F-N Criterion lines

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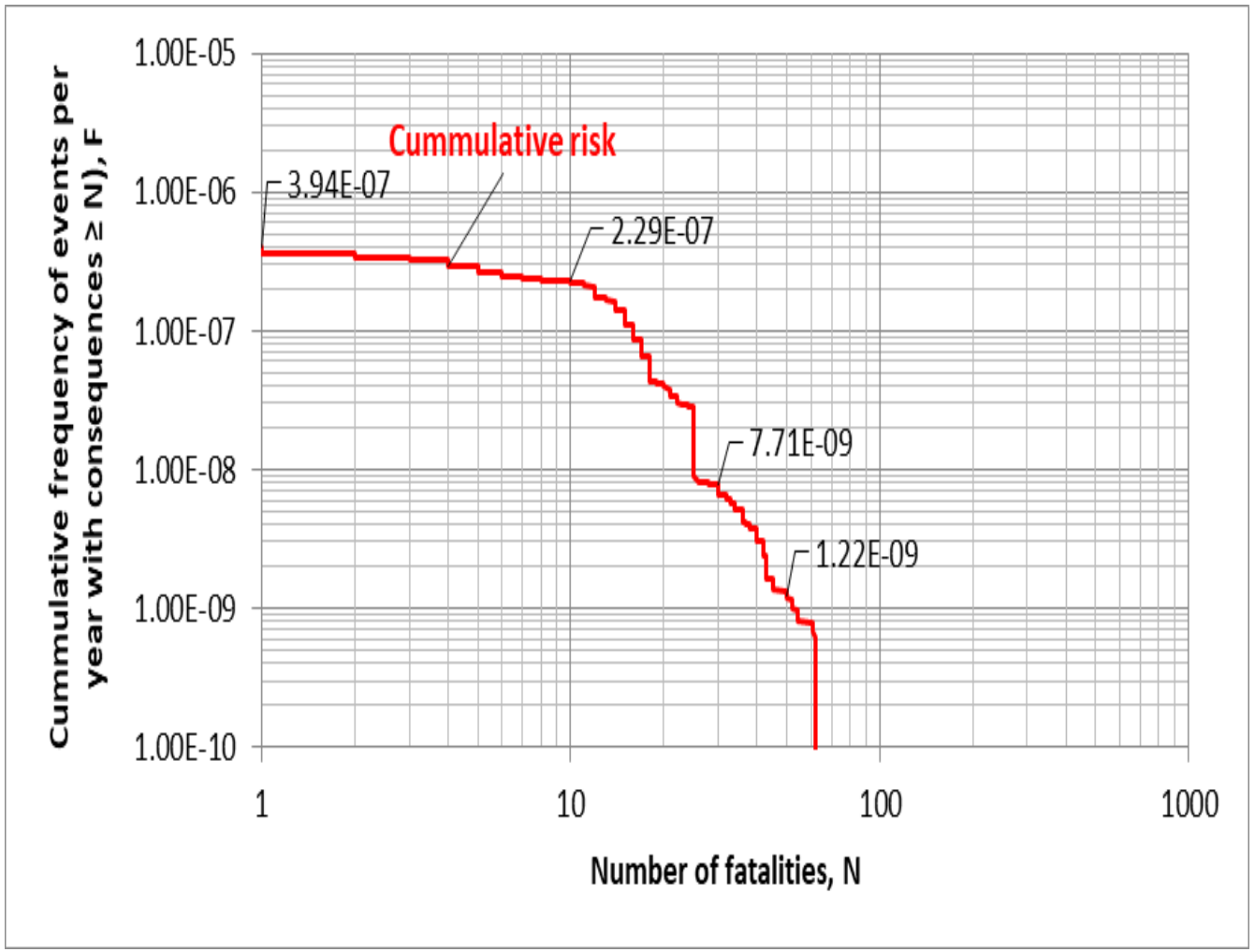
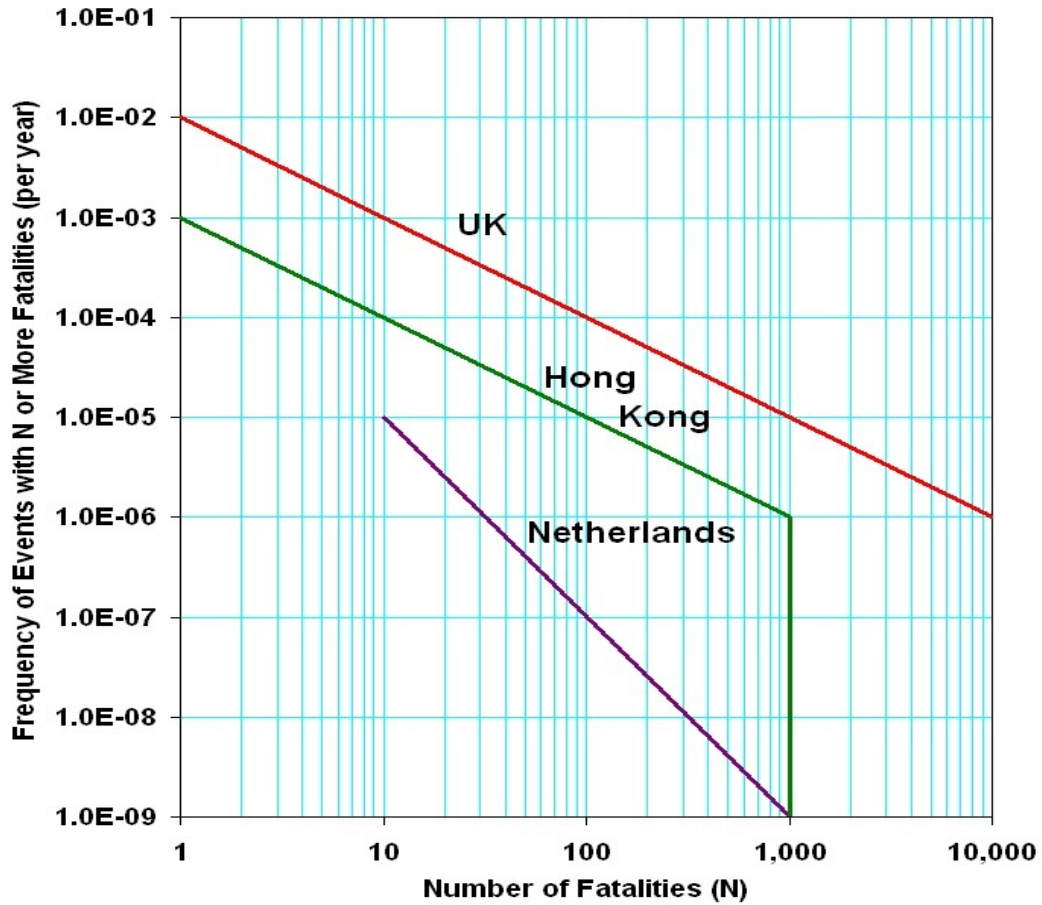
F-N curves and F-N criterion lines – what is it?





