TOPOLOGY AND APPLICATIONS

NOVEMBER 30 – DECEMBER 2, 2016

Schedule

November 30	Speaker
10:00 – 10:10	Erik Kjær Pedersen: *Welcome!*
10:15 – 11:15	Dennis Sullivan: *Manifolds with singularities*
11:30 – 12:30	Arnold Levine: *Cancer: The path from Evolutionary Changes to the Three Dimensional Structures of Proteins and Their Functions*
12:30 – 14:15	Lunch
14:15 – 15:15	Edvard Moser: *Grid cells and our sense of space*
15:15 – 16:00	Coffee
16:00 – 17:00	Kathryn Hess: *Topology meets neuroscience*

December 1	Speaker
09:30 – 10:30	Ib Madsen: *Real algebraic K-theory*
10:45 – 11:45	John Rognes: *The Davis–Mahowald spectral sequence*
11:45 – 13:00	Lunch
13:00 – 14:00	Søren Galatius: *Simplicial deformation rings and the homology of arithmetic groups*
14:15 – 15:15	Gerd Laures: *Trolls and other characteristic classes in TMF*
15:15 – 16:00	Coffee
16:00 – 17:00	Ralph Cohen: *Comparing topological field theories: The string topology of a manifold and the Floer theory of its cotangent bundle*
19:00	Conference dinner

December 2	Speaker
09:30 – 10:30	Gunnar Carlsson: *Mayer–Vietoris and Products for Persistent Homology of Finite Metric Spaces*
10:45 – 11:45	Herbert Edelsbrunner: *Distances and divergences in topological data analysis*
11:45 – 13:00	Lunch
13:00 – 14:00	Marcel Bökstedt: *Spaces of vector sequences*
14:00 – 14:30	Coffee
14:30 – 15:30	Ulrike Tillmann: *From moduli spaces of manifolds to K-theory*

Abstracts

Marcel Bökstedt: Spaces of vector sequences

Abstract: We consider sequences of vectors $v_1, \dots, v_n$ in $\mathbb{C}^m$. These vectors are subject to the restrictions that for each i the subsequence $v_i, v_{i+1}, \dots v_{i+m-1}$ forms a basis for $\mathbb{C}^m$. We also fix the first m and the last m vectors in the sequence.

I will show that this approaches the space $\Omega(U_m/T^m)$ as n goes to infinity, and use the limit case to study the cohomology of the unstable approximations. I will also try to argue that this is an interesting family of spaces. This is a mixture of work by me and by Simon Stolze, and much of it will appear in his PhD thesis.

Gunnar Carlsson: Mayer–Vietoris and Products for Persistent Homology of Finite Metric Spaces

Abstract: In this talk we will discuss methods for understanding the persistent homology of finite metric spaces which are decomposed into unions and products.

Ralph Cohen: Comparing topological field theories: The string topology of a manifold and the Floer theory of its cotangent bundle

Abstract: I will describe recent work of Costello, Lurie, and Kontsevich and his collaborators, which describe how 2D topological field theories are classified. In particular we describe two notions of duality among A_∞-algebras and categories, and show how they give rise to field theories. These are referred to as "Calabi–Yau" structures. I'll then describe joint work with S. Ganatra that uses these results to establish an equivalence between two chain complex valued topological field theories: the String Topology of a manifold and the Symplectic Field theory of its cotangent bundle. This generalizes results of Viterbo, Abbondandolo and Schwarz, Abouzaid, and others.

Herbert Edelsbrunner: Distances and divergences in topological data analysis

Abstract: Given a finite set in a metric space, the topological analysis assesses its multi-scale connectivity quantified in terms of a 1-parameter family of homology groups. Going beyond metrics, we show that the basic tools of topological data analysis also apply when we measure dissimilarity with Bregman divergences. A particularly interesting case is the relative entropy whose infinitesimal version is known as the Fisher information metric. It relates to the Euclidean metric on the sphere and, perhaps surprisingly, the discrete Morse properties of random data behaves the same as in Euclidean space.

Søren Galatius: Simplicial deformation rings and the homology of arithmetic groups

Abstract: A classical result of Mazur asserts that any representation ρ of a group (or pro-group) over a field admits a universal deformation. This is a representation over a complete local ring, whose reduction modulo the maximal ideal is identified with ρ, and which is in a suitable sense universal with these properties. I will discuss joint work with Akshay Venkatesh, in which we enrich this picture to simplicial rings. The classical deformation ring is recovered by taking π_0, but some higher homotopy groups may be non-trivial. Subject to some (partially established) conjectures, we prove that homology of certain arithmetic groups admit the structure of a free module structure over the homotopy groups of the simplicial deformation ring for certain

Galois representations.

Kathryn Hess: Topology meets neuroscience

Abstract: I will present an overview of applications of topology to neuroscience on a wide range of scales, from the level of neurons to the level brain regions. In particular I will describe in detail collaborations in progress with the Blue Brain Project on topological analysis of the structure and function of digitally reconstructed microconnectomes and on topological classification of neuron morphological types. I will then briefly sketch applications of topology to the analysis of brain imaging data.

Gerd Laures: Trolls and other characteristic classes in TMF

Abstract: Cannibalistic classes were first studied by Bott for real K-theory in the context of spherical fibrations. I will introduce the analogous classes for string bundles with values in various forms of TMF. I will provide some tools to compute them and will talk about possible applications.

