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Viewpoint

Zero-value problems of the logarithmic mean divisia index decomposition method

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Abstract

Recently, the Logarithmic Mean Divisia Index (LMDI) approach to energy decomposition has been espoused as the preferred indexing method. Whilst the LMDI method provides perfect decomposition, and is time-reversal invariant, its strategy to handle zero-values is not necessarily robust. In order to overcome this problem, it has been recommended to substitute a small value $\delta = 10^{-10} - 10^{-20}$ for any zero values in the underlying data set, and allow the calculation to proceed as usual. The decomposition results are said to converge as δ approaches zero. However, we show that under this recommended procedure the LMDI can produce significant errors if applied in the decomposition of a data set containing a large number of zeroes and/or small values. To overcome this problem, we recommend using the analytical limits of LMDI terms in cases of zero values. These limits can be substituted for entire computational loops, so that in addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times.

Keywords: Logarithmic mean divisia index; Structural decomposition analysis; Zero values

1. Introduction

In recent articles by Ang and colleagues (Ang et al., 1998, 2003; Ang, 2004a,b), the Logarithmic Mean Divisia Index (LMDI) approach to energy decomposition analysis has been espoused as the preferred indexing method. Whilst the LMDI provides perfect decomposition, and is time-reversal invariant, its strategy to handle zero-values is not necessarily robust. In order to overcome this problem, it has been recommended to substitute a small value $\delta = 10^{-10} - 10^{-20}$ for any zero values in the underlying data set, and allow the calculation to proceed as usual (Ang et al., 1998; Ang, 2004b, p. 4). The decomposition results are said to converge as δ approaches zero (Ang and Choi, 1997, p. 68).

In this article we show that this procedure can lead to significant errors in decompositions of data sets containing a large number of zeroes and/or small values.[1] In the following, we first briefly review the LMDI methodology, then demonstrate the zero-value problem, and finally offer a remedy.

2. Methodology

The basic approach to additive[2] structural decompositions of a function $y(x_1, x_2, \ldots, x_n)$ of n determinants is through its total differential

$$\mathrm{d}y = \frac{\partial y}{\partial x_1}\,\mathrm{d}x_1 + \frac{\partial y}{\partial x_2}\,\mathrm{d}x_2 + \cdots + \frac{\partial y}{\partial x_n}\,\mathrm{d}x_n. \quad (1)$$

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[1] This statement can refer to either Index Decomposition Analysis (IDA) or Structural Decomposition Analysis (SDA). We note that the LMDI was originally conceived for applications in IDA, where the number of zero values is generally less than in SDA.

[2] For a comparative review see Hoekstra and van den Bergh (2003) and Choi and Ang (2003).

In case $y(x_1, x_2, \ldots, x_n) = x_1 \cdot x_2 \cdot \ldots \cdot x_n$ (with the x_i being scalars, vectors or matrices),

$$\mathrm{d}y = \prod_{j=1, j\neq 1}^{n} x_j \,\mathrm{d}x_1 + \prod_{j=1, j\neq 2}^{n} x_j \,\mathrm{d}x_2 + \cdots + \prod_{j=1, j\neq n}^{n} x_j \,\mathrm{d}x_n = \sum_{i=1}^{n}\left(\prod_{j=1, j\neq i}^{n} x_j \,\mathrm{d}x_i\right). \quad (2)$$

Analysing discrete time series with a Divisia decomposition approach, differences Δy are obtained by integrating infinitesimal changes $\mathrm{d}y$:[3]

$$\begin{aligned}\Delta y &= \int_{y_0}^{y_1} \mathrm{d}y = \int_{x_{1,0}}^{x_{1,1}} \prod_{j=1, j\neq 1}^{n} x_j \,\mathrm{d}x_1 \\ &\quad + \int_{x_{2,0}}^{x_{2,1}} \prod_{j=1, j\neq 2}^{n} x_j \,\mathrm{d}x_2 + \cdots + \int_{x_{n,0}}^{x_{n,1}} \prod_{j=1, j\neq n}^{n} x_j \,\mathrm{d}x_n \\ &= \sum_{i=1}^{n}\left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j\neq i}^{n} x_j \,\mathrm{d}x_i\right) \\ &= \sum_{i=1}^{n}\left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1}^{n} x_j \,\frac{\mathrm{d}x_i}{x_i}\right) \\ &= \sum_{i=1}^{n}\left(\int_{x_{i,0}}^{x_{i,1}} y(\ldots, x_i, \ldots) \,\frac{\mathrm{d}x_i}{x_i}\right) \\ &= \sum_{i=1}^{n} \bar{y}([x_{i,0}, x_{i,1}]) \ln \frac{x_{i,1}}{x_{i,0}}. \end{aligned} \quad (3)$$

