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Advanced Control
Robust Control of Pipeline-riser Slugging

A thesis submitted in fulfillment
of the requirements for the degree of Sivil Ingeniør

by

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Preface

This assignment was given to me by professor Sigurd Skogestad, he was also one of the supervisors. The other supervisor were the Ph.D. student Espen Storkaas. Professor Morten Hovd agreed to be the one responsible for the assignment. I would like to thank these three individuals for excellent supervision and a challenging and interesting diploma assignment.

Trondheim, Norway, June 2004 BJØRGULF HAUKELIDSÆTER

Abstract

The idea behind this thesis was to control slugging in riser pipelines using topside measurements. These measurements are not particularly suited for SISO control, so a multivariable H_∞ control design was proposed. Designing advanced control schemes for multi-phase systems is no trivial task as the multi-phase flow is described by a partial differential equation. However, Espen Storkaas and Sigurd Skogestad (Storkaas & Skogestad 2004) have made a simplified model of the slug-flow in riser pipelines. With this simplified model came the opportunity for advanced control design, provided the simplified model was valid for such undertakings. It was found that such is the case and a MISO H_∞ controller using the pressuredrop DP over the choke and the volumetric flow $\dot{Q}_{mix}$ through the choke was designed. The pressuredrop was chosen to be the main controlled variable to ensure process gain. The stabilzation of slugging in OLGA 2000 were the test that validated the controller and the simplified model. The basis for the design were many simpler control designs amongst them a cascade design using the same outputs. The cascade design is another suitable control solution, but it was found that the MISO H_∞ controller was a better control solution. The H_∞ controller achieved nominal performance and robust stability and it was found that it could handle 85% of uncertainty in the actuator before going unstable. Other multivariable designs were inspected and it was found that there are many possible control structures. The outputs considered in this thesis were DP, $\dot{Q}_{mix}$, mass flow $\dot{W}_{mix}$, gas and liquid holdup α_{gas} and $\alpha_{liq}$and the density ρ_{mix}.

Chapter 1

Introduction

The control of slugging in risers has not always been considered a control problem. The reason for this was the pressure measurement at the bottom, being suited for control. Controllers made with this output achieved stability. If this output is not available, the problem becomes more difficult and the need to regard it as a control problem arises. In the absence of the pressure measurement downside, control must be achieved with the less favorable outputs, in terms of controllability, topside. These outputs include the pressuredrop over the choke, the volumetric flow through the choke, the mass flow through the choke, the density of the fluid through the choke and the liquid- or the gas holdup. None of these are particularly good outputs for SISO control. They all have different negative points like RHP zeros and small zeros close to the imaginary axis.

This fact introduces the need for more advanced control schemes. The sad fact then is the limitations due to the very complex and large (the number of states) PDE model that describes the multi-phase flow and the slug behavior in the riser. Design of advanced control schemes in models of complexity this high is a difficult problem (Storkaas & Skogestad n.d.). To overcome it, a simplified bulk model that describes the slug-flow has been developed by Espen Storkaas and Sigurd Skogestad (Storkaas & Skogestad 2004). This simplified model makes the design of advanced control schemes a much easier problem. The behavior and dynamics of the model has been compared and verificated to the industrial renowned OLGA model and the similarities are apparent (Storkaas & Skogestad 2004). The only matter then is whether the simplified model describes the slug-flow well enough for advanced control design? This question will be answered in this thesis. A H_∞ controller that uses some of the unfavorable outputs topside of the riser will be designed. While other simpler control configurations may control the slugs in small riser systems with very limited disturbances, this one will be applicable for a broader range of plant perturbations. The controller will be tested on a OLGA model that describes the same system as the simplified model. This will be the verification and test of the simplified model and the control design.

A alternative control design, for antislug control, has been successfully implemented. This is a cascade PI design. This thesis will therefore answer another important question. Can a advanced controller that uses measurements topside outperform the cascade controller?

Chapter 2

Theory

In the theory part what is needed for the advanced H_∞ control scheme and the analysis of the design will be developed. The sensitivity functions, why they are important for control and what the minimizing of them implies will be addressed. Next a general structure for control, called P, will be developed. This structure contributes a transfer function from all inputs to all output, the structure from which the H_∞ controller is found. The closed loop system with and without uncertainty and perturbations of the nominal model will be developed. The closed loop system with uncertainty and perturbation of the nominal model will be the basis of the robustness analysis of the controller. Thereafter a small treatise about scaling will be presented. Next the mixed sensitivity stacked H_∞ problem will be formulated, a formulation which will be the one used in the thesis. Finally the criteria for robust control will be presented.

2.1 The sensitivity functions S, T and KS

In this section the sensitivity functions will be developed. They will be very important later in the design of the H_∞ controller. The derivation of the sensitivity functions are as follows. The plant model is written as (Skogestad & Postlethwaite 2001)

$$y = Gu + G_d d \tag{2.1}$$

The control signal u is

$$u = K\left(r - y - n\right) \tag{2.2}$$

The expression for the control signal is substituted into the expression for the plant

$$y = GK\left(r - y - n\right) + G_d d \tag{2.3}$$

The closed loop response becomes (Skogestad & Postlethwaite 2001)

$$\begin{aligned} y &= \underbrace{\left(I + GK\right)^{-1} GK}_{T} r + \underbrace{\left(I + GK\right)^{-1}}_{S} G_d d - \underbrace{\left(I + GK\right)^{-1} GK}_{T} n \\ y &= Tr + SG_d d - Tn \end{aligned} \tag{2.4}$$

The control error is

$$e = y - r$$

The relation between S and T is given as

$$S + T = \left(I + GK\right)^{-1} + GK\left(I + GK\right)^{-1} = \left(I + GK\right)\left(I + GK\right)^{-1} = I \tag{2.5}$$

The expression for the plant y is substituted into the expression for the control error e

$$e = -Sr + SG_d d - Tn \tag{2.6}$$

The expression for the plant input u then becomes (Skogestad & Postlethwaite 2001) if we substitute e into equation 2.2

$$u = KSr - KSG_d d - KSn \tag{2.7}$$

Then it becomes clear that by putting bounds on KS a bound is put on u, by putting bounds on T, the sensitivity to noise n is restricted and by putting bounds on S, good disturbance rejection is achieved.

2.2 MIMO Systems

If outputs are put together into a multivariable system the new system get a set of properties of whom many is not shared by the separate outputs it consist of. If one of the outputs for instance has a RHP zero, this zero will disappear in the multivariable system as the total system does not have this zero anymore. The RHP zero will have a particular direction given to it. In this direction the RHP zero comes into effect in the multivariable system. In SISO systems RHP zeros and RHP poles put a restriction on the bandwidth. In a multivariable system this will only be a problem if the directions of the two coincide. This is the strength of the multivariable design, being able to make a whole new system from several outputs. More about multivariable systems and multivariable control systems can be found in (Skogestad & Postlethwaite 2001).

2.3 The systems P, N, M and F

P is a system that is open around the controller K. The control input v is a output in the P system and the control output u is a input. The system P is given as

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{2.8}$$

where v is the actuator input, u is the actuator output, z is the exogenous outputs, a vector that contains all the output signals and w is the exogenous inputs, a signal that contains all the input signals. The signal w often consists of the disturbances, the reference signals and the measurement noise.

N is a system that is closed around the controller K. To close the loop one substitutes out v and u from the equation 2.8 with the relation $u = Kv$. The resulting system N is the transfer function from the exogenous inputs to the exogenous outputs, $z = Nw$. N is given by what is called a lower fractional linear transformation as

$$N = P_{11} + P_{12}K\left(I - P_{22}K\right)^{-1} P_{21} \tag{2.9}$$

The equation given in 2.9 is actually a small part of a larger system given as

$$\begin{bmatrix} y_\Delta \\ z \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} u_\Delta \\ w \end{bmatrix} \tag{2.10}$$

where N_{22} is the matrix N given in equation 2.9. The system N_{22} is a system without any uncertainty, called the nominal system, whereas the system described in equation 2.10 is a system that incorporates uncertainty and model perturbation. The system described in equation 2.10 forms the basis for the robustness analysis of the system. y_Δ is the input to the model perturbation block Δ that represents a perturbation of the nominal model. u_Δ is the output of the perturbation block and is to be added to the control signal u to form uncertainty in the control signal. If the loop around the uncertainty block is closed the system F is found. F is given by what is called a upper linear fractional transformation of the uncertain system N. F is found from equation 2.10 by making the substitution $u_\Delta = \Delta y_\Delta$. F is given as

$$F = N_{22} + N_{21}\Delta\left(I - N_{11}\Delta\right)^{-1} N_{12} \tag{2.11}$$

where Δ is a perturbation of the model. F is important in the robustness analysis of the system as it forms the basis for determining robust performance. M is actually the same as N_{11} in equation 2.10 and is the transfer function from the input y_Δ to the output u_Δ of the perturbation block Δ. M forms the basis for determining robust stability of the system. Note that apart from the nominal model N_{22}, N_{11} is the only source of instability in equation 2.11.

2.4 Scaling

In the advanced design that are to be done a scaled model is needed. It is important for comparisment that the same scaling is used for all the controllers that are to be designed. In addition it is important to use realistic values for the different scales, because the value of the scales will affect the peak values for the different sensitivity functions. The scaling of the system is a important part of the design process and should not be dealt with lightly. The scalings for the control signal, for the measurements and for the disturbances has to be defined. The states is just internal variables in the input output model, so they don't have to be scaled. The scale D_x is just included for completeness. The new scaled signals are defined as follows

$$x = D_x x_s;\; y = D_y y_s;\; u = D_u u_s;\; d = D_d d_s \tag{2.12}$$

where D is a matrix containing the largest deviation from the point of operation and the subscript s means the scaled signal. The new scaled system is given as

$$\dot{x}_s = D_x^{-1} A D_x x_x + D_x^{-1} B D_u u_s + D_x^{-1} E D_d d_s \tag{2.13}$$
$$y_s = D_y^{-1} C D_x x_s + D_y^{-1} D D_u u_s + D_y^{-1} F D_d d_s \tag{2.14}$$

This scaled system form the basis for the H_∞ design. It is important that all the signals are scaled between 0 and 1 for the weight of the signals in the design to be meaningful. To begin with the $MaxDev$ parameter from the tables 3.25 and 3.27 is chosen. In the OLGA simulations the scalings for the outputs will be adjusted with the signals in that model.

2.5 Mixed Sensitivity H_∞ design

A controller with certain properties regarding the peaks of the sensitivity functions S and T is the goal of the design. In addition large inputs from the controller to the process should be penalized, through a bound on KS. To achieve this the stacked H_∞ problem (Skogestad & Postlethwaite 2001) is formulated

$$\min_K \|N(K)\|_\infty , N = \begin{bmatrix} W_u KS \\ W_T T \\ W_P S \end{bmatrix} \tag{2.15}$$

where K is a stabilizing controller, W_u is a bound on KS, W_T is a bound on T and W_P is a bound on S. Bounds are put on S to increase performance for some selected outputs. The bounds on T is for robustness and to avoid sensitivity to noise. Lastly bounds are put on KS to restrict the input to the controller. The closed loop nominal system is given as

$$z = Nw \tag{2.16}$$

The H_∞ controller is to minimize the norm from w to z. The controller should fulfill certain criteria in order to ensure robust control. These criteria are given next.

2.5.1 Well posedness of the problem formulation

To be able to solve the minimization problem formulated in equation 2.15 the input P has to meet certain requirements if one is to find a solution. The generalized plant P can be expressed as a state space realization (Skogestad & Postlethwaite 2001)

$$P \overset{s}{=} \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{2.17}$$

To be able to solve the H_∞ minimization problem the following criteria has to be achieved (Skogestad & Postlethwaite 2001)(p 363)

$$(A, B_2, C_2) \text{ is stabilizable and detectable}$$

$$D_{12} \text{ and } D_{21} \text{ has full rank}$$

$$\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \text{ has full column rank for all } \omega$$

$$\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} \text{ has full row rank for all } \omega$$

$$D_{11} = 0 \text{ and } D_{21} = 0$$

More about the different requirements and other possible requirements and what they imply can be found in (Skogestad & Postlethwaite 2001).

2.6 Robustness criteria

It is critical that the choice of appropriate weights are made so as to achieve the performance and robustness properties required. For the controller to work satisfactory robust stability and preferably robust performance and avoidance of saturation in the actuators is required. These requirements are (Skogestad & Postlethwaite 2001) (p.303)

$$NS \overset{def}{\Leftrightarrow} N \text{ is internally stable} \tag{2.18}$$

$$NP \overset{def}{\Leftrightarrow} \|N_{22}\|_\infty < 1; \text{ and } NS \tag{2.19}$$

$$RS \overset{def}{\Leftrightarrow} F \text{ is stable } \forall\Delta, \|\Delta\|_\infty \le 1; \text{ and } NS \tag{2.20}$$

$$RP \overset{def}{\Leftrightarrow} \|F\|_\infty < 1, \quad \forall\Delta, \|\Delta\|_\infty \le 1; \text{ and } NS \tag{2.21}$$

where Δ is the family of perturbations of the model. The perturbation block can be a real number or a complex one. To find robust performance the Δ structure representing the family of possible perturbations of the model must be included in the F structure and equation 2.21 must be achieved. Δ has to be stable and satisfy

$$\|\Delta\|_\infty \le 1 \tag{2.22}$$

Another name for the norm given in equation 2.19 is γ (gamma). γ will be used throughout the thesis.

2.6.1 Another way of expressing robust stability

There is another easier way to find out if robust stability is satisfied using the infinity norm. From equation 2.11 it can see that with nominal stability there is only one source for instability, the term

$$(I - N_{11}\Delta)^{-1} \tag{2.23}$$

Therefore by inspection of N_{11} instead F robust stability can be determined. The criteria is given as

$$\|N_{11}\|_\infty < 1 \tag{2.24}$$

This criteria will be used in place of the one given in equation 2.20.

2.6.2 The uncertainty weight W$_I$

To find out whether robust stability is achieved a given amount of uncertainty is to be added to a uncertain signal in the system. The uncertain signal were chosen to be the control signal u. The uncertainty weight is normally given as (Skogestad & Postlethwaite 2001) (p 268)

$$W_I = \frac{\tau s + r_0}{\left(\frac{\tau}{r_\infty}\right) s + 1} \tag{2.25}$$

where s is the Laplace operator, r_0 is the low frequency uncertainty, $\frac{1}{\tau}$ is the frequency where the relative uncertainty reaches 100% and r_∞ is the uncertainty at higher frequencies. This is the kind of uncertainty weight that will be used.

Chapter 3

Model analysis and practical considerations

In this section everything needed for the different control designs will be developed. Also the model will be analyzed.

3.1 Expressions for S, T and L

The model for the main controller is a MISO model, with one actuator and two measurements. It will only have a reference for one of the outputs, the other output will be used for feedback information only. The sensitivity functions and the loop transfer function for it have to be developed.

If the inner loop with the added measurement is considered a part of a new G', the outer open loop transfer function L, from u_1 to y_1, may be obtained. First the expression for the inner loop from y_2 to u_1 is derived

$$y_2 = (1 - G_2K_2)^{-1}\, G_2u_1 \tag{3.1}$$

Then the expression for the outer loop from y_1 to u_1 is derived and the expression for y_2 is substituted into it

$$y_1 = G_1\,(u_1 + K_2y_2) = G_1\left(1 + K_2\,(1 - G_2K_2)^{-1}\, G_2\right) u_1 \tag{3.2}$$

The new G' is then given as

$$G' = \left(G_1 + G_1K_2\,(1 - G_2K_2)^{-1}\, G_2\right) \tag{3.3}$$

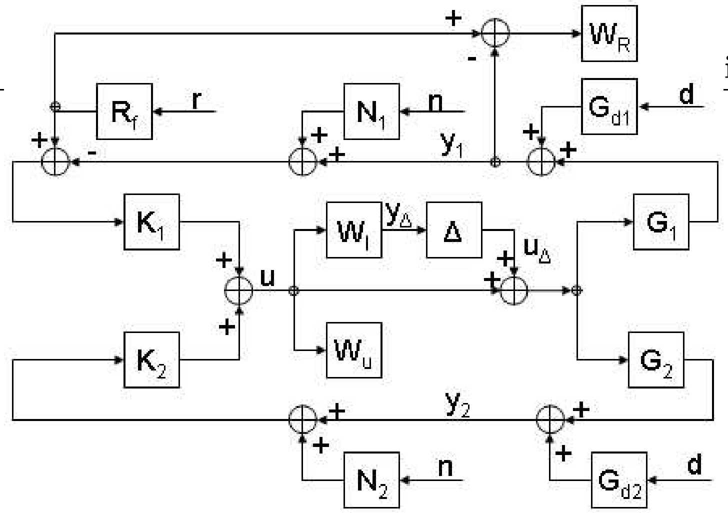

Figure 3.1: Block diagram of the system with uncertanties and perturbation.

The expression for the L from u_1 to y_1 becomes

$$L = G' K_1 \tag{3.4}$$

From the expression of L we can obtain S and T

$$S = (1 + L)^{-1} = (1 - G_2 K_2)\,(1 + G_1 K_1 - G_2 K_2)^{-1} \tag{3.5}$$

$$T = 1 - S = G_1 K_1\,(1 + G_1 K_1 - G_2 K_2)^{-1} \tag{3.6}$$

3.2 Choice of H_∞ control design

The H_∞ mixed sensitivity design is chosen for the robust control design. The reason for this is the simplicity and intuitivity of the design. It is a loop shaping technique were the singular values of the transfer functions for the sensitivity function S or its complementary function T, or both, is shaped over all frequencies. The transfer function KS may also be shaped. In the theory part these transfer functions were developed, and their importance for the overall performance of the system were examined. By shaping these transfer functions' singular values they may be bounded below a given bound, usually 2, to ensure robust stability. The boundedness of the transfer function KS ensure bounded control inputs. A aggressive controller is not wanted, as the actuators then may go into saturation. The controller has a small area of operation, a stationary unstable point. The actuator should operate close to and around this point. In the mixed sensitivity design all these requirements are expressed through weight functions. It's a flexible and straightforward design where most of the effort lies in finding appropriate weights for the transfer functions.

3.2.1 Mixed Sensitivity H_∞ problem formulation

The formulation of the H_∞ design forms the basis for the matrices to be developed next. The formulation is given as

$$\min_K \|N(K)\|_\infty, N = \begin{bmatrix} W_u K S \\ W_R S \end{bmatrix} \tag{3.7}$$

A weight on the control output u to avoid heavy actuator usage and a weight on the control error $(r - y_1)$ to achieve integral action and good reference tracking properties in the controller is chosen. The specific weight on the model output y_1 is disregarded as this variable is included in the weight of the control error. This proved to be a good problem formulation for our slugcontrol problem. Next the matrices that forms the basis of the design is developed.

3.3 Expressions for P, N and F

3.3.1 Derivation of P and N_{22} for control synthesis

The expressions for the systems P, N and F have to be calculated for the robustness analysis of the main control design. First the P that are to be used in the controller design is derived. This is the nominal design, so it will be without uncertainty weights. Later another P with uncertainty must be derived. This structure will form the basis for the derivation of the N for the robustness analysis. In the derivation of P the exogenous inputs w and exogenous outputs z have to be defined. The exogenous inputs w were decided to be the reference signal r for the pressure DP over the choke, the disturbance d, the measurement noise n. The output u from the controller is considered a input in P. The inputs are gathered in a vector as $\begin{bmatrix} d & r & n & u \end{bmatrix}^\top$, or in short as $\begin{bmatrix} w & u \end{bmatrix}^\top$. As outputs in the design, the weights W_u, W_R, for the signals u and $(r - y_1)$ were decided upon. The input to the controller, called v, is also considered a output in P. Lastly a weight N_n for the measurements noise was included. This is done to weigh the measurement noises' impact on the system. Then the matrix P can be obtained

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} W_u u \\ W_R (r - y_1) \\ v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & W_u \\ -W_R G_{d,1} & W_R & 0 & -W_R G_1 \\ -G_{d,1} & 1 & -N_{n,1} & -G_1 \\ G_{d,2} & 0 & N_{n,2} & G_2 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{3.8}$$

Note that there isn't any reference signal for y_2 therefore the terms regarding this variable are positive. The matrix in equation 3.8 is the one that forms the basis for the controller design. Matlab's mu toolbox is used to synthesize the controller. The matrix P is given in to the function $hinfsyn()$ in Matlab to develop the controller. To inspect the properties of the controller the closed loop system N has to be developed. N can be expressed by P as given in equation 2.9. The different elements in the equation can be found by inspection of equation 3.8. The elements P_{11}, P_{12}, P_{21}, P_{22} are

$$\begin{aligned} P_{11} &= \begin{bmatrix} 0 & 0 & 0 \\ -W_R G_{d,1} & W_R & 0 \end{bmatrix};\ P_{12} = \begin{bmatrix} W_u \\ -W_R G_1 \end{bmatrix} \\ P_{21} &= \begin{bmatrix} -G_{d,1} & 1 & -N_{n,1} \\ G_{d,2} & 0 & N_{n,2} \end{bmatrix};\ P_{22} = \begin{bmatrix} -G_1 \\ G_2 \end{bmatrix} \end{aligned} \tag{3.9}$$

The next step is to obtain N, the closed loop nominal control system, from the transformation $u = Kv$ and equation 2.9 thereby closing the loop around the controller K. As mentioned earlier N in equation 2.9 actually is N_{22} in equation 2.10, therefore N_{22} will be used in place of N

$$\begin{aligned} N_{22} = &\begin{bmatrix} 0 & 0 & 0 \\ -W_R G_{d,1} & W_R & 0 \end{bmatrix} \\ &+ \begin{bmatrix} W_u \\ -W_R G_1 \end{bmatrix} K \left(1 - \begin{bmatrix} -G_1 \\ G_2 \end{bmatrix} K\right)^{-1} \begin{bmatrix} -G_{d,1} & 1 & -N_{n,1} \\ G_{d,2} & 0 & N_{n,2} \end{bmatrix} \end{aligned} \tag{3.10}$$

$$N_{22} = \begin{bmatrix} -W_u SZ\left(K_1 G_{d,1} - K_2 G_{d,2}\right) & -W_R S\left(G_{d,1} + G_1 K_1 Z G_{d,2}\right) \\ W_u SZK_1 & W_R S \\ -W_u SZ\left(K_1 N_{n,1} - K_2 N_{n,2}\right) & W_R SZ\left(G_2 K_1 N_{n,1} - G_1 K_1 N_{n,2}\right) \end{bmatrix}^{\top} \tag{3.11}$$

where Z is the expression $(1 - G_2 K_2)^{-1}$ and S is the sensitivity function given in equation 3.5.The infinity norm of the matrix described in equation 3.11 forms the basis of nominal performance analysis as given in equation 2.19.