Arnold Levine: Cancer: The path from Evolutionary Changes to the Three Dimensional Structures of Proteins and Their Functions

Abstract: The origins of cancers in humans derive from both inherited and somatic mutations in genes that are selected for growth and survival of cells under conditions where normal cells are controlled. Sequencing of the genetic information (DNA) from normal cells and cancerous cells, permits a catalog of mutations in those genes that contribute to cancerous growth. Genes encode the information for producing two-dimensional structures of proteins. Inherent in the sequence of amino acids in a protein is the information to fold or to be folded into a three–dimensional structure that provides a specific function for that protein. Mutations alter the three dimensional structure of proteins. Algorithms have been developed to predict the structural changes brought about by mutations, and the functional consequences of these changes in structure. Based upon this, drugs have been developed to restore the proper structure of a mutant, or altered, protein so as to restore function and destroy the cancer. These points will be illustrated by examining a large cohort of individuals who inherit a mutated cancer-causing version of the p53 gene. The natural history of individuals in this cohort over a lifetime will be explored and elucidated.

Ib Madsen: Real algebraic K-theory

Abstract: Real algebraic K-theory associates to an exact category with duality, e.g. the category of of projective modules with the usual duality, a $\mathbb{Z}/2$-equivariant spectrum (a real spectrum) and hence groups $K_{p,q}$ for each pair of integers. If the category is the category of projective modules over the ring of continuous functions on a compact space one obtains Atiyah's real K-groups. The talk will outline the construction and some of the theorems associated with it, including a Real version of the cyclotomic trace. At present, little is known in terms of calculations, so there is a lot to be done in making the Real cyclotomic trace an effective calculational tool. One would also like to generalize the whole theory to Waldhausen's A-theory. The lecture represents joint work with Lars Hesselholt.

Edvard Moser: Grid cells and our sense of space

Abstract: The entorhinal cortex and the hippocampus are elements of the brain's circuit for spatial navigation and memory. Interest in the functions of these brain areas was raised half a century ago, when a brain surgery affecting these areas left patient H.M. with a severe loss of episodic memory as well as an inability to navigate in space. This incidence motivated attempts to study the activity of neurons in the hippocampus of experimental animals and led, 15 years later, to the discovery of place cells — cells that fire if and only if animals are at certain locations. Over the past 15 years, we have explored the wider circuit of the mammalian positioning system. I will show that the entorhinal cortex contains grid cells — cells with firing fields that tile environments in a periodic hexagonal pattern, like an internal coordinate system — as well as cells that monitor direction, speed and local borders. Collectively these cells form the elements of a positioning system that dynamically monitors our changing location in the environment, and that may provide the spatial component of all episodic memories. Deficiencies in the function of this map may be at the core of neurological diseases where spatial orientation is affected, such as Alzheimer's disease.

John Rognes: The Davis–Mahowald spectral sequence

Abstract: I will report on joint work with Robert R. Bruner. The cohomology of the subalgebra $A(2)$ of the mod 2 Steenrod algebra, generated by Sq^1, Sq^2 and Sq^4, was calculated by May in his unpublished Ph.D. thesis, and in published form by Shimada and Iwai (1967). Davis and Mahowald (1982) gave a different approach to this calculation, using the more familiar cohomology groups of the subalgebra $A(1)$, generated by Sq^1 and Sq^2, as an intermediate step. We revisit the work of Davis and Mahowald to clarify the multiplicative structure present in their approach. As a result, we obtain a manageable understanding of the E_2-term of the Adams spectral sequence converging to the 2-completed homotopy of the topological modular forms spectrum. A similar approach has proved to be useful in the analysis of the topological Hochschild homology of topological modular forms, as a step on the way to the topological periodic homology of that ring spectum.

Dennis Sullivan: Manifolds with singularities

Abstract: In 1953 Rene Thom showed that not all integral homology classes could be represented as images of top classes of closed manifolds. Classes up to six could be so represented by manifolds. But in dimension $7, 8, 9, \ldots$ manifolds with singularities would be required. It was fascinating to see what the singularities had to look like in the representatives of general homology classes.

An indirect argument 1970 lead to the picture that any homology class in dimension $7, 8, 9$ and 10 could be represented by a map of a closed manifold with a codimension five sub manifold S of non manifold points. The normal neighborhood of S was a product of S and the cone over $\mathbb{C}\mathrm{P}^2$, the complex projective plane.

Furthermore the homology class of S represented a non zero mod three homology class inside the carrier of the cycle corresponding to a specific homology operation applied to the class being represented- the dual of the cohomology operation: Bockstein of Steenrods first pth power for the prime three.

Each consecutive set of four dimensions required adding a new singularity to those already needed. The picture was rich, pictorially understandable, and only sketched in the literature. The iterated singularity construction with many variants was developed carefully and correctly in nontrivial early work of Nils Baas. Once this machinery was in place one could rigorously construct many generalized homology theories with explicit geometric cycle representatives. To get a picture of Thom's remarkable

discovery one only had to build an example with the correct homology groups for a point.

Ulrike Tillmann: From moduli spaces of manifolds to K-theory

Abstract: For mapping class groups of surfaces it is well-understood that their homology stability is closely related to the fact that they give rise to an infinite loop space. Indeed, they define an operad whose algebras group complete to infinite loop spaces.

In recent work with Basterra, Bobkova, Ponto and Yaekel we define operads with homology stability (OHS) more generally and prove that they are infinite loop space operads in the above sense. The strong homology stability results of Galatius and Randal-Williams for moduli spaces of manifolds can be used to construct examples of OHSs. As a consequence the map to K-theory defined by the action of the diffeomorphisms on the middle dimensional homology can be shown to be a map of infinite loop spaces.