What is wrong with Criterion FN-Lines for Judging the Tolerability of Risk?

- **Evans, A.W**. and Verlander, N.Q. (1997), Risk Analysis, Vol. 17, No. 2

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What Is Wrong with Criterion FN-Lines for Judging the Tolerability of Risk?

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This paper is concerned with the intellectual framework in which judgments are made about the tolerability of so-called *societal risk*. The current practical approach is based on the position of the FN-curves representing the risks from hazardous systems in relation to criterion FN-lines. The objections to FN-criteria are that they can give unreasonable conclusions and that they are inconsistent. Statistical decision theory suggests an alternative and preferable rule of minimising the expected disutility, that is average harm, from accidents.

KEY WORDS: FN-curves; societal risk; tolerability; expected utility.

1. INTRODUCTION

Engineering systems such as industrial plant or public transport systems inevitably pose some risk of accidents, which may cause death or injury to staff, passengers, or other people, together with damage, disruption, and other costs. The risks may be estimated by using historical data, or by quantified risk models, or by a combination of these. After the risks have been estimated, judgments must be made by the managers of the system and by the safety regulatory authority on behalf of society (usually the Health and Safety Executive (HSE) in Britain) as to what further safety measures, if any, are desirable. This latter process is labeled *risk appraisal* or *risk evaluation*.

Three general types of quantified criteria are in use for risk appraisal. These are labeled (1) Individual risk criteria, (2) cost-benefit analysis, and (3) societal risk criteria. Individual risk criteria are concerned with the risk to representative or specified individuals, in contrast to the other two criteria, which are concerned with aggregate risk over groups of people. The usual form of individual risk criterion places an upper limit on the tolerable risk of death to individuals as a result of a specified activity: common upper limits to the tolerable risk of death used in Britain are 1 in 1000 per year for employees and 1 in 10,000 per year for third parties.[1] The rationale for having such limits is that they protect any single individual or group from bearing too large a share of risk, or in other words they promote equity in the distribution of risk. In what follows we shall assume that all individual risk tolerability limits are met, and therefore not consider them further.