3.3.2 Derivation of N and F for robustness analysis

The input signals are $\begin{bmatrix} u_\Delta & d & r & n \end{bmatrix}^\top = \begin{bmatrix} u_\Delta & w \end{bmatrix}^\top$ where u_Δ the output from the perturbation block Δ, which is considered a input to the system under the N structure. The output signals are as before except for y_Δ which is the input to the perturbation block Δ, considered a output under the N structure, which is open around the perturbation block. The transfer function from the output signals to the inputsignals are

$$\begin{matrix} y_\Delta \\ z \end{matrix} = \begin{bmatrix} y_\Delta \\ W_u u \\ W_R\left(r - y_1\right) \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} u_\Delta \\ w \end{bmatrix} \tag{3.12}$$

where N_{11} is the transfer function from y_Δ to u_Δ. The resulting matrix is found from the relation given in equation 3.12. The whole matrix will not be written out, by the different terms are given as

$$N_{11} = -W_I SZ\left(G_1 K_1 - G_2 K_2\right) \tag{3.13}$$

$$N_{12} = \begin{bmatrix} -W_I SZ\left(K_1 G_{d,1} - K_2 G_{d,2}\right) \\ W_I SZK_1 \\ -W_I SZ\left(K_1 N_{n,1} - K_2 N_{n,2}\right) \end{bmatrix}^\top \tag{3.14}$$

$$N_{21} = \begin{bmatrix} -W_u SZ\left(G_1 K_1 - G_2 K_2\right) \\ -W_R S\left(G_1 + G_1 K_1 Z G_2\right) \end{bmatrix} \tag{3.15}$$

where W_I is the uncertainty weight and Z is short for the expression $(1 - G_2 K_2)^{-1}$. The elements N_{11}, N_{12}, N_{21} and N_{22} is needed for the derivation of the final closed loop perturbed system with uncertainty, the matrix F. The final matrix N_{22} that complete F through the relation given in equation 2.11 is the same as the one given in equation 3.11. F is now easily found. The expression is not derived as it is calculated numerically in Matlab using the elements presented.

3.4 The simplified slug model

The model used to describe the riser slugging behavior is not a partial differential equation system, but a simplified bulk model developed by Espen Storkaas and Sigurd Skogestad. The model is described in (Storkaas & Skogestad 2004). The model has only three states, the mass of gas behind the slug, the mass of liquid in the slug and the mass of the gas in front of the slug. Riserslugging as described by the simplified model can be seen in figure

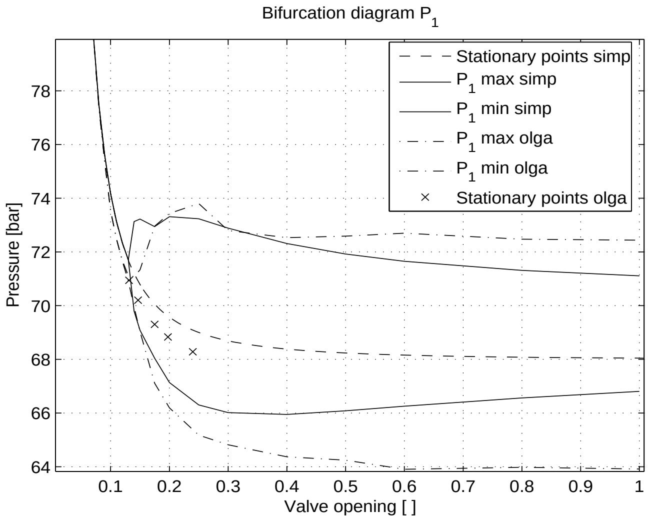

Figure 3.2: The unstable stationary points are shown between the upper and lower values for the downside pressure during slugging.

3.4 (Storkaas & Skogestad 2004).The model is stable up to a choke opening of $u = 0.13$ (Storkaas & Skogestad 2003). This becomes clear when the bifurcation diagrams in figure 3.2 and figure 3.3 are studied. After about $u = 0.13$ the plant enters the slug flow regime. The dotted line that continues in the unstable area is a series of unstable stationary points. The control design will be based on two such points.

3.4.1 Model mismatch

In this thesis the simplified slug-flow model will be used for the control system design. This model is different from the reknown OLGA model at some points, due to simplification. These points has to be considered if a successful design is to be achieved. The final test and validation of the control design is successful stabilization of slug-flow in a OLGA simulation.

Prediction of gain in the choke

In the simplification of a model there is always model mismatch. Later in the thesis two points of operation are to be chosen. These will be the points upon which the control design will be based. In the design of the controllers in the simplified model, model mismatch becomes a concern at one of these points, the point farthest out on the bifurcation diagram. This point will be referred to as the low pressure operating point.

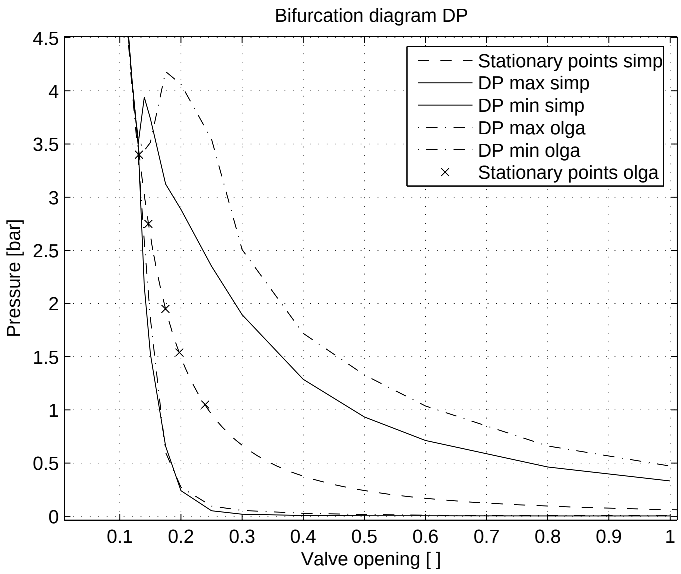

Figure 3.3: The unstable stationary points are shown between the upper and lower points of the pressuredrop over the choke during slugging.

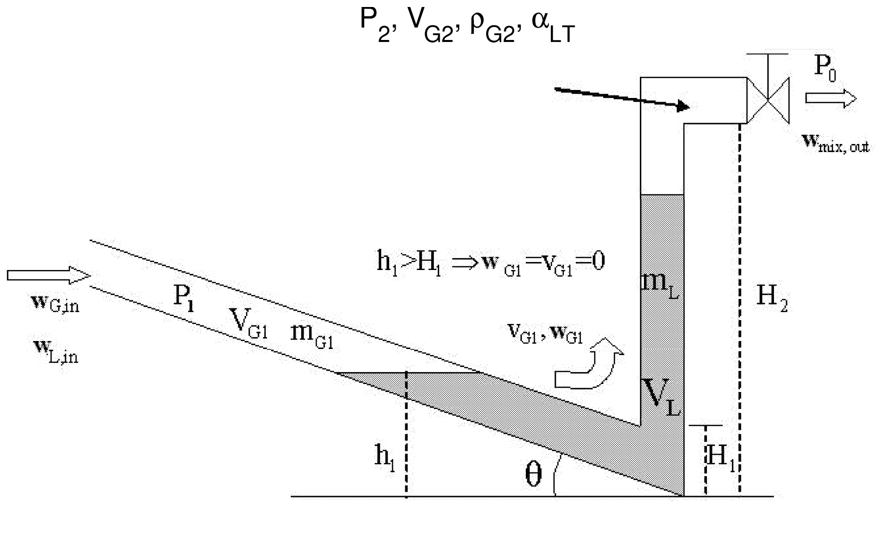

Figure 3.4: Riser slugging as modelled in the simplified model.

The other point will be referred to as the high pressure operating point. At the low pressure operating point the simplified model predicts too low a gain in the actuators. To overcome this problem a constant k is multiplied with the control signal. This constant was found to be about 2 at the low pressure operating point.

Amplitudes of the pressuredrop over the choke during slugging

If figure 3.3 is studied closely it becomes apparent that the amplitude of the pressuredrop are larger in OLGA than in the simplified model. This is due to model mismatch as the simplified model is a simplification of the OLGA model. But also because the simplified model is tuned around the point where $u = 0.13$, the point where the slugging start. This model mismatch can be observed in the other bifurcation diagram as well.

3.5 The operating points of the controllers

Two points of operation were chosen. The two points can be found along the stippled line in figure 3.3, called unstable stationary points. At the two points there are slugging in the system, but through active manipulation of the valve around the points, the slugging can be removed altogether. The model was linearized around the points that are characterized with the stationary values given as

$$\begin{bmatrix} P_1 \\ DP \\ \rho_T \\ \dot{Q}_{mix} \\ \dot{W}_{mix} \\ \alpha_{Lm} \end{bmatrix} = \begin{bmatrix} 70.0438 \\ 1.9403 \\ 541.6647 \\ 0.0166 \\ 9.002 \\ 0.9598 \end{bmatrix} \tag{3.16}$$

at the high pressure point of operation. And

$$\begin{bmatrix} P_1 \\ DP \\ \rho_T \\ \dot{Q}_{mix} \\ \dot{W}_{mix} \\ \alpha_{Lm} \end{bmatrix} = \begin{bmatrix} 68.6887 \\ 0.6671 \\ 537.5202 \\ 0.0167 \\ 9.002 \\ 0.9598 \end{bmatrix} \tag{3.17}$$

at the low pressure point of operation. The points of linearization in table 3.16 and table 3.17 will be used as setpoint for the different controllers in the thesis. The opening of the choke for the two points were

$$u = 0.175 \tag{3.18}$$

at the high pressure point of operation. And

$$u = 0.3 \tag{3.19}$$

at the low point of operation. The opening of the choke is a number between 0 and 1, where 0 means that the valve is closed. The poles of the model at the two points of linearization were

$$p = \begin{bmatrix} -1.9613 & 0.0007 \pm 0.0063i \end{bmatrix}^\top \tag{3.20}$$

at the high pressure point of operation. And

$$p = \begin{bmatrix} -2.0154 & 0.0038 \pm 0.010i \end{bmatrix}^\top \tag{3.21}$$

at the low pressure point of operation, where i is the imaginary operator. There are clearly different bandwidth requirements for the two models when considering their RHP poles. The higher operating point's RHP poles are far quicker than for the lower one.

3.5.1 Bandwidth limitations from RHP poles

Both linearized plants has a complex conjugated pole in the right half plane. This put a lower limit on the bandwidth for the sensitivity function T. (Skogestad & Postlethwaite 2001)(p. 185) states that $\omega_{BT} > 1.15\,|p|$. This means that a lower bound on the bandwidth of T is approximately ensured if

$$\begin{aligned} \omega_c &> 0.0073; \; p = 0.0007 \pm 0.0063i \\ \omega_c &> 0.0123; \; p = 0.0038 \pm 0.010i \end{aligned}$$

There are also bandwidth requirements due to RHP zeros. From (Skogestad & Postlethwaite 2001)(p. 175) we have the limitation on the bandwidth given as

$$\omega_B < \frac{z}{2} \tag{3.22}$$

where z is the RHP zero.

3.6 Controllability analysis

3.6.1 The measurements and their zeros

The different measurements or outputs are the upstream pressure P_1, the pressure difference over the valve (the actuator) DP, the density ρ_T of the mixed product that exits the valve, the massflow $\dot{W}_{mix}$ of the mixed product that exits the valve, the volumetric flow $\dot{Q}_{mix}$ of the mixed product that exits the valve and the liquid (and gas) holdup α_{Lm} at the exit of the riser. There is one downside measurement, the downside pressure P_1. The other measurements are topside measurements. All the measurements can be collected in a measurement vector

$$y = \left[P_1, DP, \rho_T, \dot{W}_{mix}, \dot{Q}_{mix}, \alpha_{Lm}\right]^\top \tag{3.23}$$

The downside measurement P_1 is not to be considered for the main control design. It is included for comparisment only. Only the topside measurements are to be considered.

There are zeros and poles associated with the different outputs. The measurements and the zeros associated with the high pressure operating point are

$$\begin{array}{cccccc} P_1 & DP & \rho_T & \dot{W}_{mix} & \dot{Q}_{mix} & \alpha_{Lm} \\ -0.0031 & 0.0143 & -0.0004 & -2.4725 & -1.4322 & 0.0038 \\ - & 0.9587 & 0.0041 & -0.0004 & -0.0029 & 6.3773e-11 \\ - & - & - & -0.0000 & -0.0004 & - \end{array} \tag{3.24}$$

The outputs and their value at the stationary point, their stationary gain and their deviation during slugging is given as follows

$$\begin{array}{ccccccc} & P_1 & DP & \rho_T & \dot{W}_{mix} & \dot{Q}_{mix} & \alpha_{Lm} \\ Value & 69.326 & 1.8843 & 541 & 9.002 & 0.0166 & 0.9598 \\ |G(0)| & 23.2578 & 21.8756 & 69.9961 & 2.5357e-7 & 0.0021 & 2.6196e-9 \\ MaxDev & 3.4075 & 1.5669 & 211.4038 & 4.7643 & 0.0066 & 0.0816 \end{array} \tag{3.25}$$

The selection of outputs looks poor considering that the downside measurement P_1 is not to be considered. In SISO control only $\dot{Q}_{mix}$ is usable and possibly DP if there is any bandwidth of operation for it, considering the bandwidth requirement in equation 3.22. ρ_T is obviously not suited for SISO control as there is no bandwidth of operation for it due to the RHP zero. $\dot{W}_{mix}$ is not usable due to the very low LHP zero, which could cause a problem with input constraints at low frequencies, because the steady-state gain is small (Skogestad & Postlethwaite 2001)(p. 181), and the near zero stationary gain. In addition a small LHP zero in an uncertain plant can pass over to the RHP. α_{Lm} is not suitable for SISO control for the same reasons as $\dot{W}_{mix}$.

The measurements and the zeros associated with them for low pressure operating point are

$$\begin{array}{cccccc} P_1 & DP & \rho_T & \dot{W}_{mix} & \dot{Q}_{mix} & \alpha_{Lm} \\ -0.0031 & 0.0127 & -0.0004 & -2.5237 & -1.4543 & 0.0038 \\ - & 1.0605 & 0.0041 & -0.0004 & -0.0029 & 5.4916e-11 \\ - & - & - & -0.0000 & -0.0004 & - \end{array} \tag{3.26}$$

The outputs and their value at the stationary point, their stationary gain and their deviation during slugging are given as follows

$$\begin{array}{ccccccc} & P_1 & DP & \rho_T & \dot{W}_{mix} & \dot{Q}_{mix} & \alpha_{Lm} \\ Value & 69.326 & 1.8843 & 541 & 9.002 & 0.0166 & 0.9598 \\ \lvert G(0) \rvert & 4.7140 & 4.4249 & 14.6524 & 8.1942e-8 & 4.5652e-4 & 4.5727e-10 \\ MaxDev & 5.0650 & 2.3979 & 429.3700 & 11.3187 & 0.0197 & 0.8655 \end{array} \tag{3.27}$$

There could be problems with $\dot{Q}_{mix}$ at the low pressure operating point due to the low steady state gain. DP is not possible to use in SISO control at the low pressure operating point considering the bandwidth requirements from the RHP zero.

In the thesis a multivariable controller will be made. For the multivariable design all of the arguments used against the outputs will become obsolete. The reason for this being the properties of the multivariable system being different for the outputs it consist of. If there are RHP zeros in one of the outputs for instance, they will disappear as the total system doesn't have any RHP zeros anymore. The arguments used above will only hold in SISO control.

3.7 Control structure

In the controllability analysis it was found that for SISO control the only outputs usable topside was DP and $\dot{Q}_{mix}$. These two were therefore chosen for the multivariable control design. There is only one actuator, the valve topside. It is impossible to control two outputs with one actuator, only one output can be satisfactory controlled. The pressuredrop was chosen to be the main controlled output. There are several reasons for this. The most important one being the process gain requirement. A certain pressuredrop over the choke is required to ensure process gain. Tight control of DP will ensure this process gain. Even at the low pressure operating point the steady state process gain of DP is

4.4249 as can be seen in table 3.27. $\dot{Q}_{mix}$ has a very low process gain. In addition there is a lot of noise in the volumetric flow measurement. The volumetric flow measurement will be used for feedback information only. This will be clear later when the P structure for the H_∞ problem is defined. The outputs are put together into a measurement vector as

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} DP \\ \dot{Q}_{mix} \end{bmatrix} \tag{3.28}$$

3.8 Practical considerations

The low pressure operating point is a very aggressive one that would never be considered for a real, full scale system. The reason for this is the low pressuredrop over the valve at the operating point, and the resulting low steady state process gain. The pressure over the choke will be about 0.7 bar at the point. In a real, full scale system the pressure over the choke shouldn't be below about 2 bar . The purpose for the analysis and design done at this operating point is robustness testing, controllability analysis and testing of the design.

3.9 How to choose the weights W_u, W_R and N

The choice of weight functions is critical for establishing the required robustness properties in the plant. The plant has two outputs. This is the pressure drop over the actuator and the volumetric flow through the actuator. The pressure drop over the actuator should be as low as possible, but not at the cost of stability. In general this will ensure that there is a large throughput through the actuator, or in other words as high a production as possible. The volumetric flow is used entirely for feedback information.

The difference between the pressuredrop measurement DP and the reference for it r is to be the input to the weight function for performance W_R. This weight also introduces integral action in the controller. The control signal is input to the weighting function W_u to limit the usage of the actuator. And lastly a weight N is put on the measurement noise. This weight will not be an output, but will be included in the output v, which is the input to the controller (a output in the P structure). The exogenous outputs z of the plant is then defined, in a P reference frame, as

$$z = \begin{bmatrix} W_u \\ W_R \end{bmatrix} \tag{3.29}$$

As a first choice for weight functions constant weights are chosen, not transfer functions. Later the weight functions can be extended to include transfer functions. The first choice for weights is

$$\begin{aligned} W_u &= 1 \\ W_R &= 0.5 \\ N &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.002 \end{bmatrix} \end{aligned} \tag{3.30}$$

How the weights are chosen will depend upon the design objectives. These objectives will be derived later in the thesis based upon the experience gained from the simpler designs.

3.10 Reference filtering

The reference of the controlled output is filtered with a first order filter $RF(s)$ to avoid saturation and steps in the controller. This makes the transition from one setpoint to another smooth. The filter used is given as

$$RF(s) = \frac{0.001}{s + 0.001} \tag{3.31}$$

The addition of the reference filter will help ensure better stability properties in the controlled plant, and it will enable the system to make larger steps in the setpoints.

3.11 About the simulations

3.11.1 The simplified model

There are made a lot of simulations in the thesis. Almost all the simulations have the same environment, except where otherwise stated. There are constant disturbances. The disturbances are mass of gas that enters the pipe $m_{gas,in}$, mass of liquid that enters the pipe $m_{liq,in}$ and the pressure P_0 downstream of the valve

$$d = \begin{bmatrix} \dot{m}_{gas,in} \\ \dot{m}_{liq,in} \\ P_0 \end{bmatrix} = \begin{bmatrix} 0.362\frac{kg}{s} \\ 8.64\frac{kg}{s} \\ 50\,\text{bar} \end{bmatrix} \tag{3.32}$$

The disturbances are the same at each operating point. These are also the disturbances that go into the linearization of the model. The reference is filtered to make setpoint changes more fluent, using the filter given in 3.31. There is only a time delay of 1 second in the measurements and no uncertainty is added to the control signal or the measurements. There is a saturation on the actuator that keeps it between 0 and 1, should it want to do otherwise. The simulations are done with the nonlinear simplified model, whereas the controllers are made with a linearization of the simplified model. The initial state of the simplified nonlinear slug-flow model is in the slugging regime, an unstable mode. The simulations in Matlab was done in Simulink with the $ode23t()$ method.

3.11.2 The OLGA 2000 model

The simulations done in OLGA 2000 had the same environment as the simplified simulations, except that there was no time delay added to the measurement. There is the sampletime in the link between OLGA 2000 and Matlab. This sampletime is 10 seconds, and should be considered a time delay. Apart from that the case was exactly the same as in the simplified simulations.

Chapter 4

Design considerations

The controllers presented in this chapter form the benchmark for the final control design.

4.1 The simple controllers

A set of simple P SISO controllers for the outputs $\dot{Q}_{mix}$ and P_1 were designed. It was not possible to design a SISO P controller for the output DP at any of the operation points.The simple controller for $\dot{Q}_{mix}$ weren't able to control the plant at the low pressure operating point. The simple controller from the output P_1 performed very well at both operating points. The controllers can be found in the appendices.

4.2 The cascade controllers

There are already successfully implemented cascade controllers for slug control. There are two controllers, one for each operating point. The controllers use the output $\dot{Q}_{mix}$ in the inner loop and the output DP in the outer loop. DP is the controlled variable. The first one, designed for the high pressure operating point is given as

$$K_{DP} = 9\left(1 + \frac{1}{40s}\right);\ K_{\dot{Q}_{mix}} = -0.0006\left(1 + \frac{1}{1000s}\right) \tag{4.1}$$

The second cascade controller, designed at the low pressure point is given as

$$K_{DP} = 15\left(1 + \frac{1}{30s}\right);\ K_{\dot{Q}_{mix}} = -0.0025\left(1 + \frac{1}{900s}\right) \tag{4.2}$$

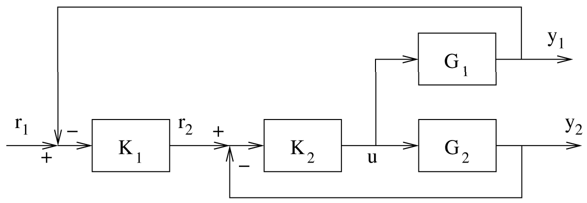

Figure 4.1: Block diagram of the cascade controller.

4.2.1 Peaks in sensitivity functions and robustness properties

For comparisment purposes the peaks in the sensitivity functions and the robustness properties of the cascade controllers have to be calculated. The block diagram in figure 4.1 (Storkaas & Skogestad 2003) are used in the derivation. The open loop transfer function from the main controlled output y_1 to it's reference r_1 is given as

$$L = G_1 K_2 \left(1 + G_2 K_2\right)^{-1} K_1 \tag{4.3}$$

With L the expressions for the sensitivity functions S, T and KS are easily developed. The peaks for the high pressure operating point were

$$\|S\|_\infty = 1.4591; \;\; \|T\|_\infty = 1.3877; \;\; \|KS\|_\infty = 0.0613 \tag{4.4}$$

The peaks for the low pressure were point

$$\|S\|_\infty = 3.1615; \;\; \|T\|_\infty = 2.1945; \;\; \|KS\|_\infty = 0.2946 \tag{4.5}$$

The peaks for KS are low. This is the KS that is seen from the control signal u into the plant to the disturbance d in the primary controlled output y_1. The cascade controller ensures small use of the actuators. The peaks of the sensitivity functions S and T at the higher operating point goes above 2.

