GEOMETRY AND LIE THEORY
APPLICATIONS TO CLASSICAL AND QUANTUM MECHANICS

DEDICATED TO ELDAR STRAUME ON HIS 70TH BIRTHDAY

Schedule

November 3	Speaker	November 4	Speaker
10:00 – 10:10	Welcome!	09:30 – 10:20	Burkhard Wilking
10:10 – 11:00	Alain Chenciner	10:30 – 11:20	Claudio Gorodski
11:10 – 12:00	Valentin Lychagin	11:30 – 12:00	Vladimir Tkachev
12:00 – 13:30	Lunch	12:00 – 13:30	Lunch
13:30 – 14:20	Boris Doubrov	13:30 – 14:20	Sigbjørn Hervik
14:30 – 15:20	Dennis The	14:30 – 15:20	Andrew Swann
15:20 – 16:00	Coffee	15:20 – 16:00	Coffee
16:00 – 16:50	Wu-Yi Hsiang	16:00 – 16:30	Henrik Winther
17:00 – 17:30	Bjørn Jahren & Lars Sydnes	16:40 – 17:30	Arnfinn Laudal
19:30	Conference Dinner		

Abstracts

Alain Chenciner: Angular momentum and Horn's problem

Abstract: The central configurations of n point masses are those configurations $x = (\vec{r}_1, \cdots, \vec{r}_n)$ which admit periodic rigid motions when submitted to Newtonian attraction. Such rigid motions necessarily take place in an euclidean space E of even dimension $2p$ and, the initial configuration x_0 being given, they are of the form $\vec{r}_i(t) = e^{\omega t J}\vec{r}_i(0)$, where J is a complex structure on E compatible with the euclidean structure, that is an isometry such that $J^2 = -\mathrm{Id}$ ([C]). Studying the angular momenta of the set of relative equilibria of some n-body central configuration leads to the following purely algebraic question: Given a symmetric $2p \times 2p$ non negative matrix S_0 (the inertia matrix of the configuration), characterize the image of the mapping which, to each complex structure J, associates the ordered spectrum $\{\nu_1 \geq \nu_2 \geq \cdots \geq \nu_p\}$ of the J-hermitian matrix $S_0 + J^{-1}S_0J$, considered as a complex $p \times p$ matrix. On the other hand, Horn's problem asks for the possible spectra of matrices $C = A + B$, where A and B are hermitian (or real symmetric) with given spectra. Introducing two Horn's problems, one in dimension p and one in dimension $2p$, one proves that the image of $\mathcal{F}$ is a convex polytope which can be described. Moreover, this polytope comes equipped with a partition into subpolytopes whose boundaries correspond to periodic relative equilibria which could bifurcate into a family of quasi-periodic relative equilibria of balanced configurations, which are a natural generalization of central configurations.

References

[C] A. Chenciner, *The Lagrange reduction of the N-body problem: a survey*, Acta Mathematica Vietnamica (2013) 38:165–186.

[C–JP] A. Chenciner and H. Jiménez-Pérez, *Angular momentum and Horn's problem*, Moscow Mathematical Journal, Volume 13, Number 4, October–December 2013, 621–630.

[HZ] G. Heckman and L. Zhao, *Angular Momenta of Relative Equilibrium Motions and Real Moment Map Geometry*, to appear in Inventiones Mathematicæ.

Boris Doubrov: The classification of three-dimensional homogeneous spaces with non-solvable transformation groups

Abstract: Sophus Lie classified all 1- and 2-dimensional homogeneous spaces and outlined the ideas of classifying 3-dimensional spaces in volume 3 of "Transformation groups" by him and F. Engel. We show how these ideas can be formulated in the modern language and, as example, present the full classification of all 3-dimensional homogeneous spaces with non-solvable transformation group. We also show that the same problem in the nilpotent case does not admit a parametrization by a finite number of independent parameters.

