Finding self-optimizing control variables using plant data

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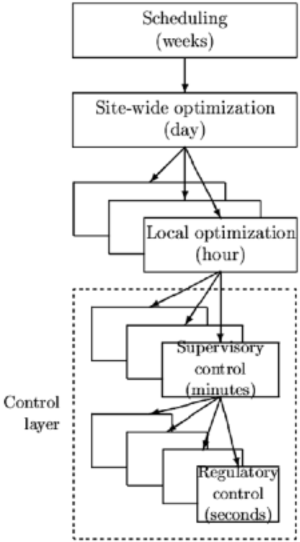
Main focus of the project

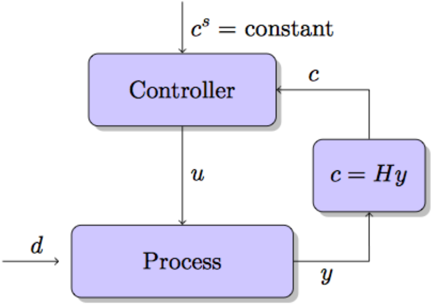
- Use historical plant data
- Combinations of measurements
- Self-optimizing variables

The bigger picture



- Different time scales
- Lower layers operate at a shorter time scale
- Bring the system back to optimal operation after a disturbance
- Self-optimizing variables

Self-optimizing variables



Achieve near optimal operation by controlling the control variables $\mathbf{c} = \mathbf{H}\mathbf{y}$ at constant set points.

How to find the optimal H

- Null space
- Exact Local method
- In this project: Data-based method

The data-based method

- Express the cost function in terms of measurements

$$\boxed{J(u,d)} \approx \boxed{J(y)}$$

The data-based method

- Express the cost function in terms of measurements

$$J(u,d) \approx J(y)$$

- Use available measurements data to estimate a quadratic cost function
- Use parameters from the estimated cost function to calculate H

$y_{data} = [y_1 \; y_2 \; \dots \; y_n]$ $\Longrightarrow$ Estimation $\Longrightarrow$ Quadratic cost function, J(y)

$\Downarrow$

H-matrix

The data-based method

- Express the cost function in terms of measurements

$$\boxed{\text{J(u,d)}} \approx \boxed{\text{J(y)}}$$

Approximated cost function around the nominal point:

$$J(\Delta u, \Delta d) \approx J^* + \begin{bmatrix} J_u^* & J_d^* \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \Delta u^T & \Delta d^T \end{bmatrix} \begin{bmatrix} J_{uu}^* & J_{ud}^* \\ J_{du}^* & J_{dd}^* \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix}$$

Linearized measurement model

$$\Delta y = \tilde{G}^y \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix}$$

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Linearized measurement model

$$\Delta y = \tilde{G}^y \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix}$$

or

The data-based method

- Express the cost function in terms of measurements

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Linearized measurement model

$$\Delta y = \tilde{G}^y \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix} \qquad \text{or} \qquad \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix} = \left[\tilde{G}^y\right]^\dagger \Delta y$$

The cost function as a function of measurements

- Express the cost function in terms of measurements

$$\boxed{\text{J(u,d)}} \approx \boxed{\text{J(y)}}$$

Approximated cost function around the nominal point:

$$J(\Delta y) = J^* + \underbrace{\begin{bmatrix} J_u^* & J_d^* \end{bmatrix} \left[\tilde{G}^y\right]^\dagger}_{J_y^*} \Delta y + \frac{1}{2}\Delta y^T \underbrace{\left[\tilde{G}^y\right]^{\dagger T} \begin{bmatrix} J_{uu}^* & J_{ud}^* \\ J_{du}^* & J_{dd}^* \end{bmatrix} \left[\tilde{G}^y\right]^\dagger}_{J_{yy}^*} \Delta y$$

$$= J^* + J_y^* \Delta y + \frac{1}{2}\Delta y^T J_{yy}^* \Delta y$$

Look at J^*_{yy}

$$J^*_{yy} = \left[\tilde{G}^y\right]^{\dagger T} \begin{bmatrix} [J^*_{uu} \quad J^*_{ud}]\tilde{G}^{y\dagger} \\ [J^*_{du} \quad J^*_{dd}]\tilde{G}^{y\dagger} \end{bmatrix}$$

Look at J^*_{yy}

$$J^*_{yy} = \left[\tilde{G}^y\right]^{\dagger T} \begin{bmatrix} [J^*_{uu} & J^*_{ud}]\tilde{G}^{y\dagger} \\ [J^*_{du} & J^*_{dd}]\tilde{G}^{y\dagger} \end{bmatrix}$$