4.2.2 The structure N for the cascade controllers and NP

The cascade controllers robustness properties needs to be derived. In order to inspect these, the nominal closed loop system N_{22} for the cascade control system will be developed. The input signals are chosen as $w = \left[\begin{array}{cc} d & r_1 \end{array}\right]^\top$ and the output signal $z = y_1$. N_{22} becomes the transfer function from the output to the inputs given as

$$z = \begin{bmatrix} (1 + G_1K_2K_1 + G_2K_2)^{-1} \left(G_{d,1} \left(1 + G_2K_2\right) - G_1K_2G_{d,2}\right) \\ (1 + G_1K_2K_1 + G_2K_2)^{-1} G_1K_2K_1 \end{bmatrix}^{\top} w \tag{4.6}$$

$$N_{22} = \begin{bmatrix} (1 + G_1K_2K_1 + G_2K_2)^{-1} \left(G_{d,1} \left(1 + G_2K_2\right) - G_1K_2G_{d,2}\right) \\ (1 + G_1K_2K_1 + G_2K_2)^{-1} G_1K_2K_1 \end{bmatrix}^{\top} \tag{4.7}$$

Then it is calculated whether the cascade controller fulfills the nominal performance criteria. The criteria in equation 2.19 is used and the norm is calculated using Matlab. First the norm for the high pressure point is calculated

$$\|N_{22}\|_{\infty} = 27.7534 \tag{4.8}$$

To achieve nominal performance the norm should have been below 1. The result is as expected, a PI controller isn't tight, it's takes some time for it to settle at the reference. Next the norm for the low pressure point is calculated

$$\|N_{22}\|_{\infty} = 32.2012 \tag{4.9}$$

The cascade controllers don't achieve robust performance, but they may still achieve robust stability. The transfer function from the signal y_{Δ} to the perturbed control signal u_{Δ} has to be developed. The system N_{11} is found to be

$$N_{11} = W_I T \tag{4.10}$$

where T is the transfer function developed using L in equation 4.3. The uncertainty weight W_I that will be used later in the H_{∞} analysis is applied here also. The weight is given as

$$W_I = \frac{s + 0.2}{\left(\frac{1}{2}\right) s + 1} \tag{4.11}$$

Robust stability is checked for at the high pressure point.

$$\|N_{11}\|_{\infty} = 0.2776 \tag{4.12}$$

The cascade controller achieves robust stability. The uncertainty could be brought as high as to 70% (at low frequencies) and still robust stability was achieved. Next the cascade controller at the low pressure point is checked for robust stability

$$\|N_{11}\|_\infty = 0.4393 \tag{4.13}$$

The uncertainty at low frequency were taken as high as 45% and still robust stability was achieved. The impression from this analysis is that the cascade controller is a slow, robust controller. Now it remains to see whether the advanced design can achieve better robustness properties than the cascade design.

4.2.3 Simulations

The cascade controllers are simulated in Matlab and OLGA 2000. The advanced controller should outperform the cascade design. The performance of the cascade controller in these simulations will be the least we can expect from the advanced control scheme. The advanced controller will be designed in the simplified model. The following simulations will tell something about the differences in OLGA and in the simplified model. The knowledge of these differences will be a help in the work with the design of the H_∞ controllers.

The cascade controller at the high pressure point of operation

Performance of the controller in OLGA 2000 The controller managed to stabilize the plant. This can be seen in figure 4.2. The pressuredrop gets rather high before the plant is stabilized. The reason for this is that the valve is closed when the controller is turned on. To maintain the same mass flow, the volumetric flow increases, as does the pressuredrop over the choke. Had the controller been faster could the peak in DP in figure 4.2 been lower. The sluggishness in the controller agrees with the high gamma found. The controller is turned on after 70 minutes. It takes 180 minutes before the reference is reached. The overall performance of the controller is satisfactory. It could have been a bit tighter, but this would affect how much uncertainty the controller could handle before going unstable.

Actuator usage In figure 4.4 the actuator usage in the OLGA simulation can be seen. There is only a perturbation of 0.1 in the actuator. In figure 4.5 the actuator usage in both simulations can be seen. The controller as seen in the simplified model is fast and aggressive. It is clear that the actuator behaves less aggressive in OLGA than in the simplified model. The perturbation during startup of the controller in the simplified model is as high as 0.175. After the startup the perturbations is larger in the OLGA simulation though. This becomes clear during the setpoint change.

Setpoint changes The way the setpoint change is made in the simulations can be seen in figure 4.3. The controller behaves similar in the two simulations. The main difference is the controller as simulated in the simplified model gives a tighter response. There is also a difference in the resulting valve opening. This can be seen at the end of the simulation in figure 4.5.

Differences in the behavior of the controllers The two controllers behave differently in the simulation. The controller as simulated in the simplified model is much more aggressive and tight than as simulated in OLGA.

Similarities in the behavior of the controllers They both show good tracking properties as seen in figure 4.3. Apart from that they both show some overshoot in their general performance.

Differences in the models The amplitude of DP in OLGA are obviously bigger than what is predicted in the simplified model. This can be seen in figure 4.2. The difference makes it more difficult to stabilize the plant in OLGA than in the simplified model. It takes much longer time. This could be improved by a better tuning of the simplified model. But this has not been done. The model had to be tuned around the low pressure operating point to achieve this.

Similarities in the models The setpoint change looks very similar in both simulations as can be seen in figure 4.3. The frequency of the slugging looks also very similar, there is a small phase difference only as can be seen in figure 4.2. In (Storkaas & Skogestad 2004) it is stated that the frequency in the simplified model is about $10 - 15\%$ higher in the simplified model in OLGA.

The cascade controller at the low pressure point of operation The differences and similarities in the model and controller in the two simulations are the same as for the latter operating point. It will not be repeated here.

Performance of the controller in OLGA 2000 The perturbations in DP are smaller at the higher operating point, so is the process gain, the frequency of the slugs is faster though. The controller handles itself fine as seen in figure 4.6. The controller is turned on after 70 minutes. It takes only 50 minute before the setpoint is reached. The actuator usage is more severe though.

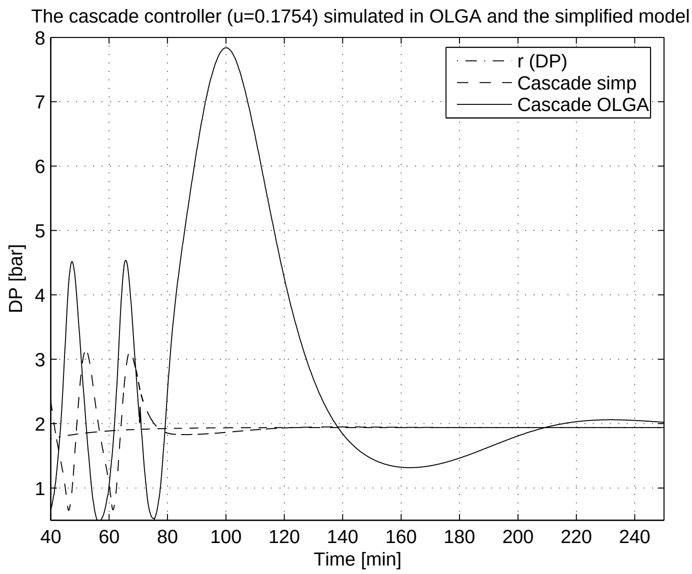

Figure 4.2: Simulation of the cascade controller in OLGA and in the simplified model as seen in the output DP at the lower operating point.

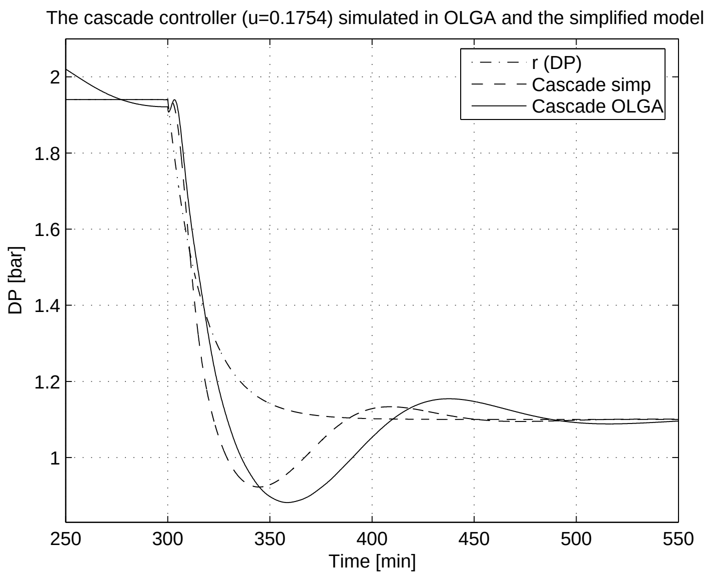

Figure 4.3: The cascade controller at the lower operating point simulated in OLGA and in the simplified model as seen from the output DP. The second part of the simulation.

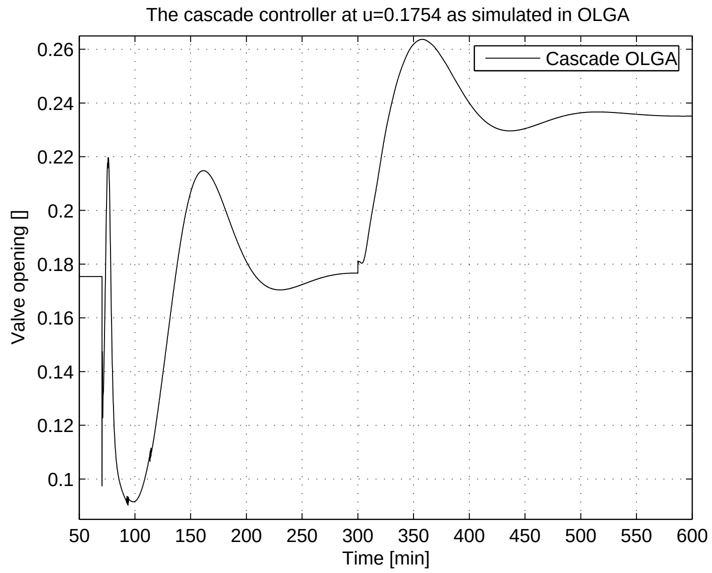

Figure 4.4: The cascade controller at the operating point $u = 0.1754$ as simulated in OLGA 2000.

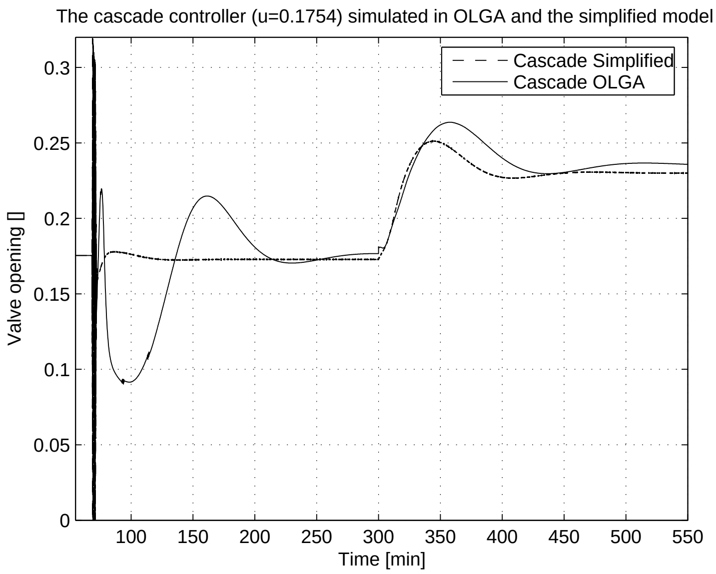

Figure 4.5: The cascade controller at the lower point of linearization simulated both in OLGA and in the simplified model.

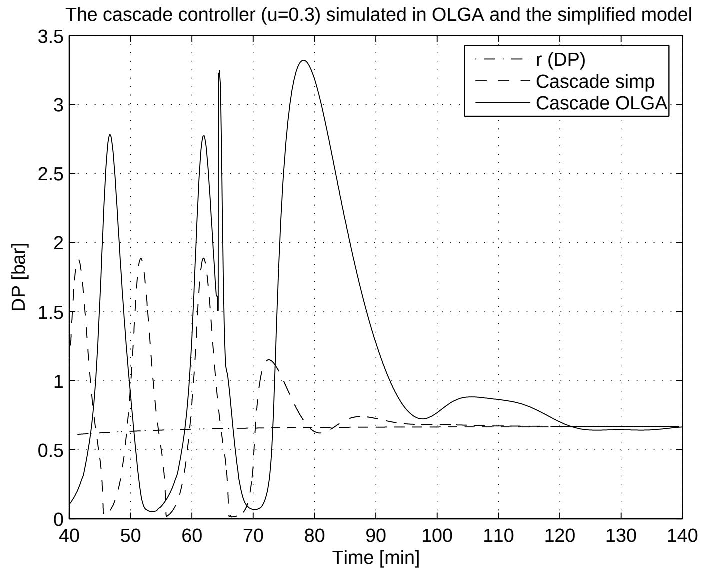

Figure 4.6: Cascade control at the higher operating point as seen in output DP sumlated in OLGA and in the simplified model.

Actuator usage The actuator usage in the two simulations are similar, but again the controller as simulated in the simplified model uses the actuators more severely. The largest perturbation in the simplified model is almost 0.45 as can be seen in figure 4.6. The largest perturbation in the controller in OLGA during startup is 0.3 as can be seen in figure 4.8.The overshoot during the setpoint change is larger in the simulation done in the simplified model as can be seen in figure 4.7 and figure 4.10.

Setpoint changes The setpoint change in the OLGA model can be seen in figure 4.7. The control is very tight. The tracking property of the controller is good. In figure 4.10 it is seen that it takes a larger valve opening in OLGA to achieve the same pressuredrop over the valve as in the simplified model.

4.3 Advanced control of slugging

In the following sections a analysis of SISO H_∞ controllers designed for the different outputs DP, $\dot{Q}_{mix}$ and P_1 will be done. All the details regarding these controllers can be found in the appendices. Only the most important results is presented here.

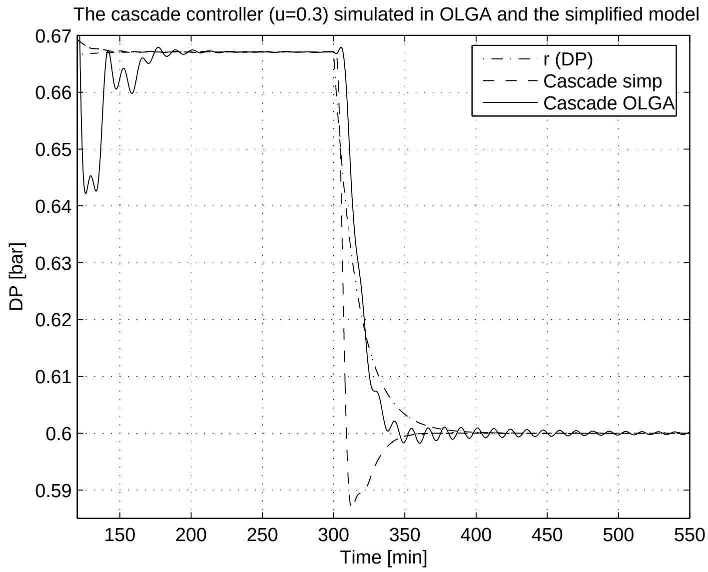

Figure 4.7: Cascade control at the higher operating point as seen in DP simulated in OLGA and in the simplified model. The second part of the simulation.

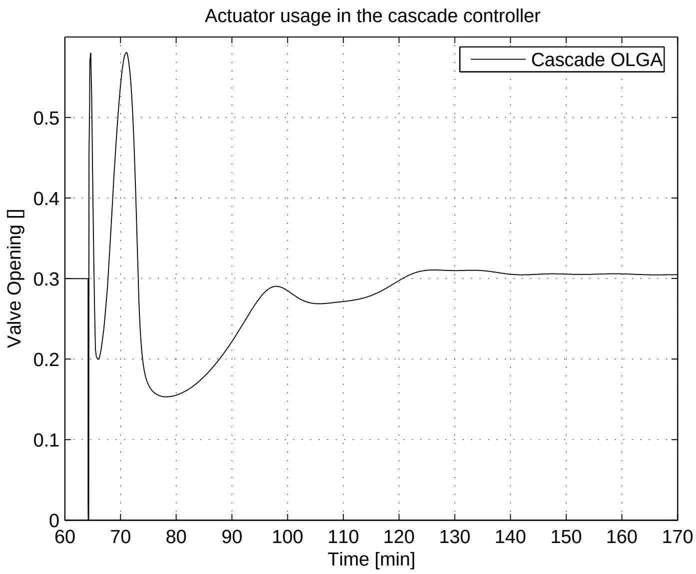

Figure 4.8: Simulation of the cascade controller at the higher operating point in OLGA.

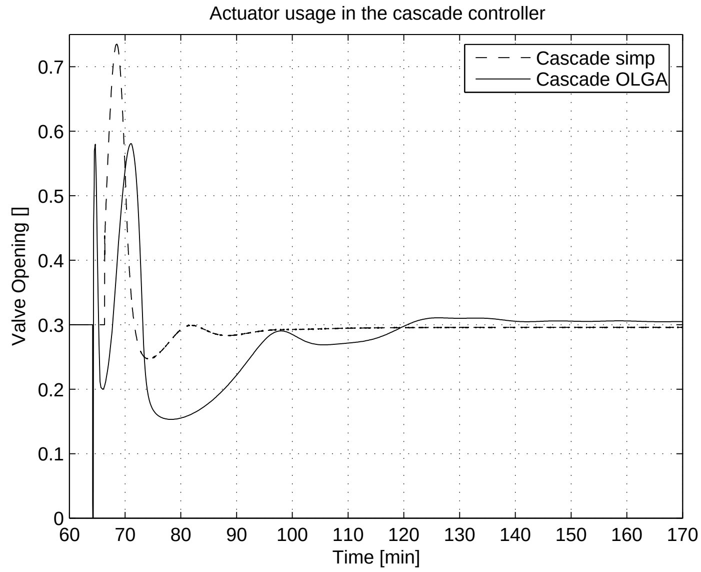

Figure 4.9: Cascade control at the higher operating point simulated in OLGA and in the simplified model. As seen when the controller is turned on.

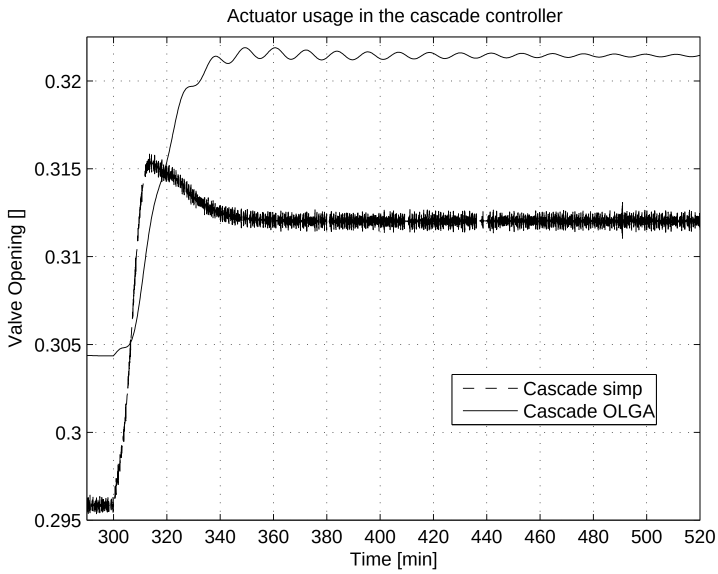

Figure 4.10: Cascade control at the higher opertaing point. Setpoint change simulated in OLGA and in the simplified model.

4.3.1 Design of SISO H_∞ controllers

H_∞ control of the output DP

At the high pressure operating point this output achieved nominal stability and nominal performance ($\|N_{22}\|_\infty = 0.8670$). $\|T\|_\infty = 1.4496$ so it had some sense of integral action. The peaks of the sensitivity functions were all below 2. $\|KS\|_\infty = 0.4745$ which seems to be fairly good. The gain margin was 2.0025 and the phase margin was $-46.7504\,^\circ$. The slugging ended 10 minutes after the controller was turned on.

At the low pressure operating point this controller didn't stabilize the plant. This is due to the overlapping bandwidth requirements.

H_∞ control of the output $\dot{Q}_{mix}$

At the high pressure operating point this output achieved nominal stability and nominal performance ($\|N_{22}\|_\infty = 0.7259$). $\|T\|_\infty = 1.2934$ so it had integral action. The peaks of the sensitivity functions were all below 2. $\|KS\|_\infty = 0.5514$ which seems to be good. The slugging ended twenty minutes after the controller was turned on. The gain margin was 2.1767 and the phase margin was $56.1289\,^\circ$.

At the low pressure operating point nominal stability was achieved, but not nominal performance ($\|N_{22}\|_\infty = 5.7768$). $\|T\|_\infty = 2.4261$ so it had less integral action. All of the peaks of the sensitivity functions were above 2. $\|KS\|_\infty = 4.5203$ which is a bit large. After the setpoint change it took twenty minutes for the slugging to end. The gain margin was 1.5792 and the phase margin was $28.4785\,^\circ$.

H_∞ control of the output P_1

At the high pressure operating point this output achieved nominal stability and nominal performance ($\|N_{22}\|_\infty = 0.8852$). $\|T\|_\infty = 1.1760$ so it had integral action. The peaks of the sensitivity functions were all below 2. $\|KS\|_\infty = 0.4895$ which is good. The slugging ended twenty minutes after the controller was turned on. The gain margin was 2.9884 and the phase margin was $79.1194\,^\circ$.

At the low pressure operating point nominal stability was achieved, but not nominal performance ($\|N_{22}\|_\infty = 3.3121$). $\|T\|_\infty = 1.7690$ so it had less integral action. All of the peaks of the sensitivity functions were below 2, except for KS. $\|KS\|_\infty = 2.5232$ which is a bit large. After the setpoint change it took thirty minutes for the slugging to end. The gain margin was 0.4346 and the phase margin was $63.0202\,^\circ$.