There is widespread agreement that individual risk tolerability criteria are not sufficient on their own, because there are many circumstances in which it is sensible to reduce risk, even though no one may be individually at high risk. Therefore it is usual practice to use either or both of cost-benefit analysis (CBA) and societal risk criteria alongside individual risk criteria. Both are concerned with the aggregate risk to groups of people rather than to individuals. CBA starts with given possible safety measures, and compares their costs and benefits in monetary terms. The decision criterion is that a safety measure should be adopted if and only if the benefits exceed the costs. The benefits of safety measures usually include reductions in risk to groups of people, and CBA requires these risk reductions to be valued and summed over all the individuals affected. In contrast to CBA, societal risk criteria do not place values on the costs and benefits of safety measures, but place limits

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Formulation of the F-N criterion line

- The criterion line can be described by its intersection with the n=1 axis, L and the slope of the line, α_{FN}, in the double logarithmic representation.

- The steeper the line the more "risk averse" is the attitude. The formula for the acceptance is

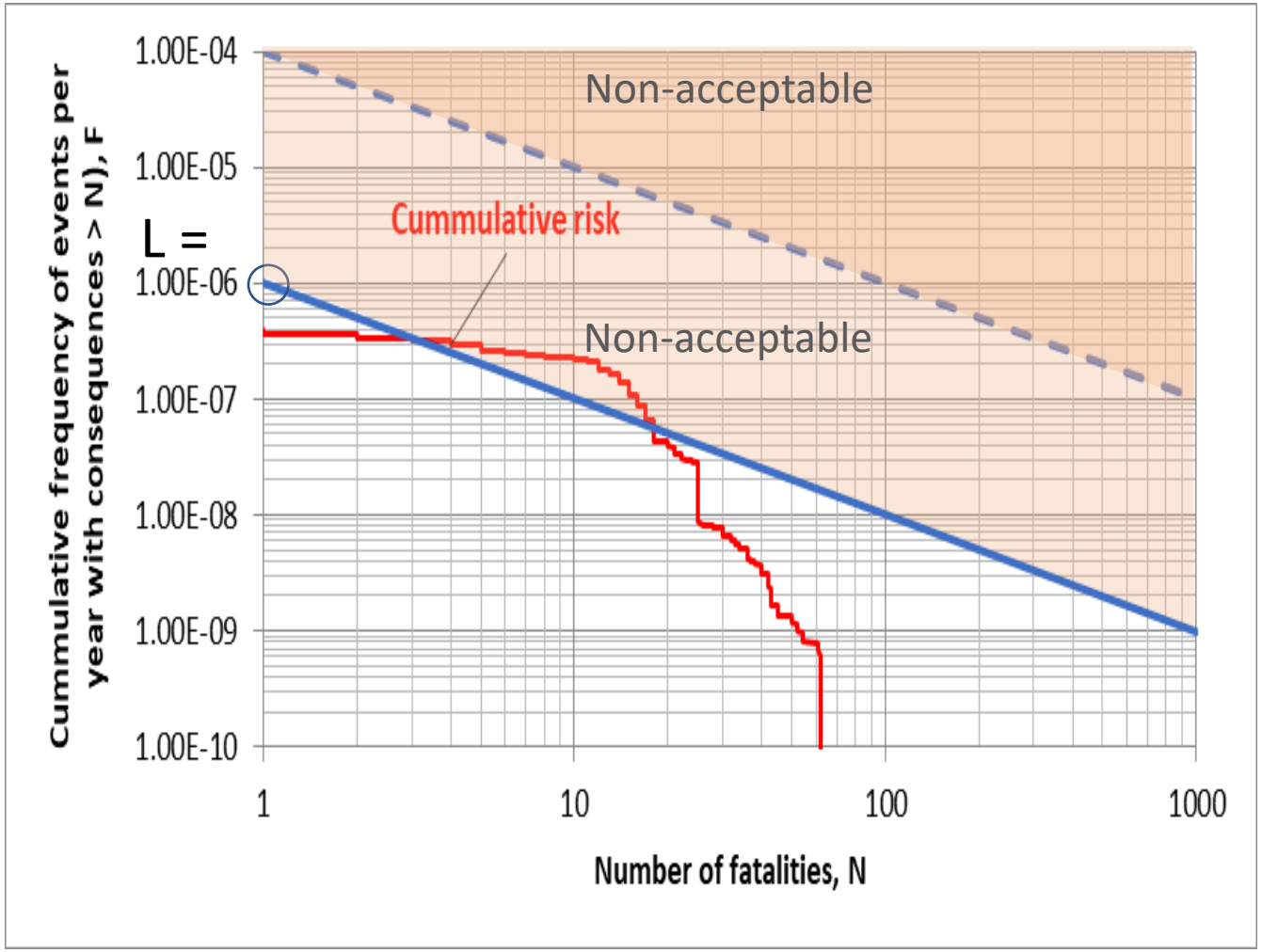
$$\log(F) \leq \log(L) - \alpha_{FN} \cdot \log(n), for\ all\ (F,n)$$

