, QG, ACCLIQ )  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-4, SECTION=1, TIME=0 s,  
VARIABLE=AL  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-5, SECTION=2, TIME=0 s,  
VARIABLE=AL  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, TIME=0 s, VARIABLE=( NINTGR,  
VOLGBL )  
TREND BRANCH=BRAN-1, DELETEPREVIOUS=OFF, PIPE=PIPE-5, SECTION=1, TIME=0 s,  
VARIABLE=PT BARA
```

```
!  
|*****  
!- PROFILE Definition  
!-----  
PROFILE DELETEPREVIOUS=OFF, DTPLOT=400 s, VARIABLE=HOL  
!  
ENDCASE
```