4.4 Concerns regarding the final design

The advantage of the H_∞ design is the prospect of being able to decide upon a set of properties that the control system is to achieve. Throughout the analysis of the simple controllers, the cascade controllers and the SISO H_∞ controllers experiences have been made. Most importantly it has been discovered that the cascade controllers handles uncertainty well. This will be one of the main objectives for the design. The controller should also be able to control the plant in the presence of a changing environment. To achieve this successfully the controller should have good reference tracking, but not at the cost of stability. This means that the $\|T\|_\infty$ should be as close to 1 as possible. Further the norms of S and KS should be small to achieve good disturbance rejection and low usage of actuators. For overall robustness the peaks of all three sensitivity functions should be below 2. Performance is not very important, especially not at the higher operating point, where none of the SISO H_∞ controllers achieved this. Nominal performance should be achieved at the high pressure operating point, but not at the cost of the other objectives. Robust performance is of course only a bonus. It will not be stressed. It is much more important for the controller to be stable and handle uncertainty. This will also help the system reach lower setpoints and to successfully control the plant in the presence of a broader range of disturbances.

4.4.1 The design objectives

From the experience gained thus far a set of design objectives have been decided upon. These objectives represent the way to best control the plant. The objectives are given with the most important one a the top

$$1.\ RS \Rightarrow \ \|N_{11}\|_\infty < 1 \tag{4.14}$$

$$2.\ \|T\|_\infty \approx 1 \tag{4.15}$$

$$3.\ \|S\|_\infty < 2; \ \|KS\|_\infty < 2 \tag{4.16}$$

$$4.\ NP \Rightarrow \ \|N_{22}\|_\infty < 1 \tag{4.17}$$

4.5 Improvements with advanced control

Simple and advanced design have been studied. It will be examined whether the advanced design improve the performance and the robustness of the system. The peaks of the norms says much about the robustness of the controllers. The controllers have been gathered in tables for the sake of comparisment.

4.5.1 Table of the sensitivity peaks the high pressure operating point

Of the controllers presented in the table 4.18 the SISO H_∞ controller for P_1 is the best one in terms of robustness. This is no surprise as this output is the one that is best suited for control. Next we have the SISO H_∞ controller for $\dot{Q}_{mix}$ that is almost as good as the latter, it is in fact better in terms of performance. From this analysis the conclusion is that the design of a advanced MISO controller looks promising considering what the SISO controllers for the two outputs achieves. The MISO controller should be better than the SISO advanced controller for P_1, considering the better performance of the output $\dot{Q}_{mix}$ and the assumption that the multivariable controller will be better than the two outputs it consist of. Lastly the thing to notice is the improvement in robustness properties after introducing a advanced design. In the output P_1 which is well suited for control the improvement in robustness is small. In the output DP however the improvement is large since we did not manage to control it with a simple control solution. In the output $\dot{Q}_{mix}$ the improvement is also large, the norms for the different sensitivity functions are more than halved.

$$
\begin{array}{ccccc}
 & \|S\|_\infty & \|T\|_\infty & \|KS\|_\infty & \gamma\left(\|N_{22}\|_\infty\right) \\
K_{DP} & - & - & - & - \\
K_{\dot{Q}_{mix}} & 3.6036 & 4.0365 & 1.2613 & - \\
K_{P_1} & 1.0038 & 1.2627 & 1.5559 & - \\
H_{\infty,DP} & 1.9975 & 1.4496 & 0.4745 & 0.8670 \\
H_{\infty,\dot{Q}_{mix}} & 1.8603 & 1.2934 & 0.5514 & 0.7259 \\
H_{\infty,P_1} & 1.5029 & 1.1760 & 0.4895 & 0.8852
\end{array} \tag{4.18}
$$

4.5.2 Table of the sensitivity peaks for low pressure operating point

For the higher operating point things are much like for the lower one. The improvements from simple to advanced control are just bigger. Another difference is that the performance is better for the output P_1 this time. The usage of control input is the biggest advantage with the advanced design. $\|KS\|_\infty$ for the output $\dot{Q}_{mix}$ with the advanced design was reduced by a factor of about six. It would seem more difficult to outperform the output P_1 at this operating point, considering that of the two outputs that are going in to the MISO controller only one of them actually works and it's performance is poor compared to P_1. These results may not be accurate due to model mismatch at the operating point, but the qualitative results should be correct. One final observation is that neither controller achieves nominal performance at this operating point, not even the SISO H_∞ controller for P_1. This design objective will therefore be disregarded at the higher operating point.

$$\begin{array}{ccccc} & \|S\|_\infty & \|T\|_\infty & \|KS\|_\infty & \gamma\left(\|N_{22}\|_\infty\right) \\ K_{DP} & - & - & - & - \\ K_{\dot{Q}_{mix}} & 1.9960 & 2.4579 & 29.9404 & - \\ K_{P_1} & 1.0660 & 1.8090 & 5.5965 & - \\ H_{\infty,DP} & - & - & - & - \\ H_{\infty,\dot{Q}_{mix}} & 2.7345 & 2.4261 & 4.5203 & 5.7768 \\ H_{\infty,P_1} & 1.4497 & 1.7690 & 2.5232 & 3.3121 \end{array} \tag{4.19}$$

Chapter 5

The final H_∞ controller

In this section the final H_∞ controller will be derived. This controller will be multivariable and use the outputs DP and $\dot{Q}_{mix}$ exclusively. The controller should achieve the design objectives and it should avoid saturation. The first controller to be designed is the one with the least peaks in the sensitivity functions S, T and KS. The weight functions used in design will include transfer functions as well.

5.1 Aspects regarding robust design

In the design of robust controllers a set of criteria that are hard enough to be regarded as robust has to be decided upon. In the case of robust stability this includes choosing proper values for the uncertainty the model should handle and still be stable. The uncertainty weight from 2.25 is given as

$$W_I = \frac{s + 0.2}{\left(\frac{1}{2}\right) s + 1} \tag{5.1}$$

The perturbation block Δ is chosen to be

$$\Delta = \frac{\frac{40}{2}s - 1}{\frac{40}{2}s + 1} \tag{5.2}$$

where the perturbation function is a time delay of 40 seconds. Normally the perturbation block represents the family of possible perturbations complex and scalar that fulfils the criteria given in equation 2.22. The perturbation in this system is scalar though, as u is the uncertain variable, and it is assumed that the time delay is the worst perturbation possible for the system. Under these assumptions robust performance is achieved if the criteria given in equation 2.21, with the Δ given in equation 5.2, is achieved.

5.2 The controller with integral action

This controller is the one who is supposed to fulfil the design objectives. The previous analysis has shown what is possible and what is not. In the design objectives robust stability was stressed as the most important objective. It was found that both robust performance and robust stability could be achieved, the only cost being the controller handling somewhat less uncertainty. Therefore two controllers were designed. One that achieved robust performance (under the assumptions mentioned earlier) and robust stability and one that only achieved robust stability. The latter is presented first.

5.2.1 Designed at the high pressure stationary point

The controller that achieves robust stability

The controller is designed with the output weights W_u, W_R and N which is given as

$$W_u = \frac{0.1667s + 0.034}{s + 0.51};\ W_R = \frac{0.1667s + 0.00015575}{s + 1.8690e - 005};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.002 \end{bmatrix} \tag{5.3}$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -0.2032 \\ -0.0159 \pm 0.0090i \\ -0.0035 \pm 0.0018i \\ -0.0009 \\ -1.9614 \\ -1.9613 \\ -1.9613 \\ -1.4506 \\ -1.4506 \\ -1.4506 \end{bmatrix} \tag{5.4}$$

NS is achieved. This controller achieved the gamma

$$\gamma = 0.8299 \tag{5.5}$$

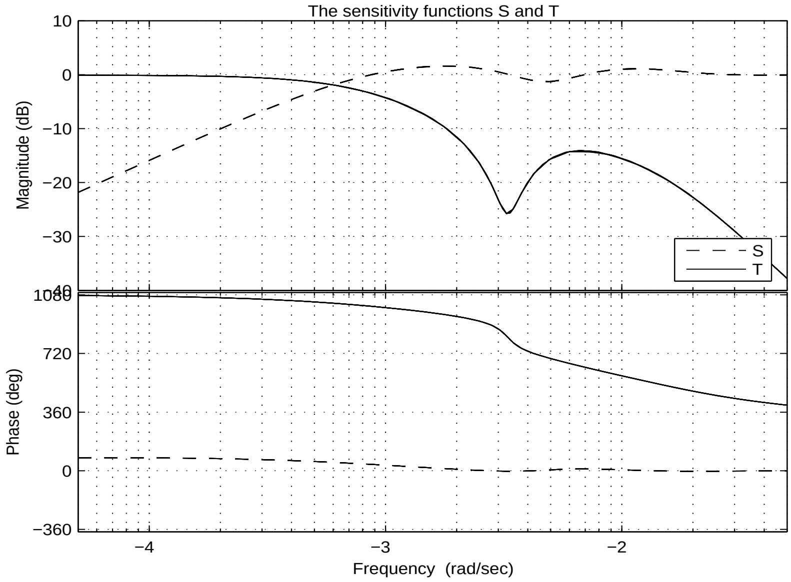

Figure 5.1: Bode plot of the sensitivity functions S and T for the RS controller.

This ensures nominal performance, NP. The gamma is actually lower than the gamma for P_1 at this operating point. The sensitivity peaks were

$$\|S\|_\infty = 1.1905; \|T\|_\infty = 0.9895 \, \|KS\|_\infty = 0.4090 \tag{5.6}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small as is $\|S\|_\infty$. All the peaks are lower than the ones for the SISO H_∞ controller for P_1. The plots for S and T can be seen in figure 5.1.

$$\|N_{11}\|_\infty = 0.2322 \tag{5.7}$$

The peak is well below 1, robust stability, RS, is ensured. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 85% and still robust stability was achieved. Remember that the cascade controller could handle uncertainties of up to 70% and still achieve robust stability so there is a improvement in the robustness with the advanced design.

$$\|F\|_\infty = 3.0951 \tag{5.8}$$

Clearly robust performance is not achieved, but the performance properties are all the same much better than for the cascade controller. All the design objectives are ensured, and the design is complete. The simulations will be dealt with in the chapter regarding the validation of the design.

The controller that achieves robust performance

The controller is designed with the output weights W_u, W_R and N which is given as

$$W_u = \frac{0.1667s + 0.034}{s + 1.02};\ W_R = \frac{0.1429s + 0.00015575}{s + 3.4421e - 005};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.002 \end{bmatrix} \tag{5.9}$$

This controller achieved the gamma

$$\gamma = 0.4505 \tag{5.10}$$

This ensures nominal performance, NP. The sensitivity peaks were

$$\|S\|_\infty = 1.1772; \|T\|_\infty = 0.9889\ \|KS\|_\infty = 0.5185 \tag{5.11}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small, but it is bigger than for the controller that only achieved robust stability. It takes more usage of the actuators to obtain better performance properties. The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -1.4461 \\ -1.9613 \\ -0.2022 \\ -0.0210 \pm 0.0086i \\ -0.0035 \pm 0.0018i \\ -1.020 \\ -0.0011 \\ -3.4807e - 005 \end{bmatrix} \tag{5.12}$$

All the poles are stable, nominal stability, NS, is achieved. With nominal stability and performance ensured robust stability is checked for

$$\|N_{11}\|_\infty = 0.2381 \tag{5.13}$$

The criteria is ensured, the peak is well below 1, robust stability is achieved. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 80% and still robust stability was achieved. There is also a improvement in the robustness with this design, as this is also better than the cascade design in terms of robustness. Next robust performance is checked for

$$\|F\|_\infty = 0.9506 \tag{5.14}$$

Robust performance is achieved under the assumptions made earlier. The uncertainty could be taken as high as 55% and still robust stability and robust performance was achieved. The simulations will be presented in the chapter that deals with the validation of the design.

5.2.2 Designed at the low pressure stationary point

Having learned from previous designs and from the cascade design that robustness in respect of stability and not in terms of performance is the key to successful antislug control, especially at aggressive operating points, this controller should handle as much uncertainty as possible. Nominal performance will not be stressed. As seen earlier the SISO H_∞ controller for P_1 didn't achieve this criteria, therefore it is considered too difficult. The disturbances were kept out of the design. The weight function for the noise were also changed.

$$W_u = 6;\ W_R = \frac{0.1s + 0.00012}{s + 1.3500e - 006};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{5.15}$$

This controller achieved the gamma

$$\|N_{22}\|_\infty = 8.8451 \tag{5.16}$$

Nominal performance, NP is not achieved. This is not surprising and was not a design goal. The sensitivity peaks were

$$\|S\|_\infty = 1.0049; \|T\|_\infty = 1.0043\, \|KS\|_\infty = 1.3434 \tag{5.17}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small, as it is lower than 2. The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -2.0154 \\ -2.0154 \\ -0.0038 \pm 0.0100i \\ -0.0038 \pm 0.0099i \\ -2.7548e-005 \\ -1.3500e-006 \end{bmatrix} \tag{5.18}$$

All the poles are stable, nominal stability, NS is achieved. With nominal stability and performance ensured the next to check for is robust stability

$$\|N_{11}\|_\infty = 0.6617 \tag{5.19}$$

The criteria is ensured, the peak is well below 1, robust stability is achieved. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 30% and robust stability was still achieved. The cascade controller could handle uncertainties of up to 45% and still achieve robust stability at the point. The cascade controller is therefore better at this operating point in terms of stability than the advanced controller. The advanced controller has better tracking properties though, and somewhat better performance properties. Next robust performance is checked for

$$\|F\|_\infty = 26.6557 \tag{5.20}$$

Clearly robust performance is not achieved, but the performance properties are all the same better than for the cascade controller, even though this was not stressed in the design. Next is the simulations of the controller. These will be handled in the next chapter that deals with the validation of the design.

5.3 Alternative multivariable controllers

The output DP and $\dot{Q}_{mix}$ were combined into a multivariable system. Now other outputs are to be combined with DP. The outputs are the massflow $\dot{W}_{mix}$, the gas fraction α_{gas} and the density ρ_{mix} of the mixture topside. These multivariable controllers are to be compared with the H_∞ design made out of DP and $\dot{Q}_{mix}$, but only at the high pressure operating point. It will be harder to say anything about the similarities at the low pressure operating point due to the model mismatch. Remember that the H_∞ controller for DP and $\dot{Q}_{mix}$ was not fine-tuned, but made as robust as possible to make it work. The design at the high pressure operating point are more valid for comparisment. These controllers will not be simulated.

5.3.1 H_∞ controller made out of DP and α_{gas}

The controller is designed with the output weights W_u, W_R and N which is given as

$$W_u = \frac{0.1667s + 0.034}{s + 0.51};\ W_R = \frac{0.2s + 0.00022575}{s + 2.7090e - 005};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} \tag{5.21}$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -3.1020 \\ -1.9614 \\ -0.2033 \\ -0.0154 \pm 0.0098i \\ -0.0032 \pm 0.0032i \\ -2.7090e - 005 \\ -0.0011 \\ -0.5100 \end{bmatrix} \tag{5.22}$$

NS is achieved. This controller achieved the gamma

$$\gamma = 0.8301 \tag{5.23}$$

This ensures nominal performance, NP. The sensitivity peaks were

$$\|S\|_\infty = 1.9468; \|T\|_\infty = 0.9889\ \|KS\|_\infty = 0.4584 \tag{5.24}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small, $\|S\|_\infty$ is a bit large, but it is beneath 2. With nominal stability and performance ensured the next to check for is robust stability and robust performance

$$\|N_{11}\|_\infty = 0.4127 \tag{5.25}$$

The peak is well below 1, robust stability, RS, is ensured. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 45% and still robust stability was achieved. Next robust performance is checked for

$$\|F\|_\infty = 2.6247 \tag{5.26}$$

Clearly robust performance is not achieved. All the design objectives are ensured, and the design is complete. It is possible to use α_{gas} as a added measurement and it has good robustness properties. It might get even better if it were to be fine-tuned.

5.3.2 The other designs

The other two designs, made with $\dot{W}_{mix}$ and ρ_{mix}, gave almost the same results as the one above. The details can be found in the appendix. They both achieved NS and NP. The gamma was the same for all three control designs. They both achieved RS. The norms were $\|N_{11}\|_\infty = 0.2660$ for $\dot{W}_{mix}$ and $\|N_{11}\|_\infty = 0.3508$ for ρ_{mix}. The multivariable controller made with $\dot{W}_{mix}$ could handle 75% uncertainty, whereas the controller made with ρ_{mix} could only handle 55%. The sensitivity peaks were

$$\|S\|_\infty = 1.6130; \|T\|_\infty = 0.9890 \, \|KS\|_\infty = 0.4583 \tag{5.27}$$

for $\dot{W}_{mix}$ and

$$\|S\|_\infty = 2.0000; \|T\|_\infty = 1.0096 \, \|KS\|_\infty = 0.4592$$

for ρ_{mix}. None of them achieved RP.

5.3.3 Evaluation of the alternative designs

In terms of the design objectives, none of the above alternative controllers achieved them better than the original design with DP and $\dot{Q}_{mix}$. With that said, they all look suited for control. All of them fulfils the design objectives. The multivariable controller that combines DP and $\dot{W}_{mix}$ achieves the objectives the best. This is not surprising since the massflow and volumetric flow are closely related. This controller is also the only one of the above that beats the cascade design in terms of how much uncertainty it can handle, before going unstable. The controllers might be better if fine-tuned. As presented here they were tuned until they achieved the design objectives, nothing more.

Chapter 6

Validation and testing of the control design

To validate the design done in the simplified model, the advanced controller was tested in a simulation in OLGA 2000. If the controller performed well in this simulation, that would prove the validity of the simplified model as a basis for advanced controller design. Also the simulation in OLGA 2000 was compared to the simulations done in the simplified model. The differences and similarities would also help us in the validation of the design.

6.1 The high pressure operating point

6.1.1 The controller that achieved robust stability

Performance of the controller in OLGA 2000 The controller manages to stabilize the plant. This can be seen in figure 6.1. The overall performance of the controller is satisfactory.

Actuator usage In figure 6.3 the actuator usage of the controller in the two simulations can be inspected. In the OLGA 2000 simulation the controller has to use the actuators much more to stabilize the plant. It takes more time to reach the reference in OLGA as well. The controller was turned on after about 70 minutes. The reference is reached after about 250 minutes in the OLGA simulation, 180 minutes later. In the simplified simulation it took 1 hour to reach the reference, that is 3 times as short. The actuator usage after stabilization is very similar. There is only a perturbation of 0.08 in the actuator as can be seen in figure 6.1. This agrees with the norm of KS which was found to be low.

Setpoint changes The way the setpoint change is made in the simulations, as it looks from the actuators point of view, can be seen in figure 6.3. It is nicely done. The controller behaves almost exactly the same in the two simulations. There is only a difference in the resulting valve opening. The setpoint change is neatly done. The tracking properties are apparently good.

Differences in the behavior of the controllers The controller has a harder time stabilizing the plant in OLGA than in the simplified model. This can be seen in the usage of actuators in figure 6.3 and in figure 6.1.

Similarities in the behavior of the controllers Tight control is achieved in both simulations as can be seen in figure 6.2. After stabilization the two controllers behave the same.

Differences in the models The amplitude of DP in OLGA are obviously bigger than what is predicted in the simplified model. This can be seen in figure 6.1. The difference makes it more difficult to stabilize the plant in OLGA than in the simplified model. The reason for this has been mentioned earlier. In short it is due to the choke closing, the volumetric flow increasing to keep the same mass flow, which results in a higher pressuredrop over the choke.

As was stated in the part about setpoint changes there was a difference in the valve openings after the setpoint change. There was a difference in the valve opening before the setpoint change also, but the difference was smaller. There is a difference in the models here and from what have been seen it grows with increasing valve opening.

Similarities in the models The setpoint change looks very similar in both simulations as can be seen in figure 6.2. The frequency of the slugging also looks similar, there is a small phase difference though.

Validation at the lower point (RS)

The control design is validated. There is a difference in the way the plant is stabilized in the simulations, but apart from that the overall behavior is similar and satisfactory.

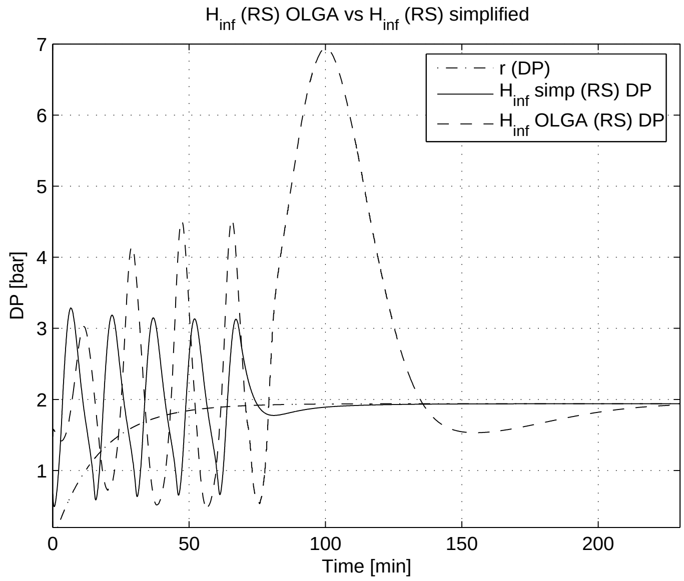

Figure 6.1: The beginning of the simulation in OLGA 2000 and in the simplified model of the H_∞ controller that achieves RS as seen in output DP.

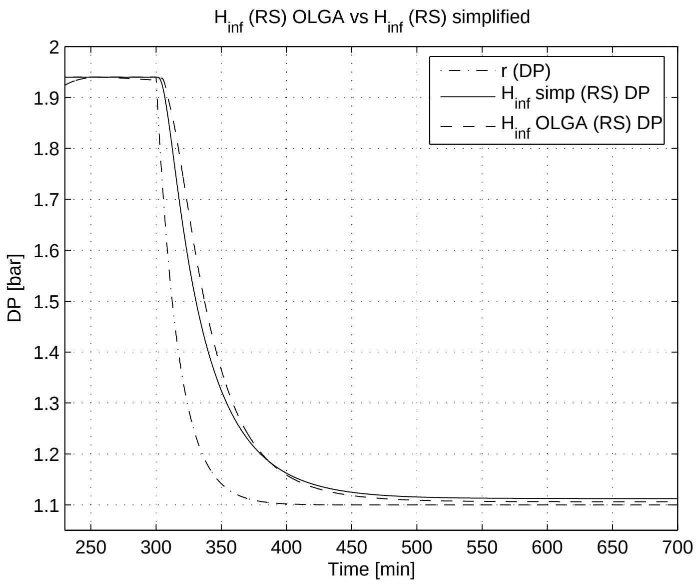

Figure 6.2: The setpoint change in the simulation of the H_∞ controller that achieves RS in OLGA 2000 and in the simplified model.

