Workshop on Rough Paths

and

Combinatorics in Control Theory

University of California

San Diego

July 25 - 27, 2011

Schedule: Monday to Wednesday LECTURE ROOM: **AP&M 2402**

09:00am – 10:30am:	**Lecture** (2×40min.)
10:30am – 11:00am:	COFFEE-TEA
11:00am – 12:00am:	**1st talk** (55min.)
12:00am – 02:00pm:	LUNCH
02:00pm – 03:00pm:	**2nd talk** (55min.)

MONDAY, July 25, 2011

– Terry LYONS

Lecture: **The expected signature of a stochastic process. Some new PDE's**

Abstract: How can one describe a probability measure of paths? And how should one approximate to this measure so as to capture the effect of this randomly evolving system. Markovian measures were efficiently described by Strook and Varadhan through the Martingale problem. But there are many measures on paths that are not Markovian and a new tool, the expected signature provides a systematic way of describing such measures in terms of their effects. We explain how to calculate this expected signature in the case of the measure on paths corresponding to a Brownian motion started at a point x in the open set and run until it leaves the same set. A completely new (at least to the speaker) PDE is needed to characterise this expected signature. Joint work with Ni Hao.

– Fabrice Baudoin

Some aspects of differential equations driven by fractional Brownian motions

Abstract: The purpose of this talk will be to review some recent results on properties of the solution of stochastic differential equations driven by a fractional Brownian motion with a Hurst parameter $H \in (1/3, 1)$. We shall in particular discuss upper Gaussian bounds for the density of the solution and functional inequalities satisfied by the distribution of the solution. The talk is based on works with Cheng Ouyang (Purdue University) and Samy Tindel (IECN, Nancy).

– Bronislaw Jakubczyk

Hamiltonian vector fields, observables and Lie series

Abstract: Given a family f_u of vector fields on a manifold M and a family of functions h_u on M, both parametrized by the same parameter $u \in U$ (control), they define the control system $\dot{x} = f_u(x)$ with observation $u = h_u(x)$. We may ask when such a system is Hamiltonian, that is, if there is a symplectic or Poisson structure on M such that f_u are Hamiltonian vector fields, with h_u as their Hamiltonians. We will show that an answer to this question leads to formal Lie series and formal momentum maps.

The second question we will discuss is when a given Lie series comes from a Hamiltonian control system on a symplectic manifold M. We will specify criteria for this to be true and show that the Lie series determines the minimal symplectic manifold uniquely, up to a symplectomorphism. Any real analytic, connected symplectic manifold together with a controlled and observed system can be constructed from a Lie series.

TUESDAY, July 26, 2011

– W. Steven GRAY

Lecture: **A formal power series approach to nonlinear system interconnections**

Abstract: An analysis is presented of the radii of convergence for the parallel, product, cascade and unity feedback interconnections of analytic nonlinear input-output systems represented as Fliess operators. Such operators are described by convergent functional series, indexed by words over a noncommutative alphabet. Their generating series are therefore specified in terms of noncommutative formal power series. Given growth conditions on the coefficients of the generating series for the component systems, the radius of convergence of each interconnected system is computed assuming the component systems are either all locally convergent or all globally convergent. It is shown definitively that local convergence is preserved under unity feedback, which had been an open problem.

– Makhin Thitsa

On the radius of convergence of interconnected analytic nonlinear systems

Abstract: We show the relationship between a constrained Lagrangian system and a free Hamiltonian one through the following tools: Lagrangian submanifolds and Tulczyjews Triples. The formalism is also extensible to discrete Mechanics. Regarding the discrete case, we present some examples of exact matching between symplectic numerical integrators applied to Hamiltonian systems and variational integrators applied to constrained Lagrangian ones.

– Fernando Jimenez Alburquerque

Lagrangian Submanifolds and Constrained Variational Calculus: Continuous and Discrete Settings (30min. talk)

Abstract: We show the relationship between a constrained Lagrangian system and a free Hamiltonian one through the following tools: Lagrangian submanifolds and Tulczyjews Triples. The formalism is also extensible to discrete Mechanics. Regarding the discrete case, we present some examples of exact matching between symplectic numerical integrators applied to Hamiltonian systems and variational integrators applied to constrained Lagrangian ones.

– Elisa Lavinia Guzmán Alonso

A Tulczyjew Triple in Field Theory (30min. talk)

Abstract: In 1976, W. Tulczyjew introduced different canonical isomorphisms between the spaces T^*TQ, TT^*Q and T^*T^*Q of a smooth manifold Q. These mappings are of furthermost importance since they relate the canonical symplectic structures of each space and they highlight the strong relation between the Lagrangian and Hamiltonian formalisms for autonomous mechanical systems. Here, I present a generalization of the Tulczyjew triple to Field Theory. The framework will be the one of jet bundles and multisymplectic geometry.