It can be shown that with

$$H = [J^*_{uu} J^*_{ud}]\tilde{G}^{y\dagger}$$
$$HF = 0$$

Like in the null space method

Look at J^*_{yy}

$$J^*_{yy} = \left[\tilde{G}^y\right]^{\dagger T} \begin{bmatrix} [J^*_{uu} & J^*_{ud}]\tilde{G}^{y\dagger} \\ [J^*_{du} & J^*_{dd}]\tilde{G}^{y\dagger} \end{bmatrix}$$

$$\boxed{H \equiv \left[J^*_{uu} \quad J^*_{ud}\right] \left[\tilde{G}^y\right]^\dagger}$$

$$J^*_{yy} = \left[\tilde{G}^y\right]^{\dagger T} \begin{bmatrix} & H & \\ [J^*_{du} & & J^*_{dd}]\tilde{G}^{y\dagger} \end{bmatrix} \rightarrow \left[\tilde{G}^y\right]^T J^*_{yy} = \begin{bmatrix} & H & \\ [J^*_{du} & & J^*_{dd}]\tilde{G}^{y\dagger} \end{bmatrix}$$

Only interested in the H-matrix

$$[G^y \quad 0_{n_y \times n_d}]^T \; J^*_{yy} = \begin{bmatrix} H \\ 0_{n_y \times n_d} \end{bmatrix}$$

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In Summary:
If J^*_{yy} and G^y is known, we can calculate the optimal measurement combination H

Summary

1. Fit cost function to measurements

$J = J^* + J_y^* \Delta y + \frac{1}{2} \Delta y^T J_{yy}^* \Delta y$

2. Find the measurement gain

$G_y = \frac{\Delta y}{\Delta u}$

3. Determine H

$[G^y \quad 0_{n_y \times n_d}]^T \; J_{yy}^* = \begin{bmatrix} H \\ 0_{n_y \times n_d} \end{bmatrix}$

4. Control

$c = Hy$

Obtaining J^*_{yy}

- Plant measurement data: $y_{data} = [y_1 \; y_2 \; \dots y_n]$
- Measurements of the cost function: $J = [J_1 \; J_2 \; \dots J_n]$

Augmenting the data with two measurements, y_1 and y_2:

$$\boxed{y_{aug} = [y_1 \; y_2 \; y_1^2 \; y_1 y_2 \; y_2^2]}$$

Using Partial Least Square regression to predict the model $J = [1 \; y_{aug}^T]\beta$

In the case with two measurements

$$J = \begin{bmatrix} 1 & y_{aug}^T \end{bmatrix} \beta$$

Or written out:

$$J = \beta_1 + y_1\beta_2 + y_2\beta_3 + y_1^2\beta_4 + y_1y_2\beta_5 + y_2^2\beta_6$$

Or on a quadratic form

$$J = \beta_1 + \begin{bmatrix} \beta_2 & \beta_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} 2\beta_4 & \beta_5 \\ \beta_5 & 2\beta_6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

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Generating data

- Biodiesel plant in Chemcad
- Evaporator process
- Two distillation columns in series

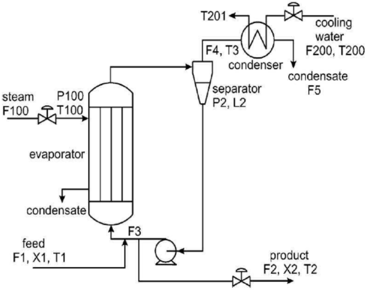

Figure : Flowsheet of the evaporator process

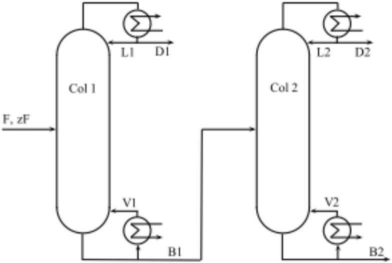

Figure : Flowsheet of the distillation columns in series

So far . . .

- Understanding the method to be used
- Two Matlab scripts that generates data for the evaporator process and the two distillation columns in series.
- A matlab script to use the data-based method that is *almost* working.
- Learned how to simulate in Chemcad

Next . . .

- Get Chemcad to work with Matlab
- Debug the Matlab script for the data-based method

Once a H-matrix is found:

- Control the process with the calculated H-matrix.
- Find the truly optimal operation.
- Find the H-matrix with the exact local method.
- Compare the losses.

Questions?

$$\begin{bmatrix} G^y & 0_{n_y \times n_d} \end{bmatrix}^T J_{yy}^* = \begin{bmatrix} H \\ 0_{n_y \times n_d} \end{bmatrix}$$

In Summary:
If J_{yy}^* and G^y is known, we can calculate the optimal measurement combination H