All the abstracts in alphabetical order

(v. 22.05.2023)

Time/Day	Wednesday	Thursday	Friday
09:00–09:50		Kassmann	
09:50–10:40		del Teso	Fjordholm
10:40–11:00		*Coffee break*	*Coffee break*
11:00–11:50		Serea	Mæhlen
11:50–12:40		Chowdhury	Vázquez
12:40–13:30	*Opening (13:00–13:30)*	*Lunch*	*Closing and lunch*
13:30–14:20	Quirós	*Lunch*	*Lunch*
14:20–15:10	Krupski	Cirant	
15:10–15:30	*Coffee break*	*Coffee break*	
15:30–16:20	Rutkowski	Silva	
16:20–17:10	Gómez-Castro	Solem	
19:00–23:59		*Social dinner*	

Speaker: Indranil Chowdhury

Title: *Numerical methods for fractional HJB equations: Improved error bounds and weakly degenerate equations*

Abstract:

We discuss monotone numerical methods for fractional and nonlocal HJB equations in strongly and weakly degenerate cases. These equations are dynamic programming equations of optimal control of SDEs driven by pure jump Levy processes. We discuss very precise (fractional) error bounds for diffusion corrected difference-quadrature methods. By precise fractional rate we mean, the convergence rates depend optimally on the order of the nonlocal operators. For weakly degenerate nonlocal equations, we obtain stronger regularity type estimates for both the viscosity solutions and numerical approximations. As a result, we show the improvement in error bound for the schemes compared to strongly degenerate cases.

Speaker: Marco Cirant

Title: *Hölder and maximal regularity for Hamilton-Jacobi equations*

Abstract:

I will discuss some results on the regularity of solutions to semi-linear equations of Hamilton-Jacobi type. I will in particular focus on the Hölder regularity of solutions, and the problem of maximal regularity in L^p spaces, developing two different approaches based on the Bernstein method and a blow-up analysis. Both the elliptic and the parabolic settings will be discussed.

Speaker: Félix del Teso

Title: *A convergent discretization of the porous medium equation with fractional pressure*

Abstract:

We carefully construct and prove convergence of a numerical discretization of the porous medium equation with fractional pressure,

$$\frac{\partial u}{\partial t} - \nabla \cdot \left(u^{m-1} \nabla (-\Delta)^{-\sigma} u \right) = 0, \tag{FPE}$$

for $\sigma \in (0,1)$ and $m \geq 2$. The model, introduced by Caffarelli and Vázquez in 2011, is currently one of two main nonlocal extensions of the local porous medium equation. It has finite speed of propagation, but as opposed to the other extension, it does not satisfy the comparison principle. We exploit the fact that the *cumulative density* $v(x,t) = \int_{-\infty}^{y} u(y,t)dy$ satisfies

$$\frac{\partial v}{\partial t} + |\partial_x v|^{m-1} (-\Delta)^s v = 0, \quad s = 1 - \sigma,$$

which is a nonlocal quasilinear parabolic equation in non-divergence form that can be analyzed through viscosity solution methods.

The numerical method consists in discretizing this equation with a difference quadrature scheme with upwinding ideas and then compute the solution u of (FPE) via numerical differentiation. Our results cover both absolutely continuous and Dirac or point mass initial data, and in the latter case, machinery for discontinuous viscosity solutions are needed in the analysis.

This is a joint work with E. R. Jakobsen.

Speaker: Ulrik Fjordholm

Title: *Continuity equations, particle paths and nonlinear conservation laws*

Abstract:

Nonlinear, scalar conservation laws have been thoroughly studied over the past few decades. We give a new interpretation of these PDEs as (nonlinear) continuity equations, and couple the Kruzkhov uniqueness theory with the well-posedness of associated particle paths – ODEs which govern the mass transportation of the solution. This is joint work with Magnus Ørke and Ola Mæhlen (both at UiO).

Speaker: David Gómez-Castro

Title: *Newtonian vortex equations with non-linear mobility*

Abstract:

In this talk we will consider conservation equations of the form

$$\begin{cases} u_t = \operatorname{div}(u^\alpha \nabla v) \\ -\Delta v = u \end{cases}$$

This system can equivalently be written as one equation $u_t = \operatorname{div}(u^\alpha \nabla W * u)$ where W is the Newtonian potential. For linear mobility, $\alpha = 1$, the equation and some variations have been proposed as a model for superconductivity or superfluidity. In that case the theory leads to uniqueness of bounded weak solutions having the property of compact space support, and in particular there is a special solution in the form of a disk vortex showing a discontinuous leading front.

