Aggregate Measures of Complex Economic Structure and Evolution

A Review and Case Study

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Summary

It is perhaps in the nature of complex systems that they call for aggregate measures that enable analysts to grasp their structure and evolution without being overwhelmed by their very complexity. Complex interindustry theory and models are a typical case, where the underlying database—an input–output table—routinely contains thousands of data points for a single year. Within input–output analysis, quantitative measures have been developed that describe and characterize interindustry interactions and that have been used to compare economies, both in a static taxonomy and through their evolution over time. First, we review and critically discuss a number of concepts that have been proposed and applied to interindustry systems, such as interconnectedness, interrelatedness, linkages, and economic landscapes. Second, we apply these concepts to a case study of the Australian economy between 1975 and 1999 in terms of environmental headline indicators. Our results enable the reader to judge the usefulness and ability of the measures in capturing the key structural elements and evolutionary processes governing the interaction between the economy and the environment. For the Australian case study, the measures showed a diversifying economy occurring together with a specialization of environmental flows.

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Introduction

Industrial ecology examines interactions among the environment, the economy, and technology. Although particular interactions or life cycles are often a focus, holistic measurements are also key to understanding the metabolism of our societies. Researchers have, over the past 5 decades, sought measures that describe in a condensed form the structure and functioning of national economies (see the classical work by Chenery and Watanabe [1958]).

The subdiscipline within economics that perhaps deals most with the complex structure of economies is *interindustry*, or *input—output* theory. Based on work by Wassily Leontief (1953), input—output analysis examines the interdependence of productive sectors amongst each other as well as their dependence on primary inputs, such as labor and capital, and their output for final demand. Leontief was always interested in aspects of natural resource use and repercussions of economic activity for the environment, so it is no surprise that his input—output framework is ideally suited to integrating the economic system with resource and pollution variables. By generalizing the input—output theory to incorporate links between the economy and the environment, we are able, in one way, to analyze the symbiosis of productive industries, an analogy that sits at the heart of industrial ecology.

Because of its analytical elegance, input—output analysis is widely applied to economic and environmental issues. For example, every modern general equilibrium model has an input—output model at its core.

Today, most input—output tables break down the economy into at least 50 but up to 500 sectors of production. Thus, they routinely contain thousands of data points for a single year, with each data point representing a monetary transaction between industries. These monetary tables have been complemented with satellite accounts describing resource and environmental impacts. Because of this wealth of information, researchers have developed quantitative measures that encapsulate the structure of complex intersectoral interactions into a few descriptors, which, in turn, have been used to characterize economies, both in a comparative taxonomy and through their evolution and state of development over time.

Our aim in this article is twofold: First, we critically discuss a number of concepts that have been proposed and applied to input—output systems, such as (inter)connectedness, (inter)relatedness, linkages, and economic landscapes. The article thus updates older reviews by Hamilton and Jensen (1983), Szyrmer (1985a), and Jensen and West (1985). These measures are designed to give an aggregate indication of complexity and, when applied in a temporal context, can elucidate trends toward or away from complexity of the economic system. Generally, we expect the measures to show an evolution toward an increasingly complex economic system. This might be thought of as an evolution toward more circuitous or longer production paths, more reliance of primary industries on secondary and tertiary industries, and so forth. Second, we want to interpret complexity in terms of environmental indicators. We analyze both density and absolute quantity of environmental flows so that we can interpret increasing complexity for both economically and environmentally important flows.

We apply these concepts to the Australian economy for 1975–1999 for environmental headline indicators. Our results enable the reader to judge the usefulness and ability of aggregate measures of economic structure in capturing at a glance the key structural elements and evolutionary processes governing the interaction between industries and the environment.

Review of Concepts to Measure Complexity in Economic Systems

A Taxonomy of Measures

Numerous measures of complexity have been proposed in the literature, and they can be classified in a number of ways (Szyrmer 1985a, 1985b). First, we distinguish between measures of *connectivity* and *connectedness*. By *connectivity*, we are referring to the Boolean existence or presence of connections within the system. In comparison, *connectedness* refers to metrics that consider the magnitude of these connections. A comparison

Table 1 Classification of measures of economic complexity

		Connectivity	*Connectedness*
Transactions	Inputs		ω, I^{m}, $\det(\mathbf{I}\text{-}\mathbf{A})$, σ, κ, $\lambda_{\max}$, τ, H^{T}
	Outputs		
Multipliers	Inputs	K^{**}	$U_{\cdot j}$, $\mathbf{S}$, γ^{*}, H^{L}, I^{c}, I^{s}
	Outputs		$U_{i\cdot}$, $\mathbf{S}$,

of both types of measure can give an indication of whether the system is diversifying in structure and whether the system is becoming more reliant on key sectors.

Second, we distinguish measures that deal with direct flows in the form of a *transactions* matrix ($\mathbf{T}$ or $\mathbf{A}$; see the *Input–Output Theory* section below for a definition of variables) or that also consider indirect flows by using a *multiplier* matrix ($\mathbf{L}$ and $\mathbf{G}$). If the complexity measures are consistent, we should see similar results between these categories. A third distinction is made between measures dealing with *inputs* ($\mathbf{T}\hat{\mathbf{x}}^{-1}$, $\mathbf{L}$) and those dealing with *outputs* ($\hat{\mathbf{x}}^{-1}\mathbf{T}$, $\mathbf{G}$).

Most of the measures we review have been defined for analysis of quantities of inputs through models such as the Leontief (1953) inverse (see table 1), because the Ghosh calculation and interpretation are less well known (Ghosh and Sarkar 1970). The majority of measures describe connectedness, because of its greater explanatory power. Many of the measures proposed in the literature have been defined for only one particular combination of characteristics, but corresponding alternative definitions pertaining to different categories are readily possible.

Within the following review and subsequent case study, we have unearthed all the complexity measures we could find. Occasionally, we slightly modify the measures to use a common notation. These modifications do not alter the essential characteristics of the measure and are merely incorporated to facilitate comparison. Because Szyrmer's categorization succeeds well in structuring all approaches, we have adopted his taxonomy in our case study (Szyrmer 1985a, 1985b). The literature has not always followed Szyrmer's terminology. Therefore, we present the review in this section as it has evolved historically.

Input–Output Theory

The following review deals with concepts and measures that were initially founded in input–output theory but that have since had numerous applications in industrial ecology (Duchin 1992). It is therefore appropriate to briefly recapitulate the basic input–output relationships. The main ingredient is a transactions matrix $\mathbf{T}$ with elements $(T_{ij})_{i=1,\ldots,N}$ describing the intermediate demand by sector *j* of commodities produced by sector *i*. The transactions matrix connects primary inputs $\mathbf{v}$ and final demand $\mathbf{y}$ in the *national accounting identity*,

$$v_k + \sum_i T_{ik} = \sum_j T_{kj} + y_k = x_k$$

where $\mathbf{x}$ is called gross output. Rearranging the righthand side of the above equation to

$$x_k = \sum_j \frac{T_{kj}}{x_j} x_j + y_k =: \sum_j A_{kj} x_j + y_k$$

$$\Leftrightarrow \mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}$$

where elements of the matrix $\mathbf{A}$ are called *direct requirements*, yields the well-known input–output relationship

$$(\mathbf{I} - \mathbf{A})\,\mathbf{x} = \mathbf{y} \Leftrightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{y} = \mathbf{L}\mathbf{y},$$

where $\mathbf{L}$ is the famous *Leontief inverse*, also referred as the *total requirements* matrix.

Alternatively, in the Ghosh model, $\mathbf{A^*}$ is the *direct sales* matrix, such that $A^*_{ij} = T_{ij}/x_i$ allows the interpretation of supply side effects:

$$\mathbf{x}'\,(\mathbf{I} - \mathbf{A}^*) = \mathbf{v} \Leftrightarrow \mathbf{x}' = \mathbf{v}'\,(\mathbf{I} - \mathbf{A}^*)^{-1} = \mathbf{v}'\mathbf{G},$$

with $\mathbf{G}$ being the Ghosh inverse and the prime denoting transposition.