- With α_{FN} = 1, all sets of (F,n) on the line gives the same product (L).

$$\log(F) = \log(\mathrm{L}) - \log(n) \Rightarrow \log(F) + \log(n) = \log(\mathrm{L}) \Rightarrow$$

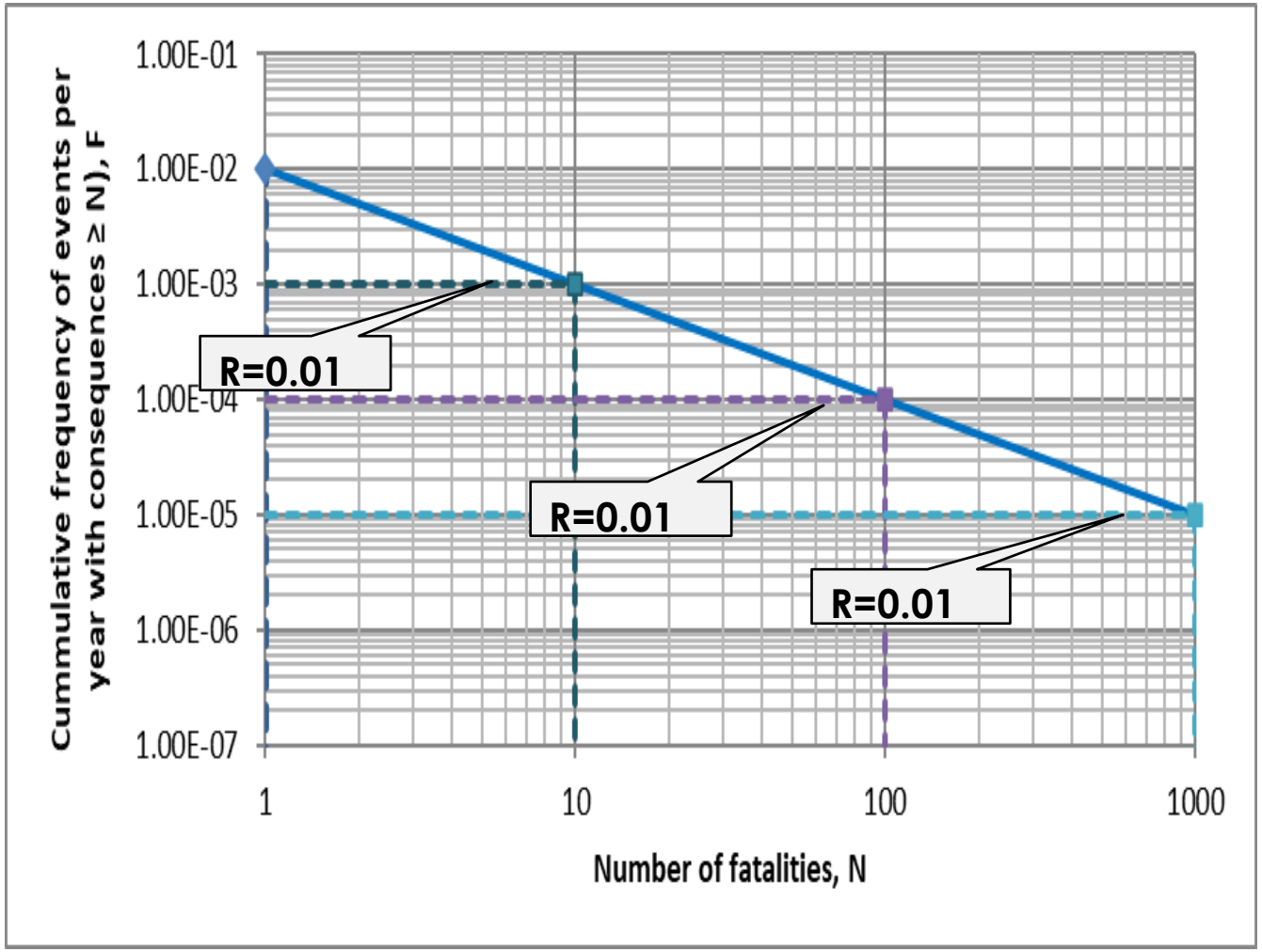
$$\log(F \cdot n) = \log(\mathrm{L}) \Rightarrow F \cdot n = L$$

