NTNU – Trondheim
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Department of Hydraulic and Environmental Engineering

Examination paper for TVM 4155 Numerical modelling and hydraulics

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Problem 1.

A river is considered wide and it has a longitudinal slope of 1/900. The discharge pr. width is 4.52 m^2/s. The bed is covered with gravel of size 25 mm. We assume that there are no bedforms in the river.

a) Assuming d_{90} is twice the average size of the gravel, what is the Manning-Stricklers friction coefficient?

b) Compute the water depth and velocity assuming uniform flow.

c) Will the gravel be eroded or not?

d) Compute the sediment discharge using Engelund-Hansens formula.

e) Explain the correspondence between the answers on questions c) and d)

f) Compute the sediment discharge using Mayer-Peter and Müllers formula.

g) Explain the correspondence between the answers on questions c) and f)

h) Describe the physics of the movement of antidunes.

Problem 2.

An outlet from a hydropower plant is located at the bottom of a lake. The tunnel discharges water in the vertical direction. The tunnel diameter is 5 meters. The temperature of the water in the lake is 8 degrees C. The water discharge from the tunnel is 20 m^3/s. The temperature of the water from the hydropower plant is 20 degrees C.

a) What is the densimetric Froude number of the plume?

b) What is the water velocity 40 meters above the outlet of the hydropower tunnel?

c) What is the water discharge of the plume 40 meters above the outlet of the hydropower tunnel?

d) If the water temperature of the hydropower plant had been 4 degrees instead of 20 degrees, what would have happened with the water after it had passed the hydropower tunnel outlet?

Problem 3.

a) Derive the weighting coefficients for the second-order upwind scheme, used for the solution of the convection-diffusion equation.

b) Describe two disadvantages and one advantage with this scheme, compared with the first-order upwind scheme.

c) Why would the central scheme be unstable?

Problem 4.

In the class notes and in the lectures, we have derived the equation for the 1D kinematic wave using Mannings equation. Derive the equation for the kinematic wave, based on the Chezy equation instead of the Mannings equation. Why do we get a different result?

Problem 5. Free water surface algorithms

a) Give the names of at least 6 computer programs that can be used to compute the coefficient of discharge for a spillway.

b) Describe the theory for the Volume of Fluid method.

c) Give the partial differential equation that is used to solve the VOF fraction in the Volume of Fluid method.

d) What is the difference between the VOF method and the Level Set Method?

e) What is the advantage of using an adaptive grid to compute the location of the free water surface?

f) How accurate can a CFD program compute the coefficient of discharge for a spillway?

Problem 6.

ERCOFTAC gives seven main sources of errors and uncertainties in CFD.

a) List all the errors and uncertainties given by ERCOFTAC

b) What type of error is false diffusion from the list in question a)? From which term in which equation does false diffusion come from?

c) Describe three methods to reduce false diffusion, together with their advantages/disadvantages.

d) To evaluate uncertainties in CFD results it is possible to vary the input parameters and see how their values affect the results. Give examples on parameters to vary if you are to assess the errors and uncertainties from question a).

Formulas and tables:

$$U = \frac{1}{n} r_h^{\frac{2}{3}} I^{\frac{1}{2}} \qquad r_h = \frac{A}{P} \qquad M = \frac{1}{n} \qquad M = \frac{26}{d_{90}^{\frac{1}{6}}}$$

$$a = -fU = -\left(\frac{4\pi}{T}\sin\phi\right)U, \qquad f\text{=1.26*10}^{-4}\text{ (Norway)}$$

$$g(I_0 - I_f) - g\frac{dy}{dx} = U\frac{dU}{dx} + \frac{dU}{dt}$$

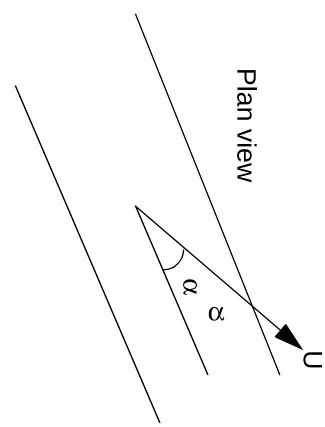
$$\tau = c_{10}\rho_a U_a^2 \qquad \Gamma = 0.11u_* y$$

