Linear Theory of Regular Waves Review

Wave property	**SHALLOW WATER** $(d / \lambda < 1/20)$	**INTERMEDIATE WATER** $(1/20 < d/\lambda < 1/2)$	**DEEP WATER** $(d/\lambda > 1/2)$
Velocity potential ($\boldsymbol{u} = \nabla\phi$)	$\phi = \frac{ag}{\omega}\frac{\cosh k(z+d)}{\cosh kd}\cos(\omega t - kx)$	$\phi = \frac{ag}{\omega}\frac{\cosh k(z+d)}{\cosh kd}\cos(\omega t - kx)$	$\phi = \frac{ag}{\omega} e^{kz}\cos(\omega t - kx)$
Dispersion relation	$\omega^2 = g k^2 d$	$\omega^2 = gk \tanh kd$	$\omega^2 = gk$
Wave length - wave period relation	$\lambda = T\sqrt{gd}$	$\lambda = \frac{g}{2\pi}T^2 \tanh\frac{2\pi d}{\lambda}$	$\lambda = \frac{g}{2\pi}T^2 \quad (\approx 1.56\ T^2)$
Wave profile	$\eta = a\sin(\omega t - kx)$	$\eta = a\sin(\omega t - kx)$	$\eta = a\sin(\omega t - kx)$
Dynamic pressure	$p_d = \rho g a \sin(\omega t - kx)$	$p_d = \rho g a \frac{\cosh k(z+d)}{\cosh kd}\sin(\omega t - kx)$	$p_d = \rho g a\, e^{kz}\sin(\omega t - kx)$
Horizontal particle velocity	$u = \frac{\omega a}{kd}\sin(\omega t - kx)$	$u = \omega a \frac{\cosh k(z+d)}{\sinh kd}\sin(\omega t - kx)$	$u = \omega a e^{kz}\sin(\omega t - kx)$
Vertical particle velocity	$w = \omega a \frac{z+d}{d}\cos(\omega t - kx)$	$w = \omega a \frac{\sinh k(z+d)}{\sinh kd}\cos(\omega t - kx)$	$w = \omega a e^{kz}\cos(\omega t - kx)$
Horizontal particle acceleration	$\dot{u} = \frac{\omega^2 a}{kd}\cos(\omega t - kx)$	$\dot{u} = \omega^2 a \frac{\cosh k(z+d)}{\sinh kd}\cos(\omega t - kx)$	$\dot{u} = \omega^2 a\, e^{kz}\cos(\omega t - kx)$
Vertical particle acceleration	$\dot{w} = -\omega^2 a \frac{z+d}{d}\sin(\omega t - kx)$	$\dot{w} = -\omega^2 a \frac{\sinh k(z+d)}{\sinh kd}\sin(\omega t - kx)$	$\dot{w} = -\omega^2 a\, e^{kz}\sin(\omega t - kx)$
Group velocity	$c_g = c$	$c_g = \frac{1}{2}c\,(1 + \frac{2kd}{\sinh 2kd})$	$c_g = \frac{1}{2}c$

$\omega = 2\pi / T,\ k = 2\pi / \lambda$

T = wave period

λ = wave length

a = wave amplitude

g = acceleration of gravity

$c = \lambda / T$ = phase speed

t = time

x = direction of propagation

z = vertical co-ordinate positive upward, origin at still water level

d = water depth

p_d = dynamic pressure

$p_d - \rho g z + p_o$ = total pressure in the water ($-\rho g z$ = hydrostatic pressure, p_o = atmospheric pressure).

$E = \frac{1}{2}\rho g a^2$ = wave energy (per unit surface area)

$P = E c_g$ = wave energy flux (per unit width along the wave crest)