Input–output analysis as introduced above has been a focus of industrial ecology for many years and usually involves the generalization of the input–output system from purely monetary terms to incorporate environmental and resource measures:

$$\mathbf{Q} = \mathbf{qLy},$$

with $\mathbf{Q}$ being total impact/resource requirements, $\mathbf{q}$ being direct intensity (e.g., kt-CO_2-eq/\$). We combine $\mathbf{q}$ with the Leontief inverse $\mathbf{L}$ to perform our complexity analysis directly on the square matrix of multipliers $\hat{\mathbf{q}}\mathbf{L}$. Most measures reviewed below were developed purely for economic measurement, and the generalization toward physical measures does not occur until we implement them in our case study.

Production Recipes and Productivity

The first attempt to reduce the complex structure of interindustry transactions to a manageable set of numbers was probably undertaken by Leontief (1953) himself, who examined input–output tables of the United States to interpret a decrease in the share of intermediate inputs in the production recipe as an increase in sectoral productivity.

Chenery and Watanabe (1958) attempted the first intercountry comparison of production. Due to low disaggregation, Leontief (1953) and Chenery and Watanabe (1958) were able to analyze each single sector. For larger databases, such as the one examined in our case study of Australia, such detail is overwhelming. Accordingly, we define a more aggregate measure

$$\omega = \frac{\sum_i v_i}{\sum_i x_i} = \frac{v}{x} = \frac{\sum_i y_i}{\sum_i x_i} = \frac{y}{x}$$

as the *proportion of value added* $v = \sum_i v_i$ within total output $x = \sum_i x_i$. The equality between the ratios of value-added v/x and final demand y/x holds because of the national accounting identity. It should be noted that in the System of National Accounts (SNA), output is dependent on the boundary of a producer unit (an establishment or enterprise) rather than a process of production. Hence, in our measurements of complexity, we are investigating the interconnectedness of establishments, not processes within the economy.

A related summary measure proposed by Hamilton and Jensen (1983) is the normalized sum of intermediate coefficients

$$I^{\mathrm{m}} = \frac{\sum_{ij=1}^{N} A_{ij}}{N}.$$

Basically, the larger the intermediate coefficients are, the higher is the degree of internal dependence in the economy, and the lower is the degree of openness, toward both exogenous (primary) inputs and exogenous outputs (final demand). A general interpretation is that the higher the degree of internal dependence is, the higher is the division of labor, which indicates, in turn, a more developed economy.[1] In other words, "an economy is more 'complex' if it churns more to achieve a given final demand" (Robinson and Markandya 1973, 119).

Distribution of Coefficients

It is not unusual for an input–output table to contain a large number of small elements and a small number of elements embodying most of the monetary flow. Peacock and Dosser (1957) suggested the number of nonzero coefficients in $\mathbf{A}$ as a measure of complexity. The larger this number is, the more connected the economy is. Modern usage of mathematical balancing techniques has resulted in tables that are entirely occupied with nonzero, albeit sometimes very small, elements, so that nonzero element counts are not really applicable anymore. Instead, we examine two measures for the distribution of coefficients: The *skewness* (σ) of a distribution characterizes the asymmetry around its mean, whereas the *kurtosis* (κ) characterizes its peakedness. The less connected an economy is, the higher is the concentration of monetary flows in a few large elements, and the higher are both skewness and kurtosis.

Linkages and Economic Landscapes

Rasmussen (1956) introduced the concepts of forward and backward interindustry *linkages* (written $U_{i\cdot}$ and $U_{\cdot j}$) as measures of structural interdependence.[2] $U_{i\cdot} > 1$ indicates strong forward linkages, or "sensitivity of dispersion," of sector i, which means that a change in all sectors' final demand would cause an above-average production increase in sector i—that is, sector i's products would be in greater demand. $U_{\cdot j} > 1$ indicates strong backward linkages, or "power of dispersion," of sector j, which means that a change in the final demand of sector j would create an above-average increase in the activity of the whole economy—that is, sector j would draw more heavily on the rest of the system.

Rasmussen's (1956) initial formulations were subsequently modified by a number of authors (for a summary, see the work by Lenzen [2003]), mostly in terms of weighting and normalization. The consensus appears to be that forward linkages $U_{i\cdot}$ are measured with the Ghosh inverse **G**, weighted by value added **v**

$$U_{i\cdot} = \frac{G_{i\cdot}}{\bar{G}} = \frac{\sum_j \frac{v_i}{v} G_{ij}}{\frac{1}{N}\sum_{ij} \frac{v_i}{v} G_{ij}},$$

and that backward linkages $U_{\cdot j}$ are measured with the Leontief inverse **L**, weighted by final demand **y**

$$U_{\cdot j} = \frac{L_{\cdot j}}{\bar{L}} = \frac{\sum_i L_{ij} \frac{y_j}{y}}{\frac{1}{N}\sum_{ij} L_{ij} \frac{y_j}{y}}.$$

The weighting ensures that the linkages describe the effects of percentage rather than unit (dollar) changes in final demand. The $U_{i\cdot}$ and $U_{\cdot j}$ are normalized by the global intensities of the Ghosh and Leontief inverses, so that $\Sigma_i U_{i\cdot}/N = 1$ and $\Sigma_j U_{\cdot j}/N = 1$.

Rasmussen's (1956) concept of interindustry linkages was further elaborated by Sonis and coworkers[3] in their studies on linkage hierarchies, fields of influences, and economic landscapes. The field of influence of a transaction A_{ij} in the direct requirements matrix **A** measures the change in the Leontief inverse **L** as a result of a change in A_{ij}. The field of influence is hence an $N \times N$ matrix for each A_{ij}, but it can be reduced to a scalar *field-of-influence intensity*. Lenzen (2003) showed that this intensity can be calculated from forward and backward linkages according to

$$S_{ij} = \frac{U_{\cdot i} T_{ij} U_{j\cdot}}{\sum_{k,l} U_{\cdot k} T_{kl} U_{l\cdot}}.$$

The matrix **S** has also been called a *multiplier product matrix* (MPM; Sonis et al. 1995a; Sonis and Hewings 1999). Sonis and colleagues (2000) showed that the MPM has minimum information properties in that it is the most homogeneous distribution of row and column multipliers of the inverse matrix. West (1999) presented an example of a time series of economic landscapes based on the MPM.

Strassert (2001) argued that conventional Rasmussen (1956) linkages exclude a number of interindustry circuits and instead suggested a hypothetical extraction method. Soofi (1992) used backward and forward linkages to define backward and forward *concentration indexes*

$$G_{\cdot j} = \sqrt{N\left(1 - \sum_i \left(\frac{L_{ij}}{L_{\cdot j}}\right)^2\right)} \quad \text{and}$$

$$G_{i\cdot} = \sqrt{N\left(1 - \sum_i \left(\frac{G_{ij}}{G_{i\cdot}}\right)^2\right)}.$$

These measures were applied by the New Zealand Treasury (Claus 2003), with the counterintuitive finding that these indexes become *small* for a *high* degree of concentration in intermediate transactions. They are similar to normalized coefficients of variation of the Rasmussen (1956) linkages, also called backward and forward *spread* (West 1999).