$$U_i\frac{\partial c}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\Gamma\frac{\partial c}{\partial x_i}\right) - kc \qquad \Gamma = 0.058\frac{Q}{IB}$$

$$\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}(-P\delta_{ij} - \rho\overline{u_i u_j})$$

$$-\rho\overline{u_i u_j} = \rho\nu_T\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\rho k\delta_{ij}$$

$$P_k = \nu_T\frac{\partial U_j}{\partial x_i}\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right)$$



$$K = -\frac{\sin\phi\sin\alpha}{\tan\theta} + \sqrt{\left(\frac{\sin\phi\sin\alpha}{\tan\theta}\right) + cos^2\phi\left[1 - \left(\frac{\tan\phi}{\tan\theta}\right)^2\right]}$$

$$\frac{\Delta}{h} = 0.11\left(\frac{D_{50}}{h}\right)^{0.3}\left(1 - e^{-\left[\frac{\tau - \tau_c}{2\tau_c}\right]}\right)\left(25 - \left[\frac{\tau - \tau_c}{\tau_c}\right]\right) \qquad k_s = 3D_{90} + 1.1\Delta\left(1 - e^{\frac{-25\Delta}{\lambda}}\right)$$

$$c_{10} = 1.1*10^{-3} \qquad \rho_a = 1.2\text{ kg/m}^3 \qquad \lambda = 7.3h$$

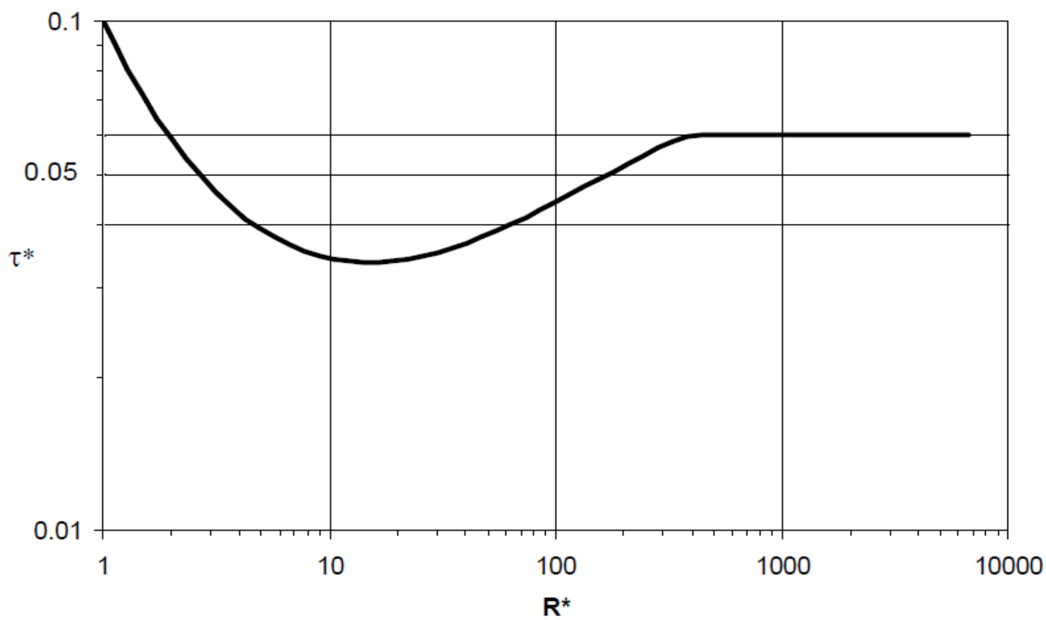
$$\frac{c(y)}{c_a} = \left(\frac{h-y}{y}\frac{a}{h-a}\right)^z \qquad z = \frac{w}{\kappa u*} \qquad \tau = \rho g y I \qquad c = U \pm \sqrt{gy}$$

$$c = \frac{5}{3}U \qquad \rho_s = 2650\text{ kg/m}^3 \qquad \rho_w = 1000\text{ kg/m}^3$$