- Hence, a single event represented by a point anywhere on the line has the same inherent risk (R=L).



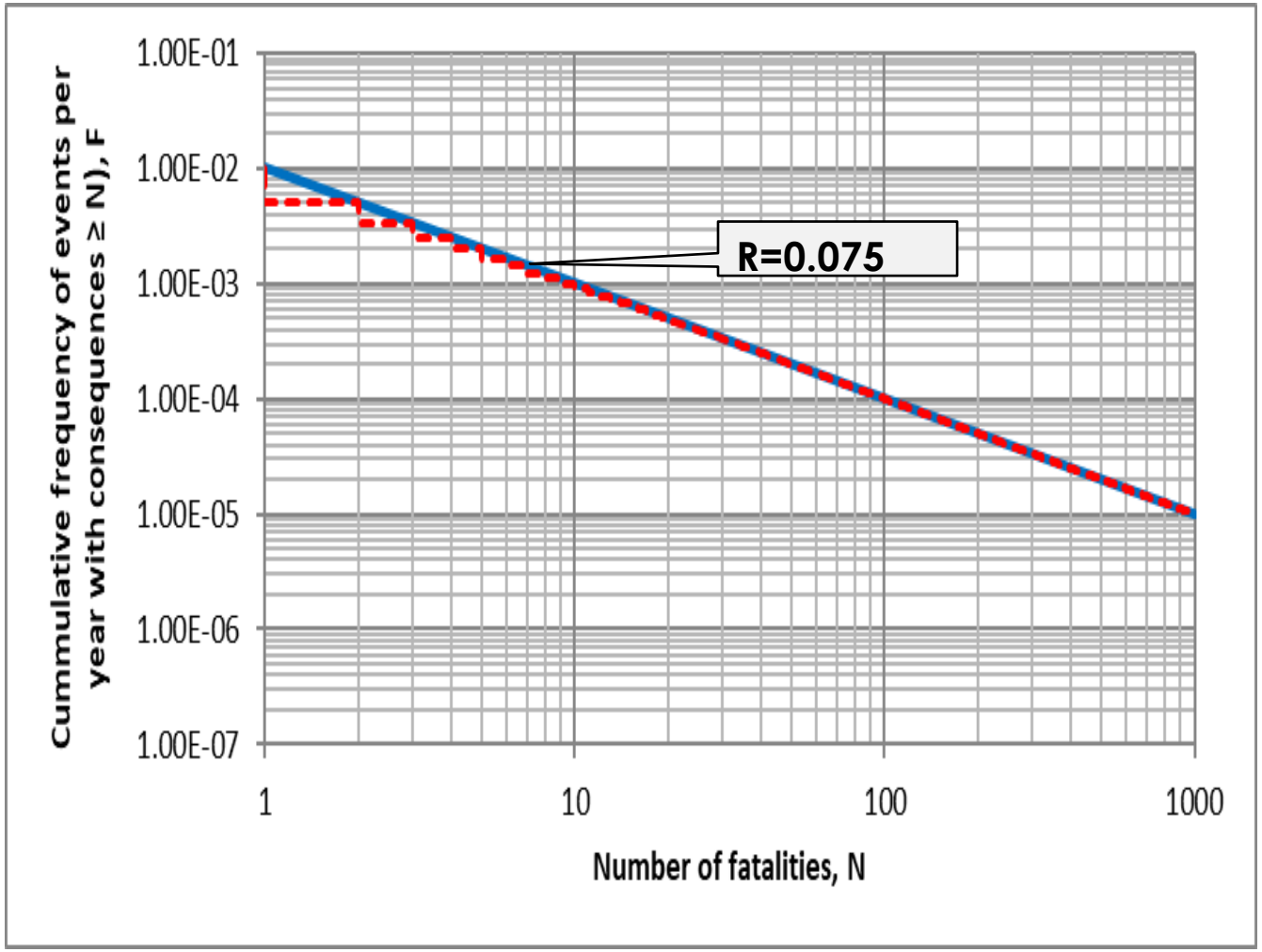
Inherent risk, accepted by the criterion line

- For “single step functions” with a single probability and consequences, things are still looking good:
  all risk curves, which are just meeting the criterion have exactly the same risk.