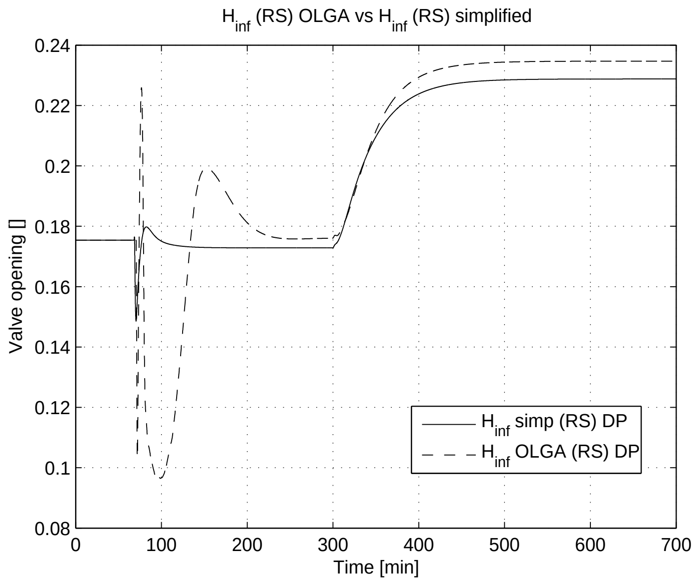

Figure 6.3: Simulation in OLGA and in the simplified model of the H_∞ controller that achieves robust stability.

6.1.2 The controller that achieved robust stability and robust performance

The parts about the differences and similarities of the controller and the model in the two simulations are the same as for the latter controller. It will not be repeated here.

Performance of the controller in OLGA 2000 The RP controller manages to stabilize the plant. This is done better than for the RS controller as can be seen in figure 6.4. It takes 40 minutes less to reach the reference for the RP controller. This becomes apparent in figure 6.4. The RS controller allowed the pressure over the choke to reach 7 bar . This can be seen in figure 6.1. The RP controller keeps the pressure below 6 bar as can be seen in figure 6.4. This is because the RP controller is more aggressive.

Actuator usage The controller is more aggressive than the RS controller. It stabilizes the plant with lesser perturbations in the actuator than the RS controller. This can be seen in figure 6.7. The setpoint change is also made with little use of the actuators. This can be seen in figure 6.5. This agrees with the low norm for KS which was achieved for the controller.

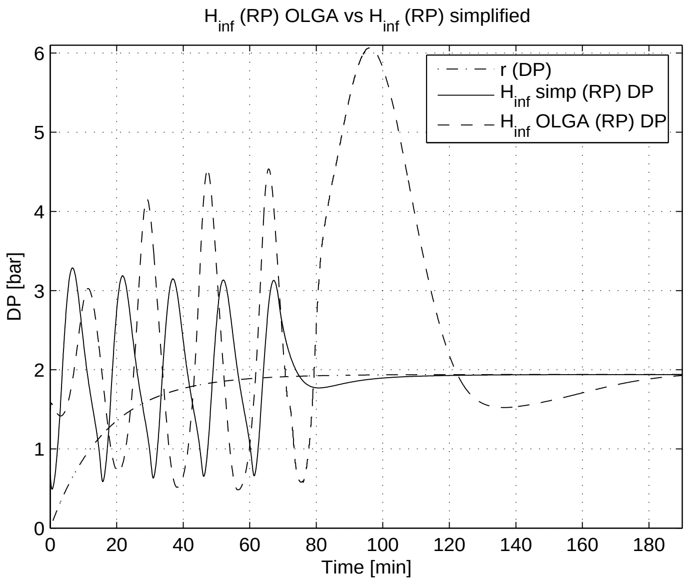

Figure 6.4: Simulation in OLGA with the H_∞ controller that achieves robust performance, zoomed in on the startup.

Setpoint changes The RP controller didn't manage the setpoint change as can be seen in 6.5. It did manage a setpoint change to $DP = 1.2\,\text{bar}$ though.

Validation at the lower operating point (RP)

The control design is not validated because the controller didn't manage setpoint change.

6.1.3 Comparisment of the two advanced controllers

As one can see of the figure 6.7 the H_∞ controller that achieved RP achieves tighter control than the controller that only achieved RS. This is achieved at the cost of stability. Clearly it is better to let forego of robust performance. It is not worth it to compromise stability. The conclusion from this section is that the controller achieving RS is the one to prefer. Later the two controllers will be tested more thoroughly.

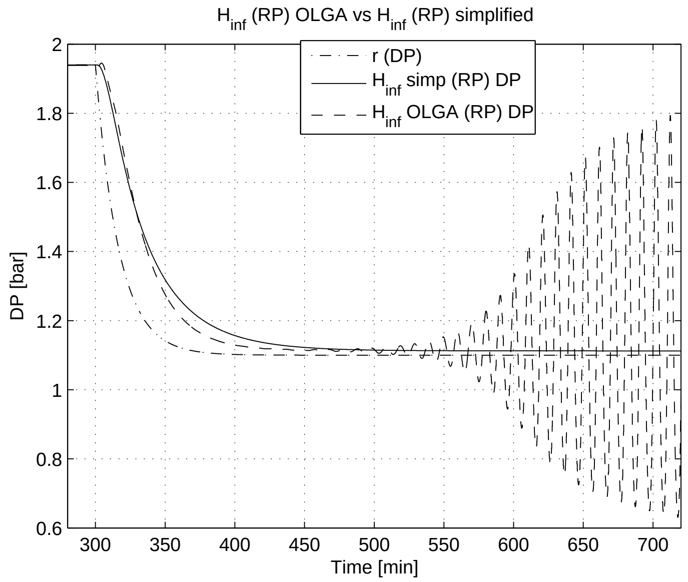

Figure 6.5: The setpoint change in DP as simulated with the H_∞ (RP) in OLGA and in the simplified model.

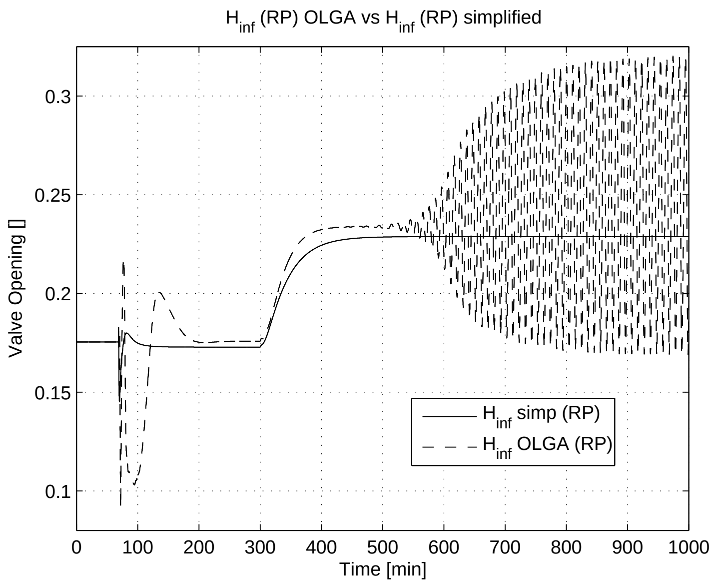

Figure 6.6: Simulation of the H_∞ controller that achieves robust perfomance in OLGA and in the simplified model.

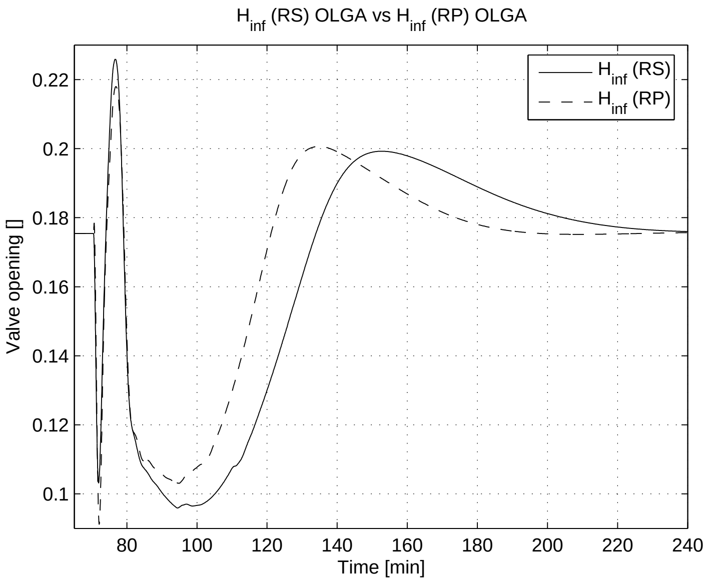

Figure 6.7: Simulation in OLGA 2000 of the two H_∞ controllers at the lower operating point.

6.2 The low pressure operating point

The model mismatch is larger at this operating point than at the other one. As discovered earlier it takes a greater valve opening in OLGA, than in the simplified model, to achieve the same pressure over the valve. It was also stated earlier that the gain in the valve was predicted too low in the simplified model. These mismatches resulted in the fine-tuning of the controller being difficult. Instead it was made as robust in terms of stability as possible. This was the design objective that would be the most appropriate considering the circumstances.

As before the differences and similarities are mostly the same as for the controller that achieved robust stability. These parts are therefore omitted from this section.

Performance of the controller in OLGA 2000 The controller manages to stabilize the plant. This can be seen in figure 6.10 and in figure 4.6. The controller calms down after the start up and goes slowly against the setpoint. It takes 40 minutes from the controller is turned on until the setpoint is reached.

Actuator usage The actuator is heavily used at the start up of the controller. This can be seen in figure 6.10. The actuator goes nearly into saturation. It still manages to

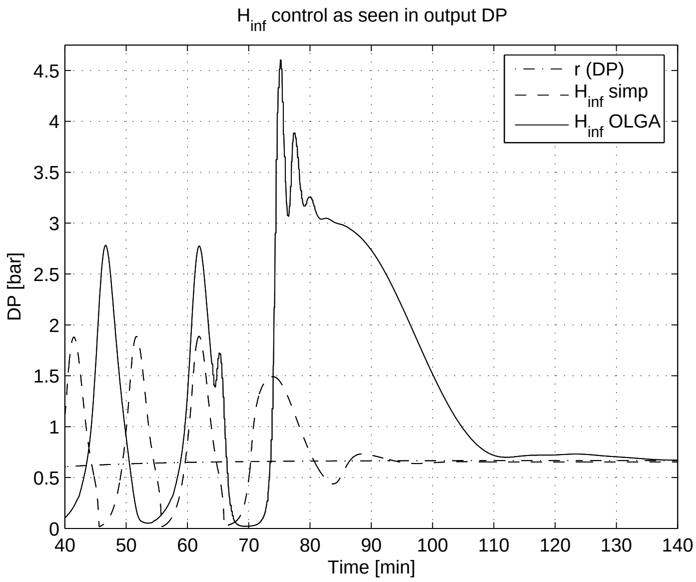

Figure 6.8: H_∞ control at the higher operating point as seen in output DP simulated in OLGA and in the simplified model.

stabilize the plant. Later the actuator aren't aggressively used.

Setpoint changes The setpoint change is neatly done. This can be seen in figure 6.11 and in figure 6.9. The performance of the controller in OLGA is actually much better than in the simplified model. The opposite would have been worse. This is a strong point for the simplified design. This means that the gamma predicted with the simplified model is too large.

6.2.1 Validation at the higher operating point

The design is validated at this operating point also. The plant is stabilized, the setpoint change is made neatly and the plant is stable after the setpoint change.

6.2.2 Validation

Stabilization of the plant was more difficult for the controller in OLGA than in the simplified model. Apart from that the controller behaved similar in both simulations. The controller managed to stabilize the plant and the setpoint changes was neatly done

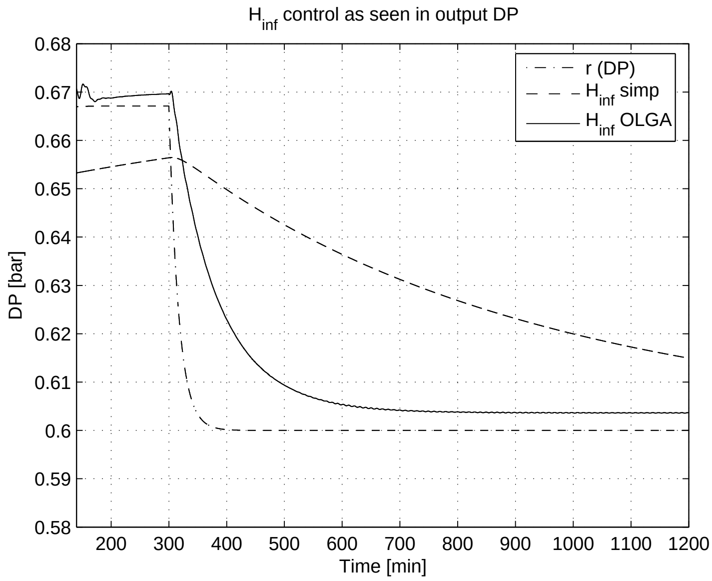

Figure 6.9: H_∞ control at the higher operating point as seen in output DP.

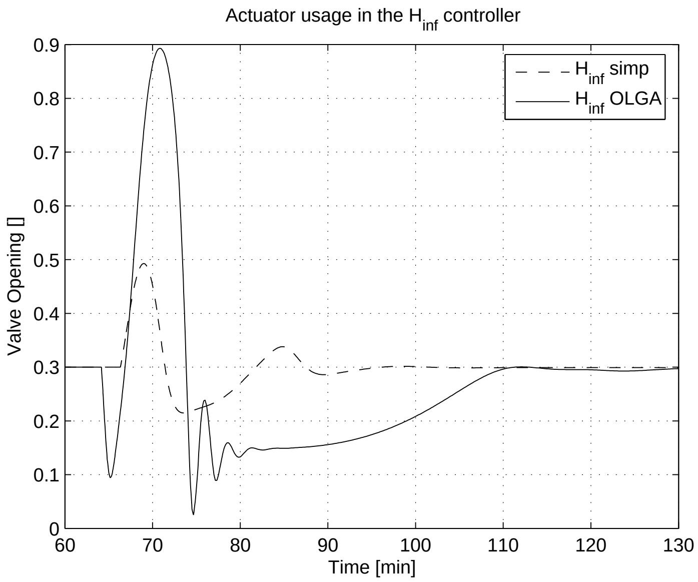

Figure 6.10: H_∞ control at the higher operating point simulated in the simplified model and in the OLGA model.

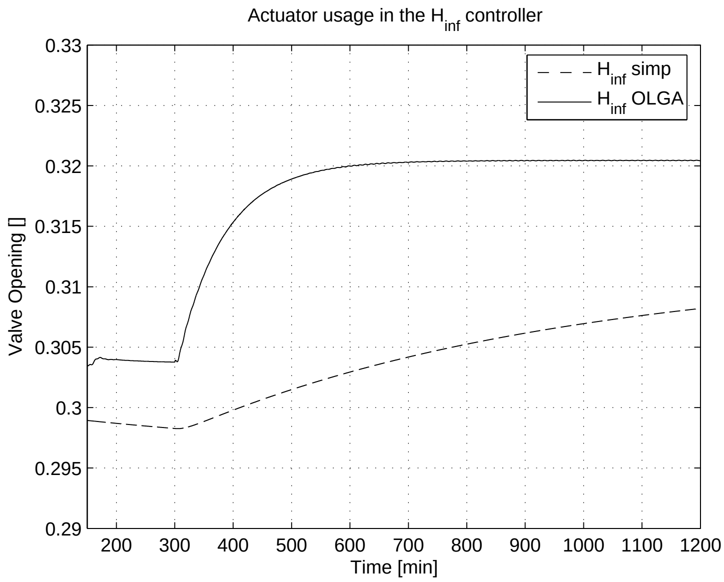

Figure 6.11: Actuator usage of the H_∞ controller at the higher operating point simulated in OLGA and in the simplified model.

at both operating points. The design is validated. The RP controller is not included in this validation.

6.3 Tracking- and disturbance rejection properties of the cascade- and the H_∞ controllers

The tracking and disturbance rejection properties of a controller says much about how well it will perform. A comparisment between the cascade controller and the more advanced control scheme H_∞ will be performed. To reach this end simulations with varying disturbances for the cascade- and the H_∞ controllers at the two different operating points have been done. The cascade controllers are the ones given in equation 4.1. The two different H_∞ controllers for the lower operating point will be tested.The simulations should tell something about the difference between the two H_∞ controllers regarding their robustness properties in general.

The disturbances are sinuses given as

$$d_1 = 0.362 + 0.005\sin(\frac{20}{180}pi + 0.00015t) \tag{6.1}$$

$$d_2 = 8.64 + 0.2\sin(\frac{10}{180}pi + 0.0001t)$$
$$d_3 = 50 + 0.5\sin(0.00005t)$$

where the bias is the value of the disturbance given in the model at the point of linearization. As can be seen in equation 6.1, the disturbances are at different frequencies and with differences in phase.

6.3.1 Simulating at the high pressure operating point

Disturbance rejection As can be seen in figure 6.12 the cascade controller is more influenced by the disturbances and yields somewhat poorer performance than the H_∞ controllers. The controller that yields the best performance is the one that satisfies the robust performance criteria. All in all it can be said that the H_∞ controllers have better properties regarding disturbance rejection than the cascade controller.

Disturbance rejection and reference tracking The disturbances are the same as before. But now the reference is a ramp starting at $DP = 1.9403$ and ending at $DP = 1.5$. Both the H_∞ controllers and the cascade controller were simulated. As we can see from figure 6.13 there is only a small difference in the tracking properties of the three controllers. It is difficult to argue whether one is better than the other, because the different controllers are all nearest the reference at one point or the other. However, it is noticed that the cascade controller is slightly more off the reference than the other two at the end of the simulation. Otherwise the three are almost identical. All yields satisfying tracking properties.

6.3.2 Simulating at the low pressure operating point

The cascade and the H_∞ design were simulated. The properties regarding disturbance rejection and tracking were inspected. In the simulations with disturbance rejection the reference was kept at $r = 0.6671$ bar throughout the simulation. In the other simulations the reference trajectory can be seen in figure 6.15.

Disturbance rejection In figure 6.14 the simulations can be seen. The cascade design keep up with the reference, the H_∞ controller does not. The advanced controllers largest deviation from the reference is 0.1 bar, whereas the deviation of the cascade controller is hardly noticeable. The cascade design has good disturbance rejection properties. The advanced design tries to counteract the disturbances, but does a bad job. It doesn't drift

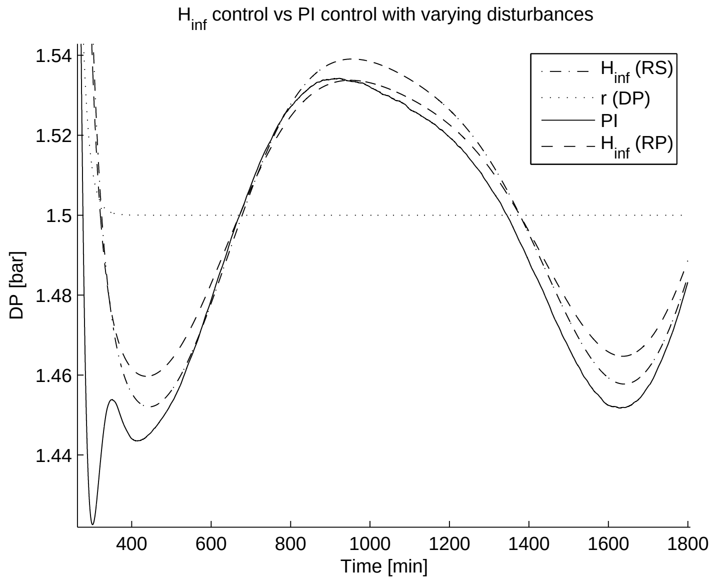

Figure 6.12: H_∞ control vs cascade control of output DP with varying disturbances

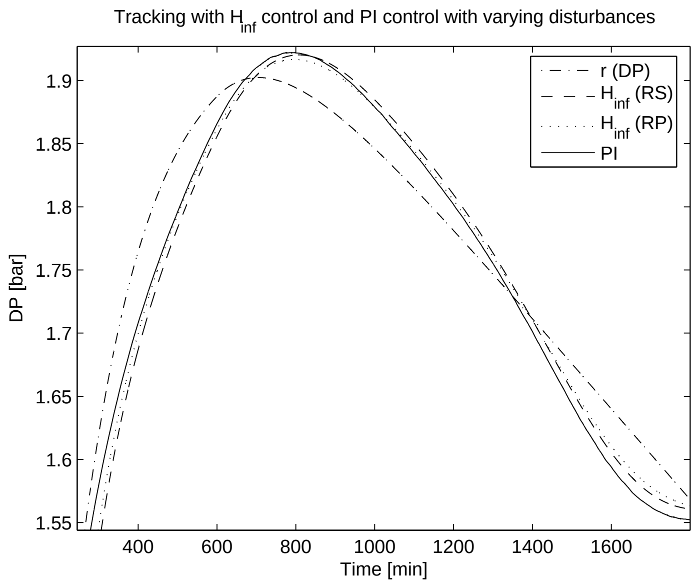

Figure 6.13: Tracking properties of the H_∞ controllers and the cascade controller.

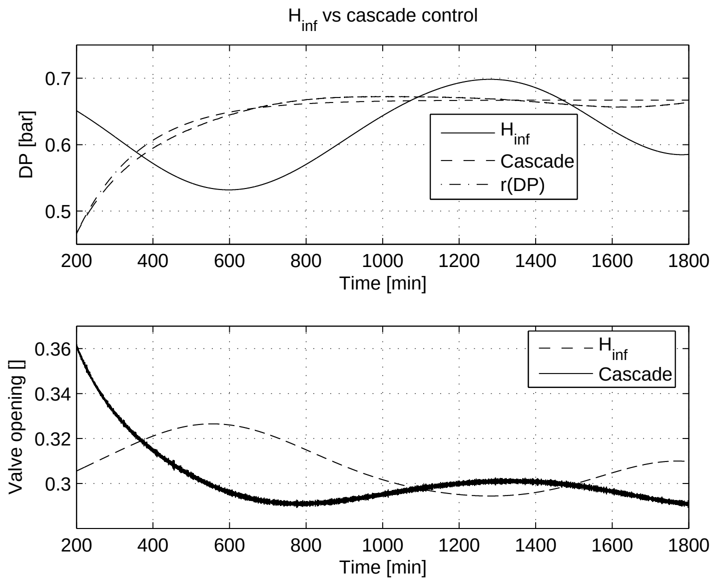

Figure 6.14: Varying disturbances simulated at the low pressure operting point in the simplified model.

away from the reference though. The disturbance rejection properties of the advanced design are poor compared to the cascade design.

Reference tracking The same argument goes for the simulation with a changing reference and varying disturbances in figure 6.27. The cascade design keeps up, the advanced design does not. The tracking properties of the cascade design are very good. The deviation of the advanced design from the reference is even larger in this simulation. The advanced controller's tracking properties are poor compared to the cascade design.