In order to compute the integral, one has to know what average values y assumes while the x_i change from $x_{i,0}$ to $x_{i,1}$ (the "integral path"). Conventional Parametrical Divisia methods assume a parametrical average $\bar{y}^{\alpha} = y_0 + \alpha(y_2 - y_1) = y_0 + \alpha\Delta y$, with $0 \leqslant \alpha \leqslant 1$. Searching for a non-parametric method, Ang and Choi (1997) and Ang and Liu (2001) propose the logarithmic mean $\bar{y}^{\lambda} = \Delta y/\Delta(\ln y)$. The resulting Logarithmic Mean Divisia (LMD) formulation

$$\Delta y^{\mathrm{L}} = \sum_{i=1}^{n} \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_{i,1}}{x_{i,0}} \quad (4)$$

is non-parametric, exact, and time-reversible (for proofs see Ang and Choi, 1997, Ang et al., 1998).[4] It is also said to be zero-robust if a small value $\delta = 10^{-10} - 10^{-20}$ is substituted for any zero values (Ang and Choi, 1997, p. 68, Ang et al., 1998; Ang, 2004b, p. 4). However, in our opinion while the latter statement is theoretically valid, it does not necessarily work under any circumstance.

Table 1
Limits of the LMDI weights for eight possible cases of zeros in the underlying data set. PN = positive number (from Ang et al., 1998, p. 492)

Case	y_0	y_1	x_0	x_1	$\Delta y^{\mathrm{L}} = \sum_{i=1}^{n} \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_1}{x_0}$
1	0	PN	0	PN	y_1
2	PN	0	PN	0	$-y_0$
3	0	PN	PN	PN	0
4	PN	0	PN	PN	0
5	0	0	PN	PN	0
6	0	0	0	0	0
7	0	0	PN	0	0
8	0	0	0	PN	0

3. Zero-value issues

Ang et al. (1998, p. 492) give eight situations in which zero values can occur in Eq. (4) (see Table 1).[5] Evaluating Ang et al.'s case 1 for the example of $y_0 = x_0 = 0$, $x_1 = 1$, and $y_1 = \{2, 10, 100\}$ shows that—because of the slow convergence of the term in Eq. (4)—errors in the Δy^{L} term are not negligible, even if δ is reduced to near the limit of double precision ($\sim 10^{-323}$), and especially if the difference between x_1 and y_1 is large (Fig. 1). Similar results can be produced for the remaining cases.

This problem may not be discernible when analysing data sets that contain only a small number of zero values. However, in structural decomposition analysis (SDA) it is often the case that the data contain a large number of zero values. Under this circumstance, discrepancies will show up as a kind of "background noise" (essentially the cumulative errors in the Δy^{L}). This noise can constitute a significant fraction of results from a correct structural decomposition, especially when

(a) zero values outnumber non-zero values;
(b) apart from the zero values, the data set contains many small values, the contribution of which gets buried in the "noise";
(c) there are large changes in the data set over the period of study (Fig. 1).

[3] We denote $y_0 = y(t = 0)$, $y_1 = y(t = 1)$, $x_{i,0} = x_i(t = 0)$, and $x_{i,1} = x_i(t = 1)$.

[4] Even compared to its main non-parametrical competitor, the Dietzenbacher/Sun/Albrecht method (Dietzenbacher and Los, 1998; Sun, 1998; Albrecht et al., 2002), it has the advantage of being computationally less demanding. The Mean-Rate-of-Change non-parametrical decomposition method suggested by Chung and Rhee (2001) is equivalent to the LMDI once logarithmic weights are used (see Lenzen, 2004).

[5] The remaining possible combinations can be excluded, since $x_0 = 0 \Rightarrow y_0 = 0$, and $x_1 = 0 \Rightarrow y_1 = 0$.