Finn's (1976) Flow Indexes

Input–output theory underwent a development in theoretical biology and ecological modeling, similar to economics (for a review, see the work by Suh [2005]). Finn (1976) defined the *cycling index* of ecosystems as

$$I^c = \frac{\sum_i \left(1 - L_{ii}^{-1}\right) x_i}{x},$$

where the numerator describes the portion of total ecosystem output that returns to the same sector one or more times. Here we apply this index to an economic system, as well as the *straight-through* *flow index*

$$I^{s} = \frac{x - \sum_{i} \left(1 - L_{ii}^{-1}\right) x_i}{x} = 1 - I^{c}.$$

Determinants and Eigenvalues

On the basis of the identity$(\mathbf{I} - \mathbf{A})^{-1} = \text{adj}\,(\mathbf{I} - \mathbf{A})/\det\,(\mathbf{I} - \mathbf{A})$, Wong (1954) suggested the *determinant* of $\mathbf{I} - \mathbf{A}$

$$\det(\mathbf{I} - \mathbf{A}) = \frac{\text{adj}\,(\mathbf{I} - \mathbf{A})_{ij}}{(\mathbf{I} - \mathbf{A})_{ij}^{-1}} \quad \text{for any } i\,,\ j$$

to measure complexity: The smaller the determinant is, the larger the Leontief multipliers are, which indicates a greater degree of complexity.

Lorenzen (1981) interpreted the dominant eigenvalue γ^* of the Leontief inverse as the *gross quota*[4]

$$\gamma^* = \frac{x^*}{y^*} = \frac{\sum_{i=1}^{N} x^*{}_i}{\sum_{i=1}^{N} y^*{}_i}$$

between the eigenvector $\mathbf{y}^*$ belonging to γ^* and the gross output $\mathbf{x}^*$ required to produce $\mathbf{y}^*$. The higher γ^* is, the higher is the interdependence of sectors in the economy. γ^* can be compared with the actual gross quota of the economy

$$\gamma_{g} = \frac{x}{y} = \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} y_i} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij} y_j}{y}$$

$$= \sum_{j=1}^{N}\sum_{i=1}^{N} L_{ij}\frac{y_j}{y} = \sum_{j=1}^{N} L_{\cdot j}\,.$$

If key sectors—as measured by $L_{\cdot j}$—are strongly represented in the actual final demand $\mathbf{y}$, then $\gamma_{g} > \gamma^*$, and vice versa. The actual gross quota is also identical to the *mean path length* used by Finn (1976) and is strongly related to the production recipe indexes already mentioned.

Dietzenbacher (1992) suggested the dominant eigenvalues of the input and output coefficients matrices to be used as a general indicator for backward and forward interindustry linkages, respectively. He presented an elegant calculus connecting the lefthand and righthand eigenvectors of the input and output matrices, respectively, to the weighted and unweighted linkages previously mentioned. Lenzen (2006) used the dominant eigenvalue $\lambda_{\max}$ of $\mathbf{A}$ to characterize feedback in ecosystem networks: The higher $\lambda_{\max}$ is, the more internal feedback exists within the system, and the longer it takes the system to adjust to an initial exogenous shock. The same logic can be applied to economic systems.

Order Matrices

Yan and Ames (1965) examined the decomposition of the Leontief inverse into production rounds (see also the work by Suh and Heijungs [2007])

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots\,.$$

The existence of zero values A_{ij} in $\mathbf{A}$ prevents changes in one sector j from directly affecting certain other sectors i. Indirect effects between such sectors can, however, propagate through the economy via a number of intermediate sectors—for example, $A_{im}\,A_{mn}\,A_{nj}$. The elements K_{ij} of an order matrix $\mathbf{K}$ measure how many production rounds are necessary to connect two sectors i and j. For example, if we assume that $\mathbf{A}^2 \ni A_{im}\,A_{mj} = 0\ \forall m$ but that $\exists m,n$ so that $\mathbf{A}^3 \ni A_{im}\,A_{mn}\,A_{nj} \neq 0$, then $K_{ij} = 3$, because at least three production rounds are necessary for a change in sector j to affect sector i. Yan and Ames (1965) suggested the reciprocal of the harmonic mean of elements of $\mathbf{K}$ as a measure of economic complexity.

Blin and Murphy (1974) objected that this measure reflects the existence of relations but not their magnitude and that it is sensitive only to the shortest existing path between two sectoral nodes (their "first meeting") but blind to all other, longer paths linking the same nodes. Blin and Murphy (1974) showed that weighting Yan and Ames's (1965) order matrix with the size of coefficients A_{ij} simply reproduces the series expansion in the above equation and that the harmonic mean becomes a backward linkage.

Furthermore, and once again, with modern tables often being filled with small but entirely nonzero elements, such an order matrix is not applicable anymore.[5] This obstacle can be circumvented by a procedure proposed by Robinson and Markandya (1973), who calculated the number of rounds k necessary for the (cumulative) effect of a change in sector j on sector i to become less (more) significant than a chosen cutoff point—that is, for A_{ij}^k ($\sum_{l=0}^{k} A_{ij}^l$) to become smaller (larger) than that cutoff. They assembled the (K_{ij}) into a *transactions round matrix* $\mathbf{K}$ showing the number of times sectors supply each other. According to Robinson and Markandya (1973, 125), the total number of transactions

$$K^{**} = \sum_{i,j=1}^{N} k_{ij}$$

"measures the total amount of information required by the system to reach equilibrium (and thus is perhaps a good measure of the complexity of the system)." This measure was also used by Bosserman (1981) to describe ecosystems and by Schnabl and coworkers (Holub and Schnabl 1985; Weber and Schnabl 1998; Schnabl 1994) in qualitative input—output analysis and minimal flow analysis.

Triangulations

Chenery and Watanabe (1958) were the first to point out the use of input—output table triangulations for determining the degree of *one-way interdependence* within total interdependence.[6] Accordingly, we write the *degree of linearity* (Helmstädter 1965b) as

$$\tau = \frac{\sum_{i>j} T_{ij}^*}{\sum_{ij} T_{ij}^*},$$

where $\mathbf{T}^*$ is a triangulated version of the transaction matrix $\mathbf{T}$. We obtain $\mathbf{T}^*$ from $\mathbf{T}$ by reordering the sectors so that the sum of the transaction on one side of the diagonal is maximized and each sector delivers more to than it receives from the subsequent sector. In a hypothetical economy where primary industries, such as agriculture, fishing, forestry, and mining, supply secondary industries (e.g., manufacturing) and tertiary industries (e.g., services) and secondary industries supply tertiary industries but not vice versa, τ would assume zero. Any departure from a zero value of one-way interdependence characterizes the dependence of primary industries on secondary and tertiary output, and so on.

Entropy

Proops (1988, 1983) used the well-known entropy

$$H^L = -\sum_{ij} p_{ij} \ln p_{ij},\ p_{ij} = \frac{L_{ij}}{\sum_{ij} L_{ij}}$$

to describe how sectors of an economic system become more enmeshed with each other over time. The larger the entropy is, the more equal are the coefficients of the Leontief inverse, and, hence, the more equally the sectors of the economy are participating in production. Zucchetto (1981) and Jackson and colleagues (1989) applied the same principle to the transactions matrix $\mathbf{T}$

$$H^T = -\sum_{ij} c_{ij} \ln c_{ij},\ c_{ij} = \frac{T_{ij}}{\sum_{ij} T_{ij}}.$$

Jackson and colleagues (1989) additionally defined the *information gain* H' between subsequent transaction matrices $\mathbf{T}$ and $\mathbf{T}_0$ as

$$H' = \sum_{ij} p_{ij}^* \ln \frac{p_{ij}^*}{p_{0,ij}^*}, \quad p_{ij}^* = \frac{T_{ij}}{\sum_{ij} T_{ij}}.$$

This would require constant price matrices, however. Ulanowicz (1980, 1981) defined a number of additional measures based on the *system uncertainty*

$$H^L = -x \sum_i g_i \ln g_i, \quad g_i = \frac{x_i}{\sum_j x_j},$$

which is further decomposed into four constituents: *ascendancy*, *tribute*, *dissipation*, and *redundancy*. These measures have only limited interpretations for economic systems (see Szyrmer 1985a).