Inherent risk, accepted by the criterion line

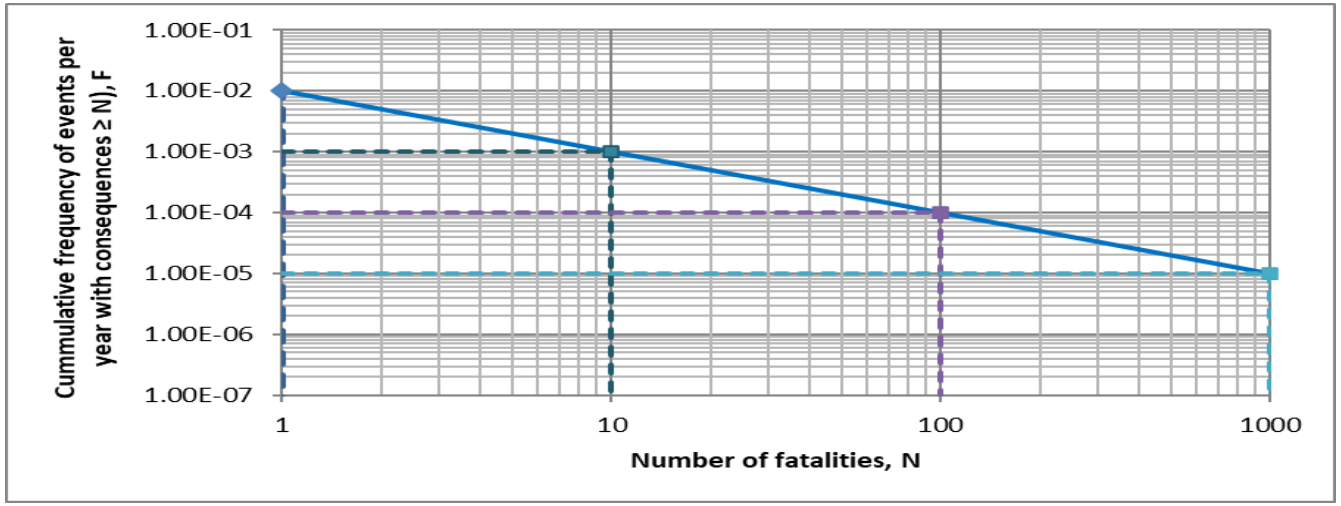
- Another FN-curve (“multi-step-line”) is exactly touching the acceptance line in all integer points of N.
- This curve is also just acceptable
- However, the total risk represented by this FN-curve can be calculated to be 0.075.