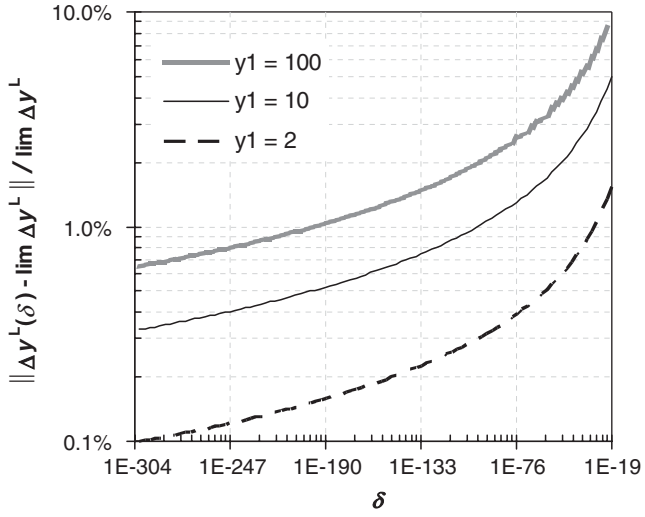

Fig. 1. Relative deviation $||\Delta y^{L}(\delta) - \lim_{\delta\to 0} \Delta y^{L}||/\lim_{\delta\to 0} \Delta y^{L} = ||\Delta y^{L}(\delta) - y_1||/y_1$, for Ang et al.'s case 1: $y_0 = x_0 = 0$, $x_1 = 1$, and $y_1 = \{2, 10, 100\}$, as a function of "small values" δ below Ang et al.'s recommended setting. Even with δ at the limit of double precision, errors can be significant.

We came across inconsistencies while carrying out an SDA of CO_2 emissions E from energy consumed in the Brazilian and Australian economies, factorised as $E = \boldsymbol{c}\mathbf{F}\mathbf{L}\boldsymbol{y}$ and $E = \boldsymbol{c}\mathbf{F}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots)\boldsymbol{y}$, where $\boldsymbol{y}$ is final demand, $\mathbf{A}$ is the direct requirements matrix calculated from the input–output tables, $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse, $\mathbf{F}$ is the industrial fuel intensity matrix, and $\boldsymbol{c}$ is the greenhouse gas content of fuels (basically a summation vector of constant weights). These and other factorisations can contain a large number of zero and small values, when

(1) industries use only a limited number of fuel types ($\mathbf{F}$),
(2) industries do not—at least directly—interact much with each other ($\mathbf{A}$), and
(3) $\mathbf{A}$ and $\mathbf{L}$ are in their make-use formulation.

The latter point refers to production and consumption of commodities and in industries being represented in one matrix, containing two separate, off-diagonal, non-zero blocks and two diagonal zero blocks (Fig. 2). In addition, the make matrix is usually similar to a unity matrix and therefore sparse.

4. An empirical example

In order to support Fig. 1 with an empirical example we will examine the effect of the zero elements in the 1994–95 and 1996–97 Australian fuel use matrices $\mathbf{F}_1$ and $\mathbf{F}_2$, and direct requirements matrices $\mathbf{A}_1$ and $\mathbf{A}_2$. For example in the $\mathbf{A}_i$, a surprisingly large number of

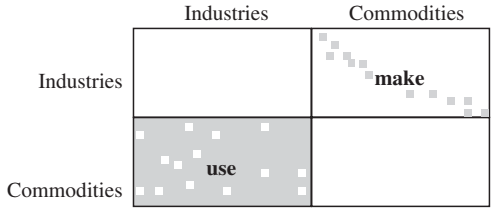

Fig. 2. Schematic of a make-use framework. White fields represent zero values.

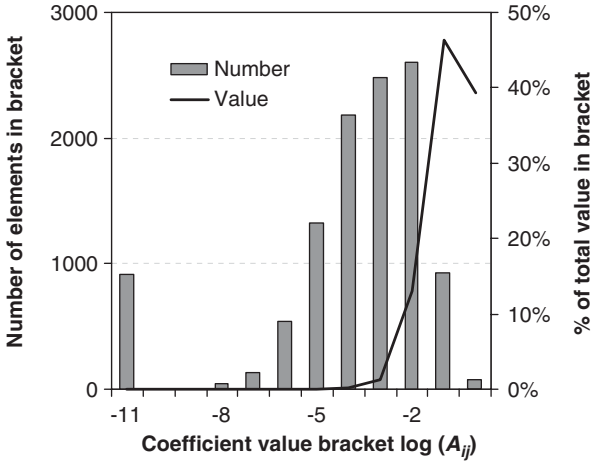

Fig. 3. Distributions of element numbers and values in the 1994–95 Australian direct requirements matrix $\mathbf{A}$, across size brackets. Almost 1000 out of 11,236 elements are smaller than 10^{-10} (vertical bars, left y-axis). Most of the total coefficient value is concentrated in about 1000 elements of size 0.01 and larger (solid curve, right y-axis).

coefficients are zero, and also most of the coefficient value is concentrated in a few elements, causing most elements in $\mathbf{A}_i$ to be considerably smaller than their average (Fig. 3).