Finally, Soofi (1992) applied the entropy formula to forward and backward linkages. In this view, a high entropy value brought about by small variations in linkages indicates a high degree of integration in the economy.

Total Flow

Szyrmer (1985b, 1992) introduced *total flow* as an alternative to the traditional concepts of direct and indirect flows as measured by **A** and **L**. The basic idea is that each flow originating from an intermediate sector i, proceeding via intermediate sectors j and k, and terminating in final demand l contributes to connectedness not only through its direct transfer ($i \rightarrow j$; **A**) or its supply to final demand ($i \rightarrow j \rightarrow k \rightarrow l$; **L**) but also via indirect intermediate paths ($i \rightarrow j \rightarrow k$). General measures for system connectedness can hence be leveraged off the total flow matrix

$$\mathbf{Z}^{\text{total}} = \mathbf{A}\mathbf{L}\hat{\mathbf{L}}^{-1}\hat{\mathbf{x}} = (\mathbf{L} - \mathbf{I})\,\hat{\mathbf{L}}^{-1}\,\widehat{\mathbf{L}\mathbf{y}},$$

where $\hat{\mathbf{L}}$ is the diagonal matrix of diagonal elements of the Leontief inverse **L**. Each element z_{ij}^{total} measures the total output between sector i and j, not just the portion destined for final demand.[7] Szyrmer labeled as *transit flow* those purely intermediate flows that make up the difference between total flow and the Leontief inverse (Szyrmer 1985a, 1985b). Total flow coefficients can be defined as

$$h_{ij} = \{\mathbf{H}\}_{ij} = \{\hat{\mathbf{x}}^{-1}\mathbf{Z}^{\text{total}}\}_{ij} = \begin{cases} \dfrac{L_{ii}x_j}{L_{jj}x_i} & i \neq j \\[2ex] \dfrac{L_{ii}x_j - x_j}{L_{jj}x_i} & i = j \end{cases}.$$

$\mathbf{Z}^{\text{total}}$ and **H** support the same range of measures as **A** and **L** (Szyrmer 1985b): sums of transactions, sums of coefficients, average coefficients, and entropies. For example, average coefficients and entropies are defined as

$$\frac{\sum_{ij} h_{ij}}{N^2} \quad \text{and} \quad -\sum_{ij} \frac{z_{ij}^{\text{total}}}{x} \ln\left(\frac{z_{ij}^{\text{total}}}{x}\right),$$

respectively.

Applications of the total flow concept can be found in the work by Szyrmer and Walker (1983) and Szyrmer and Ulanowicz (1987).

The Australian Economy 1975–1999: A Case Study

We are most interested in using the measures to analyze changes in the complexity of our economic system in terms of what we would like to call key performance indicators (KPIs). To give a practical example, we draw on 25 years of development in Australia. We begin by analyzing the economic development in Australia with reference to all measures surveyed above. We then select the most appropriate measures to interpret the evolution of complexity in Australia's economy according to our KPIs. Our KPIs include economic flows, greenhouse gas emissions (from fuel use), material flow, and employment. We include these indicators as they are used by the Australian government to cover the three tenets of economic, social, and environmental performance at the national level (Foran et al. 2005).

A survey of measures by Hamilton and Jensen (1983) found that accounting conventions and levels of aggregation affect most measures. Hence, in this study, we use consistent conventions of accounting, including indirect allocation of imports, to properly study the true structure of the economy. Also, we studied three levels of aggregation, an enlarged 344-sector database, an industry-level database of 105 sectors, and an aggregated database of 30 sectors. Ideally, we want measures of complexity that are aggregation-invariant, or at least not inconsistent at different levels of aggregation. We finally studied relative changes (by normalizing all results), as we are interested in measures that are not necessarily constant in absolute terms but that give consistent results over time. Data sources are described in the Supplementary Material on the Web.

Review of All Measures

As there is no generally accepted measure of complexity (Hamilton and Jensen 1983), to be able to interpret the range of measures surveyed, we initially give an abridged analysis of all measures. Measures that are found to be inconsistent across aggregation or that provide no discernibly useful information are subsequently excised.

To capture the temporal development, we analyze trends and normalize all results to 1999

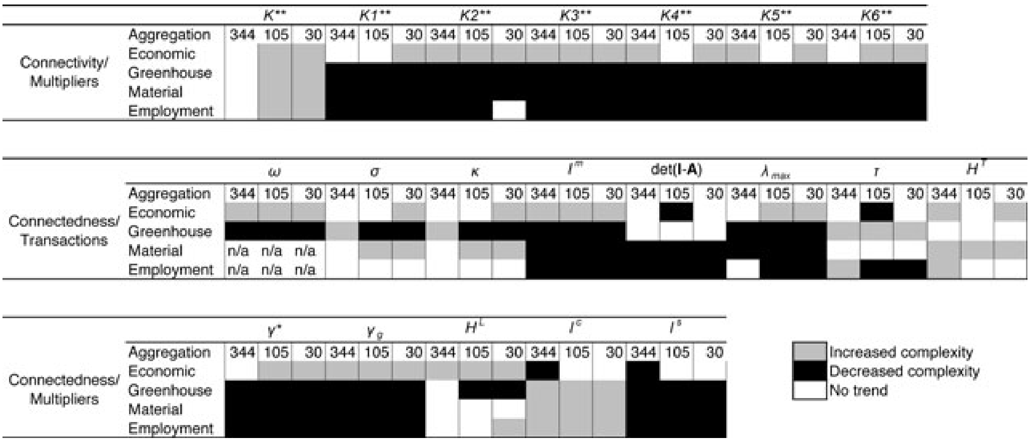

Figure 1 Summary of all measures across all indicators. Overall results are across the full time series, and depict if a trend consistent with complexity is found.

levels. In all graphs, we also weight by expected complexity, such that an upward trend is indicative of increasing complexity. For reference, a pictorial summary of the measures across all indicators (see figure 1) is provided, and it needs some initial explanation. Measures are classified by type (see the *A Taxonomy of Measures* section), according to aggregation level, and trends are color coded according to an observation in agreement with increasing complexity (gray), an observation in agreement with decreasing complexity (black), and no trend (white). Readers should refer to the literature review for our reasoning.

To provide an objective rather than subjective gauge of trends, we apply a linear regression to the results, with the independent variable being time and the dependent variable being the results of the measure. We state the existence of a trend if the calculated relative standard error of the linear coefficient is less than 50%. Although such a method is somewhat arbitrary and imprecise, we only want a reasonable reflection of the results, and the reader is referred to the subsequent figures in this section for his or her own interpretation.

We now proceed with a more in-depth review of these results.

Connectivity Measures of Multipliers

The only connectivity measure reviewed is K, the transactions rounds matrix. The reader is reminded that connectivity measures assess the Boolean existence of connections within a system. The trends found for K are classed according to different cutoff values. As previously mentioned, a cutoff of zero can put the analyst at the mercy of "manufactured" data points from computational table creation. In contrast, placing the cutoff too high in disaggregated systems will cause the measure not to pick up on changes in connectivity. In figures 1 and 2 we define the variables as follows: K** cutoff $= 0$, K1** cutoff $= 0.1$, K2** cutoff $= 0.01$, K3** cutoff $= 0.001$, K4** cutoff $= 0.0001$, K5** cutoff $= 0.00001$.

As expected, the cutoff choice had an obvious effect on the results. Around a cutoff of 0.001 (K3** in figure 2), minimal noise was found across all aggregations (see figure 1). This corresponds to just below the mean of the values of the Leontief inverse (~0.005 in 1999), whereas at 0.0001 and below, results started fluctuating to a greater extent (cf. figure 1).

