Task 1 A cross-country relay team comprises four skiers. The first two skiers must use the classical style and the last two may ski freestyle. To select two skiers for the classical legs, the coach has four skiers to choose from. For the two freestyle legs he has five other skiers to choose from. What is the number of teams that the coach can put together from the above skiers when he also considers the ordering of the different legs?

A) 18 **B)** 36 **C)** 60 **D)** 240 **E)** 400

Task 2 If A is a subset of B, what is $P(A \mid B)$?

A) $P(A) \cdot P(B)$ **B)** 1 **C)** $\dfrac{P(A)}{P(B)}$ **D)** $\dfrac{P(B)}{P(A)}$ **E)** $P(A)$

Task 3 We throw two dice. What is the probability that the sum of the two equals 11?

A) $\frac{1}{36}$ **B)** $\frac{1}{18}$ **C)** $\frac{3}{36}$ **D)** $\frac{5}{36}$ **E)** $\frac{7}{36}$

Task 4 A satelite sends signals "0" or "1" to the earth. Let B be the event that the "0" signal is sent. Let further A be the event that the "0" signal is received on the earth. Because of athmospheric disturbances there could be a shift in the signals. Based on experience one has that $P(B) = 0.7$ and $P(A|B) = P(A^c|B^c) = 0.8$, where E^c denotes the complement of an event E. What is the probability that the original signal sent from the satelite is "0", given that we observe a "1" signal on the earth?

A) 0.15 **B)** 0.37 **C)** 0.43 **D)** 0.51 **E)** 0.62

Task 5 The random variable X has probability density function

$$f_X(x) = \begin{cases} 2x+3 & \text{for } x \in [0,a] \\ 0 & \text{elsewhere} \end{cases}$$

where a is a constant. Only one value of a is valid for $f_X(x)$ to be a probability density function. Which of the following values of a is correct?

A) 0 **B)** 1 **C)** $\frac{\sqrt{13}-3}{2}$ **D)** $-\sqrt{13}-3$ **E)** $\sqrt{13}-3$

Task 6 Let X be a continuous random variable with

$$P(X \leq x) = \begin{cases} 1 - \frac{\beta}{x}, & x > \beta \\ 0, & \text{elsewhere}, \end{cases}$$

where β is a positive constant. What is the probability density $f(x)$ of X, for $x > \beta$?

A) 1 **B)** $1 - \frac{\beta}{x}$ **C)** $-\frac{\beta}{x^2}$ **D)** $\frac{\beta}{x}$ **E)** $\frac{\beta}{x^2}$

Task 7 The joint density of X and Y is

$$f(x,y) = \begin{cases} x+y, & 0<x<1, 0<y<1, \\ 0, & \text{elswhere} \end{cases}$$

What is the marginal density, $g(x)$, of X?

A) $g(x) = 1, 0 < x < 1$ **B)** $g(x) = x, 0 < x < 1$ **C)** $g(x) = x + 1, 0 < x < 1$
D) $g(x) = x + \frac{1}{2}, 0 < x < 1$ **E)** $g(x) = x^2 + \frac{2}{3}, 0 < x < 1$

Task 8 A discrete random variable X has possible values $0, 1, 2, 3, 4$ and cumulative distribution

x	0	1	2	3	4
$F(x)$	0.24	0.65	0.92	0.99	1.00

Then the expected value and variance equal:

A) $\mathrm{E}(X) = 1.2$ and $\mathrm{Var}(X) = 0.84$ **B)** $\mathrm{E}(X) = 1.0$ and $\mathrm{Var}(X) = 0.25$
C) $\mathrm{E}(X) = 1.6$ and $\mathrm{Var}(X) = 0.09$ **D)** $\mathrm{E}(X) = 1.2$ and $\mathrm{Var}(X) = 0.49$
E) $\mathrm{E}(X) = 1.2$ and $\mathrm{Var}(X) = 0.64$

Task 9 The random variable X is continuously distributed with probability density

$$f(x) = \begin{cases} n \cdot x^{n-1} & 0 < x \leq 1 \\ 0 & \text{elswhere} \end{cases}$$

where n is a constant. Then the expected value of X equals:

A) $\mathrm{E}(X) = \frac{1}{n}$ **B)** $\mathrm{E}(X) = \frac{n}{n-1}$ **C)** $\mathrm{E}(X) = \frac{1}{n+1}$ **D)** $\mathrm{E}(X) = \frac{n}{n+1}$ **E)** $\mathrm{E}(X) = \frac{1}{2}$

Task 10 Let X be a continuous random variable with probability density

$$f(x) = \begin{cases} \frac{1}{\beta^2} x e^{-\frac{1}{\beta}x}, & x > 0 \\ 0, & \text{elswhere.} \end{cases}$$

What is $\mathrm{E}(\frac{1}{X})$?