Complementing the direct requirements matrix $\mathbf{A}$ with CO_2 intensities $\boldsymbol{c}\mathbf{F}$, we carry out a structural decomposition of $E = \boldsymbol{c}\mathbf{F}\mathbf{A}\boldsymbol{y}$, which is the second term in the Taylor expansion two paragraphs above. We calculate

$$\begin{aligned}\Delta E(\mathbf{F}) &= \int \sum_{lm} \frac{\partial E}{\partial F_{lm}}\, \mathrm{d}F_{lm} \\ &= \sum_{lm} \int \frac{\partial(\sum_{ijk} c_i F_{ij} A_{jk} y_k)}{\partial F_{lm}}\, \mathrm{d}F_{lm} \\ &= \int \sum_{lmk} c_l F_{lm} A_{mk} y_k\, \frac{\mathrm{d}F_{lm}}{F_{lm}},\end{aligned}$$

which after integration is

$$= \sum_{lmk} \overline{c_l F_{lm} A_{mk} y_k}^{\lambda} \ln \frac{F_{lm,1}}{F_{lm,0}} = \sum_{lmk} \overline{E_{lmk}}^{\lambda} \ln \frac{F_{lm,1}}{F_{lm,0}}, \qquad (5)$$

and similarly

$$\Delta E(\mathbf{A}) = \sum_{imn} \overline{c_i F_{im} A_{mn} y_n}^{\lambda} \ln \frac{A_{mn,1}}{A_{mn,0}} = \sum_{imn} \overline{E_{imn}}^{\lambda} \ln \frac{A_{mn,1}}{A_{mn,0}}, \quad (6)$$

and

$$\Delta E(\boldsymbol{y}) = \sum_{ijn} \overline{c_i F_{ij} A_{jn} y_n}^{\lambda} \ln \frac{y_{n,1}}{y_{n,0}} = \sum_{ijn} \overline{E_{ijn}}^{\lambda} \ln \frac{y_{n,1}}{y_{n,0}}. \quad (7)$$

Since $\boldsymbol{c}$ is constant, $\Delta E(\boldsymbol{c}) = 0$. The total effect is $\Delta E = \Delta E(\mathbf{F}) + \Delta E(\mathbf{A}) + \Delta E(\boldsymbol{y})$.

In our analysis, we pursue two parallel strategies: First, we substitute any zero value with a "small value" δ, ranging from 10^{-10} and 10^{-50}, and calculate various ΔE_δ. Second, we shortcut all loops over i, k, l, m, and n in Eqs. (5–7) that contain any of the possibilities listed in Table 1, and directly use the analytical limit (right column of Table 1) in calculating the correct ΔE_0.

We quantify the error caused by using Ang et al.'s "small-value" strategy in terms of ten measures: four are the relative errors of the aggregate structural decomposition results ($\Delta E_{\mathrm{rel}}(\mathbf{F}) = [\Delta E_\delta(\mathbf{F}) - \Delta E_0(\mathbf{F})]/\Delta E_0(\mathbf{F})$, and analogous expressions for $\Delta E(\mathbf{A})$, $\Delta E(\boldsymbol{y})$, and ΔE). The remaining measures aim at establishing a metric of relative distance between the ΔE_δ and ΔE_0 in matrix form, defined as $\{\Delta E_\delta(\mathbf{F})\}_{lm} = \sum_k \overline{E_{lmk}}^{\lambda} \ln(F_{lm,1}/F_{lm,0})$, and analogously for the terms in Eqs. (6) and (7). According to Butterfield and Mules (1980, p. 293), "there exists no single statistical test for assessing the accuracy with which one matrix corresponds to another. Analysts working in this area have tended to use a number of [complementary] tests." Accordingly, we chose two recommended metrics (see Harrigan et al., 1980; Lenzen et al., 2004):

- the relative arithmetic mean of absolute errors

$$\mathrm{AMAE} = \frac{\sum_{ij} |\Delta E_{0ij} - \Delta E_{\delta ij}|}{\sum_{ij} \Delta E_{0ij}},$$

and

- the arithmetic mean of relative errors

$$\mathrm{AMRE} = \frac{\sum_{ij} |\Delta E_{0ij} - \Delta E_{\delta ij}| / |\Delta E_{\delta ij}|}{N},$$

where N is the number of summands with $\Delta E_{ij} \neq 0$.