Our results clearly show a trend, which is interpreted as a decrease in the overall sum of the transaction rounds matrix. This indicates a shortening of the average path length connecting two sectors and would be expected in a maturing economy. In essence, more intermediate sectors are taking on the delivery of goods and services, and a greater number of feedback loops are occurring within the economy. We found aggregation to affect results only for high and low cutoff values (see figure 1), and we judge this measure with an average cutoff value to be a good indicator of changing complexity and henceforth use K3**.

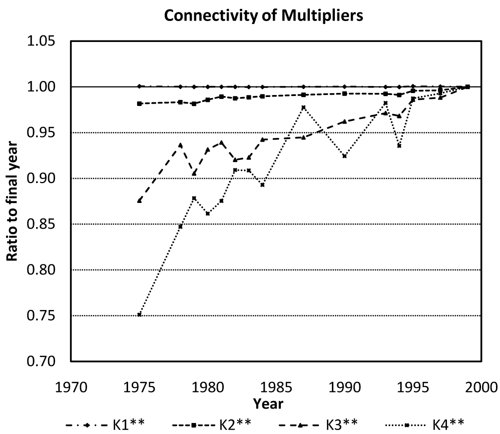

Figure 2 Evolution of relative values of order matrices, Australia 1975–1999, 344 sectors.

Connectedness Measures of Transactions

The connectedness measures of transactions measure the presence and magnitude of connections in a system. They include the proportion of value added (ω), the proportion of intermediate transactions (I^m), the determinant, the dominant eigenvalue ($\lambda_{\max}$), the degree of linearity (τ), the entropy of transactions (H^T), the skewness (σ), and the kurtosis (κ). Of these measures, the determinant gave highly variable results over time, with no consistent trend. The dominant eigenvalue of the transactions matrix showed high initial variability followed by no consistent trend, and the degree of linearity also showed no trend or consistency under aggregation. In contrast, ω, I^m, and H^T showed clear trends (see figure 3).

The dominant eigenvalue in interdependent systems determines how many rounds of interactions are needed for the system to return to a stationary state after a shock and hence determines the asymptotic behavior of the system. The lack of a trend in the results implies that there is no convergence toward an asymptotic or limited state being observed in the Australian economy.

The degree of linearity (τ) is weighted not by the total economic production but by the total industrial production. Our other measures (ω or I^m) mentioned below show increasing industrial production relative to economic production. Hence, the measure of the degree of linearity is somewhat subsumed by larger scale efficiency effects, and its exclusive interpretation is not so useful.

The proportion of value added or final demand in total output (ω) showed a small decrease over time (approximately 6%). This equates to an increasing dominance of intermediate transactions (shown also by I^m) within the economy, as one would expect with an increasingly complex production system. Chenery and Watanabe (1958) found average ω and I^m in the range of 30% to 50% in a cross-country analysis. They found an increasing share of intermediate goods in total output (I^m) with the developing economies and expected increased disaggregation to sharpen distinctions between intermediate and final use. Similar results are found for Australia, with the intermediate component of gross output growing from 35% in 1975 to 42% in 1999. In contrast,

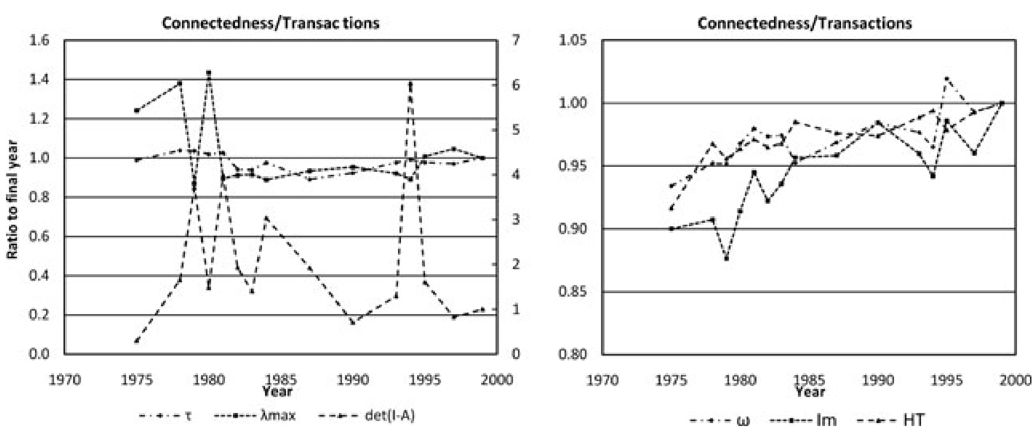

Figure 3 Connectedness measures of transactions, Australia, 344 sectors. Measures showing no trends are on the left (determinant on secondary axis), and measures showing trends are on the right.

the proportion of value added (including imports) shrank from 62% to 58%; excluding imports, this dropped from 56% in 1975 to 50% in 1999.

H^T, the entropy of the transactions matrix, has a consistent trend upward over time, reflecting an increasing enmeshing of sectors, consistent with increasing complexity. Proops (1988) generally found an increase in entropy over time, consistent with the expectation of increased complexity. Much like the findings in this study, however, increases were not always consistent, and comparisons between countries did not always find more developed countries to have higher entropies.

Skewness and kurtosis (σ and κ) within the Australian economy show relative distribution of the connections of the system. This measure was found to vary with aggregation (see figure 1) across both economic and environmental indicators. Absolute results show that the skewness for 1999 was 17.5, which indicates, as for all years, that most of the data are greater than the mean. A similar result was found for the kurtosis, which shows the data being very outlier prone ($\kappa \sim 320$, compared to a normal distribution with $\kappa = 3$). These measures are perhaps not suitable for detailed datasets with large variation in cell values. Skewness and kurtosis are sensitive to element size, and aggregation exactly influences this—an obvious shortcoming. The results for greenhouse emissions are shown in figure 4, where we found actual inversion of results under aggregation. We consider these measures inappropriate for such analysis and no longer contemplate their use.

Connectedness Measures of Multipliers

Of the range of connectedness measures of multipliers, we again saw a number of measures providing indeterminate results. Of the five aggregate measures, only γ_g and H^L showed useful trends (see figure 5) and consistency under aggregation. Gross quota γ_g generally gradually increased over time, which is in line with expected developments of complexity. γ_g was consistently less than γ^*, which shows that key sectors are not strongly represented in final demand, as would be expected in a developing economy, where key sectors are subject to greater intermediate processing. The dominant eigenvalue of **L**, γ^*, showed almost identical results to the dominant eigenvalue of **A,** λ_{max}, which implies that the measure, although consistent across transactions and multipliers, suffers the same shortcomings as identified in the connectedness measures of transactions. Finn's (1976) flow indexes (I^c, I^s) were related to each other and dependent on the diagonal of the multiplier matrix; the increasing dispersal of other flows in the economy is not captured by this index, and this is perhaps why it has not provided useful results in this context.

Summary of the Review

Not all measures of economic development reviewed were expected to provide useful

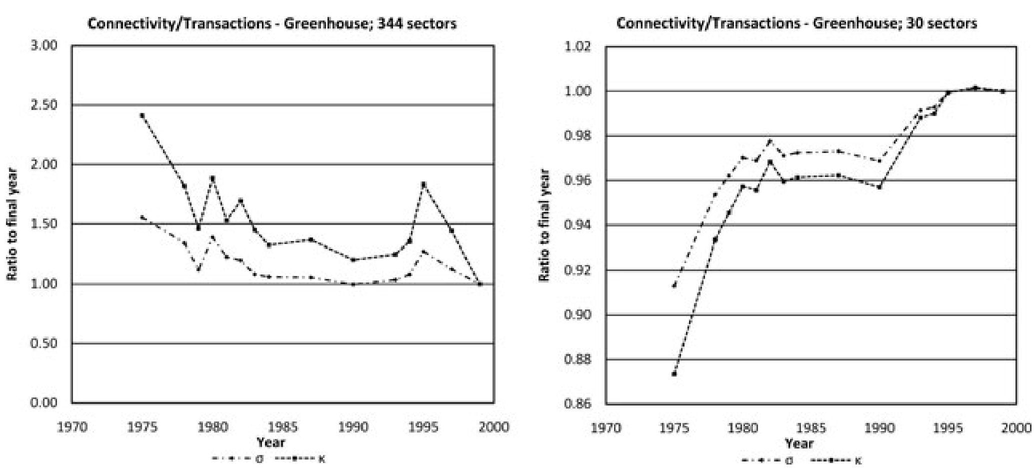

Figure 4 Skewness and kurtosis of structure of greenhouse flows, Australia, 344 sectors (left), 30 sectors (right).