$$Fr = \frac{U}{\sqrt{gh}} \qquad Fr' = \frac{u_0}{\sqrt{\left(\frac{\rho_{res}-\rho_0}{\rho_{res}}\right) g d_0}} \qquad U = C\sqrt{Ir_h}$$ Chezys formula

$$\frac{u}{u_0} = 4.3 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{-\frac{1}{3}} e^{\left[-96\frac{r^2}{z^2}\right]} \qquad \frac{\rho - \rho_{res}}{\rho_0} = 9 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{-\frac{5}{3}} e^{\left[-71\frac{r^2}{z^2}\right]}$$

$$\frac{Q}{Q_0} = 0.18 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{\frac{5}{3}} \qquad c(x,t) = \frac{c_0 L}{2\sqrt{\pi \Gamma t}} e^{-\frac{(x-Ut)^2}{4\Gamma t}}$$



Shields diagram. Vertical axis: $\tau^* = \frac{\tau_c}{g(\rho_s - \rho_w)d}$

$$R^* = \frac{u_* d}{\nu} = \frac{d\sqrt{\frac{\tau}{\rho}}}{\nu}$$

$$q_s = 0.05 \rho_s U^2 \sqrt{\frac{d_{50}}{g\left(\frac{\rho_s}{\rho_w} - 1\right)}} \left[\frac{\tau}{g(\rho_s - \rho_w) d_{50}}\right]^{\frac{3}{2}}$$

$$q_s = \frac{1}{g}\left[\frac{\rho_w grI - 0.047g(\rho_s - \rho_w)d_{50}}{0.25\rho_w^{\frac{1}{3}}\left(\frac{\rho_s - \rho_w}{\rho_s}\right)^{\frac{2}{3}}}\right]^{\frac{3}{2}}$$

$k_3 = 1.2$

$$\frac{u}{u_{max}} = \left(1 + \frac{r^2}{0.016x^2}\right)^{-2} \qquad \frac{u_{max}}{u_0} = 6.4\left(\frac{x}{d_0}\right)^{-1} \qquad \frac{Q}{Q_0} = 0.42\frac{x}{d_0}$$

$$d = k_1\sqrt{\frac{q}{N}} \qquad N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} \qquad d = k_3\left(\frac{Q^2}{B^2hN^2}\right)^{\frac{1}{3}}$$

Fall velocity of quartz spheres in water.

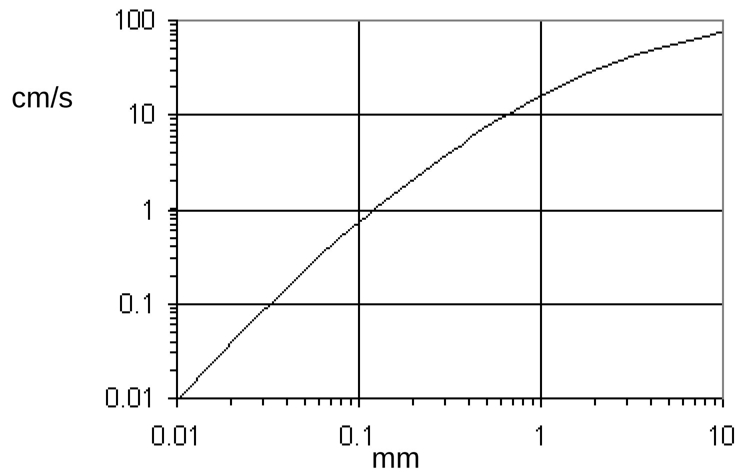
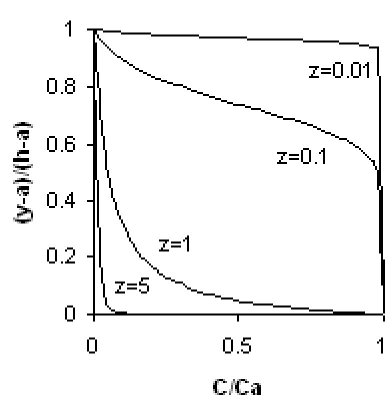


Table of water density as a function of temperature:

Temp (^{0}C)	Density (kg/m^3)
0	999.87
2	999.97
4	1000.0
6	999.97
8	999.88
10	999.73
12	999.52
14	999.27
16	998.97
18	998.62
20	998.23
22	997.80
24	997.33
26	996.81
28	996.26
30	995.68