The AMAE is still a measure of *overall* error within the ΔE_δ, since it describes the importance of all cumulative errors relative to the total effect ΔE_0. The difference between the AMAE and the aggregate relative errors ΔE_{rel} is that in the relative errors positive and negative contributions cancel out, while the AMAE accumulates the absolute value of the errors; hence $\mathrm{AMAE} > \Delta E_{\mathrm{rel}}$.

The AMRE reflects the relative error at the level of single constituents $\Delta E_{\delta ij}$, but averaged over the entire matrix. Note that we have chosen ΔE_δ as the reference in the denominator; this was done in order to be able to capture the very errors due to substituting zero values with δ. These errors would not have shown up if ΔE_0 had been the denominator, since cases with $\Delta E_0 = 0$ must be excluded from the sum.

5. Results

In 1994–95 and 1996–97 the total amounts of emissions $E = \sum_{ijk} c_i F_{ij} A_{jk} y_k$ were 83.0 Mt CO_2 and 90.2 Mt CO_2, respectively, yielding a change of $\Delta E =$ 7.15 Mt CO_2. A correct decomposition using our shortcut strategy yields $\Delta E(\mathbf{F}) = 112$ kt, $\Delta E(\mathbf{A}) = 26$ kt, and $\Delta E(\boldsymbol{y}) = 7015$ kt, so that most of the change was actually driven by final demand $\boldsymbol{y}$, and to minor degrees by changes in energy intensity and fuel mix ($\mathbf{F}$) and economic structure ($\mathbf{A}$). Inserting $\delta = 10^{-10}$ for all zeroes, the decomposition yields $\Delta E(\mathbf{F}) = 103$ kt, $\Delta E(\mathbf{A}) = 29$ kt, and $\Delta E(\boldsymbol{y}) = 7022$ kt.

Varying δ, we found that—as expected from Fig. 1—errors did not converge rapidly enough to zero, and significant errors remained even at $\delta = 10^{-50}$ (Fig. 4). The largest relative errors—around 10%—were naturally registered for the smallest change term $\Delta E(\mathbf{A})$, followed by $\Delta E(\mathbf{F})$ with ΔE_{rel} and AMAE around 1%, but AMRE near 100%. Only the largest contributor to overall change $\Delta E(\boldsymbol{y})$ is represented with reasonable accuracy, errors being between 0.001% and 0.1%.

The crucial finding is that none of these errors decreases sufficiently rapidly as δ approaches zero, so that in cases where data sets contain a sufficient number of zeroes and/or small values, Ang et al.'s "small-value" strategy will always yield flawed results, no matter how small δ.

In addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times for the LMDI, especially when zero values are numerous and/or when the function to be decomposed is a product of many factors and/or large matrices. In the latter case, many deeply nested multiple summations can be skipped whenever one of the eight cases in Table 1 occurs, and simply shortcut by using the respective limit directly. Reductions in computing time can be estimated as follows: assuming a decomposition involving n determinants, each of row size R_i, the number of loops λ in a SDA of the product of these n determinants is $\lambda = \prod_{i=1}^{n} R_i$. Inside the innermost loop, n terms (one Δ-terms for each factor) have to be