The aim of the talk is to discuss the cases $\alpha \in (0, 1)$ and $\alpha > 1$ which, as for the Porous Medium Equation, exhibit very different behaviours. First, we discuss self-similar solutions. Then, we restrict the analysis to radial solutions and construct solutions by the method of characteristics. We introduce the mass function, which solves an unusual variation of Burger's equation, and plays an important role in the analysis. We show well-posedness in the sense of viscosity solutions and construct a convergent finite-difference numerical schemes.

For sublinear mobility $0 < \alpha < 1$ nonnegative solutions recover positivity everywhere, and moreover display a fat tail at infinity. The model acts in many ways as a regularization of the previous one.

For superlinear mobility $\alpha > 1$ we show that solutions corresponding to compactly supported initial data remain compactly supported for all times leading to moving free boundaries as in the linear mobility case $\alpha = 1$. We show a waiting time phenomena.

The talk presents joint work with José A. Carrillo (U. Oxford) and Juan Luis Vázquez (U. Autónoma de Madrid) published as [1, 2].

References

[1] J. A. Carrillo, G-C, and J.L. Vázquez, A fast regularisation of a Newtonian vortex equation, *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 39 (2022), no. 3, pp. 705–747. https://doi.org/10.4171/AIHPC/17

[2] J.A. Carrillo, G-C, and J.L. Vázquez, Vortex formation for a non-local interaction model with Newtonian repulsion and superlinear mobility, *Advances in Nonlinear Analysis*, vol. 11, no. 1, 2022, pp. 937-967. https://doi.org/10.1515/anona-2021-0231

Speaker: Moritz Kassmann

Title: *Robust nonlocal trace and extension theorems*

Abstract:

We prove trace and extension results for Sobolev-type function spaces that are well suited for nonlocal Dirichlet and Neumann problems including those for the fractional p-Laplacian. The main focus is on optimal choice of exterior data. The results are robust with respect to the order of differentiability. In this sense they are in align with the classical trace and extension theorems. The talk is based on a recent joint work with Floran Grube, see arXiv:2305.05735.

Speaker: Miłosz Krupski

Title: *Lévy processes, controlled time rate and mean field games*

Abstract:

In the original formulation of *mean field games*, agents control the drift of a Wiener process describing their movement and collect gains based on their position within the aggregate population. This can be seen as the ability to control the *spatial* direction of their advancement, to find and follow the optimal location.

If the "undisturbed" movement of agents is instead governed by a general *Lévy process* (one or multidimensional), due to its stochastic nature the control of direction is no longer available. The agents can only control the *time rate* at which they choose to disperse. In case of *self-similar* processes (like drift), this approach can still be reinterpreted as spatial control.

Such model justifies the study of more general mean field games. Establishing the well-posedness of their solutions requires a revision of classical proofs which involves some novel technical observations.

Speaker: Ola Mæhlen

Title: *Nonlocal equations of mixed hyperbolic-parabolic type on bounded domains*

Abstract:

We provide an entropy-formulation of hyperbolic-parabolic equations, posed on bounded domains, that feature nonlocal degenerate diffusion. The nonlocal diffusions considered are those represented by symmetric Lévy operators (including all fractional Laplacians). These operators are not well defined when applied to functions on bounded domains, which we tackle by posing Dirichlet 'boundary conditions' in the full complement of our domain of interest.

I will give an overview of the arguments for the existence and uniqueness of corresponding entropy solutions and explain how these arguments differ from the local case. Some theory on Lévy operators will also be discussed. This is joint work with

- Espen R. Jakobsen (Norwegian University of Science and Technology),
- Jørgen Endal (Norwegian University of Science and Technology) and
- Nathaël Alibaud (Université de Bourgogne Franche-Comté).

Speaker: Fernando Quirós

Title: *Asymptotic profiles for inhomogeneous classical and nonlocal heat equations*

Abstract:

We study the large-time behaviour of solutions to inhomogeneous heat equations in the whole space, with a diffusion operator which may be local or nonlocal both in space and time. We find that the asymptotic profiles depend strongly on the space-time scale and on the time behaviour of the spatial L^1 norm of the forcing term. Some of our results are surprising even for the classical heat equation in the somewhat studied case in which the right-hand side is globally integrable in space and time. On the other hand, our assumptions on the source term allow for the space integral to grow to infinity as time goes to infinity.

This is joint work with Noemí Wolanski (IMAS-UBA-CONICET, Argentina) and Carmen Cortázar (PUC, Chile).