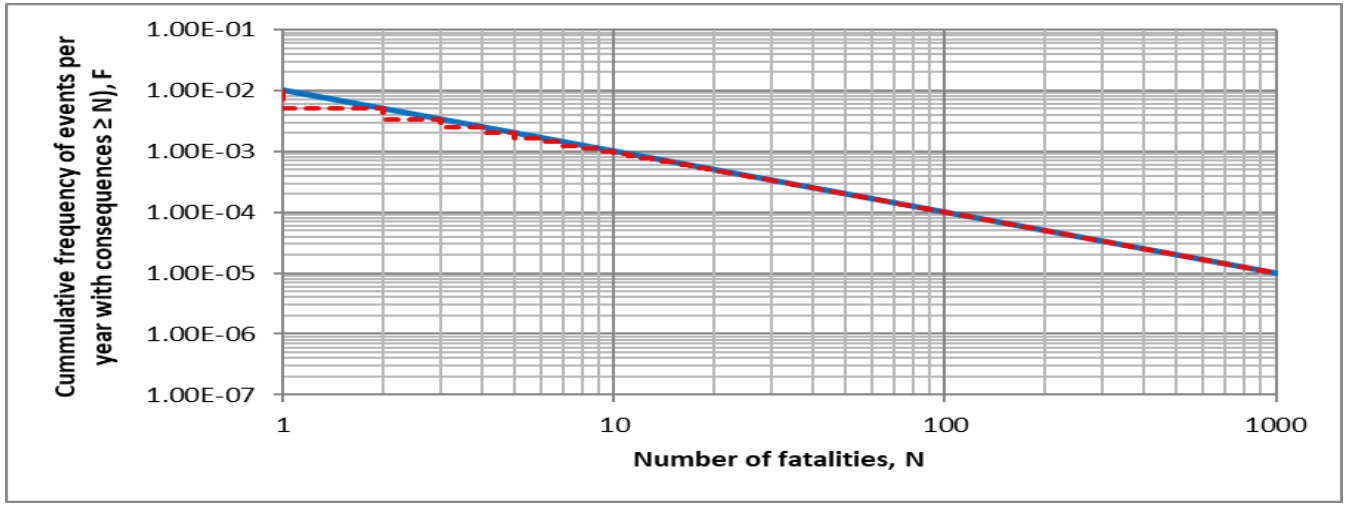


Inherent risk, accepted by the criterion line

- Two systems, which are equally acceptable by use of F-N criterion lines, may differ in risk by a factor 7.5!

- FN-criterion lines do not support decisions based on maximum utility and cannot be recommended as a tool for risk evaluation





Similar conclusion by Evans

- The paper by Evans and Verlander conclude that
- *“There are two objections to the […] FN-criterion. First, by concentrating on just one extreme feature of a statistical distribution, the […] criterion ignores other features which are relevant to a decision.*
- *In this way, the […] criterion can lead to decisions that appear unreasonable.*
- *Second, and more seriously, […] criteria are, in the language of decision theory, incoherent. That is, they give inconsistent preferences, or, when applied in the present context* ***they give inconsistent judgments about tolerability of risk***
- *Statistical decision theory suggests an alternative and preferable rule of minimizing the expected disutility, that is average harm, from accidents*”.

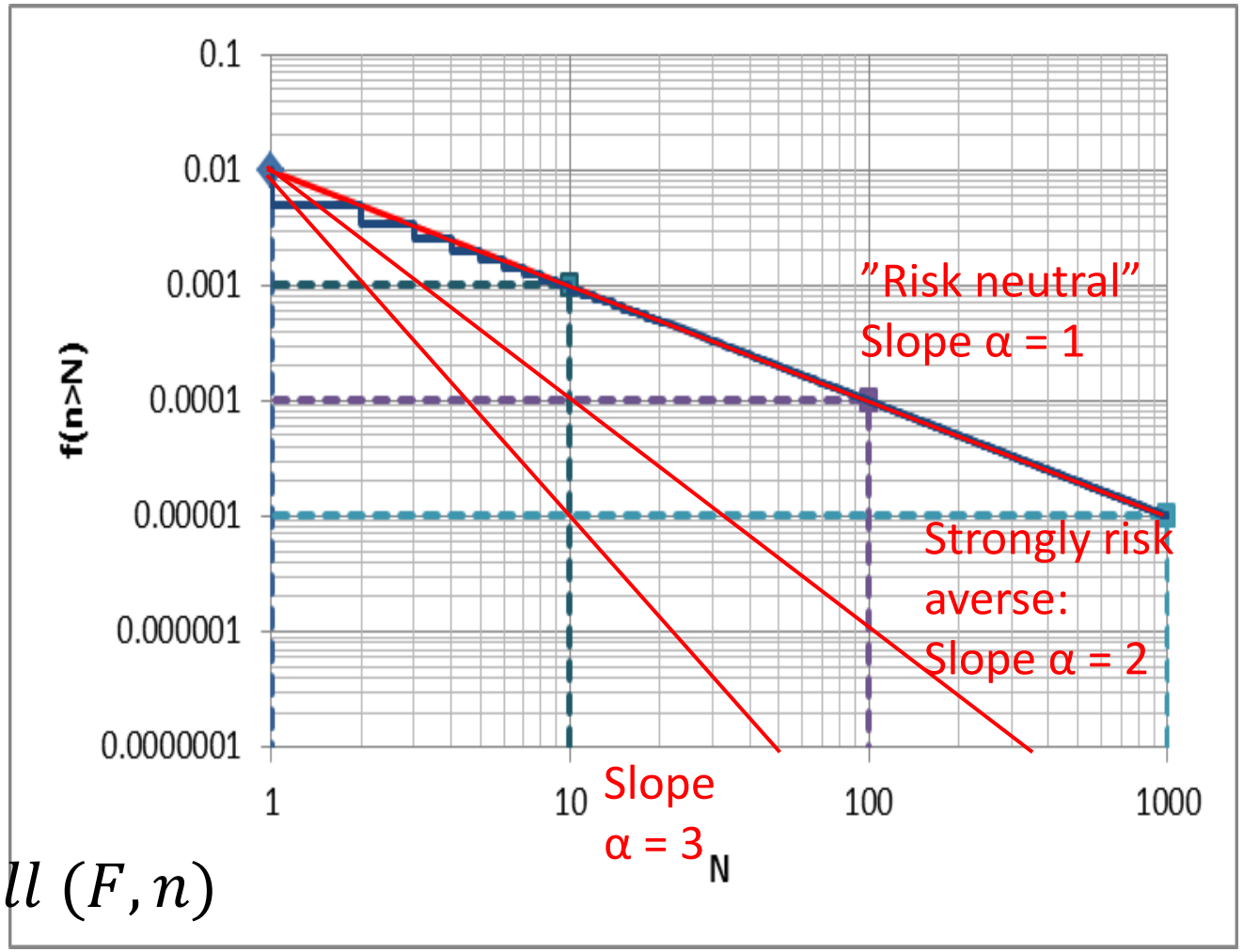
So what about the slope?