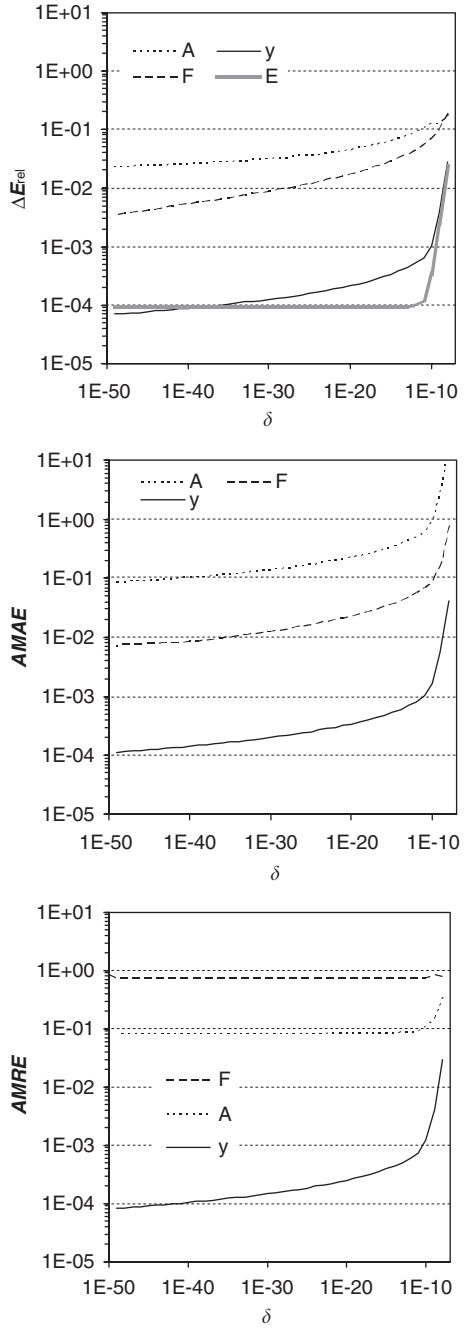

Fig. 4. ΔE_{rel}, AMAE and AMRE of decomposition terms for **F**, **A** and ***y***, as a function of the "small value" δ.

evaluated. Each Δ-term requires $2(n-1)+8$ operations,[6] so that the total number of operations is $\omega = \lambda \times n \times 2(n-1)+8$. For example in this paper, $n = 3$, $R_1 = 30$, $R_2 = 107$, and $R_3 = 107$, so that $\lambda = 343,470$ and $\omega = 12,364,920$. Note that this calculation neglects accesses to memory, and sub-operations for evaluating logarithms. Assuming that a desktop computer carries out about 10^8 such operations per second, the computing time for an SDA as in this paper is below 1 s. However, SDAs involving more factors and larger matrices are not uncommon: In recent case studies of decomposing a CO_2 calculation involving Brazilian Social Accounting Matrices (Wachsmann, 2005) or Australian make-use matrices (Wood, 2003), $\omega_{\mathrm{Br}} \approx 9 \times 10^{10}$ and $\omega_{\mathrm{Au}} \approx 10^{17}$, resulting in computing times of 1/2 h and 10^5 h, respectively. Considering that in CO_2 and energy compositions, fuel mix, energy use and final demand matrices are sparse, skipping zero loops can reduce these computing times significantly. We calculated that if in three matrices 50% of elements are zero, the Brazilian case reduces by a factor of almost 10, and if in five matrices 90% of elements are zero, the Australian case reduces by a factor of 10^5.

[6] In terms of Eq. (5): $n-1=3$ multiplications for $E_{lmk} = c_l F_{lm} A_{mk} y_k$, and $2(n-1)$ operations for evaluating $E_{lmk,0}$ and $E_{lmk,1}$. One operation each for the difference ΔE, the logarithms $\ln E$, and the difference of the logarithms $\Delta \ln E$ increases the total to $2(n-1)+4$. Calculating $\overline{E_{lmk}}^{\lambda}$ requires a further division, calculating $\ln F_{lm,1}/F_{lm,0}$ requires two operations, and including the final multiplication, the total number of operations is $2(F-1)+8$.

6. Conclusions

The Logarithmic Mean Divisia Index (LMDI) decomposition—at least under the recommended procedure—may be theoretically zero-value robust, but cannot handle zero values if the data set to be decomposed contains a large number of zeroes and/or small values. Substituting a small value $\delta = 10^{-10}-10^{-20}$ for any zero values, and allowing the calculation to proceed as usual can lead to significant errors.

To overcome this problem, we recommend using the limits published by Ang et al. (1998) for the eight possible cases of zero values (see Table 1, right column). These limits can be substituted for entire computational loops representing multiple summations as in Eqs. (5–7). In addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times for the LMDI, especially when zero values are numerous and/or when the function to be decomposed is a product of many factors and/or large matrices.

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