– Joris Vankerschaver

A geometric approach to particle manipulation using symplectic connections

Abstract: Many problems in nature and engineering involve controlling rigid bodies and/or massless particles submerged in a fluid. In the case where the fluid is inviscid, the combined solid-fluid system has a Hamiltonian description, to which the techniques from geometric control theory can be applied. I will discuss the geometric aspects of interacting fluids and solids, with a particular emphasis on controlling individual fluid particles, and I will show how geometry gives rise to a qualitative understanding of particle dynamics using Magnus expansions. Time permitting, I will show that the particle manipulation system fits into a new class of control systems, which we have termed symplectic control systems.

WEDNESDAY, July 27, 2011

– Luis DUFFAUT ESPINOSA

Lecture: **On the well-posedness of cascades of analytic nonlinear input-output systems driven by noise**

Abstract: It was shown recently that the probabilistic characterization of input processes (controls) is an obstacle to having well-posed interconnections of analytic nonlinear input-output systems. For example, the cascade connection of two such systems is only known to be well-posed when a certain independence property is preserved by the first system in the connection. However, the description of these inputs can be done in a deterministic manner via T. Lyons' construction of a "rough path". This concept employs Chen's identity to extend the notion of the Young integral. It then exploits the underlying combinatorics of the signature of a path making it consistent with the known setting but with the advantage that the independence property can be dropped from the analysis.

– Melvin Leok

General Techniques for Constructing Variational Integrators

Abstract: The numerical analysis of variational integrators relies on variational error analysis, which relates the order of accuracy of a variational integrator with the order of approximation of the exact discrete Lagrangian by a computable discrete Lagrangian. The exact discrete Lagrangian can either be characterized variationally, or in terms of Jacobi's solution of the Hamilton–Jacobi equation. These two characterizations lead to the Galerkin and shooting-based constructions for discrete Lagrangians, which depend on a choice of a numerical quadrature formula, together with either a finite-dimensional function space or a one-step method. We prove that the properties of the quadrature formula, finite-dimensional function space, and underlying one-step method determine the order of accuracy and momentum-conservation properties of the associated variational integrators. We also illustrate these systematic methods for constructing variational integrators with numerical examples.

This approach can also be generalized to the case of degenerate Hamiltonian systems, which allows for the construction of variational integrators for Hamiltonian systems which do not have a Lagrangian analogue. When extended to the case of Hamiltonian field theories, this also provides a systematic framework for constructing geometric integrators that are automatically multisymplectic.

Joint work with James Hall, Cuicui Liao, Tatiana Shingel, Joris Vankerschaver, and Jingjing Zhang. Supported in part by NSF DMS-1001521 and DMS-1010687.

– Maria Barbero

Geometric interpretations of the symmetric product in affine differential geometry

Abstract: The symmetric product of vector fields for Levi-Civita connections appeared for first time in [1] in the study of gradient systems. This product appeared again in [2] where it was used to characterize the controllability of a large class of mechanical control systems. Since then, the symmetric product has been widely used to solve control theoretic problems for mechanical systems, such as motion planning [3], trackability [4], and so on.

A geometric description of the symmetric product will be given in this talk using parallel transport, along the lines of the flow interpretation of the Lie bracket [5]. This geometric interpretation of the symmetric product is used to provide an intrinsic proof of the fact that the distributions closed under the symmetric product are exactly those distributions invariant under the geodesic flow.

References:

[1] P.E. Crouch. Geometric structures in systems theory. Institution of Electrical Engineers. Proceedings. D. Control Theory and Applications, 128(5):242-252, 1981.

[2] A.D. Lewis and R.M. Murray. Controllability of simple mechanical control systems. SIAM Journal on Control and Optimization, 35(3):766-790, 1997.

[3] F. Bullo, N.E. Leonard, and A.D. Lewis. Controllability and motion algorithms for underactuated Lagrangian systems on Lie groups. Institute of Electrical and Electronics Engineers. Transactions on Automatic Control, 45(8):1437-1454, 2000.

[4] F. Bullo and A.D. Lewis. Geometric Control of Mechanical Systems: Modeling, Analysis, and Design for Simple Mechanical Systems. Number 49 in Texts in Applied Mathematics. Springer-Verlag, New York-Heidelberg-Berlin, 2004.

[5] M. Barbero-Liñán and A.D. Lewis, Geometric interpretations of the symmetric product in affine differential geometry. Preprint 2011. arXiv:1104.1208v1 [math.DG]