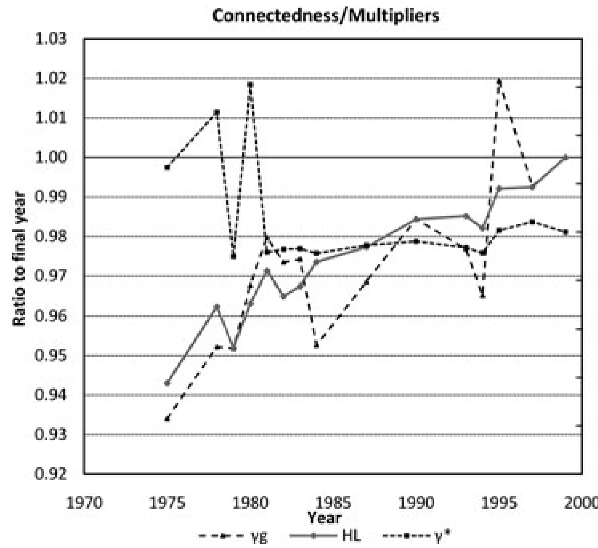

Figure 5 Connectedness measures of multipliers, 344 sectors.

application in a study of complex system evolution. A number were found to be either too greatly influenced by aggregation (σ and κ) or not holistic in their coverage (τ, I^c, I^s). The determinant can be interpreted as a measurement of the volume of the economic system but has not been found to provide useful insight into the interrelatedness of the system.

The measures that were found to be useful in tracking the evolution of the Australian economy were as follows: transactions rounds matrices K^{**}, the proportion of value added (ω), the normalized sum of intermediate transactions (I^m), the entropy of transactions (H^T), the gross quota (γ_g), and the entropy of multipliers (H^L). The entropy measures could be considered similar to skewness and kurtosis (σ and κ), as both seek to estimate the equality of the system variables. The entropy, however, is weighted by element size, which results in greater invariance under aggregation. The dominant eigenvalue γ^* did not show clear trends in the economic analysis, but this may be due to its construction and interpretation. Hence, we maintain its use in the study.

In their survey of six of the measures, Hamilton and Jensen (1983) found all measures but ω to be affected by aggregation. We do not find as great effects here. They also found I^m to be the most useful measure, consistent with findings here.

We further group the measures into those related to overall economic and environmental *efficiency* (ω, I^m, γ_g). These measures operate on a summation of system variables and generally relate to impacts occurring inside or outside the economic system, compared to the overall economic output. Second, we group entropy measures and transactions rounds matrices (H^T, H^L, K^{**}) to signify *enmeshing* and equality of system variables. These measures relate less to efficiency and more to how connected the system is. Finally, we have the measure of *resilience*—the dominant eigenvalue of the multiplier matrix ($\gamma*$), which can be interpreted as a measure of asymptotic behavior of the system.

Analysis of Complexity With Key Performance Indicators

With a reduced set of measures, we now turn to analyzing the implications of increased complexity for industrial ecology through our KPIs. Each economic measure of complexity that showed a trend was consistent with the hypothesis of increased complexity over time in Australia. We now focus on determining whether this is mirrored in terms of greenhouse gas emissions, material flow, and employment. We would like to determine whether we are seeing more enmeshing of flows of KPIs in our economy and whether this is happening in line with or in opposition to the economic development. Trends for the six aggregate measures are presented in figure 6. Enmeshing measures (H^T, H^L, K^{**}) are presented on

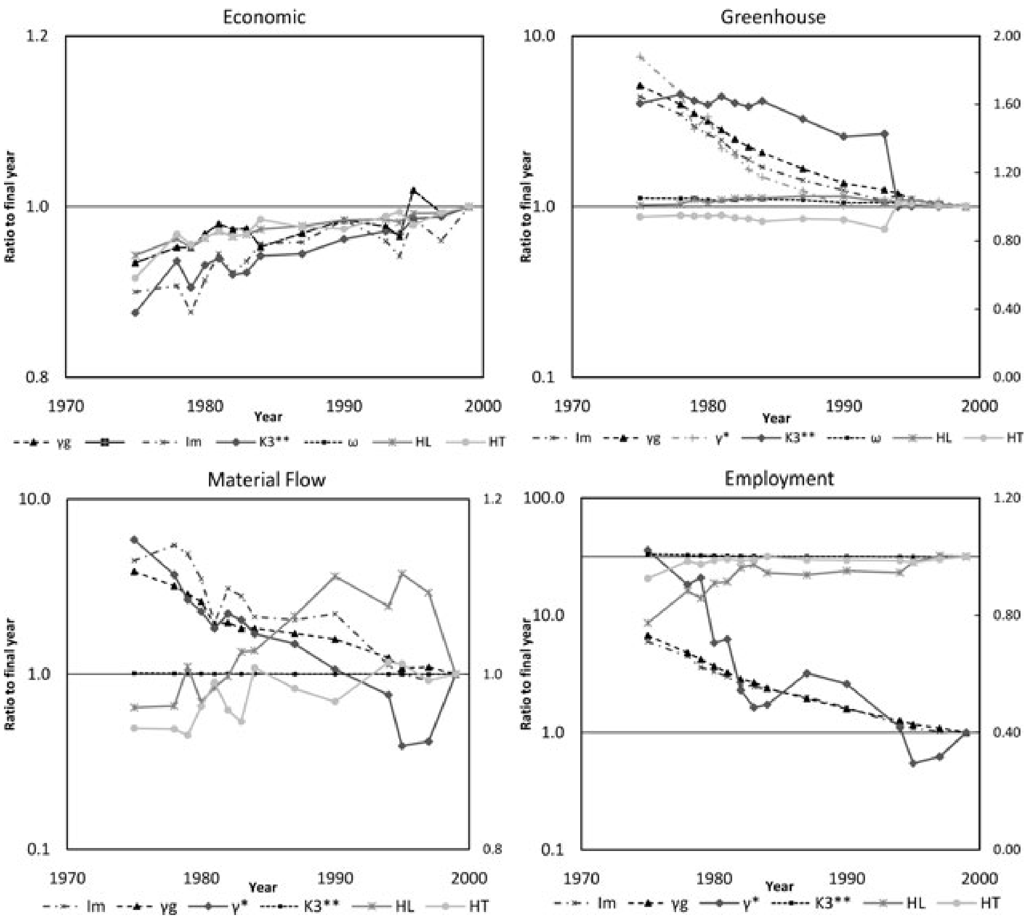

Figure 6 Evolution of measures of complexity for Australia, 1975–1999 (logarithmic scale).

a secondary axis for the KPIs due to the relatively higher changes in the other measures, which are presented on a logarithmic scale.

For greenhouse gas emissions, we see trends of decreasing complexity. This is mainly due to efficiency measures, however, with lower greenhouse impacts occurring in the industrial structure (measured by I^m, γ_g). ω for greenhouse emissions measures the weighted residential greenhouse gas emissions, which have been fairly constant over time. Our indicators of enmeshing do not show significant changes, despite a rapid change in the mid-1990s, which might have been due to the early 1990s electricity liberalization in Australia. Excluding this event, there was a slight reduction in the enmeshing over time, as indicated by (H^T, K^{**}), which was not reflected in H^L (entropy of multipliers). This indicates that specialization in greenhouse-gas-emitting activities occurred within the economy such that direct links to emissions were reduced but indirect links were maintained.