A) 2β **B)** β **C)** $\frac{1}{\beta}$ **D)** $\frac{1}{2\beta^2}$ **E)** $\frac{1}{\beta^2}$

Task 11 Marius and Martin play a game of karaoke. They compete with a randomly selected song. Experience shows that the score Marius gets on a random song, X, has expected value 4000 points and standard deviation 1500 points. Similarly, the score Martin gets on a random song, Y, has expected value 2500 points and standard deviation 1000 points. We assume that the scores of Marius og Martin are positively correlated with correlation coefficient $\rho(X, Y) = 0.5$. What is the covariance between the scores of Marius and Martin, $\text{Cov}(X, Y)$?

A) 750000 **B)** 1125000000 **C)** 75000 **D)** 1125000 **E)** 0.5

Task 12 Martin (6 years) is a starter in snowboarding, and he finds it difficult to take the ski-lift alone. One day he decides to try alone $n = 12$ times. Assume that the results of his attempts are independent and that the probability of falling off is constant for all tries, equal to $p = 0.7$. Let X denote the number of times Martin falls of the ski-lift this day. What is $P(X \geq 6)$?

A) 0.961 **B)** 0.893 **C)** 0.328 **D)** 0.322 **E)** 0.033

Task 13 In a lake there are 40 fish, and 16 of the fishes have a weight below 100 gram. You are out fishing and catch 3 fish. What is the probability that at least one of the fish you catch has a weight below 100 gram?

A) 0.795 **B)** 0.205 **C)** 0.652 **D)** 0.292 **E)** 0.8846

Task 14 We assume that errors occuring in an fiber optic communication system follows a Poisson process with an error rate of 1 error per 1 minute. What is the probability that more than 3 errors occur within 4 minutes?

A) 0.4335 **B)** 0.5665 **C)** 0.6472 **D)** 0.8153 **E)** 0.3428

Task 15 A petroleum company is going to drill for oil in a particular region. They will drill one well at the time. The probability of finding oil in a randomly selected well is $p = 0.3$, for all wells. The results of the drilling is independent among the different wells. The company decides to drill new wells until they find oil the first time. What is the probability that they have to drill more than two wells?

A) 0.70 **B)** 0.49 **C)** 0.34 **D)** 0.30 **E)** 0.24

Task 16 An electronic chip has a diameter which is normal distributed with expectation $\mu = 6.52$ mm and standard deviation $\sigma = 0.03$ mm. The specification for a chip is a diameter in the range 6.50 ± 0.05 mm.
What is the probability that a randomly selected chip will have a diameter inside this specification?

A) 0.68 **B)** 0.76 **C)** 0.83 **D)** 0.95 **E)** 0.99

Task 17 A machine for filling bottles with softdrinks is adjusted such that the volume filled is assumed to be normal distributed with expected value 510 ml and standard deviation σ. The producer wants a probability of 0.99 for the event that a random bottle contains at least 500 ml. What is then the standard deviation of the filled volume?

A) 2.1 **B)** 3.3 **C)** 4.3 **D)** 5.0 **E)** 5.1

Task 18 Bus number 5 goes from the City Center to Dragvoll. Suppose that the buses pass the bus stop at Gløshaugen according to a Poisson process with intensity $\lambda = 6$ buses per hour, i.e. the number of buses that passes during an hour is Poisson distributed with expectation $\mu = \lambda \cdot 1 = 6$.

Suppose you run to catch the bus, but just misses it. What is the probability that you must wait more than 20 minutes for the next bus?

A) 0.812 **B)** 0.167 **C)** 0.135 **D)** 0.003 **E)** 0.865

Task 19 Let X be a continuous random variable with cumulative distribution

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases}$$

Let $Y = 2X$. What is the probability density $g(y)$ of Y?

A) $g(y) = \begin{cases} \frac{1}{2}, & y \in [0,2] \\ 0, & \text{elswhere} \end{cases}$ **B)** $g(y) = \begin{cases} 0, & y < 0 \\ \frac{1}{4}y, & y \in [0,2] \\ \frac{1}{2}, & y > 2 \end{cases}$ **C)** $g(y) = \begin{cases} \frac{1}{4}y, & y \in [0,2] \\ 0, & \text{elswhere} \end{cases}$

D) $g(y) = \begin{cases} 0, & y < 0 \\ 2y, & y \in [0,\frac{1}{2}] \\ \frac{1}{2}, & y \in [\frac{1}{2},2] \end{cases}$ **E)** $g(y) = \begin{cases} 2, & y \in [0,\frac{1}{2}] \\ 0, & \text{elswhere} \end{cases}$

Task 20 An elementary school with 18 pupils has a reading competition where the kids are to read as many books as possible within a period of 4 weeks. Assume the number of books a randomly selected pupil reads within this period is Poisson distributed with expectation $\mu = 12$. Assume further that all pupils read books independently of each other. What is the probability that the pupil that wins the competition has read at least 20 books?

A) 0.19 **B)** 0.02 **C)** 0.38 **D)** 0.81 **E)** 0.32