Speaker: Artur Rutkowski

Title: *The master equation for the mean field games with Lévy diffusions*

Abstract:

We prove existence and uniqueness of solutions to the master equation associated with the mean field game system

$$\begin{cases} -\partial_t u - \mathcal{L}u + H(x,u,Du) = F(x,m(t)) & \text{in } (t_0,T)\times\mathbb{R}^d, \\ \partial_t m - \mathcal{L}^* m - \operatorname{div}\left(m D_p H(x,u,Du)\right) = 0 & \text{in } (t_0,T)\times\mathbb{R}^d, \\ m(t_0) = m_0, \qquad u(T,x) = G(x,m(T)), \end{cases} \tag{1}$$

where $\mathcal{L}$ is a Lévy operator, roughly speaking, of order greater than 1. For example, we allow $-(-\Delta)^{\alpha/2}$ with $\alpha \in (1,2]$, mixed local-nonlocal operators, and strongly anisotropic stable operators.

In order to obtain our result we establish several well-posedness and regularity results for linear and semilinear equations with Lévy diffusions, which are of their own interest. Furthermore, because of working in the whole space, we need a new compactness argument for the linearized system associated with (1).

The master equation can be applied to show that the mean field games are a good approximation of games with a large number of players. This was first done in the seminal work of Cardaliaguet, Delarue, Lasry and Lions [1].

Based on a joint work with Espen R. Jakobsen (NTNU).

References

[1] P. Cardaliaguet, F. Delarue, J.-M. Lasry, P.-L. Lions. *The master equation and the convergence problem in mean field games*, volume 201 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2019.

Speaker: Oana Serea

Title: *Zubov's method for Lyapunov functions*

Abstract:

Zubov's method is a classical method for computing Lyapunov functions and domains of attraction for differential equations

$$\dot{x} = f(x), \quad x \in \mathbb{R}^N$$

with a locally asymptotically stable equilibrium $x^* \in \mathbb{R}^N$. Zubov's main result states that under appropriate conditions and for a suitable function $g : \mathbb{R}^N \to \mathbb{R}$ the Zubov equation

$$\nabla W(x) f(x) = -g(x)(1 - W(x))\sqrt{1 + \|f(x)\|^2},$$

a first order partial differential equation, has a unique differentiable solution $W : \mathbb{R}^N \to [0, 1]$ with $W(x^*) = 0$, which characterizes the domain of attraction $\mathcal{D}$ of x^* via $\mathcal{D} = \{x \in \mathbb{R}^N \,|\, W(x) < 1\}$ and which is a Lyapunov function on $\mathcal{D}$. Firstly, we provide generalizations of Zubov's equation to differential games without Isaacs' condition. We show that both generalizations of Zubov's equation (which we call min-max and max-min Zubov equation, respectively) possess unique viscosity solutions which characterize the respective controllability domains. As a consequence, we show that under the usual Isaacs condition the respective controllability domains as well as the local controllability assumptions coincide. Secondly we study generalizations of Zubov's equation to algebraic differential equations.

Speaker: Francisco Silva

Title: *A Lagrange-Galerkin scheme for first order mean field games systems*

Abstract:

In this work, we consider a first order mean field games system with non-local couplings. A Lagrange-Galerkin scheme for the continuity equation, coupled with a semi-Lagrangian scheme for the Hamilton-Jacobi-Bellman equation, is proposed to discretize the mean field games system. The convergence of solutions to the scheme towards a solution to the mean field game system is established in arbitrary space dimensions. The scheme is implemented to approximate two mean field games systems in dimension one and two.

Speaker: Susanne Solem

Title: *Asymptotic stability of stationary states of a stochastic neural field in the form of a PDE*

Abstract:

A Fokker–Planck-like partial differential equation was recently proposed to represent certain stochastic neural fields. So far, the PDE has been rigorously derived from a stochastic particle system and its noise-driven pattern-forming bifurcations have been characterized. However, due to its nonlinear and non-local nature, it is far from obvious how to determine the stability of the stationary states for different noise strengths. In this talk, I will present some recent results in this direction.

Speaker: Juan Luis Vázquez

Title: *Nonlinear Diffusion Equations driven by Fractional Operators*

Abstract:

We will review some basic models of nonlinear diffusion driven by nonlocal operators. Then we will present some lines of recent progress in the study of p-Laplacian evolution equations both of local or nonlocal type. The evolution equation driven by a nonlocal infinity Laplacian is presented.