Claudio Gorodski: Representations of compact Lie groups and their orbit spaces

Abstract: Let $\rho\colon G \to \mathbf{O}(V)$ be a faithful orthogonal representation of a compact Lie group G on an Euclidean space V. The space of orbits $X = V/G$ has a natural structure of metric space. In this talk, we would like to address the following question:

How much of ρ can be recovered from X?

The prototypical example of such situation is the adjoint representation of a compact connected Lie group on its Lie algebra, in which case X is identified with a Coxeter Riemannian orbifold. Our work generalizes some results about adjoint representations to other representations; namely, we propose to hierarchize representations in terms of the complexity of their orbit spaces. Such questions can also be considered in the realm of non-linear isometric actions and singular Riemannian foliations. (Based on joint work with A. Lytchak (Köln)).

Sigbjørn Hervik: Wick rotations and holomorphic Riemannian geometry

Abstract: In quantum field theories (QFT) it is common to perform Wick rotations of the Minkowskian space to a Riemannian space in order to compute certain integrals. If we want to study QFT in curved spaces, it is certainly not obvious whether such a Wick rotation to a Riemannian space can be done, indeed, generically, we expect this to not be possible. In order to address this question we will define Wick rotations using holomorphic Riemannian geometry. The structure group of the frame bundles will then be related through the real forms of the complex orthogonal group, $O(n, \mathbb{C})$. In this formalism, we are able to study this problem using results from real invariant theory and the classical theory of Lie groups.

Wu–Yi Hsiang: A new local invariant of sphere packings and clean-cut solutions of sphere packing problems

Abstract: This new local invariant was motivated by a problem Eldar posed to me about ten years ago: How to study sphere packings of two sizes? As it turns out, such a new type of local invariant also provides a wonderful key in the study of one-size sphere packings.

For example, the optimality of global packings of various kinds of sphere packings can be simply deduced from the optimality of this new local invariant defined in terms

of single layer local packing. Namely, its optimal upper bound is $\sqrt{\pi}/18$ and it is equal to $\sqrt{\pi}/18$ when and only when the local packing is the f.c.c. or the h.c.p.

I shall briefly outline the key lemmas of the proof of the above optimal estimate in the setting of spherical geometry.

Arnfinn Laudal: Noncommutative Algebraic Geometry, Topology, and Physics

Abstract: The relationship between algebraic geometry, topology, and physics, is well documented, and the field is very popular. I shall, in my talk (do my best to) introduce an extension of the methods used up to now, to include my version of noncommutative algebraic geometry.

The interest would be, to explain the relationship between notions like Ghost fields, Chern–Simons classes, Maxwell and Borch's equations, for electromagnetism, resp. time development of spin structures and entanglement, and Seiberg–Witten's monopole equation. I shall, maybe, just vaguely touch upon the beautiful results in low-dimension algebraic topology, based upon these methods, due to Donaldson and his followers.

References

[1] O.A. Laudal (2007) *Phase Spaces and Deformation Theory*, Preprint, Institut Mittag-Leffler, 2006-07. See also the part of the paper published in: Acta Applicanda Mathematicae, 25 January 2008.

[2] O.A. Laudal (2011) *Geometry of Time Spaces*, World Scientific, (2011).

[3] O.A. Laudal (2013) *Cosmos and its Furniture*. Mathematics in the 21st Century, Springer Proceedings in Mathematics and Statistics, ed. P. Cartier et al. Springer Basel 2014.

Valentin Lychagin: Quotients

Abstract: The quotients of solution spaces of differential equations by actions of Lie (pseudo)groups of symmetries will be discussed. It will be shown that under some conditions the Lie–Tresse theorem is valid and the quotients itself could be realized as new differential equations (diffietes). Applications to classical problems in theory of algebraic invariants, relativity theory and differential geometry will be given.