The material flow and employment indicators showed similar results—large efficiency improvements (I^m, γ_g) of the system, with smaller changes in enmeshing. K^{**} showed almost no change over time for both indicators, with the entropy measures being mixed but generally showing a trend of increased connectedness—that is, there was an increased equality in the distribution of the flows of these KPIs.

The resilience measure (γ^*) had high variability and an indeterminate trend for economic flows. This was not the case for our KPIs, where a strong downward trend was observed. γ^* indicates the system's asymptotic behavior; hence, we would expect it to reduce as we converge on a limiting factor of growth. For the KPIs, we see a clear trend toward reduced γ^*, which indicates that the system is converging toward a more stable state, with the expectation that this implies that there is less abundant or excess use of the KPIs.

Linkages and Economic Landscapes

The preceding measures are all aggregate measures that amass the changes in an economy into a single figure. At a more detailed level, we can look at linkages and "economic landscapes." These measures retain the sectoral breakdown and, as such, do not provide the single measure that is desirable when one is interpreting the evolution of complex systems. They are, however, useful in gaining an impression of the structural changes occurring in an economy over time and, as such, can inform and validate the aggregate results of the previous measures. We present these results separately here to assess and corroborate the findings in the previous section.

Due to the complexity of the figures, we only present results for the economy (see figure 7) and greenhouse gas emissions (see figure 8) and for the beginning and end years of the analysis.

We can immediately see from figure 7 that the evolution of the economy results in both a much greater dispersion of significant backward linkages and an increase in the size of backward linkages. This is consistent with our aggregate measures of complexity, where we saw an increase in the complexity of the economy over time. Forward linkages also show an evolution to a greater dispersion of key sectors, particularly in the trade, transport, and services section of the economy. Thus, in terms of forward linkages, we see a flattening in the relative importance of sectors over time.

For greenhouse gas emissions (see figure 8), changes are less evident, but we see fewer sectors that have large backward linkages and a decrease in the relative size of most important forward linkage sectors. The importance of electricity supply is most evident. Manufacturing (middle sectors of the graph) has decreased in both

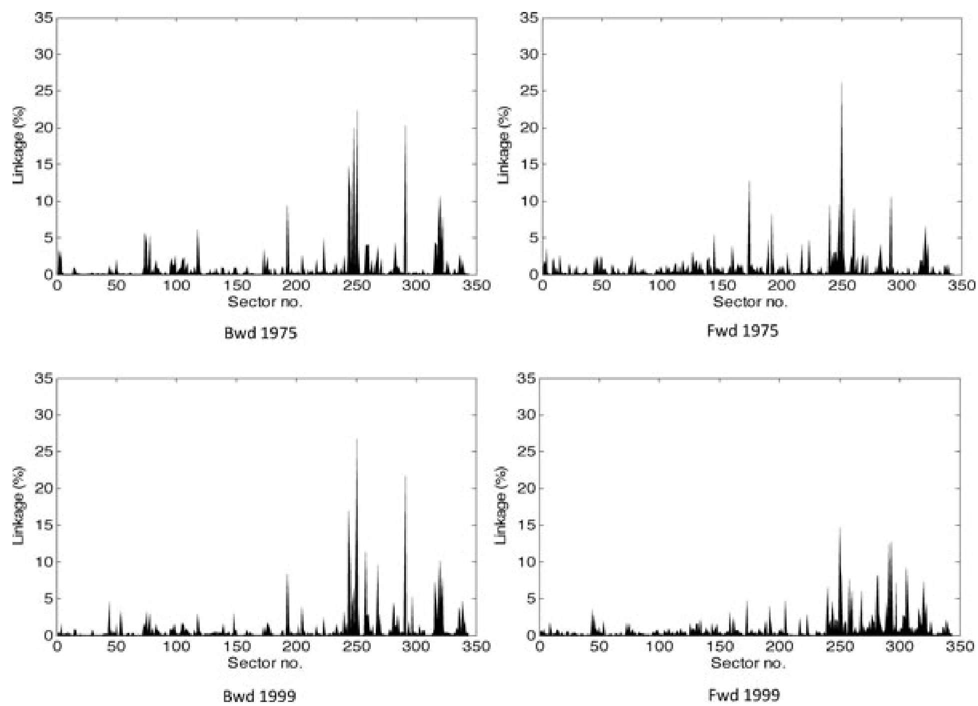

Figure 7 Backward (Bwd) and forward (Fwd) linkages, Australia, 1975 and 1999, 344 sectors. x-axis shows primary industries on the left and service industries on the right.

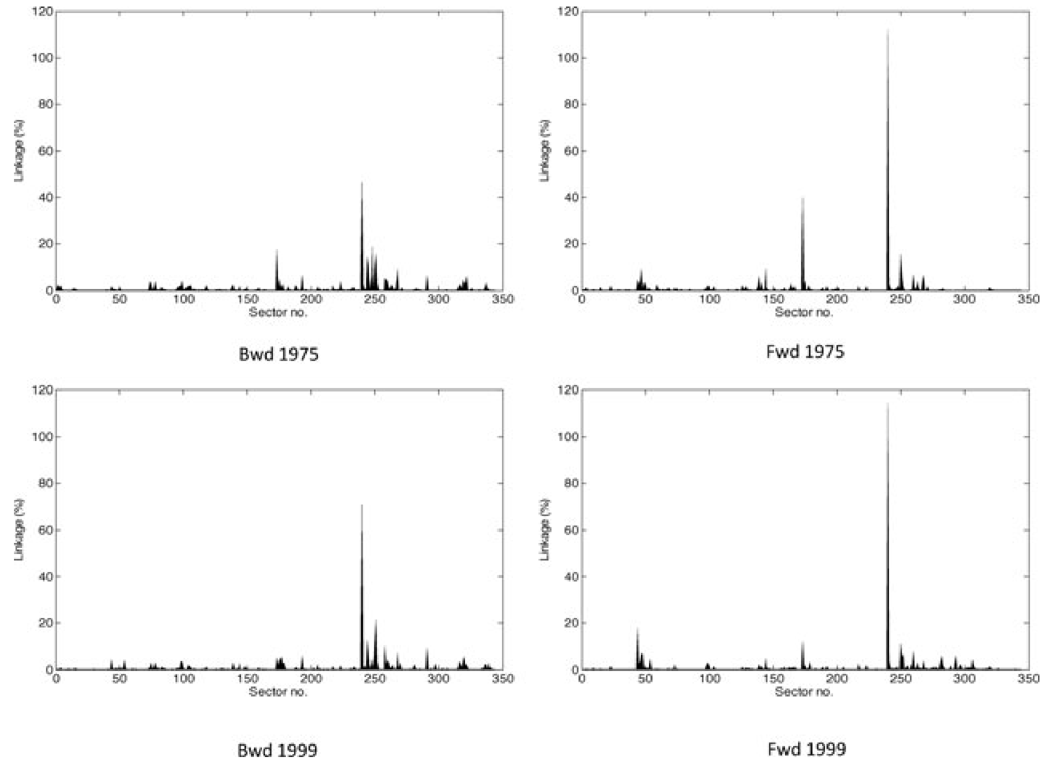

Figure 8 Backward (Bwd) and forward (Fwd) linkages of greenhouse gas emissions, Australia, 1975 and 1999, 344 sectors. *x*-axis shows primary industries on the left and service industries on the right.

forward and backward linkages, which implies that we are seeing fewer direct and indirect resource flows coming and going out of this sector.