Solution:

Problem 1.

a) d_{90} = 2*0.025 m * 0.05 m. Mannings factor: 26/(0.05)^1/6 = 42.8

b) Velocity from Mannings formula: U=42.8 * (1/900)^0.5 * y^2/3

Depth from continuity: y = 4.52/U. Uses this to eliminate y from the equation above. This gives:

U^5/3 = 42.8 * (1/900)^0.5*^4.52^2/3 = 3.9 -> U = 2.26 m/s

The depth is then y = 4.52/U = 2 m

c) Compute the bed shear and the critical shear for erosion of a particle.

Shear = 1000*9.81*2*1/900 = 22 Pa

Particle Reynolds number is $Re = \frac{\sqrt{\frac{22}{1000}}x0.025}{10^{-6}} = 3691$

Looking at Shields diagram, we see that the Shields factor of 0.06 is correct for this case.

The critical shear stress is:

Shear, critical = 0.06 * 9.81 * (2650-1000) * 0.025 = 24 Pa

Since the critical shear stress is above the actual shear stress, there should be no erosion.

d) Engelund-Hansens formula:

$$q_s = 0.05x2650x2.26^2\sqrt{\frac{0.025}{9.81\left(\frac{2650}{1000}-1\right)}}\left[\frac{22}{9.81(2650-1000)0.025}\right]^{\frac{3}{2}} = \underline{\underline{0.33\ \mathrm{kg/m/s}}}$$

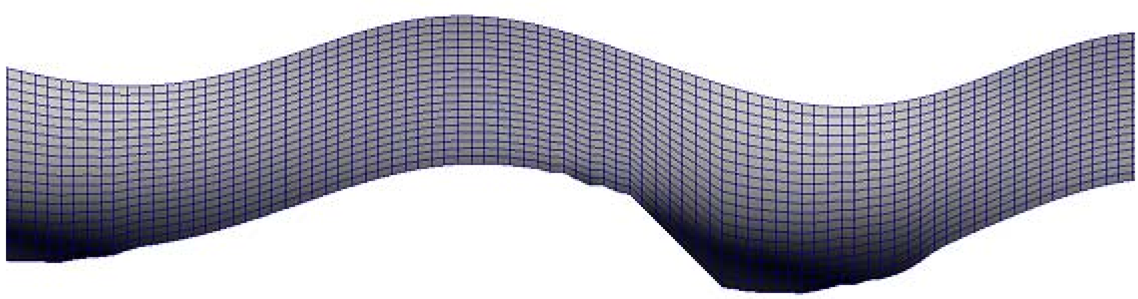
e) Engelund-Hansens formula will always give a positive sediment transport, also if the critical shear stress is larger than the actual shear stress. The formula has no term (actual shear stress - critical shear stress).

f) Mayer-Peter and Müllers formula:

$$q_s = \frac{1}{9.81}\left[\frac{1000x9.81x\frac{2}{900} - 0.047x9.81(2650-1000)0.025}{0.25x1000^{\frac{1}{3}}\left(\frac{2650-1000}{2650}\right)^{\frac{2}{3}}}\right]^{\frac{3}{2}} \qquad = 0.19 \text{ kg/m/s}$$

g) The critical shear stress in Mayer-Peter and Müllers formula assumes a Shields coefficient of 0.047. This is too low for our case, and therefore we get a finite sediment transport. Had we used the value 0.06 (which is the correct one), the Mayer-Peter and Müller formula would have given zero sediment transport.

h) Antidunes form when the Froude number is close to 1, critical flow. We can have erosion on the downstream side of the dune and deposition on the upstream side. The water profile follows the bed. A hydraulic jump may form at the antidune. A drawing can be given, similar to the figure below:



Problem 2:

a)