6.3.3 The results from the disturbance rejection and tracking simulations

The cascade controller performs well at both operating points. It has good disturbance rejection and tracking properties. The H_∞ controller has good tracking and disturbance rejection properties at the high pressure operating point, but poor properties at the low pressure operating point. The best controller overall in these simulations is the cascade controller.

If the shortcomings of the H_∞ design at the low pressure operating point due to model mismatch is considered the best controller is the H_∞ design.

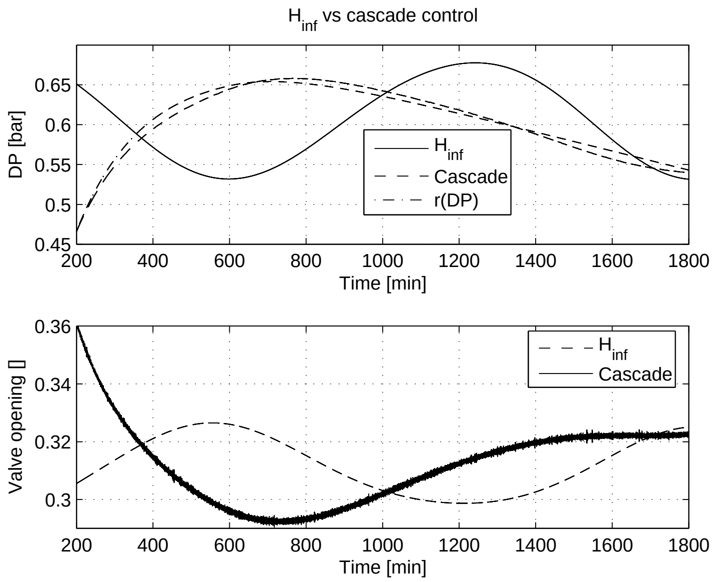

Figure 6.15: Tracking under influence of varying disturbances simulated in the simplified model.

6.4 Influence of uncertainty and time delay in the H_∞- and cascade controllers

Simulations where a uncertainty of 40% and a time delay of 40 seconds were added to the control signal were performed. The choice of these specific quantities were made because the robust controller should handle it.

6.4.1 Simulating at the high pressure operating point

In figure 6.16 the startup of the two H_∞ controllers can be seen. Clearly they both handle the uncertainty and delay. The *RS* controller handles it best though. It is seen that this controller has lass perturbations in the actuator, and has a easier time dealing with the situation. In figure 6.17 the setpoint change is seen. The two controllers are both very tight. The *RP* controller is a a bit closer to the reference though. Both controllers does the setpoint change nicely considering the circumstances.

In figure 6.18 one can see the startup of the cascade controller. It hasn't got any problems stabilizing the plant. It's actuator usage is more severe than for the advanced controllers though. It stabilizes the plant faster than the advanced designs. The setpoint change in figure 6.19 isn't done as nice as for the advanced controllers. There is a overshoot in *DP*

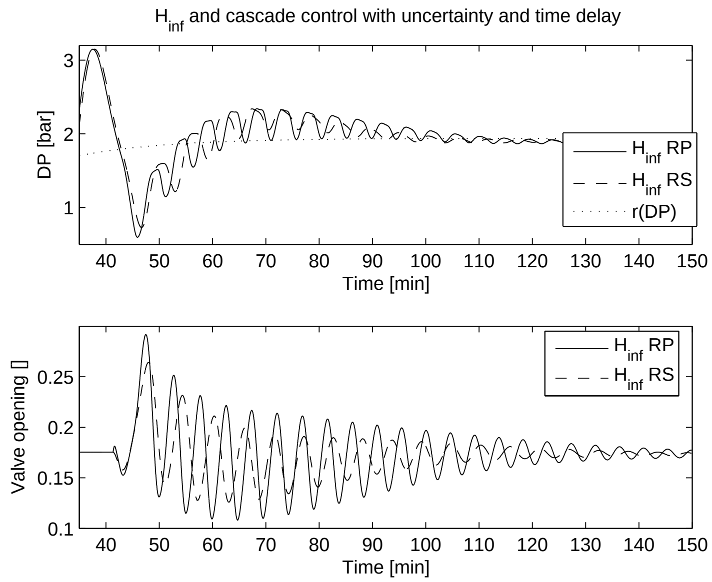

Figure 6.16: Simulating the H_∞ controllers in the simplified model with a uncertainty of 40% and a time delay of 40 seconds added to the control signal. The startup of the controller.

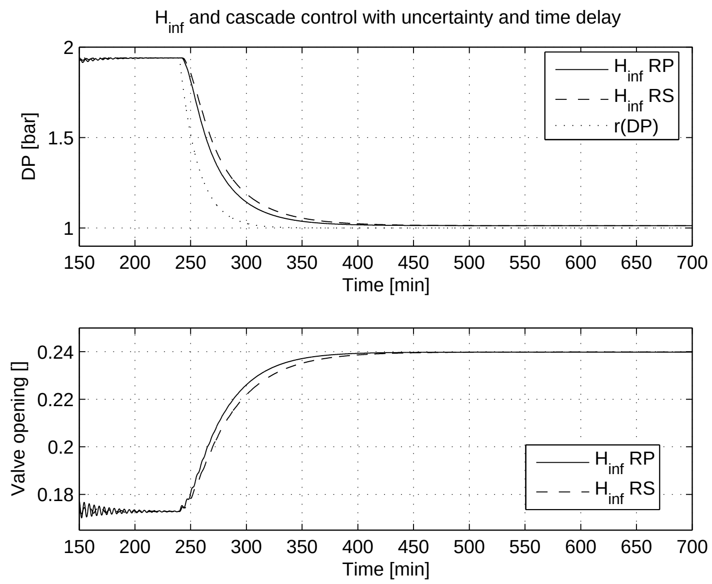

Figure 6.17: Simulation in the simplified model of the H_∞ controllers were a uncertainty of 40% and a time delay of 40 seconds is added to the control signal. The setpoint change.

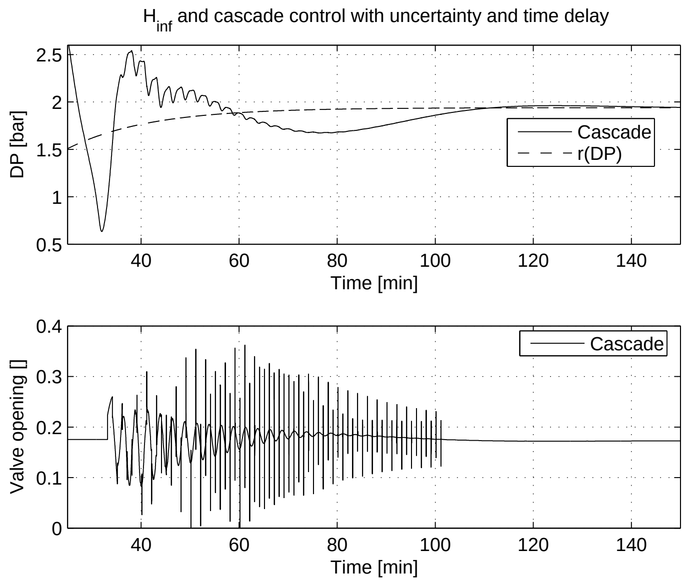

Figure 6.18: Simulation in the simplified model of the cascade design at the high pressure operating point where 40% uncertainty and 40 seconds time delay is added to the control signal. Startup of the controller.

in the setpoint change.

6.4.2 Simulating at the low pressure operating point

At the low pressure operating point it can be seen in figure 6.20 that the advanced controller has no problems stabilizing the plant. It doesn't take particularly long time either.

The cascade controller in figure 6.21 doesn't manage to fully stabilize the plant. There are high frequent fluctuations in the output DP. The actuator usage is severe also.

The advanced controller does the setpoint change rather slowly, but it doesn't go unstable as can be seen in figure 6.22. The plant is kept stable and it is clear that the controller will make the output DP converge to the reference. The actuator usage controlled and minimal.

The cascade controller has problems making the setpoint change. The cascade controller seems to be too aggressive to stabilize the plant. The high frequent fluctuations doesn't stop and the actuators are severely used throughout the simulation.

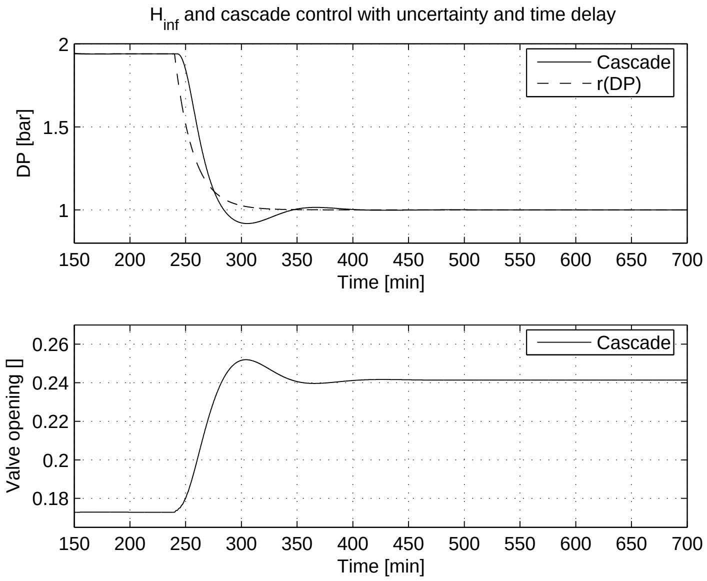

Figure 6.19: Simulation in the simplified model of the cascade controller where a uncertainty of 40% and a timedelay of 40 seconds are added to the control signal. The setpoint change

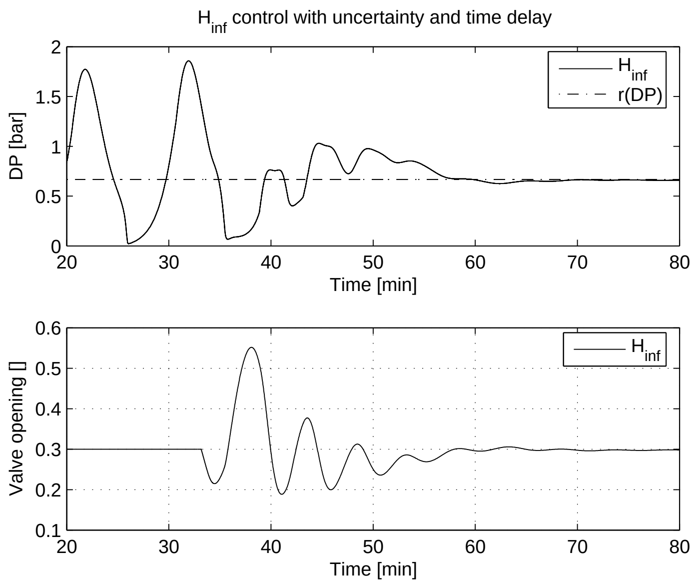

Figure 6.20: Simulation with 40% uncertainty and 40 seconds time delay in the control signal. Stratup of the controller at the low pressure operating point.

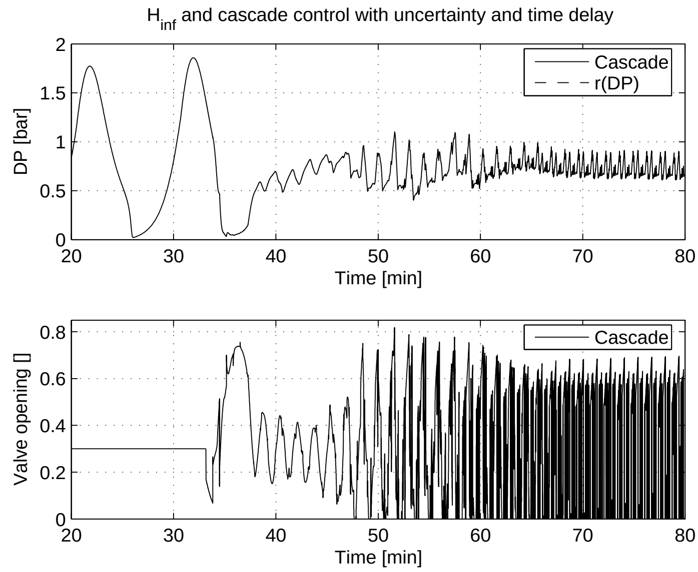

Figure 6.21: Simulation of the cascade design at the low pressure operating point in the simplified model with 40% uncertainty and 40seconds of time delay added to the control signal. Startup of the controller.

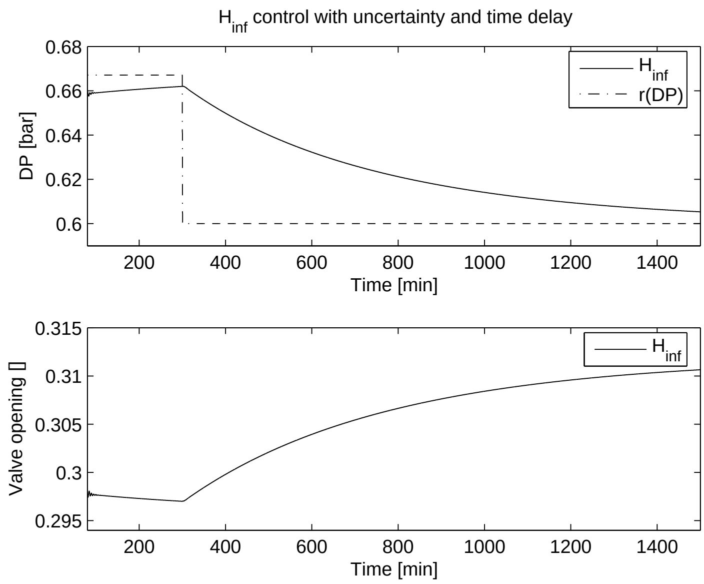

Figure 6.22: Simulation of the advanced controller at the low pressure operating point with 40% uncertainty and 40 seconds time delay added to the control signal. The setpoint change.

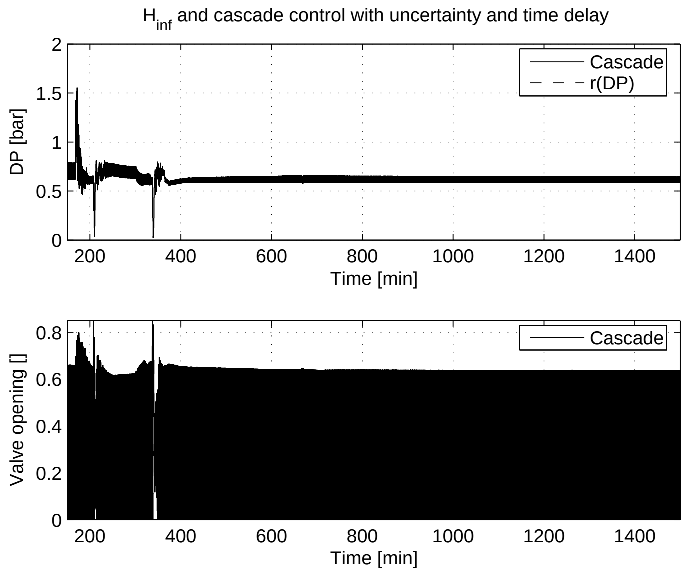

Figure 6.23: Simulation of the cascade controller at the low pressure operating point in the simplified model with 40% uncertainty and a time dealy of 40 seconds added to the control signal. The setpoint change.

6.4.3 Results from the uncertainty and time delay simulations

In the simulation at the high pressure operating point the cascade controller stabilized the plant before the advanced controllers, but the actuator usage was more severe. At the higher operating point it became clear that the cascade controller was too aggressive. It didn't manage to stabilize the plant. It used the actuators severely. The conclusion is that the advanced design handles uncertainty and time delay better than the cascade design.

6.5 The cascade design and the advanced design

The simulations in OGLA were compared. In the analysis it was found that at the high pressure operating point the H_∞ is the most robust controller both in terms of stability and performance. The advanced controller didn't come to it's right at the low pressure operating point due to model mismatch.

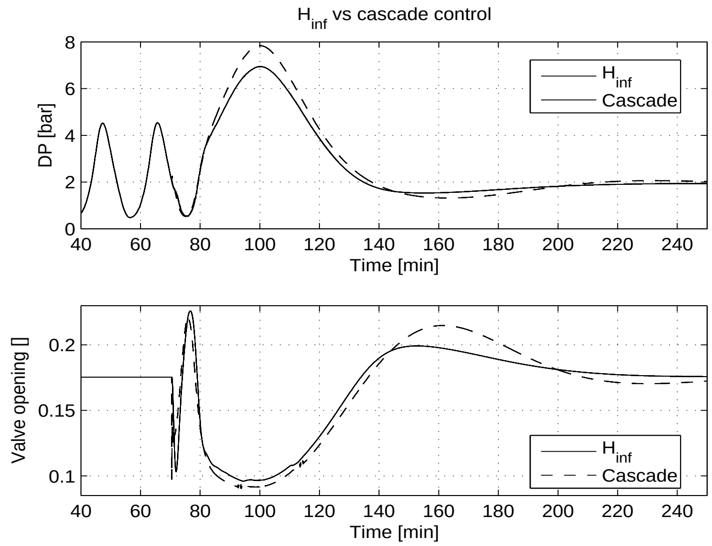

Figure 6.24: Simulation of the H_∞ and the cascade controller in OLGA 2000. Startup of the controller.

6.5.1 Performance in OLGA at the high pressure operating point

Stabilization of the plant The response of the advanced controller is tighter than the cascade design. This can be seen in figure 6.24. The controllers stabilize the plant in about the same time though. The actuator usage is also much the same.

The setpoint change In figure 6.25 it is seen that the advanced controller does the setpoint change better than the cascade controller. The cascade controller shows some overshoot, whilst the advanced controller shows good tracking properties. The reference is reached in about the same time though. The actuator usage is less for the advanced controller.

6.5.2 Performance in OLGA at the low pressure operating point

Stabilization of the plant In figure 6.26 the stabilization of the plant can be seen. The cascade controller stabilizes the plant before the advanced controller. In addition the pressuredrop over the choke is kept lower. It is also seen that the advanced controller uses the actuator much more than the cascade controller. The advanced controller seems to be smoother.

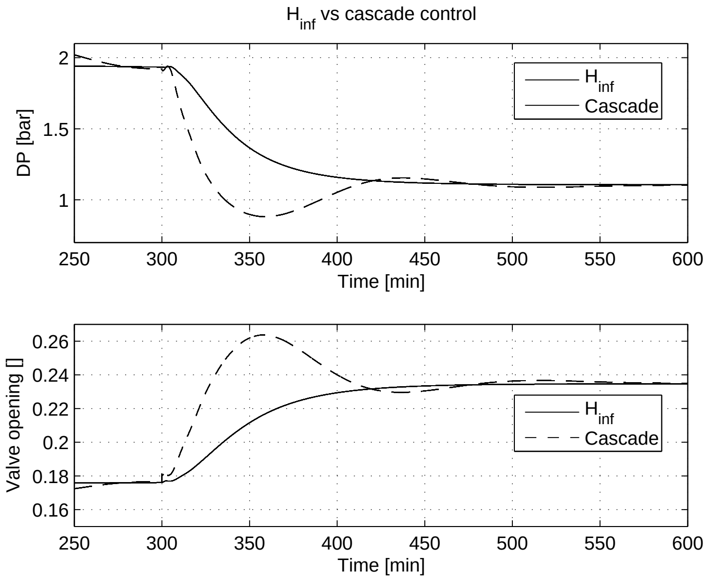

Figure 6.25: Simulation of the cascade- and the advanced controller in OLGA. The set-point change.

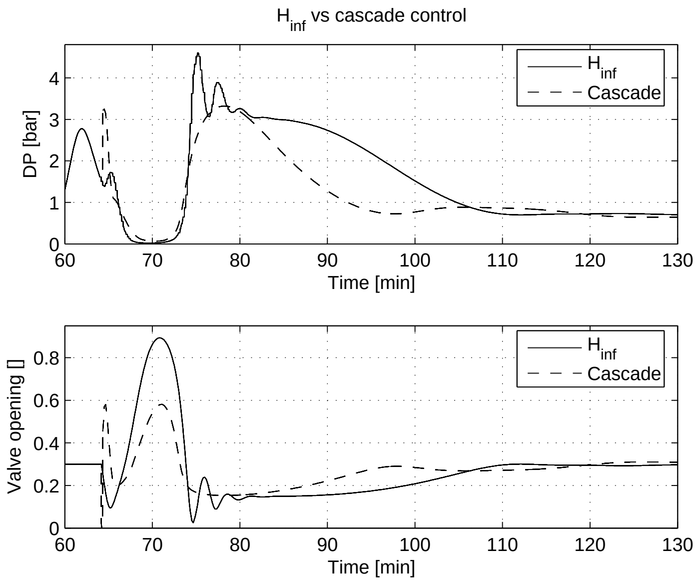

Figure 6.26: Simulation of the cascade and the advanced controller in OLGA. The startup of the controller.

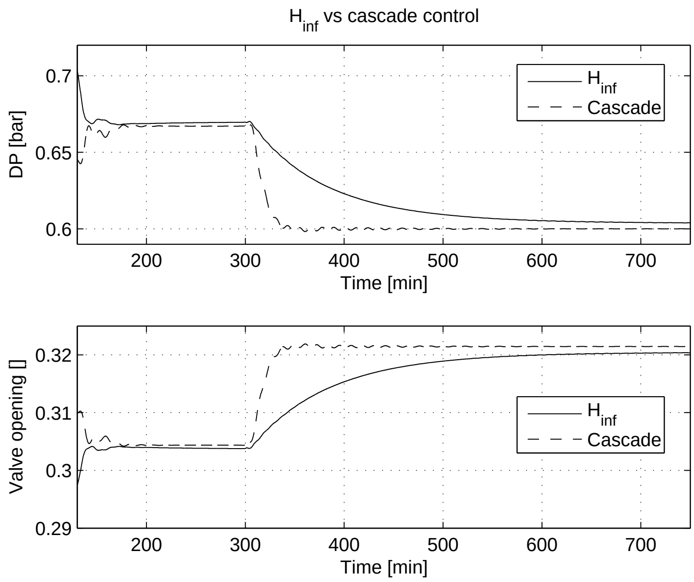

Figure 6.27: Simulation of the cascade and advanced controller in OLGA. The setpoint change.

The setpoint change The setpoint change is done much faster by the cascade controller than by the advanced controller as can be seen in figure 6.27. The advanced controller seems to be smoother and to use less actuators though.

6.5.3 Results from the OLGA simulation

At the high pressure operating point the advanced controller had the best overall performance. But the cascade design was quite good too. At the low pressure operating point the cascade design was the best. The advanced design used less actuators though.

