MA3203 - Problem sheet 6
bonus sheet

Problem 1. Let $\varphi\colon \Lambda \to \Gamma$ be a surjective algebra homomorphism of two finite dimensional algebra.

(a) Show that a left Γ-module M is a left Λ-module via the action of Λ on M defined by
$$\lambda \cdot m \stackrel{\text{def}}{=} \varphi(\lambda)m$$
for $\lambda \in \Lambda$ and $m \in M$.

(b) Let M and N be two Γ-modules. Show that
$$\operatorname{Hom}_\Gamma(M, N) = \operatorname{Hom}_\Lambda(M, N)$$
when we view M and N as Λ-modules above.

(c) Show that ${}_\Gamma M \simeq {}_\Gamma N$ if and only if ${}_\Lambda M \simeq {}_\Lambda N$ for two left Γ-modules M and N.

(d) Define $F\colon \operatorname{mod}\Gamma \to \operatorname{mod}\Lambda$ by letting
$$F(M) = {}_\Lambda M$$
and
$$F(f) = f$$
for a Γ-module M and a Γ-homomorphism $f\colon M \to N$. Show that $F\colon \operatorname{mod}\Gamma \to \operatorname{mod}\Lambda$ is an exact full and faithful functor. Is F a dense functor?

(e) (Challenge) Let $\Lambda = kQ/I$ for a quiver Q, a field k and an admissible ideal I in kQ. Show that if Λ has finite representation type, then there are no double arrows in Q.