It can be discussed, whether this is "rational":
Is the aversion a model for the "indirect consequences" of the events?

- The slope of the FN-criterion line may supposedly be used to model risk aversion:
- The term "*risk aversion*" is often associated with the notion, that

<u>1 event with 100 fatalities is worse than 100 events with each 1 fatality,</u>

even though the total risk is the same.



- F-N criterion lines with slopes α_{FN}:

$$\log(F) \leq \log(L) - \alpha_{FN} \cdot \log(n), for\ all\ (F, n)$$

Risk aversion

- Comparison of FN Criterion lines with risk aversion with utility functions used for risk evaluation in terms of cost efficiency:

Risk neutral:

- Linear (dis)utility function $u(n) = n \cdot K$,
  where K is a weighting of n, the fatalities

Risk adverse

- It can be shown that the slope α_{FN} is equivalent with an exponent in the utility function. Hence: the (dis)utility function is

$$u(n) = n^{\alpha_{FN}} \cdot K = (n \cdot K) \cdot n^{(\alpha_{FN}-1)}$$

- This means that the weighting factor for each fatality is depending on n and α_{FN},
  and that large-consequence accidents are given more weight
- Please note that the weighting can become quite extreme.

	Weighting factor on 1 Fat. In Mill CHF (example VOSL = 7 Mill CHF)		
n	α_{FN}=1	α_{FN}=2	α_{FN}=3
1	7	7	7
10	7	70	700
100	7	700	70'000
1000	7	7'000	7'000'000

	Combined additional disutility of accdent in Mill CHF (= indirect consequences?)		
n	α_{FN}=1	α_{FN}=2	α_{FN}=3
1	0	0	0
10	0	630	6'930
100	0	69'300	6'999'300
1000	0	6'993'000	6'999'993'000

It should be carefully considered if this is really corresponding to the aims of the decision → →

Slope of criterion line as model for indirect consequences

It is recommended instead to carefully consider the indirect consequences of large accidents and model these consequences

The additional secondary (indirect) disutilities would be 0 for single accidents where individuals are killed alone (n=1).

α_{FN} = 1: additional secondary (indirect) disutilities would be 0 for all accidents

α_{FN} = 2: For an accident with 100 fatalities the apparent secondary disutilities would be 69 Billion CHF and for accidents with 1000 fatalities nearly 7 Trillion CHF .

α_{FN} = 3: the values become even more extreme,
For an accident with 1000 person killed the additional secondary disutilities would be 7000 Trillion CHF.

	Combined additional disutility of accident in Mill CHF (= indirect consequences?)		
n	α_{FN}=1	α_{FN}=2	α_{FN}=3
1	0	0	0
10	0	630	6'930
100	0	69'300	6'999'300
1000	0	6'993'000	6'999'993'000

- For comparison: GNP of all countries in the World of < 100 Trillion CHF/yr

Something is wrong with Criterion FN-Lines for Judging the Tolerability of Risk!

- F-N criterion lines give inconsistent judgments about tolerability of risk
- F-N criterion lines are unsuited for modelling a reasonable risk aversion or indirect consequences

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