Chapter 7

Discussion

7.1 Validity of the simplified model as a model for advanced control design and model mismatch

There are two concerns that has to be considered. The first is whether a fine-tuned simplified model will be valid for advanced control design? Will the simplifications make it too inaccurate? If the first is fulfilled, the second has to be considered. To what extent must the simplified model be tuned to be suitable for advanced control design?

The model mismatch between OLGA and the simplified model were one of the concerns before starting the design process. It had been stated in papers that the macroscopic behavior of the two were similar. But would the little details not modelled in the simplified model prove to be a hindrance for advanced control design? In the design process it was found that this was not a big concern at the high pressure operating point. Here the controller could handle 85% uncertainty and still be stable and the controller had very good tracking properties. With these two obtained the controller performed very nicely in the OLGA simulation. The properties of the controller that we found in the robustness and controllability analysis was recognized in the OLGA simulation. This proves that the simplified model is valid for control design. The details not modelled are in no way a hindrance to the design. This means that it should be possible to design a good controller at the low pressure operating point also. With the first concern fulfilled it's the second concern.

The cascade design outperformed the H_∞ design at the low pressure operating point. The design made at the high pressure operating point was very successful though. At this point the H_∞ design outperformed the cascade design. The reason for this is the validity of the simplified model at this point. As may be remembered the simplified model is tuned at the point where the slugging starts. This point is not too far off the high pressure operating point. It is clear that the simplified model is valid for advanced control design considering the design made at the high pressure operating point. If the simplified model were tuned at the low pressure operating point it should increase the performance of the controller designed at the point greatly. The reason for this is the notion that a advanced design should outperform the cascade design as was the case at the high pressure operating point. If it was possible there it should be equally possible at the low pressure operating point. In general the simplified model should be tuned at the intended operating point to maximize robustness and performance.

The model was not fine-tuned at the low pressure operating point. The two problems with the controller handling less uncertainty at this operating point and the mismatch between the models being rather large resulted in it being impossible to fine-tune the controller. This made it very difficult to design a controller that fulfilled the robust performance criteria. It adds up in us having to make a controller as robust in terms of stability as possible. With this said the controller at the higher operating point performed rather well in OLGA.

7.2 Improving performance by adding measurements

Introducing multivariable control seems to be the right thing to do when the downside pressure P_1 is not available. To be able to quantify the improvement when introducing multivariable control table 7.1 has been made. It consists of the different norms for the SISO and the MISO controllers that have been designed. It should be noted that the SISO controller haven't been tested in OLGA. Their design is therefore not validated. The same goes for the alternative controllers. A robustness analysis has been performed on these latter though. They should therefore be realistic. The comparisment is only performed with controllers made from the outputs available topside.

The best controller is the H_∞ with DP and $\dot{Q}_{mix}$. The improvement when adding a measurement considering the only simple controller available topside is large. The peaks are more than halved. If the best controller is compared with one of the advanced SISO controllers the improvement is less, but still it is quite an improvement. The tracking properties becomes better. The disturbance rejection properties becomes better. The actuators sensitivity to noise, disturbances and reference changes have become better. The best controller handles a lot of uncertainty also.

	$\|S\|_\infty$	$\|T\|_\infty$	$\|KS\|_\infty$	$\gamma(\|N_{22}\|_\infty)$	$\|N_{11}\|_\infty$	$\|F\|_\infty$
$K_{\dot{Q}_{mix}}$	3.6036	4.0365	1.2613	–	–	–
$H_{\infty,DP}$	1.9975	1.4496	0.4745	0.8670	–	–
$H_{\infty,\dot{Q}_{mix}}$	1.8603	1.2934	0.5514	0.7259	–	–
$H_{\infty,DP,\dot{Q}_{mix}}$	1.1905	0.9865	0.4090	0.8299	0.2322	3.0951
$H_{\infty,DP,\alpha_{gas}}$	1.9468	0.9889	0.4584	0.8301	0.4127	2.6247
$H_{\infty,DP,\dot{W}_{mix}}$	1.6130	0.9890	0.4583	0.8301	0.2660	2.7665
$H_{\infty,DP,\rho_{mix}}$	2.0000	1.0096	0.4592	0.8301	0.3508	2.6078

(7.1)

At the low pressure operating point there are only two controllers in the table. This is the main design and the SISO H_∞ controller for $\dot{Q}_{mix}$. The design for this latter has not been validated, so it may be too optimistic. Still there is a indication of a large improvement in robustness when adding a measurement. The peaks the sensitivity functions are more than halved. It is clear that there is quite an improvement when adding a measurement. This is as expected.

	$\|S\|_\infty$	$\|T\|_\infty$	$\|KS\|_\infty$	$\gamma(\|N_{22}\|_\infty)$	$\|N_{11}\|_\infty$	$\|F\|_\infty$
$H_{\infty,\dot{Q}_{mix}}$	2.7345	2.4261	4.5203	5.7768	–	–
$H_{\infty,DP,\dot{Q}_{mix}}$	1.0049	1.0043	1.3434	8.8451	0.6617	26.6557

(7.2)

7.3 Design objectives

The design objectives were based upon the experiences gained from simpler designs. The objectives proved to be the right ones both in terms of the choices and their priority. This is claimed based on the validity of the designs. As trivial as it may seem it was important to prioritize the design objectives in order to get the best control system possible.

7.4 Multivariable control

In the control design multivariable control has been utilized. This methodology is the obvious solution to the control problem studied in this thesis. It outperforms SISO control as the outputs in SISO terms have poor controllability properties. All the multivariable controllers studied in the thesis achieved robust stability. There are many possible choices for control structures, provided the controller is MISO.

7.5 Advantages with advanced control

Earlier it was pointed out that since the advanced controller at the high pressure point of operation works, it should be possible to achieve a much better controller at the low pressure operating point if the simplified model were fine-tuned at that point. Therefore the properties of the advanced controller at the low pressure operating point are not considered to be representative of the possibilities of advanced design. This implies that in this treatise only the controllers at the high pressure operating point will be considered. The cascade design and the advanced design will be evaluated in terms of the design objectives.

Both the advanced design and the cascade design could handle a lot of uncertainty before going unstable. This was the main design objective, robust stability. The advanced controller were found to be more optimal in terms of stability than the cascade design as it could handle 85% of uncertainty, whereas the cascade design could only handle 70%.If the simplified model were fine-tuned at the operating point, maybe it could handle a little more. In terms of tracking the designs were similar. The $\|T\|_\infty$ of the advanced design were closest to 1 though and probably has a bit better tracking properties than the cascade design. The third design objective was as small norms for the other sensitivity functions as possible. This was also achieved the best by the advanced design. The last design objective was nominal performance. This was only achieved by the advanced design.

Another strong point to the advanced multivariable design is that it is possible to design controllers from a large set of outputs. The choice with the cascade design is more limited.

Chapter 8

Further work

8.1 Alternative control design

There is a alternative controller that could perform better than the one proposed, especially at the low pressure operating point. This controller uses a fast and simple controller in a inner loop. The choice for output here would be either DP or $\dot{Q}_{mix}$, preferably $\dot{Q}_{mix}$. The controller would never have to bring the plant out of the slug regime, as the plant would be shut down if slugging were to occur. At the higher operating point DP couldn't be used as stated earlier. In the outer loop we would have one of the multivariable controllers proposed in this thesis. To the multivariable controller the inner loop would be considered a part of the system. The fast inner controller would stabilize the system and the outer controller would then have a easier time controlling it. This could increase robustness in terms of both stability and performance.

8.2 Using other outputs for control design

There are some combinations of outputs that were not tested in this thesis. The main reason for choosing DP as a main output was to ensure enough process gain in the system. In a multivariable design ρ_{mix} could probably also be used as a main output, as this output also has a high process gain. This output could be combined with the same outputs that were tested together with DP. Another option would be to add a third output to see if this can improve robustness.

8.3 Validation of control designs

The alternative multivariable H_∞ controllers and the SISO H_∞ controllers designed in the thesis should be validated. A validation would tell how well they perform in comparisment with the main design presented here. The SISO H_∞ controllers should be tested for robustness in order to find out how much uncertainty they could handle before going unstable. This analysis would tell how much the uncertainty handling improves by adding a measurement.

8.4 Tuning the simplified model at the low pressure operating point

The simplified model used in this thesis were tuned around the point were the slugging started. The point can be seen in figure 3.2. The low pressure operating point used in this thesis were so far away from this point that the model mismatch turned out to be a problem. The model should be tuned around the low pressure operating point used in this thesis. The design made at the high pressure operating point were successful. This validates the simplified model as a tool for design of advanced control schemes. It would be possible to make much better controllers at the high operating point if the simplified model were tuned at this particular point. In general it would be preferable if the simplified model were tuned at the operating point where to controller were to operate.

Chapter 9

Conclusion

Several issues were addressed in the thesis. The first was whether the simplified model was valid for advanced control design. The simplified model was found to be valid for this purpose. The advanced controller designed stabilized the plant in a OLGA simulation. To maximize robustness and performance in the controller, the model should be fine-tuned at the controller's intended operating point.

There were too large a model mismatch between simplified model and OLGA model to achieve a optimal design at the low pressure operating point. The controller performed good anyhow, but it is assumed that the controller will be much better if the model were to be fine-tuned at the operating point.

It was found that the topside measurements combined into multivariable control systems was the best solution to the control problem. There were many possible control structures. The ones that were investigated in the thesis and found to be usable was the pairs DP and $\dot{Q}_{mix}$, DP and α_{gas}, DP and $\dot{W}_{mix}$, DP and ρ_{mix}. In all these designs DP was chosen to be the main controlled variable to ensure process gain. The main design was made with the control structure DP and $\dot{Q}_{mix}$.

The main controller achieved robust stability ($\|N_{11}\|_\infty = 0.2322$) and could handle 85% uncertainty in the actuator before going unstable at the high pressure operating point. This was better than the cascade design which was found to handle 70% of uncertainty before going unstable. It was also found that the cascade design was aggressive as it didn't handle uncertainty and time delay equally well as the advanced design. In terms of tracking properties and disturbance rejection the cascade- and advanced design performed equally well. The difference being the cascade showing some overshoot in it's general behavior. The cascade controller didn't achieve nominal stability either, but the advanced controller did ($\gamma = 0.8299$). The sensitivity peaks were also better for the advanced design ($\|S\|_\infty = 1.1905$, $\|T\|_\infty = 0.9865$, $\|S\|_\infty = 0.4090$). It was found that the advanced design was the better one. The cascade solution was a valid solution also though.

There was a improvement to the overall performance when a measurement was added. If compared to the SISO H_∞ controller made with the $\dot{Q}_{mix}$ the improvement in the peaks were about 0.7 in S, 0.3 in T and 0.15 in KS. The values are probably better as the SISO H_∞ design hasn't been validated in OLGA simulation. The SISO design can probably not handle as much uncertainty either. At the low pressure operating points the peaks were more than halved.

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Appendix A

Appendix

A.1 Simple control of slugging

In the following sections simple P controllers will be designed. These controllers are made for later comparisment with the H_∞ controllers.

A.1.1 Simple control with the DP measurement

This output, as seen earlier, puts a upper limit on the bandwidth of the system. The upper bound at the high pressure operating point is

$$\omega_B < \frac{z}{2} = \frac{0.0143}{2} = 0.007\,15; \; u = 0.1754$$
$$\omega_B < \frac{z}{2} = \frac{0.0127}{2} = 0.0063; \; u = 0.3$$

With these bandwidth requirements the system should be in the frequency band given as

$$0.0073 < \omega_B < 0.007\,15 \tag{A.1}$$

which is clearly not possible. This will therefore not be pursued any further.

A.1.2 Simple control with the $\dot{Q}_{mix}$ measurement

The flow measurement has good high frequency properties, but the low frequency properties are poor. This is due to the small LHP zero close to the origin. In addition there the steady state gain is small.

Designed at the high pressure operating point

The best simple controller, in terms of $\|KS\|_\infty$, was

$$K = 0.35$$

The closed loop poles of the system were

$$p = \begin{bmatrix} -1.6964 & -0.0003 \pm 0.0049i \end{bmatrix}^\top \tag{A.2}$$

The sensitivity peaks were

$$\|S\|_\infty = 3.6036; \|T\|_\infty = 4.0365; \|KS\|_\infty = 1.2613$$

As can be seen the peaks for S and T are rather large. The gain margin was 0.5768 and the phase margin was -32.31 °, and the bandwidth was $\omega_B = 0.0127\frac{r}{s}$.

The opening of the valve, was $u = 0.176$. As can be seen of A.1 the actuator settles itself at the intended position ($u = 0.175$). The plant is operated at the stationary point. It takes quite long for the actuator to settle at the stationary point and the slugging to end. The controller is turned on after about 40 minutes, then after about 110 minutes the valve is settled around the stationary point. As can be seen of the closed loop poles in A.2 the system is close to marginal stability. After about 420 minutes there is a small change in the reference. This change, however, goes very smoothly. As can be seen from A.2 there occurs a overshoot in the DP after the controller is turned on. This is due to the volumetric flow having to increase, to uphold the massflow, as the valve is closed shut. Therefore DP increases.

Designed at the low pressure operating point

The stationary gain of G at this stationary point, it is very close to zero. It proved too difficult to control it with a simple controller. It will not be pursued any further.

A.1.3 Simple control with the P_1 measurement

The best possible output from the selection of outputs is P_1. Good performance from a control system with this output should be expected. It will not be used in the final design

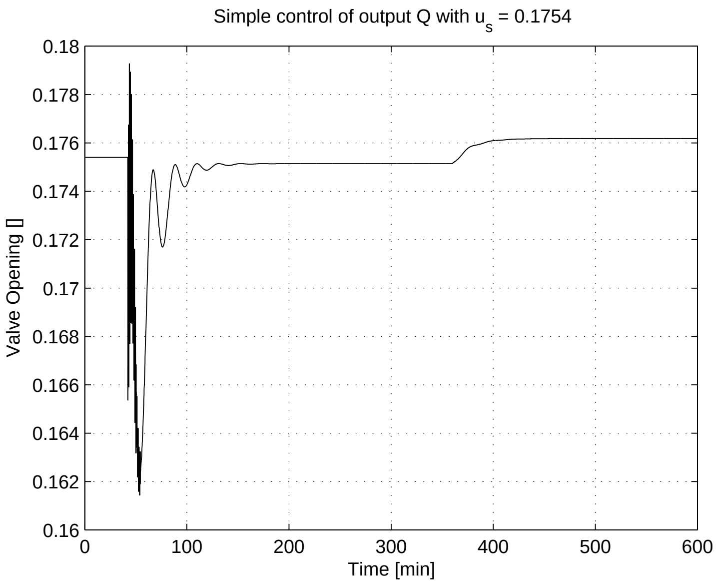

Figure A.1: Simple control of output $\dot{Q}_{mix}$ with a timedelay of $t = 30s$ added

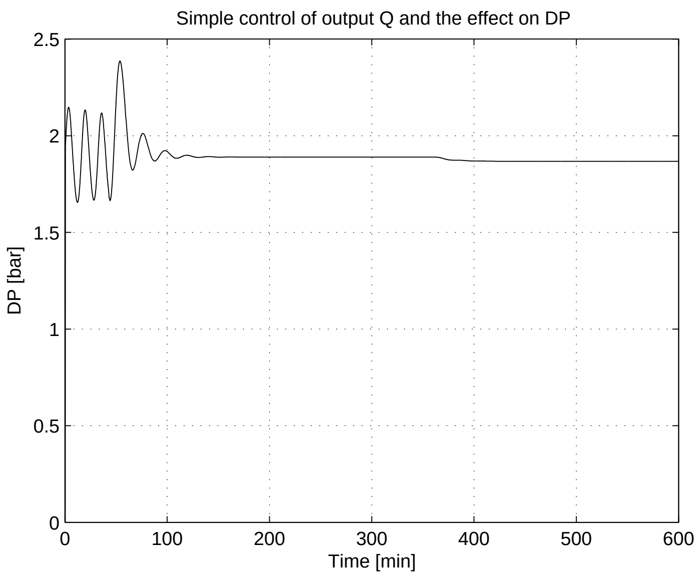

Figure A.2: Simple control of $\dot{Q}_{mix}$ and the effect on the output DP

as stated earlier. It is made for comparisment only. With this output there is only the lower bandwidth requirement from the RHP poles.

Designed at the high pressure operating point

The simple controller with a crossover frequency $\omega_c = 0.0206$ is given as

$$K = -1.55$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -1.9521 & -0.0039 \pm 0.0073i \end{bmatrix}^\top$$

The values for the different sensitivity peaks were as follows

$$\|S\|_\infty = 1.0038; \|T\|_\infty = 1.2627; \|KS\|_\infty = 1.5559$$

The peaks are low and this controller should achieve very good performance properties. The gain margin was 0.1519 and the phase margin was 67.3032 °.

As can be seen of A.3 the pressure setpoint leads to the correct valve position. In A.4 the control of how P_1 affects the output DP can be seen. The controller yields good performance.

Designed at the low pressure operating point

The lower bandwidth requirement was $\omega_c > 0.0213$. The simple controller that achieved this requirement is given as

$$K = -5.25$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -1.9976 & -0.0051 \pm 0.0120i \end{bmatrix}^\top$$

The values for the different sensitivity peaks were as follows

$$\|S\|_\infty = 1.0660; \|T\|_\infty = 1.8090; \|KS\|_\infty = 5.5965$$

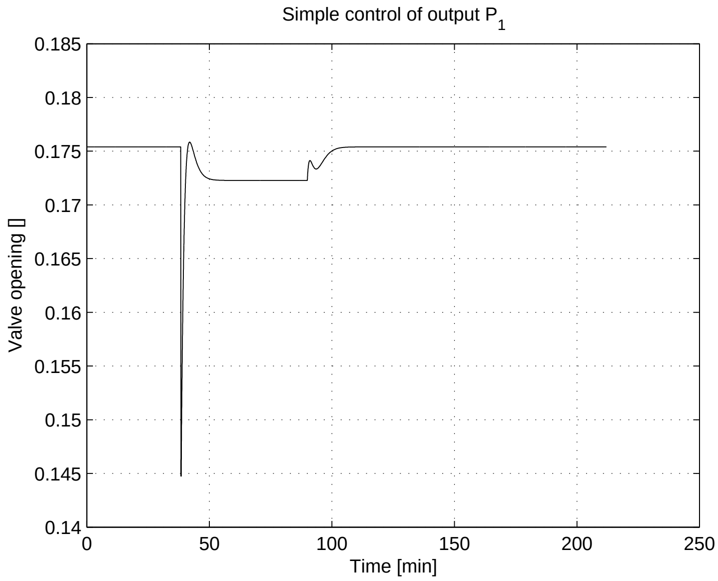

Figure A.3: Simple control of output P_1

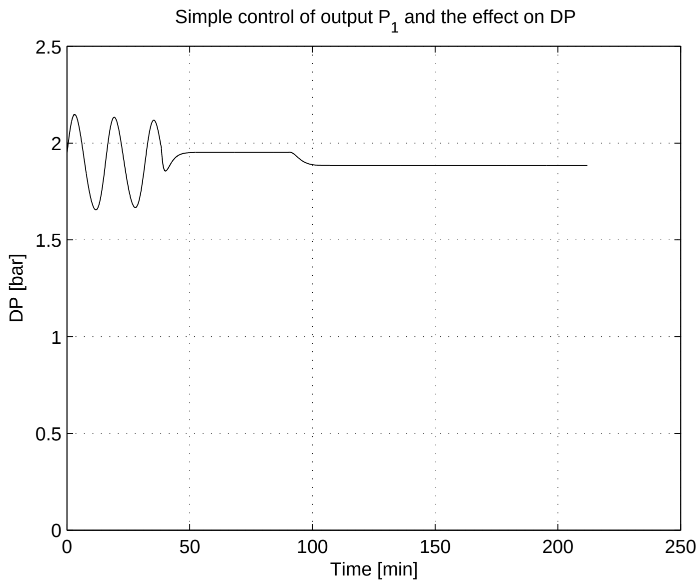

Figure A.4: How simple control of P_1 affects DP

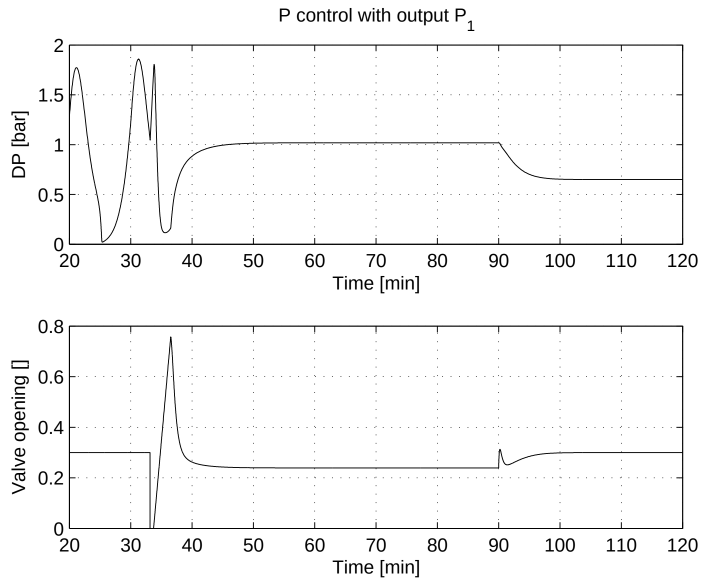

Figure A.5: Simple control of P_1 at the high pressure operating point.

The peaks are rather small, except for $\|KS\|_\infty$ which is a bit to large. This is of no practical importance as this operating point wouldn't be considered in a real, full scale system. The gain margin was 0.4306 and the phase margin was 55.9455 °.

The opening of the valve in this case was $u = 0.3$. Again it can seem as though the bifurcation diagram 3.2 was not entirely correct. According to the diagram, a choke opening of $u = 0.3$ should correspond to a pressure $P_1 = 68.69$ bar, but as can be seen this is not the case.

In figure A.5 is the simulation of the simple control of P_1 with $u_s = 0.3$. At about 40 min the controller is turned on. As can be seen the actuator handles itself fine, it doesn't go into saturation. The reason for this is the care taken when the controller is turned on and of course the fact that P_1 is an output suited for feedback control.

A.2 Design of SISO H_∞ controllers

These controllers will be designed with constant weights exclusively. An effort will not be made to make these SISO advanced controllers fulfill the design objectives. That will become more important later. The first design is for the output DP.

A.2.1 H_∞ control of the output DP

Due to the overlapping bandwidth requirements for DP, it is difficult to design a controller for it. For now we want to explore what utilization the H_∞ design can get from the different outputs single-handedly.