The economic landscape is perhaps best shown by the contour plots of the multiplier product matrix, **S** (see figure 9). From these plots, the economic evolution is apparent, with a greater number and spread of key sectors over time, particularly in the secondary and tertiary sectors of the economy. This shows an almost classic economic evolution, with the fading of primary and manufacturing sectors being replaced by a concentration of tertiary activity. The effect for greenhouse emissions is also apparent, with a decreased complexity of important greenhouse flows shown by the reduction in fill of the graphs, with concentration centering on a few key sectors. Most evident of this is the decrease in emissions from manufacturing (shown in sectors 8–15), whereas electricity requirements are becoming more widespread across the economy (sector 16).

In a study of Queensland and in a later study of Taiwan, West (1999) found a "hollowing out" of the economies using measures of relative change of intermediate requirements and key sector maps. This appears to be corroborated by the reduction in manufacturing shown in figures 8 and 9.

Discussion

Summary of Findings

Out of 14 measures of complexity, we concentrated on 7 measures, which we used to interpret the development of linkages within the economy and aspects of the environment of Australia. Our aggregate measures clearly showed increasing complexity in economic terms within the system for measures based on efficiency and measures of interindustry complexity, or enmeshing. These findings were reinforced by the disaggregate measures shown in the forward and backward linkages and the fields of influence.

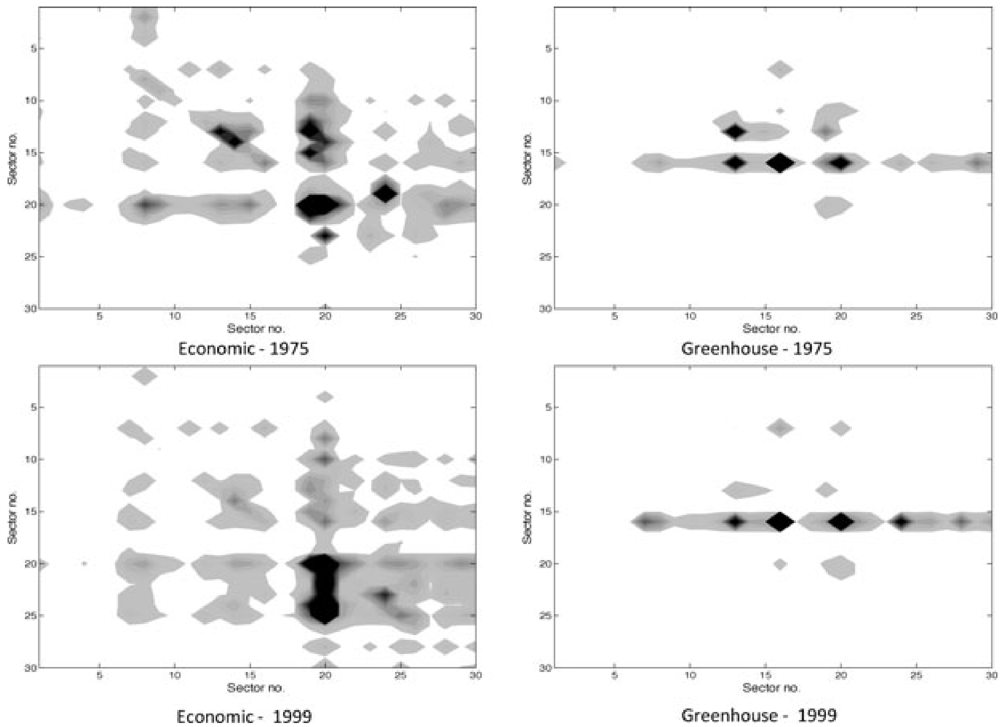

Figure 9 Contour plots showing evolution of economic structure (left) and greenhouse emissions (right) in Australia by field of influence intensity, *S*. We performed calculations at 344 sectors before aggregating to 30 sectors. Primary industries are on the upper left, and service industries are on the lower right.

Across the measures, material use and greenhouse gas emissions generally showed a reduced complexity of flows within the economy. The connectedness measures, K^{**}, showed only slight reductions in complexity, which implies that the number of environmental flows within the economy did not change significantly. More pronounced were the broad-scale efficiency measures relating resource use within the economy to gross production. As has often been the case, these measures showed reductions in the environmental intensity of production.

Both connectivity and connectedness measures of enmeshing generally showed reductions for greenhouse gas emissions and increases for material flow and employment. This indicated that Australia's economy is actually reducing the number of flows associated with greenhouse gas emissions and that the magnitudes of the remaining flows were generally reducing. When we sought to capture indirect effects by examining multipliers rather than transactions, however, we found almost no change in the measures. As a result, it appears greater specialization is occurring for greenhouse gas emissions in Australia, with overall requirements changing little. This was reinforced by the contour plots, which showed a transfer of emissions from manufacturing sectors to electricity.

The dominant eigenvalue within economic measures showed no asymptotic behavior. Within the environmental measures, however, there was a clear trend for greenhouse gas emissions and a reasonable trend for material flow. In industrial ecology terms, this means we are seeing less free and abundant use of resources and energy and a move closer to specialization and efficiency.

Conclusion

This article explores the use of measures to describe the evolving complexity of environmental—economic systems. Our analysis takes advantage of the detailed description of economic structure provided by input—output tables, generalized for environmental and social factors. We analyzed a time series of input—output tables over 25 years for Australia in terms of a variety of measures suggested in the literature.

In our case study, we find almost a classic case of economic evolution, with complexity in the Australian economic system growing reasonably consistently over time. From linkage and contour plots, we can see that this has come about from the reduction in the poorly linked primary and manufacturing sectors, to an increase in the tertiary and service sectors.

The impact this has had on environmental and social flows is almost opposite to the economic case. Although we have seen a decrease in economic specialization, there has been a corresponding increase in the efficiency of resource use and employment, which has resulted in lower levels of these factors necessary for economic function.

One of the major findings of our work is that although our economy is diversifying, our resource flows are becoming more specialized. Trade within the economy has been increasing, and the diversity of our economic sectors has increased. The industries with large environmental impacts have become more specialized and more important to the functioning of the economic system, however. In essence, the increasing diversity of our economy is coming at the expense of the outsourcing of the majority of the environmental impacts from most industries to a specialist few.

Notes

1. The same is said about ecosystems (Finn 1976; Ulanowicz 1983): The higher the degree of internal cycling of mass or energy is, the more mature the ecosystem is.
2. Their use for the identification of key sectors was subsequently suggested by Hirschman (1958), who postulated that economic development and structural change proceed predominantly along above-average linkages, so that a relatively small number of industries accelerate and amplify initially small changes, which eventually affect the whole economy.
3. Sonis and Hewings (1991), Sonis and colleagues (1995b), Sonis and colleagues (1996), and Sonis and Hewings (1999). These authors used only the Leontief inverse.
4. Lorenzen (1981) showed that $\mathbf{x}^*$ is also the eigenvector of $\mathbf{A}$ and that $(1 - 1/\gamma^*)$ is its eigenvalue.
5. The same is true for the graph-theoretic version of Yan and Ames's (1965) argument, put forward by Campbell (1975, 1972). Fontela and colleagues (2000) suggested applying a threshold criterion for the existence of a relation.
6. This concept was taken further soon after by a number of German researchers (Helmstädter 1965b; Korte and Oberhofer 1969; Lamel et al. 1972; Wessels 1981) and extended to international transactions by Helmstädter (1969) and to block-triangularity by Simpson and Tsukui (1965). Nakamura and colleagues (2007) applied triangulations in their waste—input—output approach to material flows.
7. Note that the total flow matrix does not support Leontief-type impact analyses anymore, given that, unlike in the Leontief system, where final demand components have their own mutually exclusive but collectively exhaustive supply-chain network, total flow is characterized by nonadditivity. For discussions surrounding this issue, see the work by Lenzen and colleagues (2007).

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Supplementary Material

Additional Supplementary Material may be found in the online version of this article:

Appendix S1.

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