Andrew Swann: Torus actions and Ricci-flat metrics

Abstract: This talk will consider Ricci-flat metrics defined by structures with special holonomy. The presence of closed differential forms mean that moment map techniques may be used, generalising the Delzant construction in symplectic geometry. For complete hyperKähler metrics on manifolds of dimension $4n$ with a tri-Hamiltonian action of an n-torus, we will describe how a full classification may be obtained. Time permitting, we will report on progress considering similar ideas for metrics of holonomy G_2 with a 3-torus symmetry.

Dennis The: Symmetry gaps for geometric structures

Abstract: For many (local) geometric structures, there is often a gap between the maximum and the next realizable ("submaximal") dimension of the Lie algebra of (infinitesimal) symmetries. For example, for Riemannian metrics on surfaces, 3 is the maximum, while 1 is the submaximal symmetry dimension. Many interesting geometric structures (such as conformal, CR, projective, systems of ODE, and various types of generic distributions) admit an equivalent description as so-called parabolic

geometries. The goal of this talk is to discuss how this viewpoint proved useful in the study of the gap problem for these geometries. In particular, we obtained a representation-theoretic universal upper bound on the submaximal symmetry dimension, which is sharp (and was computed) for all parabolic geometries of type (G, P), where G is a complex or split-real simple Lie group and P is a parabolic subgroup. (Joint work with Boris Kruglikov.)

Vladimir Tkachev: Hsiang algebras of cubic minimal cones

Abstract: In [1] Wu-Yi Hsiang initiated the study of algebraic minimal cones and posed a problem to classify a certain class of cubic (i.e. degree three) minimal cones. It has been very recently realized [2, 3, 4] that these cones naturally correspond to the generic norm in a special class of commutative but nonassociative algebras with associative inner product, the so-called Hsiang algebras. The key idea comes back to the Freudenthal–-Springer–-Tits construction of exceptional Jordan algebras: one replaces the study of a cubic form by the study of a certain commutative algebra recovering properties of the cubic form from the properties of the corresponding algebra, and vice versa. In this setting, the classification of all cubic minimal cones problem becomes equivalent to classification of Hsiang algebras.

References

[1] Hsiang W.–Yi, *Remarks on closed minimal submanifolds in the standard Riemannian m-sphere*, J. Diff. Geom., 1967.
[2] N. Nadirashvili, V. Tkachev and S. Vladut, *Nonlinear elliptic equations and nonassociative algebras*. Math. Surveys and Monographs, Vol. 200. AMS, 2015.
[3] V.G. Tkachev, *A Jordan algebra approach to the cubic eiconal equation*. J. Algebra 419(2014), 34–51.
[4] V.G. Tkachev, *On the non-vanishing property for real analytic solutions of the p-Laplace equation*, Proc. Amer. Math. Soc., 144(2016), 2375–2382.

Burkhard Wilking: Computing the Euler characteristic of positively curved manifolds under logarithmic symmetry assumptions

Abstract: We show that the Euler characteristic of a positively curved n manifold M coincides with the Euler characteristic of an n-dimensional compact rank 1 symmetric space provided that the rank of the isometry group of M is larger than $3\log_2 n$.

Henrik Winther: Submaximally Symmetric Quaternionic Structures

Abstract: The symmetry dimension of an almost quaternionic structure on a manifold is the dimension of its full automorphism algebra. Let the quaternionic dimension n be fixed. The maximal possible symmetry dimension is realized by the quaternionic projective space $\mathbb{HP}^n$, which has symmetry group $G = PGL(n+1, \mathbb{H})$ of dimension $\dim(G) = 4(n+1)^2 - 1$. An almost quaternionic structure is called submaximally symmetric if it has maximal symmetry dimension amongst those with lesser symmetry dimension than the maximal case. We show that for $n > 1$, the submaximal symmetry dimension is $4n^2 - 4n + 9$. This is realized both by a quaternionic structure (torsion free) and by an almost quaternionic structure with vanishing Weyl curvature. (Joint work with Boris Kruglikov and Lenka Zalabová.).