Designed at the lower operating point

The weights W_u, W_P and N which is given as

$$W_u = 1.27; W_P = 0.41; N = 0.1$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -1.9613 & -1.9613 & -0.2304 & -0.0069 & -0.0030 \pm 0.0067i \end{bmatrix}^\top$$

thereby achieving nominal stability. This controller achieved the lower gamma ($\|N_{22}\|_\infty$)

$$\gamma = 0.8670$$

thereby achieving nominal performance, and sensitivity peaks given as

$$\|S\|_\infty = 1.9975; \|T\|_\infty = 1.4496; \|KS\|_\infty = 0.4745$$

The peaks are satisfactory low. The gain margin was 2.0025 and the phase margin was $-46.7504\,^\circ$.

The opening of the valve held it's intended stationary point $u = 0.1754$. As can be seen the controller is turned on after about 40 min and the slugging ends after about ten minutes. This controller yields satisfactory performance. The reason for this is that the controller has high process gain.

Designed at the low pressure operating point

There is a impossible bandwidth of operation for this output at this operating point. The control design were therefore not performed.

A.2.2 H_∞ control of the output $\dot{Q}_{mix}$

The goal is to find out what performance it is possible to achieve with the output $\dot{Q}_{mix}$.

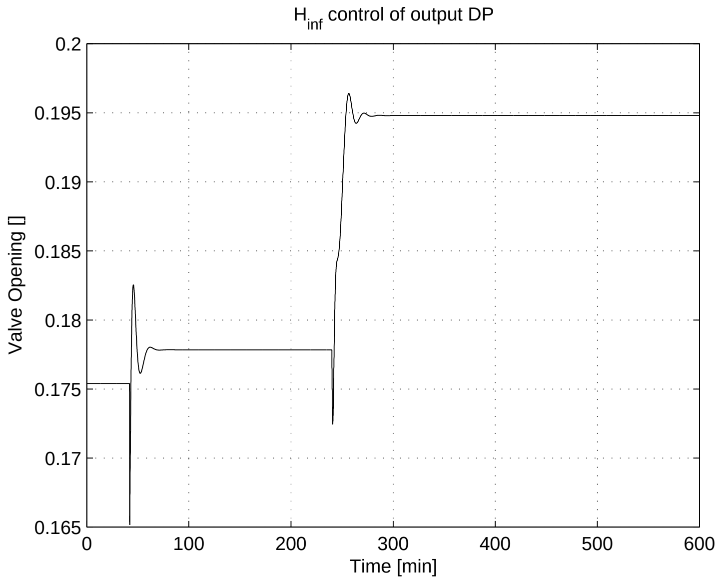

Figure A.6: H_∞ control of output DP

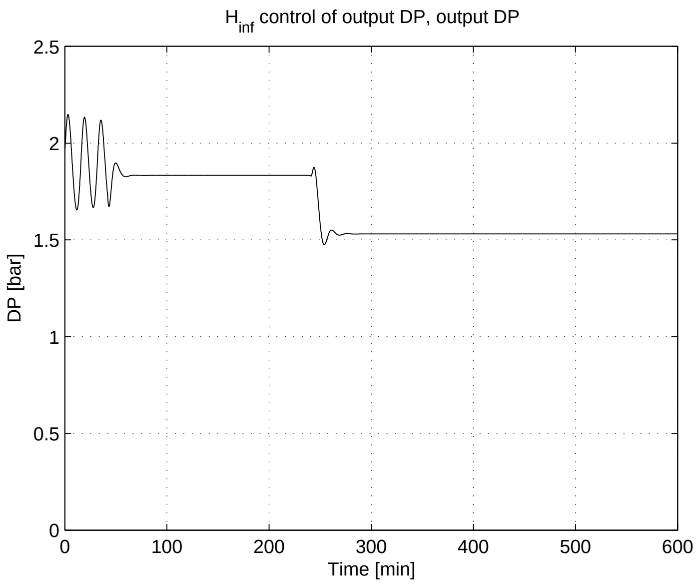

Figure A.7: The output DP controlled with a H_∞ controller using DP

Designed at the high pressure operating point

The controller is designed with the output weights W_u, W_P and N which is given as

$$W_u = 0.9;\ W_P = 0.3;\ N = 0.002$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -1.9003 & -1.7303 & -0.3620 & -0.0153 & -0.0024 \pm 0.0051i \end{bmatrix}^\top$$

Nominal stability is thereby achieved. With this controller we achieved the lower gamma

$$\gamma = 0.7259$$

thereby achieving nominal performance. The sensitivity peaks were

$$\|S\|_\infty = 1.8603; \|T\|_\infty = 1.2934; \|KS\|_\infty = 0.5514$$

The peaks are satisfactory low and the design is not violated. The only concern is the peak in S which is a bit large, it is below 2 though. $\|T\|_\infty$ is close to 1 so the second design objective is also achieved. The gain margin was 2.1767 and the phase margin was 56.1289 °.

The opening of the valve was $u = 0.177$ as can be seen of A.8. The controller is turned on after about 40 min and the slugging ends after about twenty minutes as can be seen in A.9. The actuator usage is limited.

Designed at the low pressure operating point

The controller is designed with the output weights W_u, W_P and N which is given as

$$W_u = 0.9;\ W_P = 0.3;\ N = 0.002$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -2.0211 & -1.9140 & -1.4857 & -0.0211 & -0.0042 \pm 0.0095i \end{bmatrix}^\top$$

Nominal stability is achieved. This controller achieved the gamma value

$$\gamma = 5.7768$$

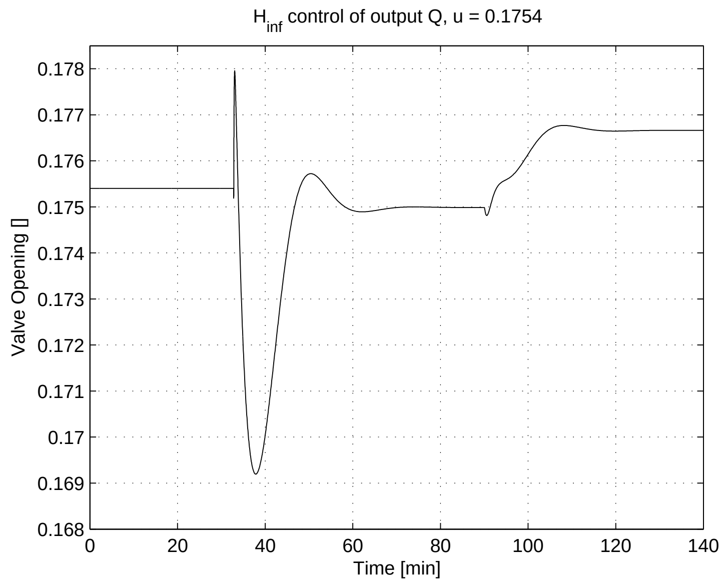

Figure A.8: H_∞ control of output $\dot{Q}_{mix}$ at the unstable stationary point $u = 0.1754$.

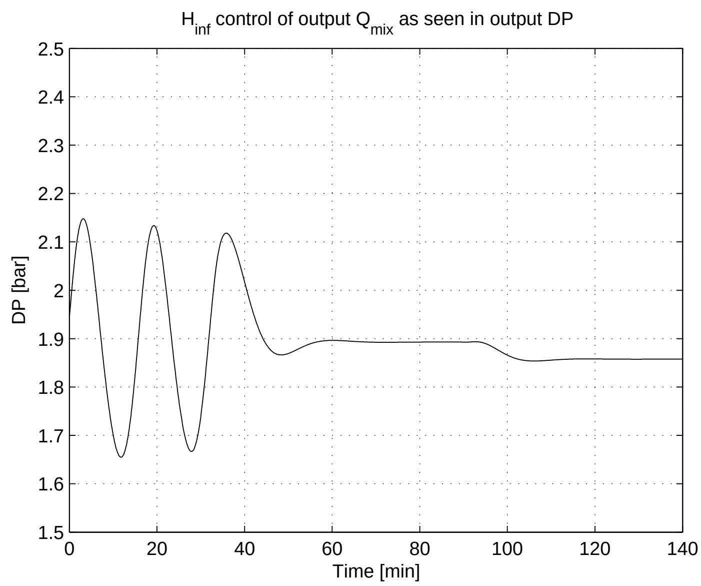

Figure A.9: H_∞ control of output $\dot{Q}_{mix}$ as seen in output DP.

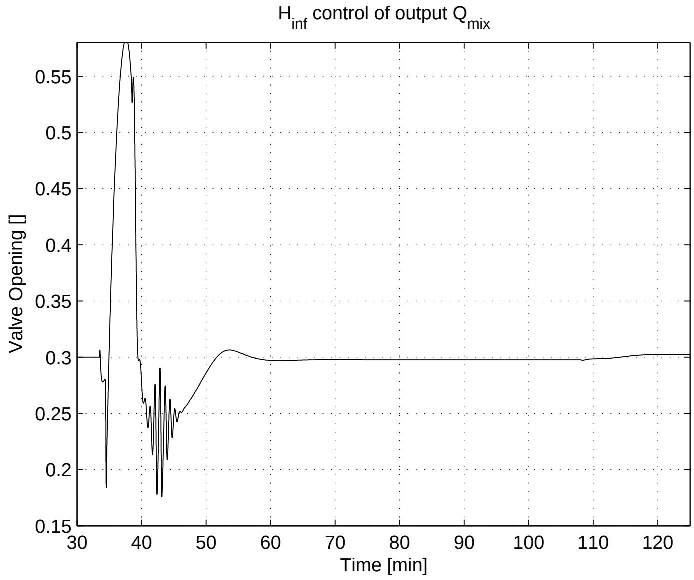

Figure A.10: H_∞ control of output $\dot{Q}_{mix}$ at the unstable stationary point $u = 0.3$.

Nominal performance is not achieved. The sensitivity peaks were

$$\|S\|_\infty = 2.7345; \|T\|_\infty = 2.4261; \|KS\|_\infty = 4.5203$$

The peaks are above what was wanted, especially the peak for KS is high. The design is violated. The gain margin was 1.5792 and the phase margin was 28.4785 °.

The opening of the valve was very close to the intended stationary point, it was $u = 0.3$ as can be see of A.10. As can be seen of A.10 the controller is turned on after about 40 min and the slugging ends after about twenty minutes as can be seen of A.11. The controller has to struggle to get the plant stabilized, but after stabilization the controller works fine.

A.2.3 H_∞ control of the output P_1

This is a good output for control so the results should be good.

Designed at the high pressure operating point

The controller is designed with the output weights W_u, W_P and N which is given as

$$W_u = 1.15;\ W_P = 0.4;\ N = 0.5$$

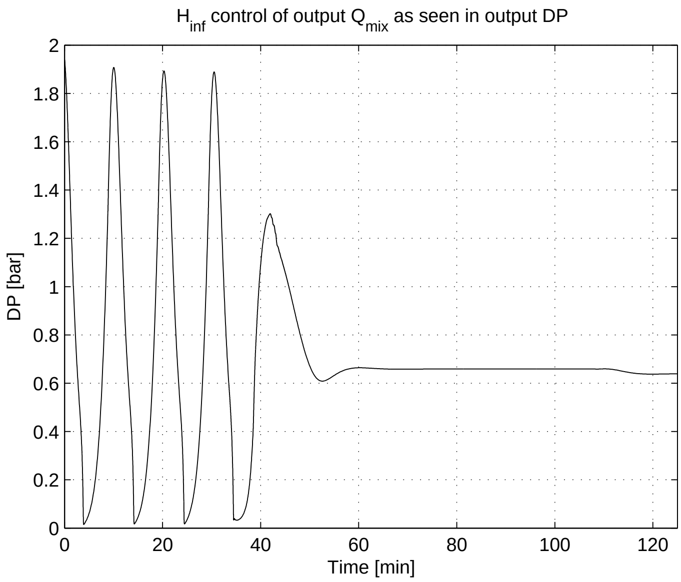

Figure A.11: H_∞ control of output $\dot{Q}_{mix}$ as seen in output DP.

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -22.2303 & -1.9614 & -1.9612 & -0.0010 & -0.0040 \pm 0.0057i \end{bmatrix}^\top$$

Nominal stability is achieved. This controller achieved the lower gamma

$$\gamma = 0.8852$$

Nominal performance is achieved. The sensitivity peaks were

$$\|S\|_\infty = 1.5029; \|T\|_\infty = 1.1760; \|KS\|_\infty = 0.4895$$

The peaks are satisfactory low. All are below 2. $\|T\|_\infty$ is also close to 1. The gain margin was 2.9884 and the phase margin was 79.1194 °.

The opening of the valve held it's intended stationary point $u = 0.175$. As can be seen in A.12 the controller is turned on after about 40 min and the slugging ends after about twenty minutes as can be seen in A.13.

Designed at the low pressure operating point

The controller is designed with the output weights W_u, W_P and N which is given as

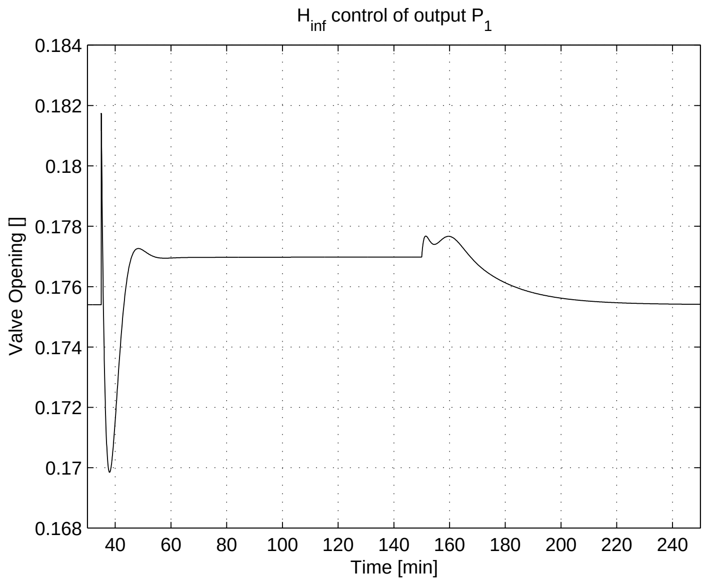

Figure A.12: H_∞ control of output P_1 at the unstable stationary point $u = 0.1754$.

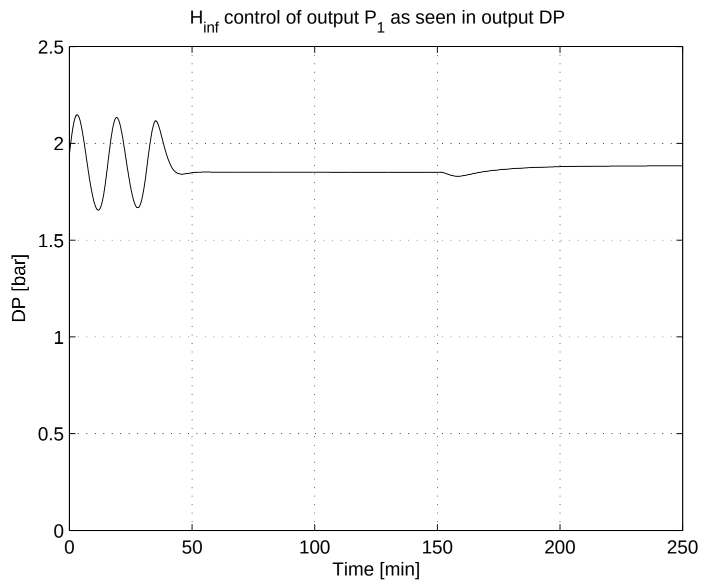

Figure A.13: H_∞ control of output P_1 as seen in output DP simulated in the simplified model.

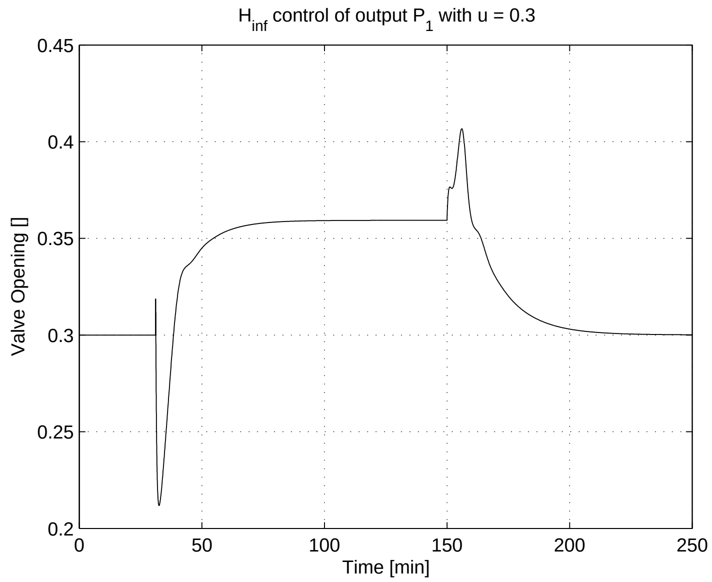

Figure A.14: H_∞ control of output P_1 at the stationary point $u = 0.3$.

$$W_u = 0.9;\ W_P = 0.25;\ N = 1$$

The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -8.9080 & -2.0155 & -2.0154 & -0.0013 & -0.0051 \pm 0.0097i \end{bmatrix}^\top$$

Nominal stability is achieved. With this controller we achieved the lower gamma

$$\gamma = 3.3121$$

Nominal performance is not achieved. The sensitivity peaks were

$$\|S\|_\infty = 1.4497; \|T\|_\infty = 1.7690; \|KS\|_\infty = 2.5232$$

As $\|KS\|_\infty$ is bigger than 2 the design is violated. The gain margin was 0.4346 and the phase margin was 63.0202 °.

The opening of the valve held it's intended stationary point $u = 0.3$ as can be seen of A.14. As can be seen in A.14 the controller is turned on after about 40 min and the slugging ends after about thirty minutes as can be seen in A.15. The actuator usage is also very limited compared to other designs. This controller has not been validated though.

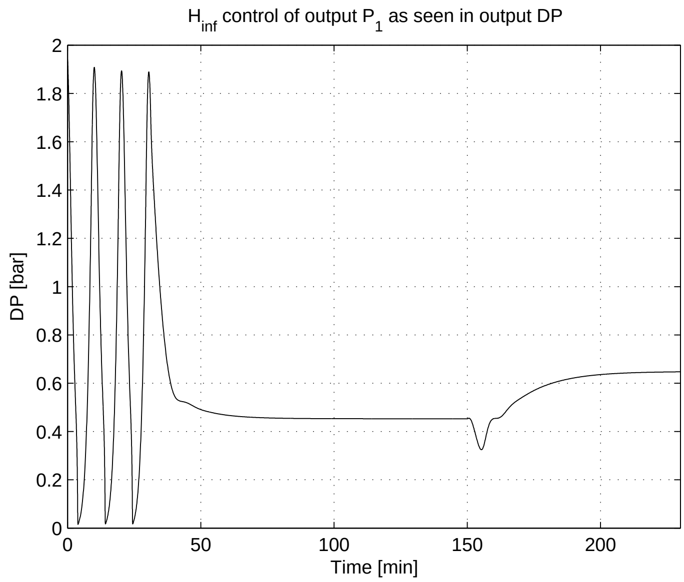

Figure A.15: H_∞ control of output P_1 as seen in output DP at the higher operating point.

A.3 Alternative multivariable controllers

The two alternative multivariable controllers that were presented in short in the main part is given here in full detail.

A.3.1 H_∞ controller made out of DP and $\dot{W}_{mix}$

The controller is designed with the output weights W_u, W_R and N which is given as

$$W_u = \frac{0.1667s + 0.034}{s + 0.51};\ W_R = \frac{0.2s + 0.00022575}{s + 2.7090e - 005};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} \tag{A.3}$$

where s is the Laplace operator. These are the same wights that were used for α_{gas}. The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -2.2277 \\ -1.9614 \\ -0.2033 \\ -0.0154 \pm 0.0100i \\ -0.0029 \pm 0.0036i \\ -0.0011 \\ -2.7090e-005 \\ -0.5100 \end{bmatrix} \tag{A.4}$$

All the poles are stable, nominal stability, NS, is achieved. This controller achieved the gamma

$$\gamma = 0.8301 \tag{A.5}$$

This ensures nominal performance, NP. This is the same gamma as was achieved by the latter controller. The sensitivity peaks were

$$\|S\|_\infty = 1.6130; \|T\|_\infty = 0.9890 \, \|KS\|_\infty = 0.4583 \tag{A.6}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small, $\|S\|_\infty$ is a bit large, but it is beneath 2. With nominal stability and performance ensured the next to check for is robust stability and robust performance

$$\|N_{11}\|_\infty = 0.2660 \tag{A.7}$$

The peak is well below 1, robust stability, RS, is ensured. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 75% and still robust stability was achieved. Next robust performance is checked for

$$\|F\|_\infty = 2.7665 \tag{A.8}$$

Clearly robust performance is not achieved. All the design objectives are ensured, and the design is complete. It is possible to use $\dot{W}_{mix}$ as a added measurement. It clearly has good robustness properties. It can handle a lot of uncertainty before going unstable. It might even be better if it were to be fine-tuned.

A.3.2 H_∞ controller made out of DP and ρ_{mix}

The controller is designed with the output weights W_u, W_R and N which is given as

$$W_u = \frac{0.1667s + 0.034}{s + 0.51};\ W_R = \frac{0.2s + 0.00022575}{s + 2.7090e - 005};\ N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} \tag{A.9}$$

where s is the Laplace operator. The closed loop poles of the system were as follows

$$p = \begin{bmatrix} -2.6052 \\ -0.2033 \\ -0.0155 \pm 0.0099i \\ -0.0011 \\ -0.0031 \pm 0.0038i \\ -1.9614 \\ -1.9613 \\ -1.9613 \end{bmatrix} \tag{A.10}$$

All the poles are stable, nominal stability, NS, is achieved. This controller achieved the gamma

$$\gamma = 0.8301 \tag{A.11}$$

This ensures nominal performance, NP. The sensitivity peaks were

$$\|S\|_\infty = 2.0000; \|T\|_\infty = 1.0096\ \|KS\|_\infty = 0.4592 \tag{A.12}$$

As one can see the peak for $\|T\|_\infty$ are close to 1 which implies good reference tracking properties. KS is also satisfactory small, $\|S\|_\infty$ is a bit large, it is exactly 2. With nominal stability and performance ensured the next to check for is robust stability and robust performance

$$\|N_{11}\|_\infty = 0.3508 \tag{A.13}$$

The peak is well below 1, robust stability, RS, is ensured. A nominal uncertainty of 20% was used, but the uncertainty could go as high as 55% and still robust stability was achieved. Next robust performance is checked for

$$\|F\|_\infty = 2.6078 \tag{A.14}$$

Clearly robust performance is not achieved. All the design objectives are ensured, and the design is complete. ρ_{mix} could be used as a added measurement. It has good properties in terms of stability. $\|S\|_\infty$ is on the limit of the design objectives, but it is still fully usable. It might be better if it were to be fine-tuned.