Density at 20 degrees: 998.23 kg/m3
Density at 8 degrees: 999.88 kg/m3
Velocity: U=Q/A = 20 m3/s / (3.14*2.5m^2) = 1.02 m/s

$$Fr' = \frac{1.02}{\sqrt{\left(\frac{999.88-998.23}{999.88}\right)9.8x5}} \qquad = \underline{\underline{3.58}}$$

b)

$$\frac{u}{u_0} = 4.3x3.58^{-\frac{2}{3}}\left(\frac{40}{5}\right)^{-\frac{1}{3}} e^{\left[-96\frac{0^2}{5^2}\right]}$$ = 0.91 or u = 0.94 m/s

c)

$$\frac{Q}{Q_0} = 0.18x3.58'^{-\frac{2}{3}}\left(\frac{40}{5}\right)^{\frac{5}{3}}$$ = 2.46, or Q = 20 m3/s * 2.46 = 49 m3/s

Problem 3

a) See Chapter 5.6 in the class notes

b) SOU has less false diffusion, longer computational time, is more complex to code and is more unstable.

c) Central scheme has some negative weighing coefficients and low ap values if there is little diffusion. This gives potentially very large negative effective weighing coefficients.

Problem 4

The formula for a kinematic wave with speed *c*, is derived from the continuity equation, looking at a similar situation as in Fig. 2.6.1 in the class notes. This gives:

$$c = \frac{dQ}{dA}$$

The differential of *Q* can be derived from the Chezy Equation for a wide, rectangular channel, differentiated with respect to the water depth:

$$\frac{dQ}{dy} = \frac{d}{dy}(AU) = \frac{d}{dy}\left(By\frac{1}{n}I^{\frac{1}{2}}y^{\frac{1}{2}}\right) = \frac{d}{dy}\left(B\frac{1}{n}I^{\frac{1}{2}}y^{\frac{3}{2}}\right) = \frac{BI^{\frac{1}{2}}}{n}\frac{3}{2}y^{\frac{1}{2}}$$

or rewritten:

$$dQ = \frac{3}{2}\frac{BI^{\frac{1}{2}}}{n}y^{\frac{1}{2}}dy$$

The formula for the area of the cross-section is then differentiated with respect to *y*:

$$dA = Bdy$$

Inserting *dA* and *dQ* from the two equations above gives:

$$c = \frac{3}{2}\frac{1}{n}I^{\frac{1}{2}}y^{\frac{1}{2}} = \frac{3}{2}U$$

which is the formula for the kinematic wave using Chezys equation.

Problem 5

a) OpenFOAM, SSIIM, Fluent, STAR-CCM+, CFX, FLOW-3D, PHOENICS. No 1D or 2D models, like HEC-RAS or MIKE11.

b) Fixed grid, two-fluids, f = volume of fluid in a cell, from 0 to 1. Differential equation solved for f.

c) $\frac{\partial f}{\partial t} + U_j\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma\frac{\partial f}{\partial x_j}\right)$ Term on right hand side may be neglected by some programs.

d) The level set method uses the distance from a cell to the surface as an unknown parameter instead of the volume fraction. The same equation is solved.

e) Advantage of adaptive grid for free water surface computations:

1. Only need to compute one phase
2. Less false diffusion since grid follow the water velocity vectors
3. Require therefore less cells
4. Can use implicit schemes which can use longer time steps.

f) Accuracy of CFD program to compute coefficient of discharge: 2-5 % for relatively simple geometries with a fine grid. With more complex geometries and coarser grids, up to 10 %. Closed conduit spillways with coarse grid, up to 20-30 %.

Problem 6

a)
1. Modelling errors
2. Errors in the numerical approximations
3. Errors due to not complete convergence
4. Round-off errors
5. Errors in boundary conditions and input data
6. Human errors due to inexperience of the user
7. Bugs in the software

b) False diffusion comes from numerical approximations. From the convective term in the convection-diffusion equation

c)

1. Finer grid. Advantages: Better resolution in the results, disadvantages: longer computational times

2. Higher order scheme: Advantages: resolve steep gradients better, disadvantages: less stability, longer computational times

3. Align the grid with the flow direction: Advantages: less false diffusion, disadvantages: Difficult to do for complex flow fields.

d)

- Grid
- Time step
- Turbulence model
- Discretization scheme
- Other numerical algorithms
- Roughness
- Boundary values of velocity distributions, k, epsilon, VOF fraction
- Geometry
- (Computer program)
- (Computer program operator :-))